

# Certification by a Monopolist

Laurent Linnemer<sup>1</sup> and Anne Perrot<sup>2</sup>

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<sup>1</sup>CREST-LEI and University of Cergy-Pontoise.

<sup>2</sup>CREST-LEI and CEME, University of Paris I.

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## **Abstract**

We consider a seller who sells a good of unknown quality to a single buyer. The seller may certify his product, that is, he may announce that the product quality is at least equal to a certification level. If he doesn't observe himself the quality of the good, he must first investigate in order to find out the quality before taking his certification decision. The buyer then revises her belief on product quality. We characterize the perfect bayesian equilibria of these games. In the "informed seller case", we show that high quality sellers certify their product while low quality sellers don't. Certification improves the profit of high quality sellers compared to the economy without certification. When the investigation decision is observable by the buyer, the seller investigates or stays ignorant according to the form of his profit function. When the investigation decision is unobservable by the buyer, the seller always investigates. An application is developed in which the seller may be either a manufacturer or a retailer involved in a vertical relationship. Various conflicts may then appear between both agents regarding certification.

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# 1 Introduction

When sellers of a good face buyers who are not informed about the quality of the product, equilibrium may break down. The original Akerlof problem of "qualitative uncertainty" (Akerlof, 1970) has given rise to a wide range of models which examine the possibility for the seller to provide information on product quality. In these papers, the role of prices as a signal and of firm's reputation (see Klein and Leffler, 1991, Shapiro, 1983, Allen 1984) or the ability to use advertising expenses and prices to reveal the product quality of a monopoly (Milgrom and Roberts, 1986-a) have been, among others, extensively analyzed. Grossman (1981) examines the role of warranties as a signal of product quality. However, another idea has been less explored in the literature, in spite of the growing interest it induces in firms : the possibility of product quality certification. For example, in Germany and in the UK, more than 80 % of the chickens are sold under certification<sup>1</sup>, and a large number of products, in particular in the food sector, are submitted to controls in order to reveal quality levels beyond the minimum level required by regulations.

Certification consists in an announcement that the product meets a minimum quality level, which is precisely specified by an appropriate packaging. This has to be distinguished from the compulsory quality standards usually defined by a public authority (see Leland 1979, Ronnen, 1991, Crampes and Hollander, 1995). For instance, in the food sector, some minimum sanitary requirements may be imposed by a public authority who aims at protecting public health, but some other characteristics of the product quality may be certified, as the vitamins content of milk or the transport conditions of animals. Usually, the characteristics that are certified stand above the minimum quality standard or more often, are even out of the field covered by the law. The agent who chooses to certify his product defines, together with a "certification body"<sup>2</sup>, the nature of the characteristics which will be certified and the precision of the information which will be conveyed to consumers. This procedure requires audits and controls of some of the production stages by the certification body. Certification may thus be viewed as a mean of signaling some aspects of the product quality to buyers. In a sense, certification plays the role of a perfect guarantee provided to consumers.

Moreover, this activity is strictly overseen by the public authority, who

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<sup>1</sup>Source : Etude comparée des systèmes de valorisation de la qualité des produits agricoles et alimentaires dans les pays de la CEE; Confédération Française de la Coopération agricole. Avril 1990.

<sup>2</sup>In France, for instance, Veritas, Qualicert, and in the UK the agency Food From Britain are examples of such certification institutes.

determines the number and the identity of the authorized certification bodies and controls their activity. This latter aspect allows to see certification as a credible announcement on product quality. Note that certification differs considerably from minimum quality standards: firms choose themselves the quality level that they reveal through certification, they can abandon it whenever they want, and the certification level chosen by one firm on the market is never imposed to its rivals.

In this paper, we study the certification choices of a monopolist who sells a good of unknown quality (following Nelson 1970, 1974, we call this good an "experience good"). The monopolist may use certification, together with prices, in order to reveal the quality of his product to the buyer. Clearly, this context is quite restrictive since it puts aside the problem of competition, but this simple framework allows to focus on the role of certification as a signal of product quality and on the coordination problem which arises between manufacturers and retailers. We examine the profitability of such strategies compared to a situation where certification is not allowed, prices being then the only remaining signal of product quality. For this purpose, we build a signaling game, developed in section 2, in which the buyer revises her belief on quality after receiving the certification-price signal. Three different informational structures are studied. In the "informed seller game" studied in section 3, the seller observes the quality of the product when he makes his choice. In section 4, he doesn't know his quality, but he can learn it (possibly at zero cost) through an appropriate investigation. Investigation may be viewed as an internal auditing procedure, through which the seller learns more precisely what his true quality is. We distinguish the cases where the buyer observes ("public investigation case") or not ("private investigation case") the investigation decision of the seller.

We show that certification improves the situation of high quality sellers compared to an economy where certification doesn't exist, but hurts low quality ones who are forced to reveal their quality more than they wish. However, according to the form of his profit function, an uninformed seller may choose to ignore the true quality of his product even in the case where investigation occurs at zero cost.

In section 5, we develop an example in which the seller is involved in a vertical relationship and may thus be a manufacturer or a retailer. In this context, we show that the retailer is more reluctant than the manufacturer to embark on certification, a stylized fact that seems to be often put forward in this sector. More generally, a conflict appears between high quality producers and retailers. This mechanism offers an additional reason why vertical restraints should be used in order to provide retailers with appropriate incentives. The section 6 provides a few conclusions.

## 2 The Model

We consider a seller  $S$  who offers an experience good of true quality  $v$  to a buyer  $B$ .  $B$  is uninformed about  $v$ . Her prior belief on  $v$  is given by a density  $g$  and a cumulative function  $G$  over the interval  $[a, b]$  with  $a > 0$  and  $v \in [a, b]$ ; we denote  $\bar{v} = E(v)$ . In what follows,  $S$  may himself be either informed or not about the value of  $v$ . In the latter case,  $S$  shares the same prior than  $B$ .

For simplicity, we assume that the good is produced at zero cost. All our results generalize to the case of costs that are independent of quality. A plausible explanation of this assumption may be for instance that quality does not correspond to an objective index – if this were the case, then a higher quality would imply a higher production cost – but refers mainly to a perception by the consumers of what quality is (see for instance Zeithaml 1988).

If the seller  $S$  knows the quality  $v$  of his product, he is able to certify it. Certification consists in a credible announcement that quality is greater than the "certification level" which may be chosen by  $S$  in the interval  $[a, v]$ . A certification strategy is a function  $c(v)$  from  $[a, b]$  into the set  $\{0, \{c\}_{c \in [a, v]}\}$  where  $c(v) = 0$  denotes no certification and  $c(v) = c \in [a, v]$  denotes certification at the level  $c$ . Certification involves a constant fixed cost  $k$ ,  $k \geq 0$ . In the real world, certification both involves fixed and variable costs. However, the fixed cost associated to a certification process is much larger than the variable cost. The fixed cost corresponds to the determination, together with the certification body, of the nature of the control that will be performed, and to the control of the production process itself; these costs may be assumed, without much loss of realism, independent of the volume of production.<sup>3</sup>

If the seller doesn't observe  $v$ , he cannot certify the product. However, in this case, we allow for the possibility of "investigation", which enables  $S$  to learn the true quality. The investigation choice is denoted by  $i \in \{0, 1\}$ . If the seller doesn't investigate,  $i = 0$ ; if he does,  $i = 1$ . The cost of investigation  $h$ , ( $h \geq 0$ ) is fixed and independent of quality. Once  $S$  has investigated, he can certify or not the quality of the product. If  $S$  has not investigated, he cannot provide any certification, since this would require the knowledge of  $v$ .

After decisions relative to certification have taken place,  $S$  chooses the price  $p$  of the product.

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<sup>3</sup>Consumers are present in the certification committees, and a part of the certification process consists in checking that the product meets the consumers' expectations about product quality. This procedure is not in contradiction with the fact that "quality" may be defined with regard to consumers' perception.

We consider three different informational contexts, all of them being common knowledge. In the first one, the seller is informed about the quality : we call this situation ”*the informed seller case*”. The corresponding game is represented in figure 1.

Figure 1: The informed seller game

In the second case, the seller is not informed about the quality but the investigation decision is observed by the buyer : this constitutes ”*the public investigation case*”. In the third one, the seller is not informed about the quality and the investigation decision cannot be observed by the buyer : we call it ”*the private investigation case*”. These two games are shown in figure 2 and 3.

Figure 2: The public investigation game

Let  $s(v)$  denote a strategy of the seller of true quality  $v$ . In the informed seller case,  $s(v) = (c(v), p(v))$  whereas in the public or private investigation cases,  $s(v) = (i(v), c(v), p(v))$ . Each information structure thus leads to a particular game.

Figure 3: The private investigation game

In each game, the buyer observes some of the seller's decisions and revises her belief accordingly. In the informed seller case and in the private investigation case, the buyer observes the price  $p$  and the certification decision  $c$ . The updated belief  $\mu$  is a function of  $p$  and  $c$  and  $\mu(c, p) = \mu(s) \in [a, b]$ .

In the public investigation case, the buyer observes in addition the investigation decision. Therefore, the updated belief  $\mu$  is also a function of  $i$  and  $\mu(i, c, p) = \mu(s) \in [a, b]$ .

In both cases, since certification is credible, we require that if the strategy  $s$  chosen by the seller involves certification (i.e. a positive certification level  $c$ ), then  $\mu(s) \geq c$ .

According to the price  $p$  and to her belief  $\mu$ , the buyer then takes her purchase decision. The demand function is noted  $D_\mu(p)$ . It is assumed to be strictly decreasing in price and strictly increasing in  $\mu$ .

If certification is impossible (for instance because it is not allowed), the strategy  $s$  of the seller reduces to the choice of a price. The seller's profit then depends only on the price and on the buyer's belief,  $\mu$ . Let  $\Pi(p, \mu) = pD_\mu(p)$  denote this profit function, which is assumed to be quasi-concave in  $p$  with a unique maximum, and increasing in  $\mu$ . Under perfect information, the profit of the seller  $v$  is  $\Pi(p, v)$ .

More generally, when certification is available, the profit function  $\Pi(s, \mu)$  of the seller is defined as follows:

- in the informed seller case,  $\Pi(s, \mu) = pD_\mu(p) - \gamma(c)$ ,
- in the public or private investigation cases,  $\Pi(s, \mu) = pD_\mu(p) - \delta(i, c)$ ,

The functions  $\gamma$  and  $\delta$  represent the investigation and certification cost functions.<sup>4</sup>

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<sup>4</sup>The functions  $\gamma$  and  $\delta$  are defined as follows :  $\gamma(0) = 0$  and  $\gamma(c) = k$  for any  $c$  in  $[a, b]$  ;  $\delta(0, 0) = 0$ ,  $\delta(1, 0) = h$ , and  $\delta(1, c) = h + k$  for any  $c$  in  $[a, b]$ . Note that since certification requires investigation, we don't need to define  $\delta(0, 1)$ .

This allows to define the perfect bayesian equilibria of these games in the following way.

**Definition 1** *A perfect Bayesian equilibrium of the game is a vector of strategies and beliefs  $\{s^*(v), \mu(s)\}$  such that:*

- (i)  $s^*(v) \in \arg \max_s \Pi(s, \mu(s))$ ;
- (ii) *For every  $s$ ,  $\mu(s)$  is consistent with the Bayesian revision, that is:*
  - if there exists  $v$  in  $[a, b]$  such that  $s^*(v) = s$ , then  $\mu(s) = E(v \mid s^*(v) = s)$ ;*
  - if there exists no  $v$  in  $[a, b]$  such that  $s^*(v) = s$ , then if  $s$  involves  $c = 0$ , any  $\mu$  is consistent and if  $s$  involves  $c \neq 0$ , any  $\mu \geq c$  is consistent.*

We now introduce some useful notation. Let  $C = \{v \mid c(v) \neq 0\}$  and  $NC = \{v \mid c(v) = 0\}$ . That is,  $C$  (resp.  $NC$ ) denotes the set of sellers who provide (resp. do not provide) certification. Denote by  $\pi(v)$  the full information equilibrium profit of a seller of true quality  $v$ , that is,  $\pi(v) = \max_p \Pi(p, v) = \Pi(p^*(v), v)$ . Since the demand function is increasing in  $\mu$ , the envelope theorem insures that  $\pi(v)$  is itself an increasing function in  $v$ . We also denote by  $v_e$  the average quality over the interval  $[a, v]$  that is,  $v_e = E(x \mid x \leq v)$ .

Before turning to the resolution of the various games, we can state a general lemma.

**Lemma 1** *In equilibrium, there exists  $\tilde{v}$  in  $[a, b]$  such that  $NC = [a, \tilde{v}]$ .*

**Proof.** First, observe that every seller  $v$  in  $NC$  obtains the same profit since any  $v$  in  $[a, b]$  can mimic a seller in  $NC$  by choosing no certification. Denote by  $\Pi^{NC}$  this common value of the profit. Consider a seller  $v \in NC$ . Then, since  $v$  is assumed not to certify in equilibrium, we have  $\Pi^{NC} \geq \Pi(s, \mu)$  for all  $s$  (with  $s = (i, c, p)$  or  $s = (c, p)$ , according to the informational context) involving a certification level  $c \leq v$ . Consider now a seller  $v'$  such that  $a \leq v' < v$ . Then we have also for any strategy  $s$  available for  $v'$ ,  $\Pi^{NC} \geq \Pi(s, \mu)$  because if  $v'$  could gain from certification, then  $v$  would also. This implies that  $v'$  belongs to  $NC$ . ■

This lemma confirms the intuition according to which high qualities are more likely to provide certification than bad ones. Note that it is valid even in the cases where the seller is not informed, since it does not exclude that no certification occurs in equilibrium (we would have in this case  $\tilde{v} = b$ ).

We now turn to the analysis of the informed seller case.

### 3 The informed seller case.

In the informed seller case, the seller first takes his certification decisions and then chooses his price. The buyer then receives a certification-price signal and updates her belief about quality accordingly.

Let us first assume that certification is not allowed (what we call the "no certification economy"). The following lemma gives the configuration of price equilibria.

**Lemma 2** *In the no certification economy, there are multiple price equilibria, among which some are semi-separating, one is separating, and one is totally pooling. The totally pooling equilibrium dominates all the others from the seller's point of view.*

**Proof.** See Appendix.

All these equilibria involve a threshold  $\hat{v}$  in  $[a, b]$  such that sellers in  $]\hat{v}, b]$  are separated (since  $p(v)$  perfectly reveals  $v$ ) whereas those in  $[a, \hat{v}]$  are not, although all sellers obtain the same profit (namely  $\pi(\hat{v}_e)$ ). The form of these separating equilibria is thus particular. The most useful result is that the pooling equilibrium, where all sellers obtain the perfect information profit of the average (prior) quality, dominates any other equilibrium. However, in this continuous type framework, no refinement criterion allows to select this equilibrium. In a discrete type version of the model, some refinement criteria could be applied, but this would require a reformulation of the model. We leave this problem aside and select exogenously the equilibrium that dominates from the sellers' point of view, that is, the pooling equilibrium.

Lemma 2 shows that without certification, no seller can obtain better than the average quality profit in the no certification economy: no pricing strategy allows a high quality seller to enjoy a larger profit because of his high quality. Hence, certification may be a valuable signal of product quality. In what follows, we will thus investigate this possibility, and compare the equilibria of the certification games (i.e., the games where certification is allowed) to the totally pooling equilibrium of the no certification economy.

We now turn to the determination of the certification strategies.

It is easy to check that when certification is possible, the seller  $v$  can guarantee at least  $\pi(v) - k$ . This outcome can be obtained by playing  $c(v) = v$  and  $p = p^*(v)$  that is, by certifying the true quality level and charging the corresponding perfect information price. This yields  $\mu \geq v$ , which in turn leads to a profit at least equal to  $\pi(v) - k$ .

We demonstrate first that no seller can benefit from choosing a certification level lower than his true quality.

**Lemma 3** *If the seller  $v$  chooses to certify ( $c \neq 0$ ), then he chooses  $c(v) = v$  and  $p = p^*(v)$ .*

**Proof.** Assume that in equilibrium, any seller whose quality belongs to some subset  $V$  of  $[a, b]$  chooses the same positive certification level  $c$  and the same price  $p$ , while any other chooses  $c(v) \neq c$ . According to the Bayesian revision of the buyer's belief, we have:  $\mu(c) = E(v \mid v \in V)$ . Consider a seller  $\hat{v}$  in  $V$  such that  $\hat{v} \geq E(v \mid v \in V)$  (i.e., close enough to  $\text{Sup}_v V$ ). Then by choosing the certification level  $\hat{v}$ , the seller  $\hat{v}$  could obtain  $\pi(\hat{v}) - k$  instead of  $\Pi(p, \mu(c))$ . Since  $\Pi(p, \mu(c)) \leq \mu(c) - k \leq \pi(\hat{v}) - k$ , the strategy  $(c, p)$  cannot be chosen by  $\hat{v}$  in equilibrium, which is a contradiction. This shows that two different qualities cannot provide in equilibrium the same positive certification level at the same price.

Suppose now that there are two qualities  $v$  and  $v'$  with  $v' > v$ , such that  $v$  chooses  $(c(v) = c, p)$  and  $v'$  chooses  $(c(v') = c, p')$  with  $c \leq v, p = p^*(v), p' = p^*(v')$ , that is,  $v$  and  $v'$  choose the same certification level, and each seller chooses its perfect information price. This implies  $\mu(c, p') = v'$  because  $v'$  is the only seller who selects the price  $p'$ . This gives to  $v'$  the profit  $\pi(v') - k$ . But then  $v$  finds it profitable to deviate by playing  $(c(v) = c, p')$  as  $\pi(v') > \pi(v)$ . It results that two different qualities cannot offer the same certification level at two different prices. This shows that if different qualities choose to provide certification in equilibrium, they choose different certification levels. In order to avoid imitation by lower qualities, seller  $v$  must not choose a certification level lower than  $v$ . Therefore in equilibrium we have  $c(v) = v$  for any seller who certifies, and the best price is then  $p^*(v)$ . ■

It results from the previous lemma that either seller  $v$  chooses to certify at level  $v$ , or he chooses no certification. If he certifies, then the buyer learns the true quality of the good: the certification of the product leads to the perfect revelation of its quality and allows the seller to charge the corresponding perfect information price. Therefore the seller's profit is then  $\pi(v) - k$ . This allows to characterize the equilibria of the complete game. We first examine the special case where certification costs are zero, and then turn to the general case.

**Lemma 4** *If there is no certification cost, the perfect Bayesian equilibrium of the game is totally separating: the seller  $v$  chooses  $(c(v) = v, p = p^*(v))$  and receives his full information profit.<sup>5</sup>*

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<sup>5</sup>Note that the seller  $v = a$  is in fact indifferent between certification and no certification and in any case he chooses  $p = p^*(a)$ , so that there are in fact two P.B.E, one in which  $a$  certifies and one where  $a$  doesn't. The seller  $v$  obtains the same profit in each equilibrium.

**Proof.** Assume that  $NC = [a, \tilde{v}]$  with  $\tilde{v} > a$ . Since by assumption  $\tilde{v}$  does not certify, we have  $\Pi(0, p(\tilde{v}), \mu(0, p(\tilde{v}))) \geq \pi(\tilde{v})$  for any price  $p(\tilde{v})$ . As for any price  $p$  we have  $\mu(0, p) \leq \tilde{v}$ , the best price of  $\tilde{v}$  is  $p(\tilde{v}) = p^*(\tilde{v})$ ; but then any  $v < \tilde{v}$  could obtain a higher profit by choosing  $(0, p^*(\tilde{v}))$  which contradicts the fact that  $\mu(0, p^*(\tilde{v})) = \tilde{v}$ . This implies that  $NC$  contains at most  $a$ . ■

According to this lemma, if certification involves no cost, any seller with  $v > a$  chooses to certify, whereas the seller  $a$  is indifferent between certification and no certification.

We are now able to compare the outcomes of the certification and no certification economies when the certification cost is zero. We restrict our attention to the equilibria which dominate from the seller's point of view.

*Ex post*, a seller whose quality is less than the average ( $v < \bar{v} = E(v)$ ) is worse off when certification is available, as his profit is reduced from  $\pi(\bar{v})$  to  $\pi(v)$ : in the certification economy, low quality sellers cannot profitably choose not to certify their products although they incur losses from certification. On the other hand, the sellers whose quality is higher than the average ( $v > \bar{v}$ ) are better off when certification is available.<sup>6</sup>

This illustrates the classical conflict of interests between low quality and high quality sellers: the former prefer a world without certification, which allows them to "hide" behind high qualities, whereas the latter prefer certification, at least when it occurs at zero cost, because it allows to extract the benefits of a high quality through signaling.

We now examine the more general case in which certification involves a positive cost  $k$ . In order to guarantee that some sellers in  $[a, b]$  benefit from certification, we assume that  $k$  is not too high, namely, we make the following assumption:  $\pi(b) - \pi(\bar{v}) > k > 0$ . Since in the no certification economy, the highest profit is  $\pi(\bar{v})$ , this assumption insures that some (high) qualities choose to certify. For any such value of the certification cost  $k$ , the equilibria are described in the following proposition.

**Proposition 1** (i) *In any equilibrium, high quality sellers certify and low quality sellers don't : at any equilibrium, there exists  $\tilde{v} > a$  such that  $NC = [a, \tilde{v}]$  and  $C = ]\tilde{v}, b]$ . However, the value of  $\tilde{v}$  is not unique and there are multiple equilibria.*

(ii) *At the equilibrium associated to  $\tilde{v}$ , the types  $v$  in  $C$  are separated: for such values of  $v$ ,  $c(v) = v$  and  $p(v) = p^*(v)$ . Types  $v$  in  $NC$  all obtain the*

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<sup>6</sup>This result is related to Milgrom and Roberts (1986-b): they examine the incentive for a seller to provide verifiable information to a buyer who may be sophisticated or not. They focus on how competition between informed parties may be a substitute for sophisticated behavior of the buyer. See also Forsythe et alli (1989).

same profit, namely  $\pi(\tilde{v}_e)$ , but the (price) strategies may be totally separating, or totally pooling, or semi-separating.

(iii) The equilibrium which dominates from the seller's point of view is such that  $\tilde{v} = \max_w \{w \mid \pi(w_e) = \pi(w) - k\}$  and  $\forall v \leq \tilde{v}, p(v) = p^*(\tilde{v}_e)$ .

**Proof.** See Appendix.

Among these equilibria, some exhibit special properties. For instance there is an equilibrium where all the sellers in a neighborhood of  $a$  choose not to certify and obtain the same profit  $\pi(a)$ , but each of them reveals his quality through his pricing strategy. By contrast, the equilibrium described in (iii) does not allow any separation of the sellers who don't certify: at this equilibrium, only the sellers in  $C$  are separated. One can check that  $\tilde{v}$  is a strictly increasing function of  $k$ . This allows to compare the dominant equilibria of the certification and no certification economies.

Recall that the equilibrium that dominates in the no certification economy requires that all sellers charge the same price  $p^*(\bar{v})$ , that is, the perfect information price corresponding to the average prior belief on quality, thus obtaining the profit  $\pi(\bar{v})$ . Consider now the best equilibrium of the certification economy above described. Certification increases the profit of a seller who certifies if  $\pi(\bar{v}) \leq \pi(v) - k$ , that is, if  $v$  is greater than some value  $v(k)$ . Note that due to the assumption  $\pi(b) - \pi(\bar{v}) > k > 0$ ,  $v(k)$  is inferior or equal to  $b$ . Sellers are now divided into three groups according to their quality. Those whose quality belongs to  $[a, \tilde{v}]$  do not certify, and obtain  $\pi(\tilde{v}_e)$  instead of  $\pi(\bar{v})$ : these (very) bad qualities lose with regard to the no certification economy because they cannot "hide" any longer behind high qualities. However, it is interesting to note that the existence of a positive certification cost benefits to some of them (namely those for whom  $v \leq \tilde{v}_e$ ) compared to the case  $k = 0$ : in this latter case, they would reveal their quality through certification. The sellers in the interval  $[\tilde{v}, v(k)]$  certify in equilibrium, but they lose compared to the no certification economy, obtaining  $\pi(v) - k$  instead of  $\pi(\bar{v})$ . Sellers belonging to  $[v(k), b]$  benefit from certification which allows them to be identified as high quality sellers.

This sheds light on multiple conflicts that may arise between high and low quality producers: first, whereas high quality producers are in favor of the existence of certification, low quality producers are not. Second, once certification takes place, low quality producers prefer a situation where certification is costly. This could lead to lobbying behavior on the part of low quality producers who have an incentive first to refuse the introduction of certification procedures in their sector and second, if this first attempt has failed, to increase the cost of the certification process by requiring a high degree of control.

In the following section, we examine the public and private investigation cases where the seller ignores the quality of his product.

## 4 The ignorant seller case

We examine first the case when the buyer observes the investigation decision.

### 4.1 The public investigation case

Assume that the seller does not know the value of  $v$ , but that he can learn it through an appropriate investigation, which costs  $h$ . In the public investigation case, the buyer observes whether the seller has decided to investigate or not. Therefore, the signal on which the buyer can update her belief now involves an investigation decision ( $i = 0$  or  $i = 1$ ) in addition to the certification and price decisions.

We solve this game by backward induction. If the seller decides not to investigate, then he plays an "ignorant seller game" with the buyer. We assume that the equilibrium which emerges in the corresponding subgame is the dominant equilibrium from the seller's point of view. We note  $\Pi_{i=0}$  the ensuing profit. If the seller chooses to investigate, he must pay the investigation cost  $h$  and then he plays an informed seller game. We note the profit in this case  $\Pi_{i=1}$  and we also assume that the equilibrium reached in the corresponding subgame is the dominant one. It follows that these subgame equilibria have already been derived:  $\Pi_{i=0}$  is the profit corresponding to the pooling equilibrium of the no certification economy, that is  $\pi(\bar{v})$ , whereas  $\Pi_{i=1}$  is the expected profit evaluated at the dominant equilibrium in the informed seller case, minus the investigation cost  $h$ . We may then derive the equilibrium investigation decision.

**Proposition 2** *If  $\pi$  is weakly concave, then for any  $k \geq 0$  and  $h > 0$  the seller always prefers to remain ignorant.*

**Proof.** See Appendix.

When the profit function is linear, and if costs  $k$  and  $h$  are zero, the ignorant seller is indifferent between investigation and no investigation. Thus as soon as one of these costs is positive, the seller prefers no investigation. It is worth noting that this result holds even if investigation involves zero cost. When the profit function is concave, the seller prefers not to investigate even if  $h$  and  $k$  are zero. Positive costs reinforce the conclusion.

The intuition behind this result is straightforward: the concavity (linearity) of the profit function with respect to  $v$  plays the same role as risk

aversion (neutrality). No investigation leads, with probability one, to the profit associated with the average quality, whereas investigation is analogous to a lottery with the same mean but higher dispersion. Thus investigation is more risky than no investigation. Conversely, it is intuitive that in the case of convexity of the profit function, the seller faces a trade off between investigation and certification costs on the one hand, and benefits due to certification on the other hand. If the costs associated to investigation and certification are not too high, the seller should investigate. The following proposition sets the result.

**Proposition 3** *If  $\pi$  is convex, then for any value of the investigation cost  $h$  such that  $E(\pi(v)) - \pi(\bar{v}) > h \geq 0$ , there exists a value  $k(h) > 0$  such that if the certification cost is lower than  $k(h)$ , the seller always prefers investigation.*

**Proof.** Since  $\pi$  is convex, Jensen's inequality implies  $E(\pi(v)) - \pi(\bar{v}) > 0$ . Let  $E(\pi(v)) - \pi(\bar{v}) > h > 0$ , and assume  $k = 0$ , then  $\Pi_{i=1} = E(\pi(v)) - h > \pi(\bar{v}) = \Pi_{i=0}$ . The result follows by continuity of  $\Pi_{i=1}$  at  $k = 0$ . ■

We have thus shown that if the buyer observes the investigation decision of the seller, the latter takes his information decision on the ground of the valuation, through the profit function, of possible quality levels. Since investigation is public, a seller who investigates must behave as an informed seller. Investigation is analogous to a lottery involving more risk than no investigation, and the ignorance of the true quality is thus preferred by any seller endowed with a linear or concave profit function, whereas a seller whose profit function is convex chooses to investigate if the related costs are not too high.

However, the assumption according to which the seller is totally ignorant of his quality is a very strong one. A more plausible situation is one in which the seller has an imperfect private signal of his product quality : for instance, he knows whether his quality is "low" or "high". In this case, one can expect that high quality sellers are more likely to investigate than low quality ones, whatever the configuration of the profit function. This would lead to less clear-cut results. In particular, the seller who receives a "high" signal is less reluctant to investigate.

We finally turn to the private investigation case.

## 4.2 The private investigation case

In this section, we examine a different information structure. The seller is still supposed to ignore the quality of the product and to have the possibility

of learning it through investigation, but the buyer is unable to observe the investigation decision. It results that if the buyer observes no certification, she cannot infer whether the seller is informed or not about the quality of the product. In either case, she must thus form the same belief about quality. This leads to a result that is in sharp contrast with the public investigation case, as shown by the following proposition.

**Proposition 4** *If the cost of investigation is not too high, then in any equilibrium, the seller investigates.*

**Proof.** Assume first that only pure strategies  $\{i = 0; i = 1\}$  are available. Let  $v^*$  be the unique solution of the equation:  $\pi(x) - k = \pi(\bar{v})$ , that is, the seller  $v^*$  is indifferent between certification and pricing at the corresponding price and the best outcome of the no certification economy. Then let  $h \leq E(\pi(v) - k | v \geq v^*) - \pi(\bar{v})$ . Suppose that in equilibrium  $i = 0$ , which implies that no certification occurs ( $c = 0$ ). Since the seller doesn't know his quality, he cannot reveal it through prices and his best price is  $p^*(\bar{v})$ . This implies  $\mu(0, 0, p^*(\bar{v})) = \bar{v}$  and the seller then obtains  $\pi(\bar{v})$ . Investigation improves the profit: indeed, after learning  $v$ , the seller can choose not to certify if  $v \leq v^*$ , which guarantees him the same profit. But if  $v > v^*$ , then certification leads to a higher profit. As  $h \leq E(\pi(v) - k | v \geq v^*) - \pi(\bar{v})$ , the expected profit conditional to investigation is higher. This shows that in equilibrium, one cannot have  $i = 0$ . Next, it is easy to check that no equilibrium mixed strategy can put a positive weight on the action  $i = 0$ . ■

Provided that the investigation cost is not too high, the seller always chooses to learn the quality of his product. Since the buyer knows this, the situation is analogous to that analyzed in the informed seller case. Investigation always improves profit because, *ex ante*, the seller knows that he might discover a sufficiently high quality to allow gains from certification.

The games that have been studied in sections 2 to 4 can be applied to many situations. In what follows, we develop an example in which the seller is involved in a vertical structure and may thus be either a manufacturer or a retailer.

## 5 Application to a vertical relationship.

In what follows, we consider a manufacturer ( $M$ ) who sells a good of true quality  $v$  to consumers through a retailer ( $R$ ). Both production and distribution costs are zero. Consumers ignore the quality of the product. According to the case, we assume that the manufacturer and the retailer themselves

may observe or not the quality of the product. The prior beliefs on  $v$ , common to all participants, are given by a uniform distribution over  $[a, b]$  with  $v \in [a, b]$ . Consumers differ in their willingness to pay  $\theta$  for the good and  $\theta$  is uniformly distributed over  $[0, 1]$ . Consumer  $\theta$  with belief  $\mu$  facing the price  $p$  buys 0 or 1 unit of the good according to  $\theta\mu - p$  being negative or positive. The demand function conditional to the belief  $\mu$  is thus  $D_\mu(p) = 1 - \frac{p}{\mu}$ .

Certification is described as above,  $k$  and  $h$  respectively denote the certification and investigation costs.

The manufacturer and the retailer play a Stackelberg game in prices where the manufacturer is the leader: the manufacturer first chooses a gross price  $w$  and the retailer then chooses a retail price  $p$ . The profits of the manufacturer and the retailer, gross of certification and investigation costs, are respectively:  $\pi_M(p, \mu) = wD_\mu(p)$  and  $\pi_R(p, \mu) = (p - w)D_\mu(p)$ .

We consider the various games described in the previous sections, and in each game, we focus on the equilibrium that gives the dominant outcome for the seller.

If the expected quality is  $\mu$  then the equilibrium prices and demand are respectively :  $p = \frac{3\mu}{4}, w = \frac{\mu}{2}, D_\mu(p) = \frac{1}{4}$ . The corresponding profits gross of certification and investigation costs are :  $\pi_M(\mu) = \frac{\mu}{8}$  and  $\pi_R(\mu) = \frac{\mu}{16}$ .<sup>7</sup>

Consider first the no certification economy. According to lemma 2, the equilibrium which dominates for the seller is that in which all sellers charge the perfect information price associated to quality  $\bar{v}$ .

A number of conflicts of various natures appears between the manufacturer and the retailer with regard to certification. These conflicts concern the choice of an environment which enables or not certification, the identity of the decision maker, and the decision itself. The first type of conflict is related to the identity of the decision maker : for instance, an agent may prefer a world without certification to a world with certification in which he would be in charge of the certification process, but a world with certification if the other one would take the decision.

In what follows, we compare the outcomes of games where the agent who takes the decisions relative to certification may be either the manufacturer or the retailer, under our various informational contexts. We examine first the case where the identity of the agent who certifies is *exogenously* given, and we then turn to the situation where the certification decisions may be delegated by the manufacturer to the retailer.

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<sup>7</sup>These profit functions are linear in  $v$ . However, most of the following results generalize to the case of concave functions. The aggregate demand function  $D_\mu(p) = \mu - p$  provides an example of profit functions which are convex with respect to  $\mu$  in equilibrium, namely  $\pi_M(\mu) = \frac{\mu^2}{8}$  and  $\pi_R(\mu) = \frac{\mu^2}{16}$ .

## 5.1 The nature of vertical conflicts

In what follows, we assume that the certification cost  $k$  is not too high (namely,  $k \leq \frac{b-a}{32}$ ), in order to ensure that both the manufacturer and the retailer find an interest to certify if the quality is high enough. It is easy to compute the threshold values  $\tilde{v}_M$  and  $\tilde{v}_R$  of the quality beyond which each agent ( $M$  or  $R$ ) certifies his product when he is in charge of the certification decision, and the values  $\hat{v}_M$  and  $\hat{v}_R$  beyond which he benefits from certification compared to a no certification economy.<sup>8</sup>

A general remark is that the manufacturer certifies the product for a much wider range of qualities than the retailer ( $\tilde{v}_M < \tilde{v}_R$ ). As a consequence, when the retailer benefits from certification, the manufacturer has a convergent interest. This could explain the success of retailers' brand names<sup>9</sup>, by which retailers define their own quality requirements. These requirements have to be met by any producer who wants to sell his product through this retailer. This procedure is quite close to certification by the latter. According to our results, a producer whose quality stands close to the standard defined by the retailer should be willing to accept a certification procedure initiated by the retailer. Conversely, these conflicting incentives could also explain that producers of medium qualities (namely, those who lie between  $\tilde{v}_M$  and  $\tilde{v}_R$ ) face retailers that are reluctant to certify themselves the product.

Beyond this feature of the model, the manufacturer and the retailer may have conflicting interests both with regard to the certification decision and to the identity of the decision maker, according to the value of the true quality.

Consider first low qualities ( $v < \tilde{v}_M$ ). No conflict appears since first, both agents agree on not certifying the product, and second, both benefit from the retailer taking this decision: since the consumers observe who chooses not to certify, they infer the quality  $E(v | v < \tilde{v}_R)$  instead of  $E(v | v < \tilde{v}_M)$ , which benefits to both agents.

For higher qualities (namely for  $\tilde{v}_M \leq v \leq \frac{\tilde{v}_M + \tilde{v}_R}{2}$ ), a conflict about the identity of the decision maker appears: on the one hand, the manufacturer prefers the retailer to be in charge of the certification decision (the retailer then chooses not to certify, the expected quality is lower but the manufacturer does not incur the certification cost). On the other hand, the retailer prefers the situation where the manufacturer certifies the product. Each agent thus favors the other one taking the decision, in spite of the fact that this decision is opposed to what he would have chosen himself.

For qualities between  $\frac{\tilde{v}_M + \tilde{v}_R}{2}$  and  $\tilde{v}_R$ , both agents prefer when the man-

<sup>8</sup>We have  $\tilde{v}_M = a + 16k$ ,  $\tilde{v}_R = a + 32k$ ,  $\hat{v}_M = \bar{v} + 8k$ ,  $\hat{v}_R = \bar{v} + 16k$ . It is worth noting that:  $\tilde{v}_M = E(v | v < \tilde{v}_R) \leq \tilde{v}_R$  and  $\hat{v}_M \leq \hat{v}_R$ .

<sup>9</sup>See for instance Mills (1995) for an analysis of retailer's private labels.

ufacturer takes the decision; in this case, certification occurs. If the retailer would have been in charge of the decision, he would have chosen not to certify and the manufacturer would then have been injured by the retailer's choice.

Beyond  $\tilde{v}_R$ , both agents choose to certify, but each one prefers when the other one is in charge of the procedure and incurs the corresponding cost.

It is also worth noting that for some low values of the certification cost (namely when  $k < \frac{b-a}{48}$ ), the manufacturer whose quality belongs to  $[\hat{v}_M, \tilde{v}_R]$  benefits from certification compared to the no certification economy, whereas a retailer in charge of the procedure chooses not to certify.

This analysis shows that conflicts may occur both about what decision should be taken and about who should take it, according to the location of  $v$  with respect to the threshold values  $\tilde{v}_M$ ,  $\tilde{v}_R$ ,  $\hat{v}_M$  and  $\hat{v}_R$ .

In order to go deeper into this issue, we examine below a situation where the identity of the agent who certifies derives from a delegation game, where at the first stage, the manufacturer may either take himself the relevant decision, or delegate this task to the retailer.

Consider now the case when at least one of the agent ignores the true quality of the product and suppose that investigation is public. Consider first the case where both agents are ignorant. Since the profit functions are linear with respect to the quality, both agents choose not to investigate as soon as one of the costs  $k$  or  $h$  is positive.

More interesting is the case where one agent is informed whereas the other is not. It is quite natural to assume that the retailer is ignorant and the manufacturer informed. In the public information game, the retailer takes the decisions relative to investigation and certification. Since these decisions take place before the price game, the manufacturer cannot reveal his quality to the retailer through pricing before the investigation decision. Thus the retailer never investigates. Of course, the producer might afterwards signal his quality through an appropriate gross price (see lemma 2) and the retailer could then learn ex post that certification would have been profitable. Hence he might have an incentive to renegotiate. If renegotiation is impossible however, there is a conflict between high quality manufacturers and retailers. Conversely, this situation is clearly beneficial to low quality manufacturers ( $v \leq \hat{v}_M$ ) who are then protected against revelation of their quality by the no investigation decision of the retailer.

Although it may seem less realistic, one can also imagine the opposite case where the manufacturer ignores the quality of the product and the retailer observes it. A good example is that of an owner of a house, who sells it through an estate agent: the latter probably evaluates better than the owner what a "good quality house" is. In the public information game, the owner is in charge of the decision and doesn't investigate. Profits are thus  $\pi_M(\bar{v})$

and  $\pi_R(\bar{v})$ . The estate agent would certify above  $\tilde{v}_R$ , he would benefit from certification above  $\hat{v}_R$ , and suffers in this case from the "no investigation-no certification" decision of the owner. Thus the conflict arises between the owner and the estate agent who knows that quality is high.

Finally, in this particular linear model, an ignorant agent is indifferent (whatever the value of  $k$ ) between being in charge of the certification process (in which case he does not certify) and certification by the other agent, whether the latter is informed or not. When the profit function is strictly concave or strictly convex we have the following results.

**Lemma 5** *When the profit function is strictly concave in  $v$ , an ignorant agent always prefers the no certification economy rather than a situation where the other agent is in charge of the certification process and incurs the corresponding cost.*

*When the profit function is strictly convex in  $v$ , an ignorant agent prefers the certification economy where the other agent is in charge of the decision.*

**Proof.** An ignorant agent compares the profit he will obtain in the no certification economy, that is  $\pi(\bar{v})$ , and his profit if the other agent (who is informed) takes the decision, that is  $\psi(\tilde{v}_j) = G(\tilde{v}_j)\pi(E(v | v < \tilde{v}_j)) + (1 - G(\tilde{v}_j))E(\pi(v) | v > \tilde{v}_j)$  where  $j$  belongs to  $\{M, R\}$ .  $\tilde{v}_j$  varies (depending on  $k$ ) between  $a$  and  $b$ . Assume  $\pi$  is strictly concave, Jensen's inequality leads to:

$\psi(\tilde{v}_j) < G(\tilde{v}_j)\pi(E(v | v < \tilde{v}_j)) + (1 - G(\tilde{v}_j))\pi(E(v | v > \tilde{v}_j))$ . The concavity of  $\pi$  insures then:

$$\psi(\tilde{v}_j) < \pi(G(\tilde{v}_j)E(v | v < \tilde{v}_j) + (1 - G(\tilde{v}_j))E(v | v > \tilde{v}_j)) = \pi(\bar{v}).$$

In the strictly convex case, the proof is symmetric. ■

In the private investigation games, any agent in charge of the decision investigates, and the results are analogous to those obtained in the informed seller games.

## 5.2 The delegation games

In what follows, we consider two different games corresponding to two informational contexts.

In the first game (game D1) the retailer is perfectly informed about product quality. In the second (game D2), the retailer ignores the product quality. Both these informational structures are common knowledge. The game D1 is illustrated by figure 4. In the game D1, where both agents are informed about the product quality, the manufacturer first decides whether to remain in charge of the decision or to delegate it to the retailer; in the former case,

Figure 4: The delegation game

the manufacturer plays an informed seller game, and in the latter, the retailer plays an informed seller game. In the game D2, the manufacturer has the same strategic choice, and according to the delegation decision, either the manufacturer or the retailer plays an ignorant seller game. The consumers may observe or not if delegation has occurred, but, as for the investigation game, the manufacturer cannot exploit the privacy of information concerning the delegation decision.

The following lemma gives the equilibrium of the game D1.

**Lemma 6** *There exists a unique equilibrium in the delegation game D1: the manufacturer certifies if  $v \in ]\tilde{v}_M, \tilde{v}_R[$ , he delegates if  $v \in [a, \tilde{v}_M] \cup [\tilde{v}_R, b]$ . In case of delegation, the retailer certifies if and only if  $v \in [\tilde{v}_R, b]$ .*

**Proof.** It is clear that for  $v \in [\tilde{v}_R, b]$  the retailer certifies if he has the lead, therefore the manufacturer is better off if he delegates than if he certifies himself.

For  $v < \tilde{v}_R$  the retailer never certifies. Observe that if  $v$  prefers delegation, then  $v' < v$  prefers delegation too : let  $v$  the expected quality when no certification is observed. Since  $\pi(v) > \pi(v) - k > \pi(v') - k$ , there exists a value  $u$  such that for  $v \in [a, u]$ , there is no certification and for  $v \in [u, \tilde{v}_R]$ , certification occurs. Then  $u$  must satisfy :  $\pi_M(u_e) = \pi_M(u) - k$ , which proves that  $u = \tilde{v}_M$ . ■

An interesting result is that the manufacturer cannot fully exploit the possibility of delegation. Indeed, as shown above, for  $\tilde{v}_M \leq v \leq \frac{\tilde{v}_M + \tilde{v}_R}{2}$ , the manufacturer prefers the situation where the retailer is *exogenously* in charge of the decision. Here, the opportunity to choose *endogenously* the decision

maker prevents him from benefiting from delegation. This is because if the retailer is exogenously in charge of the decision, he does not certify for any  $v$  in  $[\tilde{v}_M, \tilde{v}_R]$  whereas the manufacturer prefers certification to delegation for  $v$  in  $[\frac{\tilde{v}_M + \tilde{v}_R}{2}, \tilde{v}_R]$ . Then consumers revise their beliefs and infer from the observation of "no certification" that quality is lower in the delegation case than it would have been in the "exogenous case". Since the belief on quality is revised downward, this shrinks the range of qualities for which the manufacturer is able to benefit from delegation followed by "no certification" by the retailer.

Assuming either that the retailer cannot investigate or that investigation is public, the outcome of the game D2 is quite simple : if the retailer has the lead, he never certifies. Then there is no incentive to delegate. The manufacturer certifies if  $v \in [\tilde{v}_M, b]$  and delegates if  $v \in [a, \tilde{v}_M]$ .

Two more games may also be considered: assume on the one hand either that the manufacturer is ignorant or that investigation is public information and on the other hand that the retailer is either informed or ignorant. Then the manufacturer always delegates. If the retailer is informed (or if investigation is private), then he certifies for  $v \in [\tilde{v}_R, b]$  and does not for  $v \in [a, \tilde{v}_R]$ . If the retailer is also ignorant, he never certifies.

These games lead to another type of "informational" conflict which we summarize in the following corollary.

**Corollary 1** *The manufacturer (either informed or not) is better off when the retailer is informed (or when investigation is private) than when the retailer is ignorant.*

*On the opposite, the retailer is better off when he is ignorant.*

This result holds when the profit functions are weakly concave in  $v$ . In the convex case, for the relevant values of  $h$  and  $k$ , both agents prefer to be informed.

## 6 Conclusion

Our results show that the conflicts that arise between manufacturers and retailers require some kind of vertical restraints. For instance, in the case when both agents observe the product quality, a two-part tariff allows to choose the certification strategy that maximizes the profit of the vertically integrated structure and to share this profit between the agents. More generally, if complete contracts are available, all conflicts about certification disappear. However, in a world of incomplete contracts, namely if transfers

are prohibited, some of these conflicting situations should remain. In our model, one could investigate the possibility of writing contracts conditional on the certification decisions. Such contracts could bring the threshold values of qualities beyond which certification prevails closer. However, unless these values coincide, conflicts will remain.

To conclude, some interesting questions are raised by certification of product quality. In this paper, we have explored the role of certification as a signal of product quality. Many open questions remain. First, it is clear that certification may also serve strategic purposes as product differentiation, both on the part of retailers and manufacturers. In such a context, it may be worth analyzing how retailers and manufacturers compete in certification choices. Second, in the real world, certification and labels often emerge from collective agreements between producers whose products are not necessarily homogeneous in quality. This raises the problem of how these agreements are reached, and what are the resulting certification choices. A third question is whether consumers benefit from quality certification : on the one hand, they obtain a valuable information, but on the other hand, the seller then increases his price in order to exploit a higher willingness to pay. If certification involves higher marginal costs, the price also increases in response to higher costs and the overall effect on consumers surplus is a priori ambiguous. Finally, the role of certification bodies should be examined. In some countries, certification is mainly provided by competing private institutes; in others, a unique public agency is in charge of the certification : these various institutional arrangements may affect the certification choices of the agents and the global welfare.

## Appendix

**Proof of lemma 2.** Consider  $\hat{v}$  in  $[a, b]$ . The perfect information profit of  $\hat{v}_e$ <sup>10</sup> is  $\pi(\hat{v}_e)$ . For each  $v$ , define  $\hat{p}(v)$  as a solution of  $\Pi(p(v), \mu(p(v))) = \pi(\hat{v}_e)$ . Consider the following strategy : for all  $v$  in  $[a, \hat{v}]$ ,  $p(v) = p^*(\hat{v}_e)$  and for any  $v$  in  $]\hat{v}, b]$ ,  $p(v) = \hat{p}(v)$ . This strategy consists in pricing at the perfect information price associated to the quality  $\hat{v}_e$  for a seller in  $[a, \hat{v}]$ , and pricing according to the rule  $\hat{p}(v)$  for a seller in  $]\hat{v}, b]$ . We will show that  $p(v)$  is an equilibrium strategy of the seller  $v$  in the no certification economy, which is sustained by the following equilibrium beliefs :  $\mu(\hat{p}(v)) = v$  and  $\mu(p^*(\hat{v}_e)) = \hat{v}_e$ .

In addition, suppose that if there is no  $v$  in  $[a, b]$  such that  $p(v) = p$ , then  $\mu(p) = a$ , which is the "worst" possible out of equilibrium belief.

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<sup>10</sup>Recall that  $\hat{v}_e = E(v \mid v \leq \hat{v})$ .

First, if the seller  $v$  deviates and plays  $p(v')$  with  $v' \neq v$ , the buyer updates her belief in  $v'$ , but this does not benefit to the seller  $v$  since he earns the same profit  $\pi(\hat{v}_e)$ . Second, suppose that the seller  $v$  chooses a price  $p$  such that  $p(v') \neq p$  for any  $v'$  in  $[a, b]$ . Then, due to the definition of the out of equilibrium belief, the buyer thinks that the quality is in fact  $a$ . But then the profit is at most  $\pi(a)$  which is lower than  $\pi(\hat{v}_e)$ . This shows that  $p(v)$  is indeed an equilibrium price for the seller  $v$ . We have thus exhibited a semi separating equilibrium. It is clear that there are infinitely many equilibria of this type, each of them corresponding to a particular value of  $\hat{v}$  and a particular choice of  $\hat{p}(v)$ . A totally separating equilibrium is obtained for  $\hat{v} = a$ . The pooling equilibrium is given by  $\hat{v} = b$ . It gives to any seller the profit  $\pi(b_e) = \pi(\bar{v})$ , which is clearly better than what can be obtained in any other equilibrium since  $\pi$  is increasing in  $v$ . ■

**Proof of proposition 1.**

(i) It is easy to see that  $\tilde{v} > a$  : as  $\pi(a) > \pi(a) - k$ , by continuity of  $\pi$ , for an  $\varepsilon$  small enough, we have  $\pi(a) > \pi(a + \varepsilon) - k$ . This is because if a seller of quality  $a + \varepsilon$  does not certify, he earns at least  $\pi(a)$ , which implies that certification is a dominated strategy for  $a + \varepsilon$ .

Next we show the multiplicity of equilibria. Consider a seller  $\hat{v}$  in  $[a, b]$  such that  $\pi(\hat{v}_e) \geq \pi(\hat{v}) - k$ . Define  $\tilde{v}(\hat{v})$  as the unique solution of the equation:  $\pi(v) - k = \pi(\hat{v}_e)$ : that is,  $\tilde{v}(\hat{v})$  is indifferent between certification and no certification. The unicity is easy to check since the function  $\pi(v)$  is strictly increasing in  $v$ . The existence comes from the fact that  $\pi(a) - k < \pi(\hat{v}_e)$  for any  $\hat{v} \geq a$ , and from the inequality  $\pi(b) - \pi(\hat{v}_e) \geq \pi(b) - \pi(\bar{v}) > k$ . The theorem of the intermediate values then applies.

(ii) Consider now the following strategies:

if  $v \in [a, \hat{v}]$ , then  $s(v) = (0, p^*(\hat{v}_e))$ ;

if  $v \in ]\hat{v}, \tilde{v}(\hat{v})]$  then  $s(v) = (0, p_{\hat{v}}(v))$ , where  $p_{\hat{v}}(v)$  is the higher solution of the equation :  $\Pi(p, v) = \pi(\hat{v}_e)$ ;

if  $v \in ]\tilde{v}(\hat{v}), b]$ ,  $s(v) = (c(v) = v, p^*(v))$ .

That is, types in  $[a, \tilde{v}(\hat{v})]$  do not certify; among these, types in  $[a, \hat{v}]$  choose the (same) perfect information price corresponding to the average quality in this interval, that is  $p^*(\hat{v}_e)$ , whereas types in  $]\hat{v}, \tilde{v}(\hat{v})]$  reveal their types through their pricing strategies. Again, all these types obtain the same profit  $\pi(\hat{v}_e)$ .

Types in  $]\tilde{v}(\hat{v}), b]$  certify : they choose the certification level  $c(v)$  equal to their true quality and the corresponding perfect information price  $p^*(v)$ . Note that we can describe such strategies for any value  $\hat{v}$  satisfying  $\pi(\hat{v}_e) \geq \pi(\hat{v}) - k$ .

These strategies constitute an equilibrium which is sustained by the following beliefs : in equilibrium,  $\mu(0, p^*(\hat{v}_e)) = \hat{v}_e$ ;  $\mu(0, p_{\hat{v}}(v)) = v$  for

$v \in ]\hat{v}, \tilde{v}(\hat{v})]$ ,  $\mu(c(v) = v, p^*(v)) = v$  for  $v \in ]\tilde{v}, b]$  and out of equilibrium,  $\mu(0, p) = a$  for all  $p$  and  $\mu(c, p) = c$  for all  $p$  and  $c \neq 0$ . The proof is analogous to that of lemma 2.

(iii) Conversely, to any equilibrium, one can associate a value  $\hat{v}$  such that the strategies are defined as above, the common value of the profit of any seller who doesn't certify being  $\pi(\hat{v}_e)$ . In any of these equilibria, the sellers who certify (namely those in  $]\tilde{v}(\hat{v}), b]$ ) obtain their perfect information profit. Since for any  $v \leq \tilde{v}(\hat{v})$  the seller earns the same profit,  $\pi(\hat{v}_e)$ , the dominant equilibrium is defined by the value of  $\hat{v}$  that maximizes the profit of the sellers who don't certify, that is:  $\tilde{v} = \max_w \{w \mid \pi(Ew) = \pi(w) - k\}$ . ■

**Proof of Proposition 2:** On the one hand,  $\Pi_{i=0} = \pi(\bar{v})$ . On the other hand,  $\Pi_{i=1} = G(\tilde{v}) \pi(E(v \mid v < \tilde{v})) + (1 - G(\tilde{v})) E(\pi(v) \mid v > \tilde{v}) - (1 - G(\tilde{v})) k - h$ .

Assume  $\pi$  is strictly concave, Jensen's inequality leads to:

$\Pi_{i=1} < G(\tilde{v}) \pi(E(v \mid v < \tilde{v})) + (1 - G(\tilde{v})) \pi(E(v \mid v > \tilde{v})) - (1 - G(\tilde{v})) k - h$ . The concavity of  $\pi$  insures then:

$\Pi_{i=1} < \pi(G(\tilde{v}) E(v \mid v < \tilde{v}) + (1 - G(\tilde{v})) E(v \mid v > \tilde{v})) - (1 - G(\tilde{v})) k - h = \pi(\bar{v}) - (1 - G(\tilde{v})) k - h$ . ■

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