How raising the minimum wage can reduce voluntary labor market segmentation

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Abstract

This paper tries to answer to the following question: why did studies in the 1970’s and 1980’s find an adverse employment impact of the minimum wage, while several recent works fail to detect such a negative employment effect? It is argued that this has something to do with recent changes in production processes, whose result is a stronger strategic complementarity between workers. This paper analyzes how, in this context of changes in production processes, search strategies in the labor market respond to an increase in the minimum wage. The functioning of the labor market is described by a two-sided search model, where agents in both sides of the labor market search for a partner. In this framework, it is shown that, under some conditions, raising the minimum wage encourages the more educated unemployed to become less selective in their matching behavior. In other words, they accept to work with the low-skilled that they were refusing before. Then the degree of (voluntary) labor market segmentation decreases and the welfare of the low-skilled unemployed improves. However, it must be noted that raising the minimum wage is never pareto-improving: some lose and some gain. The interesting point is that the (potential) winners are the less-skilled unemployed. Note that the impact of raising the minimum wage depends on the initial degree of labor market segmentation: one necessary (but not sufficient) condition for an increase in the minimum wage to be welfare-improving is that the labor market is initially partially segmented.

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1. Introduction

Until recently, there was a broad consensus among economists regarding the negative effects of the minimum wage on employment. According to this conventional view, a minimum wage reduces employment of low-wage, young and unskilled workers. Brown, Gilroy and Kohen [1982] express this conventional point of view when they write: «Time-series studies typically find that a 10 percent increase in the minimum wage reduces teenage employment by 1 to 3 percent... We believe that the lower half of the range is to be preferred». However, this consensus has been challenged since the beginning of the 1990s by a series of empirical and theoretical works. The most influential of these studies is due to Card and Krueger [1994] and compares employment changes at fast food restaurants in New Jersey and Pennsylvania, following an increase in New Jersey’s minimum wage in 1992. It shows that the rise in the minimum wage was not followed by a reduction in fast food employment in New Jersey1. In the same line, two analyses of the 1990-1991 increases in the federal minimum wage, by Katz and Krueger [1992] and Card [1992], find no negative employment effect. But this could be caused by the low initial level of the minimum wage in the U.S.

However, in Europe, where the minimum wage is higher, several studies fail to conclude that the minimum wage has an unambiguous negative impact on employment. Dolado et al. [1996] find mixed results: recent minimum wage increases in Europe have reduced employment in some cases and increased it in others. The main conclusion is that the role of the minimum wage has certainly been exaggerated in the literature. Machin and Manning [1994] and Dickens, Machin and Manning [1998] do not find a real increase in employment in the U.K. after the abolishment of the minimum wage in 1993. Other papers give stronger evidence against the minimum wage. For example, Abowd, Kramarz, Lemieux and Margolis [1999] focus on the case of workers whose current wage will fall below the new minimum wage after the increase. They show that these individuals, «caught» by the minimum wage increase, have a lower employment probability than those who are not.

On the whole, the empirical evidence seems often contradictory and the results are difficult to interpret from a theoretical point of view. Card and Krueger [1995] summarize these recent studies in the following way: «Recent minimum wage increases have not had the negative employment effects predicted...Some of the new evidence points towards a positive effect of the minimum wage on employment; most shows no effect at all». This leads to the following question: why did studies in the 1970’s and 1980’s find an adverse employment impact of the minimum wage, while several recent works fail to detect such a negative employment effect? This

1Note that this result has been challenged by Neumark and Wascher [1995].
paper argues that this has something to do with the recent changes in production processes and, more precisely, with the stronger strategic complementarity in new organizations.

The literature suggests three reasons for which the minimum wage may have a positive effect on employment. First, there is the well-known monopsony explanation. However, the fast food example described by Card and Krueger [1994] does not seem to fit well into this explanation. The efficiency wage theory gives a second explanation: Rebitzer and Taylor [1995] show that, in an efficiency wage model, the minimum wage may have a positive impact on employment. However, once again, this framework does not seem well-adapted to the fast food industry. Lastly, it is sometimes argued that raising the minimum wage may encourage workers to acquire more education. The mechanism is the following: workers whose current productivity will fall below the new minimum wage can choose between staying unemployed (because of their lack of education) and investing in education to become more productive. Some of them decide to acquire more education, which improves the social welfare. Nevertheless, from a theoretical point of view, raising the minimum wage could also have the opposite effect. Indeed, a higher minimum wage rises the expected wage of low-skilled workers. Thus, it lowers the return of investing in education. It is difficult to know which effect (the positive or the negative) is the most important.

In short, none of these three reasons seems able to explain why studies in the 1970’s and 1980’s find adverse employment impact of the minimum wage, while several recent works fail to detect such a negative employment effect. Another argument, more closely related to our own, is proposed by Fershtman and Fishman [1994]. They are interested in the impact of the minimum wage on search strategies in a search framework. They point out that raising the minimum wage may reduce the incentive for job search and lower the average wage when agents must invest in costly search to become informed about prices.

This paper develops another mechanism, based on matching strategies, to account for the recent empirical evidence. Unlike Fershtman and Fishman [1994], the analysis takes firms into account and tries to connect recent changes in production processes with changes in search strategies in the labor market. Indeed, since twenty years, more and more firms reorganize their production processes. These firms were previously using tayloristic production processes (e.g. assembly line work). Now they use team work. The new production processes are thus characterized by a stronger technological complementarity between workers. Moreover, it is more difficult for a worker to carry out properly her tasks. As a consequence, the quality of the

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2 Indeed, it is difficult to believe that fast food industry is a good example of a monopsonistic industry (the restaurants are numerous, the products are substitutes, the workers are low-skilled).

3 See for example Cahuc and Michel [1996] and Agell and Lummerud [1997].
final output depends on the quality of all agents’ work: technological complementarity results in strategic complementarity.

This paper analyzes how, in this context of strong strategic complementarity, search strategies in the labor market respond to an increase in the minimum wage. In this aim, I assume that the production process consists of two working positions. We can think that one of the position is designed for a manager and the other is meant for a worker. Or we can think more simply of two workers. The important point is that their tasks are complementary. The functioning of the labor market is described by a two-sided search model, where agents in both sides of the labor market search for a partner. An unemployed agent defines a search strategy in two points: on the one hand she chooses a matching strategy (which types of agents she accepts as partners) and on the other hand she decides how intensively she searches for a partner. When she has initiated a successful contact or been successfully contacted by a partner, she engages in production. However, since the output produced by a partnership and workers’ earnings depend on the endowment of human capital of both partners, some contacts do not lead to a partnership. First, the minimum wage interferes: two agents cannot enter into partnership if at least one of them earns less than the minimum wage. Moreover, some unemployed agents refuse a partnership that produces an insufficient level of output. This type of behavior may result in an endogenous labor market segmentation according to education: the more educated match with other skilled agents while the less educated have to work with other low-skill agents.

In this framework, search strategies may react to changes in the minimum wage in a strikingly counterintuitive way. In particular, it will be shown that, under some conditions, raising the minimum wage encourages the more educated unemployed to become less selective in their matching behavior. In other words, they accept to work with the low-skilled that they were refusing before. Then the degree of (voluntary) labor market segmentation decreases and the welfare of the low-skilled unemployed improves. However, it must be noted that raising the minimum wage is never pareto-improving: some agents lose and some others gain. An interesting point is that the winners are the less-skilled unemployed.

Moreover, the conditions under which this phenomenon occurs are restrictive and nobody gains when these conditions are not verified. These conditions have to do with the distribution of human capital among the unemployed. The impact of raising the minimum wage depends also on

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4 See Kremer and Maskin [1996].
5 In the «real world», managers and workers differ by the type and the number of tasks they perform, but all of them are employed and payed by the shareholder of the firm. In this paper I will assume for simplicity that the two working positions consist of one task each. Thus the difference between a manager and a worker disappears, and we can call both of them «workers».
6 For example, an unemployed manager (worker) looks for an unemployed worker (manager).
the initial degree of labor market segmentation. On the one hand, we show that, when the labor market is non-segmented, a minimum wage is detrimental to all agents’ welfare. On the other hand, when the labor market is completely segmented, raising the minimum wage lowers the probability, for the agents «caught» by the minimum wage increase, of leaving unemployment. Therefore, one necessary (but not sufficient) condition for an increase in the minimum wage to be welfare-improving is that the labor market is initially partially segmented.

By connecting changes in production processes with changes in search strategies, we are thus able to explain why several recent works fail to detect a negative employment effect of the minimum wage. We are also able to understand why the empirical evidence seems so contradictory, with sometimes a positive effect, sometimes a negative one, and sometimes no effect at all.

The paper is organized as follows. In section 2 we present the basic model and define the search problem faced by an unemployed agent. Section 3 applies this analysis to the case of a simple discrete distribution of human capital with three skill categories: we describe how search strategies depend on the minimum wage and discuss the conditions under which raising the minimum wage reduces labor market segmentation. Section 4 studies the case of a uniform distribution of human capital and shows that, under certain conditions, raising the minimum wage will lead the highest-educated agents to become less selective in their search behavior. Section 5 concludes.

2. The model

We look for a production function that verifies the main characteristics of new production processes: 1) there is a strong technological complementarity between workers; 2) tasks are more difficult to carry out properly in new organizations than in tayloristic ones; 3) technological complementarity results in strategic complementarity. The «O-Ring» function proposed by Kremer [1993] verifies these three points and has the advantage of simplicity7. The hypotheses are the following: the production process consists of several tasks, any worker can commit a mistake when performing any task, and a simple mistake in any of these tasks dramatically reduces the output.

Take the case of two working positions. For example, one position can be designed for a manager and the other can be meant for a worker. Or, since in the «real world» managers and workers differ by the type and the number of tasks they perform, but are all of them employed by the shareholder of the firm, we can think more simply of two workers with more or less tasks.

7 This function is named following the space shuttle Challenger, that exploded in 1987 because one of its components, the O-Rings, didn’t endure heat.
The important point is that the tasks performed by the two agents are complementary. Then, when two agents $i$ and $j$ occupy respectively the positions number 1 and 2, the production function is the following:

$$Y_{ij} = h_i^{\beta_1} h_j^{\beta_2}$$

where $h_k$ represents type $k$ agent’s human capital, $k = \{i, j\}$, and $\beta_p$ denotes the number of tasks in post $p$, $p = \{1, 2\}$.

The probability a task is properly carried out depends logically on the worker’s human capital: when performing the same task, high-skill workers are assumed to commit less mistakes than the low-skilled. Therefore, an agent of any type is more productive when she is matched with a high-skill partner. Moreover, since the number of tasks is fixed, it is not possible to substitute several low-skill workers for one high-skill worker. These characteristics of the O-Ring function emphasize the strong technological complementarity between partners.

Note that this prevents the workers from being payed at their marginal productivity. The reason is the following. When an agent commits a mistake, the output of the partnership is partly or completely lost. Her partner’s work is also partly or completely spoiled. Therefore, the output is too small for the good partner to be payed for her «true» work. The only way to pay her for her «true» work would be to oblige the bad worker to compensate her for having lost the output. However, the bad worker would be payed a negative wage, which cannot happen\(^8\).

As a result, each partner’s earnings depend on the quality of both partners: the technological complementary results in a strategic complementary between earnings functions.

The question is now: how to share out the surplus between partners? The problems are the following: it is difficult to determine who failed to carry out properly her work; even if this was possible, a worker cannot be payed for her true work when her partner committed a mistake. An easy way to deal with these problems is to pay each worker in proportion to the number of tasks she performs\(^9\). Then, if agent $i$ occupies the post number 1 and agent $j$ the post number 2, agent $i$ is payed $w_{ij} = \frac{\beta_1}{\beta_1 + \beta_2} \left( h_i^{\beta_1} h_j^{\beta_2} \right)$ and agent $j$ is payed $w_{ji} = \frac{\beta_2}{\beta_1 + \beta_2} \left( h_i^{\beta_1} h_j^{\beta_2} \right)$. In the case where $\beta_1 = \beta_2 = 1$, the surplus is evenly shared between the two partners, whatever human capital they are endowed with\(^10\). Of course, when $\frac{h_i h_j}{2}$ is lower than the minimum wage ($w$), the

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\(^8\) More generally, monetary transfers between partners are not allowed.

\(^9\) This sharing rule can be justified in an other way. Assume that both partners are payed a fraction $\theta$ of their marginal productivity. Since agent $i$’s marginal productivity by unit of human capital is $\beta_1 h_i^{\beta_1 - 1} h_j^{\beta_2}$, she earns a wage $\theta \beta_1 Y_{ij}$. In the same way, agent $j$ earns a wage $\theta \beta_2 Y_{ij}$. Then, if the expected profit is nil at the equilibrium, $\theta = \frac{1}{\beta_1 + \beta_2}$: each worker is payed in proportion to the number of tasks she performs.

\(^10\) The particular functional form set forth exhibits increasing returns to the education of the whole work force, but much of the analysis generalizes to a production function homogenous of degree less than one, such as
meeting between agents $i$ and $j$ does not lead to a partnership\textsuperscript{11}. For the sake of tractability, in the rest of the paper, I will limit the analysis to the case where $\beta_1 = \beta_2 = 1$. Thus the difference between the manager and the worker disappears, and both agents will be called workers.

The labor market is described by a continuous-time two-sided search model in the fashion of MacNamara and Collins [1990] and Bloch and Ryder [1994]. The economy is composed of a continuum of identical agents differentiated only by their endowment of human capital. When unemployed, an agent searches for an acceptable partner on the labor market, and, when she has initiated a successful contact or been successfully contacted by a partner, she engages in production\textsuperscript{12}. A contact between two unemployed agents is successful when it leads to a partnership, that is when both agents agree to the match. However, some contacts do not lead to a partnership. There are two reasons for this. First of all, since the output produced by a partnership depends on both partners’ human capital, a high-skill unemployed agent may voluntarily refuse a partnership that produces an insufficient output. The second reason is the minimum wage. Two agents cannot form a partnership if at least one of them earns less than the minimum wage.

When meeting, both agents discover instantaneously their potential partner’s type and simultaneously decide whether they accept the match or refuse it. If both agree to the match, the partnership is concluded and the two agents leave the pool of unemployed workers\textsuperscript{13}. If one of the two agents refuses the match, they both stay unemployed and continue to search for an acceptable partner. For the sake of tractability, we assume that, after any match between two agents, two new agents with the same human capital endowments enter the economy in such a way that the distribution of human capital among the unemployed is time-invariant\textsuperscript{14}.

An unemployed agent has two decisions to take. First she must choose how intensively she will search for a partner. Let $m_i$ denote the intensity of search chosen by a type $i$ unemployed

\[ Y_{ij} = \left( h_1^{\beta_1} h_2^{\beta_2} \right)^{\varepsilon}, \text{where } \varepsilon \leq \frac{1}{\beta_1 + \beta_2}. \]

\textsuperscript{11}Note that the way in which the surplus is divided between the two partners does not affect our main results, as long as the sharing rule remains exogenous: if, in a partnership, the higher-educated worker was to receive a share $\eta$ of the output and the lower-skilled agent a share $(1 - \eta)$, labor market segmentation would occur in the same way, but for other parameters. This comes from the multiplicative form of the production function. It would be interesting to introduce Nash bargaining, but first it complicates greatly the model, and second, this is not a very realistic description of surplus sharing in firms.

\textsuperscript{12}Although the agents differ by their skill, there is one labor market for all because it is assumed that all types of workers can perform the same task, even if high-skill workers perform it better and more rapidly than low-skill workers.

\textsuperscript{13}For simplicity, we suppose there is no exogenous cause of separation: since on-the-job-search is not allowed in our economy, a match lasts for ever.

\textsuperscript{14}This assumption, although somewhat unsatisfactory, is common in the literature because this introduces some stationarity in the optimal search problem. For examples, see MacNamara-Collins [1990], Bloch-Ryder [1994], or Siandra [1994].
agent. More precisely, \( m_i \) measures the probability per unit of time that agent \( i \) contacts another unemployed agent of any type. The cost of search per unit of time \( \Gamma_i(m_i) \) is supposed to be an increasing and convex function of search intensity, and, moreover, to depend positively on human capital\(^{15}\). For the sake of tractability\(^{16}\), we assume more precisely that \( \Gamma_i(m_i) = h_i \frac{m_i^2}{2} \).

An unemployed agent must also choose with whom she accepts to form a partnership. As it has been shown in the literature\(^{17}\), the policy that maximizes the expected return from unemployment is to use a reservation strategy: accept any agent whose endowment of human capital is above a critical level and reject any one below. Let \( d_i \) denote agent \( i \)'s matching space. Then a match between two agents \( i \) and \( j \) can be concluded if and only if both \( i \in d_j \) and \( j \in d_i \).

Let \( P_i \) and \( Q_i \) denote respectively the probabilities per unit of time for agent \( i \) to initiate a successful contact and to be successfully contacted. \( P_i \) depends on \( d_i \) and \( m_i \). \( Q_i \) depends on \( d_i \) and on \( m_j \), \( \forall j \in d_i \). Lastly, let \( U_i \) denote the present discounted value of the expected income stream for a type \( i \) unemployed agent, and \( W_{ij} \) the present discounted value of the expected income stream for a worker \( i \) matched with a worker \( j \). A type \( i \) unemployed agent receives no income by assumption. Hence, \( U_i \) and \( W_{ij} \) satisfy:

\[
\begin{align*}
\frac{r}{1 - r} & = (P_i + Q_i) (W_i^e - U_i) - \Gamma_i(m_i) \\
\text{where} & \quad W_i^e = E(W_{ij} / \ i \in d_j \text{ and } j \in d_i, \text{ and } \frac{h_i h_j}{2} \geq w) \\
\text{and} & \quad r W_{ij} = \frac{h_i h_j}{2}
\end{align*}
\]

Combining these equations yields the utility of unemployment for a type \( i \) agent:

\[
\begin{align*}
\max_{\{m_i, d_i\}} r U_i = - \frac{r \Gamma_i(m_i)}{r + P_i + Q_i} + \frac{h_i}{2} \left( \frac{P_i + Q_i}{r + P_i + Q_i} \right) E(h_j / \ i \in d_j \text{ and } j \in d_i) \\
\text{where} & \quad \frac{h_i h_j}{2} \geq w
\end{align*}
\]

\(^{15}\)This last hypothesis can be justified in the following way. Higher skilled individuals face a greater search cost because they have to go through several interviews for a given job, when the low-skilled have to go through only one interview; they must be better-dressed than the low-skilled; they search on a larger geographical space, which leads them to spend more on transport.

\(^{16}\)Note that this cost specification will greatly simplify the determination of search strategies and labor market equilibrium. In particular, in the case of a uniform distribution, this multiplicative form will lead to a division of the labor market into disjoint segments. Nevertheless, the potentially positive effect of the minimum wage does not depend on this particular specification.

\(^{17}\)See Mortensen [1986] for example.
It remains to specify the distribution of human capital. The cases of a discrete (trinomial) and of a uniform distribution of this endowment will be successively studied. The case of a trinomial distribution of human capital has the advantage of simplicity and illustrates in an intuitive way the effects, on matching strategies, of raising the minimum wage. To generalize the results, it is necessary to take the case of a continuous distribution. However, to keep the analysis tractable, we will restrict the study to the case of a uniform distribution.

3. An example: the case of a trinomial distribution of human capital

Assume the economy is composed of three types of agents: high-skill (type $a$), middle-skill (type $b$) and low-skill (type $c$). Agents of type $i$ ($i = a, b, c$) are endowed with a human capital $h_i$ and represent a fraction $f_i$ of the unemployed. The number of unemployed workers is initially fixed and normalized to have measure 1, and will remain constant over time because of the assumption that any agent who finds a partner is immediately replaced.

3.1. Labor market equilibrium

3.1.1. The direct and indirect effects of the minimum wage

The workers can be affected in two ways by an increase in the minimum wage. They can become directly constrained because their matching space is reduced. They can also become indirectly concerned in the sense that, since some of their potential partners become themselves directly constrained and search less intensively, their own probability of being successfully contacted decreases.

Consider first the direct impact. Two agents cannot form a partnership if at least one of them earns less than the minimum wage ($w$). Take for example $w = \frac{h_i h_j}{2}$. Then a type $i$ agent cannot match with an agent endowed with a human capital lower than $h_j$. Assuming that $\frac{h_a h_c}{2} > \frac{h^2}{2}$ and $w < \frac{h_a h_c}{2}$, the different stages beyond which matching strategies change are the following: $\frac{h^2}{2}$, $\frac{h_a h_c}{2}$, and $\frac{h^2}{2}$. Note that in the case of a discrete distribution, raising the minimum wage between two stages has no impact on the economy.

The case where the minimum wage $w$ is lower than $\frac{h^2}{2}$ will serve as a benchmark case. When $w$ lies between $\frac{h^2}{2}$ and $\frac{h_a h_c}{2}$, two low-skilled unemployed can no longer form a partnership. For $w$ between $\frac{h_a h_c}{2}$ and $\frac{h^2}{2}$, a low-skill agent loses the possibility of matching either with another low-skill worker or with a middle-skill worker. Lastly, for $w$ between $\frac{h^2}{2}$ and $\frac{h_a h_c}{2}$, all the partnerships already quoted remain impossible and, moreover, two middle-skill unemployed can

\[18\] $
h_a > h_b > h_c.$

\[19\] A minimum wage above $\frac{h_a h_c}{2}$ has an unambiguously negative impact on the economy: thus this case will not be studied.
no longer form a partnership.

Raising the minimum wage has also an indirect impact on matching strategies and search intensities. The reason is the following. When an agent becomes directly constrained by the minimum wage, her matching space is reduced, but she stays free to choose her search intensity. Since search is less profitable, she searches less intensively\(^{20}\). But this lowers the probability for her potential partners of being contacted. The latter ones become indirectly concerned by the minimum wage. They may react by adapting their search intensity and/or their matching space.

To illustrate these definitions, assume that the minimum wage is initially lower than \(\frac{h_2}{2}\) and the labor market divided into two segments: the high-skilled on one side, the middle and low-skilled on the other side. Then the government raises the minimum wage above \(\frac{h_2}{2}\). The low-skilled become directly constrained and the middle-skilled indirectly concerned. The high-skilled stay not affected.

3.1.2. The search problem

A matching strategy \(d_i\) is feasible if, \(\forall j \in d_i, \ i \in d_j\). Since \(m_i\) measures agent \(i\)’s search intensity, the probability for a given agent \(i\) of meeting a type \(j\) agent (any one) is given by \(f_jm_i\). In the same way, her probability of being contacted by a type \(j\) agent (any one) is given by \(f_jm_j\)\(^{21}\). As a result, the probability per unit of time for agent \(i\) to initiate a successful contact is \(P_i = \sum_{j \in d_i} f_jm_i\). Her probability of being successfully contacted is \(Q_i = \sum_{j \in d_i} f_jm_j\).

Replacing \(P_i\) and \(Q_i\) with their expression in equation (2.2) gives the search problem faced by a type \(i\) agent:

\[
\max_{\{m_i,d_i\}} rU_i = \frac{\left(\sum_{j \in d_i} f_jm_ih_j + \sum_{j \in d_i} f_j\bar{m}_jh_j - rm_i^2\right)h_i}{2}
\]  

(3.1)

where \(d_i\) is a feasible matching strategy and \(\bar{m}_j\) denotes type \(j\) agents’ average search intensity.

An unemployed agent must choose, on the one hand, how intensively she searches for a partner, and, on the other hand, with whom she forms a partnership. Thus a two-steps procedure is used to solve the search problem.

\(^{20}\)This result will be proved later in the Appendix (proof of proposition 3.1).

\(^{21}\)The probability for a given agent \(i\) of being contacted by a given agent \(j\) is \(m_j\) and the fraction of type \(j\) agents among the unemployed is \(f_j\).
1. Let \( D_i = \mathcal{P}_i \{a, b, c, \emptyset\} \) denote the set of matching strategies for a type \( i \) agent. In a first step, given a matching strategy \( d_i \in D_i \), agent \( i \) determines how intensively she will search for a partner. Let \( m^*_i (d_i) = \arg \max_{m_i} \{U_i[m_i (d_i)]\} \) denote this optimal intensity of search and \( U_i[d_i] = U_i[m^*_i (d_i)] \) denote the maximum utility that agent \( i \) gets from the strategy \( d_i \).

2. In a second step, a type \( i \) agent chooses the strategy of matching that maximizes her welfare, \( d^*_i = \arg \max_{d_i} \{U_i[d_i]\} \). To solve the matching problem, it is convenient to begin with the case of the high-skilled. They must choose between three matching strategies: match only with the high-skilled (i.e. homogeneous matching), match with the high or the middle skilled, match with all types of agents. Next, the middle-skilled have to decide their search behavior towards the low-skilled. Obviously, their choice depends on the strategy chosen by the high-skilled. The reason is the following: all agents have the same preferences, whatever human capital they are endowed with. Thus they class the different alternatives in the same order. Consequently, when the low-skilled are accepted as partners by the high-skilled, they are also acceptable partners for the middle-skilled. Moreover, when the high-skilled accept the middle-skilled and refuse the low-skilled, the middle-skilled are better off when they also refuse the low-skilled. The low-skilled are the last ones to choose. In fact they do not choose, because they are ready to form a partnership with any type of agent.

3.1.3. The degree of labor market segmentation

At the equilibrium, the labor market can be completely or partially segmented, or can stay unsegmented. In the case of a partial segmentation, this segmentation can occur at the top of skill (between the high and the middle-skilled), or at the bottom (between the middle and the low-skilled). The following proposition describes in detail the conditions under which the economy is in one state or in another.

**Proposition 3.1.** Let \( C_a = 3 \left[f_a \left(\frac{h_a-h_b}{2r}\right)^2 \right] - h_b; C_b = 3 \left[f_b \left(\frac{h_b-h_c}{2r}\right)^2 \right] - h_c; \)
\[ C_{ab} = 3 \left[f_a \left(\frac{h_a-h_b}{2r}\right)^2 + f_b \left(\frac{h_b-h_c}{2r}\right)^2 \right] - h_c; \]
and \( C = C_{ab} - f_b (\frac{h_b-h_c}{r}) \left(\frac{r}{f_a} + f_a (\frac{h_a-h_b}{r}) + f_b \left(\frac{h_b-h_c}{r}\right) + \frac{h_b-h_c}{r}\right). \)

Depending on the minimum wage and on the characteristics of the labor force, the economy can be in four different cases:

(i) The labor market is divided into three homogeneous segments: on each segment, agents of type \( i \) match exclusively with other type \( i \) agents provided that they are not constrained by the minimum wage; otherwise, they stay unmatched. Thus \( d^*_i = \{i\} \) for \( i \) non-constrained and \( d^*_i = \{\emptyset\} \) otherwise. This case occurs under one of the following conditions: either \( w < \frac{h_b h_c}{2}, \)
\( C_a > 0 \) and \( C_b > 0 \); or, \( w \geq \frac{h_b h_c}{2} \) and \( C_a > 0 \).

(ii) The labor market divides into two segments: the high-skilled occupy the first segment and the middle and low-skilled occupy the second one. This case occurs under the following condition: \( w < \frac{h_b h_c}{2}, C_a > 0 \) and \( C_b < 0 \).

(iii) The labor market divides into two segments: the high and middle-skilled occupy the first segment and the low-skilled the second one. This case occurs under one of the following conditions: either \( w < \frac{h^2}{2}, C_a < 0 \) and \( C_{ab} > 0 \); or, \( w \geq \frac{h^2}{2}, C_a < 0 \) and \( C > 0 \). When \( w > \frac{h^2}{2} \), \( d^*_a = \{\emptyset\} \).

(iv) The labor market is not voluntarily segmented: the high-skilled accept agents of any type as partners. Then \( d^*_a = \{a, b, c\} \). This case occurs under one of the following conditions: either \( w < \frac{h^2}{2} \) and \( C_{ab} < 0 \); or, \( w \geq \frac{h^2}{2} \) and \( C < 0 \).

**Proof.** See the Appendix.

Proposition 3.1 first tells us that the conditions under which the economy is in one state or in an other depend on the characteristics of the labor force \((h_i \text{ and } f_i, \forall i = a, b, c)\). The four conditions \( C_a, C_b, C_{ab} \) and \( C \) give some crucial values for \( h_i \) and \( f_i \) beyond which search behaviors change. They mean more precisely that when there are many agents of a given type or when these agents are sufficiently educated, they choose a selective behavior towards the less-skilled. For example, when there are many high-skill agents among the unemployed \( (\text{at least more than } f^*_a = \left(\frac{2r}{h_a - h_b}\right) \sqrt{\frac{h_b}{3}}) \), the high-skilled have a high probability of finding a good partner. Thus they choose the homogeneous matching\(^{22}\). In the same way, when there are many middle-skill agents among the unemployed \( (\text{at least more than } f^*_b = \left(\frac{2r}{h_b - h_c}\right) \sqrt{\frac{h_c}{3}}) \), then \( C_b \) is positive and the middle-skilled refuse the low-skilled as partners. When both conditions \( C_a > 0 \) and \( C_b > 0 \) are satisfied, the labor market divides into three segments according to qualification.

The conditions \( C_{ab} < 0 \) and \( C < 0 \) have the same interpretation: they mean that the low-skilled must be educated enough and in a sufficiently high proportion among the unemployed to be accepted as partners by the more educated agents. When \( C_{ab} < 0 \) or \( C < 0 \), the labor market is not voluntarily segmented. However, these conditions are not valid on the same set of parameters. The condition \( C_{ab} < 0 \) is valid when the minimum wage is lower than \( \frac{h^2}{2} \), while the condition \( C < 0 \) is valid when the minimum wage is above \( \frac{h^2}{2} \). It is straightforward to note that the condition \( C_{ab} < 0 \) is more restrictive than the condition \( C < 0 \). In particular, a non empty set of values of \( f_i \) and \( h_i \) verify both \( C_{ab} > 0 \) and \( C < 0 \). It follows that, for the same

\(^{22}\) When \( f_a > f^*_a \), \( C_a \) is positive.
characteristics of the labor force, the low-skilled may be refused by the more educated agents\textsuperscript{23} when the minimum wage is strictly lower than $h_{i}^{2}$ and may become acceptable partners\textsuperscript{24} when it is above. Therefore, the conditions under which the economy is in one state or in an other depend also on the minimum wage. The following section will describe in detail how raising the minimum wage does affect labor market equilibrium.

3.2. What is the impact of raising the minimum wage on search strategies?

This section shows that, under some conditions, raising the minimum wage reduces (voluntary) labor market segmentation, shorters unemployment duration and improves the situation of the less-skilled unemployed. These conditions have to do with the distribution of human capital among the unemployed and will be precisely described.

Briefly, the mechanism behind this result is the following. Raising the minimum wage reduces the space of possible matches for some unemployed. Then these agents decide to search less intensively. As a result, both the probability for these less-skilled of initiating a successful contact and the probability for the more skilled of being successfully contacted decrease. The overall effect on unemployment duration depends on whether or not the more skilled adapt their search strategy to this new state. If they keep the same matching space, raising the minimum wage has the traditional negative impact on unemployment duration and the utility of the unemployed. But if the more skilled enlarge their matching space, this may greatly improve the situation of the less-skilled unemployed\textsuperscript{25}.

3.2.1. Labor market in case (i)

Assume the economy is in case (i) where the labor market is completely segmented: agents of type $i$ match exclusively with other type $i$ agents provided that they are not constrained; they stay unmatched otherwise.

Raising the minimum wage does not affect agent $i$ as long as the minimum wage stays below $h_{i}^{2}$. Consequently, her search intensity, matching space and welfare stay the same. Beyond this value $h_{i}^{2}$, agent $i$ can no longer form a partnership. Her matching space becomes empty and her probability of leaving unemployment nil. Thus her welfare decreases dramatically. It follows that, when the labor market is completely segmented, raising the minimum wage lowers the probability, for the agents «caught» by the minimum wage increase, of leaving unemployment.

\textsuperscript{23}Because $C_{a1} > 0$.

\textsuperscript{24}Because $C < 0$.

\textsuperscript{25}This result illustrates more generally the fact that second round policy effects on search behaviour may be important and lead to «perverse» behaviour.
3.2.2. Labor market in case (iv)

Assume the labor market is not voluntarily segmented: the high-skilled accept any type of agent as partner (case (iv)). If the government raises the minimum wage, the less-skilled become "caught" by the new minimum wage and their matching space is reduced. Logically, they decide to search less intensively. Then, their potential partners, the high-skilled, have a lower probability of being contacted. The latters become indirectly concerned by the minimum wage. How do they react?

The following corollary says that a worker does not voluntarily become more selective in her search behavior when the minimum wage increases.

**Corollary 3.2.** Let $i$ and $j$ denote two types of agents, where $h_i > h_j$. Assume $j \in d_i^*$ for a minimum wage $w$. Then $j \in d_i^*$ for $w' > w$, as long as $w' < \frac{h_i h_j}{2}$.

**Proof.** It is straightforward to derive this corollary from the proof of proposition 3.1.

Therefore, the high-skilled unemployed do not change their matching space, because it is not in their interest to become more selective and they cannot become less selective. However, they modify their search intensity: they search more intensively, which reduces their utility (cf. the proof of proposition 3.1). Therefore, when the labor market is non-segmented, a minimum wage is detrimental to the welfare of all the unemployed.

The reasoning is the same when the economy is initially in case (ii).

To summarize, if the economy is initially in states (i), (ii) or (iv), raising the minimum wage has the usual negative impact: the agents (directly or indirectly) affected by the new minimum wage have a lower probability of leaving unemployment.

3.2.3. The labor market is in case (iii)

It remains the case (iii). When $C_{ab} > 0$ is verified, the labor market is divided into two segments: the high and the middle-skilled belong to the upper segment, while the low-skilled occupy the bottom segment. A minimum wage above $\frac{h_2^2}{2}$ prevents these low-skilled to leave unemployment. The high and middle-skilled unemployed are not affected, provided the minimum wage is lower than $\frac{h_2^2}{2}$. However, when the minimum wage is above this value, two middle-skilled can no longer form a partnership\(^{26}\). Their matching space is reduced, but they stay free to choose their search intensity. Since search is less profitable, they search less intensively (cf. the proof of proposition

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\(^{26}\)Consequently, the middle-skilled become directly constrained by the minimum wage and the high-skilled indirectly concerned.
3.1), which lowers the probability for the high-skilled of being contacted. The question is now: how do the high-skilled react to this decrease in their probability of leaving unemployment?

They may react in adapting their search intensity and/or their matching space. They always choose to search more intensively. Moreover, when the condition $C < 0$ is verified, they decide to accept the low-skilled as partners. On the other hand, if $C > 0$, they keep the same matching strategy than before. Therefore, the high-skilled enlarge their matching space when the minimum wage is raised above $\frac{h_2^2}{2}$, provided that the distribution of human capital among the unemployed is sufficiently heterogeneous for condition $C_{ab} > 0$ to be verified, but not too heterogeneous for condition $C < 0$ to be verified. The following corollary summarizes this result.

**Corollary 3.3.** Assume both conditions are verified: $C_{ab} > 0$ and $C < 0$.

Then, the economy is in state (iii) where $d^*_a = d^*_b = \{a, b\}$ and $d^*_c = \emptyset$ when $w \in \left[\frac{h_2^2}{2}, \frac{h_2^2}{2} \right]$, and in state (iv) where $d^*_a = \{a, b, c\}$ and $d^*_b = d^*_c = \{a\}$ when $w \geq \frac{h_2^2}{2}$.

**Proof.** It is straightforward to derive this corollary from the proof of proposition 3.1.

Under the conditions $C_{ab} > 0$ and $C < 0$, the economy goes from state (iii) to state (iv) when the government raises the minimum wage above $\frac{h_2^2}{2}$. Thus the degree of (voluntary) labor market segmentation decreases. The welfare of the low-skilled unemployed is greatly improved since they are now accepted as partners by the high-skilled. They can leave unemployment and, moreover, they match with agents of good quality. The case of the middle-skilled is very different. Now they cannot match with someone of their skill. Thus they search less intensively. Both effects lead to a longer unemployment duration. Consequently, the welfare of the middle-skilled unemployed decreases perceptibly. Note that agents of types $b$ and $c$ have the same search strategy: they search with the same intensity, on the same matching space, and they have the same probability of being contacted by a high-skill agent. Thus $U^*_{rb} = U^*_{rc}$. Finally, the consequences are more balanced for the high-skilled. Since they search more intensively, the activity of search costs them more. On the one hand, their probability of being contacted by a middle-skill agent is lower, but on the other hand, they can now be contacted by a low-skill agent. This may result in a shorter unemployment duration.

To summarize, the high and the middle-skilled unemployed lose, but not much, while the low-skilled unemployed benefit greatly from this. On the whole, the welfare of the unemployed improves substantially.

In this simple example, the economy is composed of only three groups of agents, which limits the cases where an increase in the minimum wage may be welfare-improving. Of course,
it would be interesting to have more types of agents. However, the search problem becomes very
difficult to solve. To generalize the results, it is thus necessary to take the case of a continuous
distribution. However, to keep the analysis tractable, we will restrict the study to the case of a
uniform distribution.

4. The case of an uniform distribution of human capital

Assume human capital endowments in the population are uniformly distributed on the interval
$[\overline{h} - \Delta, \overline{h} + \Delta]$. $\Delta \leq \overline{h}$ measures human capital heterogeneity. The number of unemployed
workers is normalized to 1 and stays constant over time.

4.1. Labor market equilibrium

We will proceed in two steps. First we will describe the benchmark case of a non-binding
minimum wage. This will greatly simplify the general case of a constraining minimum wage.

4.1.1. The benchmark case: a non-binding minimum wage

Let $w_0$ denote any value of the minimum wage strictly lower than $\frac{(\overline{h} - \Delta)^2}{2}$. Since such a minimum
wage has no impact on matching strategies and search intensities, it will serve as a benchmark
case.

As it has been shown in the literature, the policy that maximizes the expected return from
unemployment for an unemployed worker agent is to use a reservation strategy: accept any agent
whose endowment of human capital is above a critical value and reject any one below. Let $h_{ri}$
denote this crucial value for a type $i$ agent. Inversely, let $h_{ui}$ denote the endowment of human
capital of the highest-educated agent who accepts to form a partnership with agent $i$\(^{27}\). Then,
a match between two agents $i$ and $j$ can be concluded if and only if $i$ accepts $j$ and is accepted
by $j$, i.e. if $h_j \in [h_{ri}, h_{ui}]$. $h_{ri} = R(h_i)$ and $h_{ui} = U(h_i)$ are two non-decreasing functions of
human capital. The probability for agent $i$ to initiate a successful contact can thus be rewritten
$P_i = m_i \frac{h_{ui} - h_{ri}}{2\Delta}$. In the same way, the probability for agent $i$ to be successfully contacted is
equal to $Q_i = \int_{h_{ri}}^{h_{ui}} \frac{m(h_j)}{2\Delta} dh_j$.

Replacing $P_i$ and $Q_i$ with their expression in equation (2.2) gives the search problem faced
by a type $i$ agent:

$$\max_{\{m_i, h_{ri}\}} rU_i = \frac{h_i}{2} \left( \frac{m_i \left( h_{ui}^2 - h_{ri}^2 \right)}{m_i (h_{ui} - h_{ri})} + \int_{h_{ri}}^{h_{ui}} m(h_j) h_j dh_j - 2\Delta rm_i^2 \right)$$

\(^{27}\) Any agent whose human capital is higher than $h_{ui}$ refuses a match with $i$.\n
16
Taking first order conditions with respect to, respectively, $h_{ri}$ and $m_i$ and adding these two expressions give agent $i$'s search intensity as a function of her reservation endowment of human capital $h_{ri}$:

$$m_i = \left( \frac{1}{2\Delta r} \right) \left( \frac{h_{ui} - h_{ri}}{2} \right)^2$$  \hspace{1cm} (4.2)

Next, replacing $m_i$ by its expression in the first order condition gives $h_{ri}$:

$$2\Delta r + \int_{h_{ri}}^{h_{ui}} m(h_j) dh_j \quad h_{ri} = \left( \frac{1}{2\Delta r} \right) \left( \frac{h_{ui} - h_{ri}}{2} \right)^4 + \int_{h_{ri}}^{h_{ui}} m(h_j) h_j dh_j$$  \hspace{1cm} (4.3)

The optimal reservation strategy of agent $i$, $h_{ri}$, depends on the endowment of human capital of the highest-educated agent she can match with, $h_{ui}$, and on her probability of being successfully contacted, $\int_{h_{ri}}^{h_{ui}} m(h_j) dh_j$. The important point to note is that agent $i$'s education does not appear directly in this expression. This means that, as long as the minimum wage is non-binding, any agent whose human capital belongs to $[h_{ri}, h_{ui}]$ will choose the reservation strategy $h_{ri}$ and the search efficiency $m_i$. The following proposition describes the Nash symmetric equilibrium.

**Proposition 4.1.** (MacNamara-Collins [1990])

Let $w = w_0$. At the unique equilibrium of this two-sided search model, the population, uniformly distributed on the interval $[h - \Delta, h + \Delta]$, is partitioned into a countable set of disjoint segments $[a_n, a_{n-1}]$ where $n \in \{1, \ldots, N\}$ and $N \in \mathbb{N}^+$, such that

\[
\begin{cases}
  a_0 = h + \Delta \\
  \left( \frac{a_{n+1} - a_n}{2} \right)^2 = 2\Delta \sqrt{\frac{N}{3}}, & \forall \ n \in \{1, \ldots, N-1\} \\
  a_N = h - \Delta
\end{cases}
\]

Moreover, all agents belonging to the $n$-th segment choose the same search intensity:

\[
\begin{cases}
  2\Delta m_n = \left( \frac{a_{n+1} - a_n}{2} \right)^2, & \forall \ n \in \{1, \ldots, N-1\} \\
  3m_n^2 + 2\left( \frac{a_{N-1} - a_N}{2} \right) m_N - \left( \frac{a_{N-1} + a_N}{2} \right) = 0
\end{cases}
\]

Lastly, agent $i$'s utility, when $i$ belongs to the $n$-th segment, is given by:

$$rU_i^* = \frac{h a_n}{2} = \frac{3h}{2} m_n^2.$$

**Proof.** See the Appendix.

At the equilibrium, the labor market is endogenously divided into a number $N$ of disjoint segments of human capital\(^{28}\), in such a way that an agent will match exclusively with the other unemployed of her segment. Moreover, two agents belonging to the same segment search with the same intensity. By definition, $a_0$, the upper bound of the first segment coincides with the

\(^{28}\) Note that the number of segments is finite as long as $h > \Delta$.
upper bound of the distribution of human capital, $\bar{h} + \Delta$. Next, $\forall \, n \in \{1, \ldots, N - 1\}$, the lower bound of the segment number $n$, $a_n$, is a function of the upper bound of this segment, $a_{n-1}$. Lastly, the last segment is truncated, because, by definition, the lower bound of the last segment must coincide with the lower bound of the distribution of human capital. But if it was possible, the agents of the last segment would like to be less selective in their matching strategy.

In order to understand more intuitively how the segmentation process comes about, consider the case of the agent at the top of the distribution. This agent, whose human capital is $a_o = \bar{h} + \Delta$, is accepted by every one. Thus the maximum level of human capital of her potential partner is $a_o$. From this agent’s first order conditions, it is straightforward to compute the lowest level of human capital she will accept: $a_1$. This crucial value defines both the lower bound of the first segment and the upper bound of the second segment. Then, all the agents belonging to the first segment $[a_1, a_o]$ adopt the same strategy. Consider now the agent endowed with a human capital $a_1$: the first order conditions give $a_2$ the lower bound of the second segment, etc... This downward process could be repeated successively for all segments. Figure ?? describes the segmentation process for $N = 4$.

Figure 4.1.1: Labor market equilibrium when the minimum wage is non-binding.
Some interesting properties can be derived from proposition 4.1. In particular, segment size and search intensity are two increasing functions of education. In other words, high-skill agents belong to the largest segments and choose a higher search intensity. This result comes from two opposite effects: on the one hand, search is more profitable on a larger segment \( m_n = \left( \frac{1}{2\Delta r} \right) \left( \frac{a_n - a_{n+1}}{2} \right)^2 \); on the other hand, when an agent searches more intensively, she can adopt a more restrictive matching strategy \( m_n = \sqrt{\frac{\Delta r}{\Delta \rho}} \). It seems that, on the whole, the first effect does more than to compensate for the second effect. This leads to the following result: the exit rate from unemployment in the \( n \)-th segment, \( T_n = 2m_n \left( \frac{a_n - a_{n+1}}{2\Delta} \right) = \left( \frac{\Delta \rho}{\Delta r} \right) \left( \frac{a_n - a_{n+1}}{2\Delta} \right)^3 \), is a non-decreasing function of education.

4.1.2. The general case: a constraining minimum wage

The phenomenon of segmentation into disjoint segments simplifies greatly the analysis of the effects of the minimum wage. To see why, assume the minimum wage \( w \) is between \( a^{k-1}_2 \) and \( a^{k+1}_2 \), where \( a_{k-1} \) and \( a_k \) denote respectively the upper and lower bounds of the \( k \)-th segment. This value of the minimum wage prevents some types of partnership on this segment number \( k \). In particular, two agents endowed with the education \( a_k \) can no longer enter into partnership. However, a type \( a_{k-1} \) agent may still enter into partnership with any one belonging to the \( k \)-th segment. Therefore, some agents on this particular segment are directly constrained by the minimum wage (in the sense that it reduces the space of their possible matches), while some others are indirectly concerned (in the sense that some of their potential partners are themselves directly constrained). In fact, all of them are affected by the minimum wage \( w \) and have to adapt their search behavior. Obviously, the unemployed occupying the \( n \)-th segment, \( \forall n = \{k+1, \ldots, N\} \), are also directly constrained. Inversely, the minimum wage is non-binding for the agents occupying the segments \( 1, \ldots, k-1 \).

Consequently, for a minimum wage between \( a^{k-1}_2 \) and \( a^{k+1}_2 \), we can restrict our analysis of labor market equilibrium to the description of search behaviors on the segments number \( k \) and \( k+1 \). The analysis of the other segments is uninteresting. On the one hand, the agents belonging to the segments \( k+2, \ldots, N \) stay unmatched because they are insufficiently educated (their partnership cannot produce enough to pay two minimum wages). On the other hand, the unemployed whose human capital is greater than \( a_{k-1} \) have the same behavior than in the case of a non-binding minimum wage.

As already seen, some agents belonging to the \( k \)-th segment are indirectly concerned by the minimum wage while some others are directly constrained. We divide the population into these two groups and we determine successively their optimal search strategies.
Consider the first group. Let $x$ denote the endowment of human capital of the less-educated agent indirectly affected: this means that all agents between $x$ and $a_{k-1}$ are indirectly concerned. Then let $y$ denote the endowment of human capital of the less-educated agent that can enter into partnership with a type $x$ worker: $y$ is defined by $xy = 2w$. The search problem faced by an agent $i$, $h_i \in [x, a_{k-1}]$, is still given by equation (4.1). Therefore equations (4.2) and (4.3) define agent $i$’s optimal search intensity and reservation strategy. Once more, the important point to note is that agent $i$’s education does not appear directly in these expressions. This means that all agents whose human capital belongs to $[x, a_{k-1}]$ will choose the same reservation strategy and the same search intensity. Since $y$ represents the reservation strategy for a type $x$ worker, any agent between $x$ and $a_{k-1}$ adopts this reservation strategy.

Thus search intensity is given by

$$m_x = \left(\frac{1}{2\Delta r}\right) \left(\frac{a_{k-1} - y}{2}\right)^2 \quad (4.4)$$

Replacing $m_x$ by its expression gives $y$ as a function of search intensities chosen by all agents between $y$ and $x$:

$$\left[2\Delta r + \left(\frac{a_{k-1} - x}{2\Delta r}\right)^2 \left(\frac{a_{k-1} - y}{2}\right)^2 + \int_y^x m(h_j) \, dh_j\right] y - \left(\frac{1}{2\Delta r}\right) \left(\frac{a_{k-1} - y}{2}\right)^4 \quad (4.5)$$

The maximal utility of an agent $i$ is given by:

$$r U_i^* = \frac{h_i y}{2}$$

Consider now the second group. These agents are constrained in their choice of a matching space. However, they stay free to choose the intensity of search that maximizes their welfare. More precisely, the lowest-educated agent a type $i$ agent can match with is determined by: $h_{ri} = \frac{2w}{h_i}$. Since agent $i$ is accepted as partner by the members of the first group, her matching space is given by $\left[\frac{2w}{h_i}, a_{k-1}\right]$. The search problem faced by an agent $i$, $h_i \in [b, a]$ can thus be rewritten as follows:

$$\max_{\{m_i\}} r U_i = \frac{h_i}{2} \left(m_i \left(\frac{h_{iu} - h_{ri}}{2}\right)^2 + \int_{h_{ri}}^{h_{ui}} m(h_j) \, dh_j - 2\Delta r m_i^2 \right) \quad (4.6)$$

Taking first order condition with respect to $m_i$ gives:

29 Agent $a$ is also the highest-educated agent among the unemployed directly constrained by the minimum wage.
the case of a non-binding minimum wage. Assume that Proposition 4.2.

...is between skilled of the less-educated agent indirectly a...
Proof. See Appendix.

Proposition 4.2 states that agent \( y \) belongs to the segment number \( k + 1 \). This result was not obvious. The argument is the same than in the case of a discrete distribution. When the government establishes a minimum wage, the skilled have a lower probability of being contacted by a potential partner. To compensate, they can search more intensively and/or become less selective in their search behavior. Now it has been shown that the skilled always apply the same positive relation between their search intensity and their matching space. When they search on a larger matching space, they also search more intensively. That is why they decide both to enlarge their matching space (below agent \( a_k \)) and to increase their search intensity.

4.1.3. Labor market equilibrium

It is possible now to describe the equilibrium on the labor market when the minimum wage is between \( \frac{a_k^2}{2} \) and \( \frac{a_{k+1}a_{k-1}}{2} \).

Proposition 4.3. Let \( w \in \left[ \frac{a_k^2}{2}, \frac{a_{k+1}a_{k-1}}{2} \right] \). At the unique equilibrium of this two-sided search model, the population, uniformly distributed on the interval \([a_{k+1}, a_{k-1}]\), adopts the following search strategies:

- A type \( i \) agent, where \( h_i \in [x, a_{k-1}] \), chooses a matching space \([y, a_{k-1}]\), where \( y = \frac{2w}{x} \) is given by (4.5), and searches with the intensity \( m_x = \left( \frac{1}{2\pi r} \right) \left( \frac{a_{k-1}-y}{2} \right)^2 \). Moreover \( rU_i^* = \frac{h_i y}{2} \).

- A type \( i \) agent, where \( h_i \in [y, x] \), has to search on the segment \([h_i, a_{k-1}]\), where \( h_i = \frac{2w}{h_i} \) and searches with the intensity \( m_i \) given by (4.7).

- Any agent between \( a_{k+1} \) and \( y \) is unable to find a partner.

The following figure represents this equilibrium when the minimum wage is between \( \frac{a_k^2}{2} \) and \( \frac{a_{k+1}a_{k-1}}{2} \). The dotted zones represent segments \( k \) and \( k + 1 \) when the minimum wage is non-binding. The decreasing curve represents the endowment of human capital of the lowest-educated agent a type \( i \) agent can match with: \( h_i = \frac{2w}{h_i} \). No matching is possible below this curve. The difference between this curve and the top of the square gives the matching space of an agent \( i \) when this latter is constrained by the minimum wage. All agents between \( a_k \) and \( x \) are constrained. However, all agents between \( \frac{2w}{a_k} \) and \( x \) search a partner on a larger matching space than without minimum wage (or when the minimum wage is equal to \( \frac{a_k^2}{2} \)). Lastly, agents between \( y \) and \( a_k \) find better partners than without minimum wage.
4.2. What is the impact of raising the minimum wage on search strategies?

Assume that the minimum wage is initially equal to $a^2_k$. Then, proposition 4.3 means that an increase in the minimum wage from $a^2_k$ to $w \in \left[ \frac{a^2_k}{2}, \frac{a_k+1}{2} \right]$ has the following consequences:

An agent $i$, where $h_i \in [y, a_k]$, was previously unable to leave unemployment. Now she becomes an acceptable partner for any worker between $\frac{2w}{h_i}$ and $a_k-1$, which leads to a substantial improvement of her welfare.

Agents whose endowment of human capital is between $a_k$ and $z^{30}$ are constrained by the new minimum wage and search less intensively. Raising the minimum wage is clearly detrimental to their welfare.

Agents between $z$ and $x$ are also constrained by the new minimum wage. However, they search more intensively. Moreover, the unemployed who are sufficiently educated

\footnote{where $z$ denotes the endowment of human capital of the agent such that $m_z = \left( \frac{1}{2} \right) \left( \frac{a_k-1}{\delta h} \right)^2$.}
\( h_i \geq \frac{2w}{a_k} \) will search on a larger matching space.

Agents between \( x \) and \( a_{k-1} \) increase both their search intensity and the size of their matching space. The utility of these unemployed (who are not constrained by the minimum wage) is given by \( rU_i = \frac{h_i w}{2} \). Therefore, raising the minimum wage from \( \frac{a_k^2}{2} \) to \( w \) is slightly detrimental to their welfare.

Thus raising the minimum wage from \( \frac{a_k^2}{2} \) to \( w \in \left[ \frac{a_k^2}{2}, \frac{a_k+1}{2}a_{k-1} \right] \) reduces the degree of voluntary segmentation: the highest-educated unemployed of segment \( k \) decide to accept as partners some agents (below \( a_k \)) that they were refusing before. These agents (below \( a_k \)) stayed unmatched because of their lack of education. Now they can match with good agents: thus they benefit greatly from the rise in the minimum wage. The losers are the less-skilled of the segment \( k \) (between \( a_k \) and \( x \)), because they are ”caught” by the minimum wage increase. Note that this result is consistent with the evidence pointed out by Abowd et al. [1999]: thoses who are ”caught” by the new minimum wage have lower employment probability.

If raising the minimum wage is never pareto-optimal, the interesting point is that the winners are the less-skilled unemployed of the segment \( k + 1 \). Then, if the social planer maximizes a Rawlsian welfare function\(^{31} \), raising the minimum wage can improve the social welfare of the unemployed. Moreover, the winners gain much more than the losers lose. Thus, on the whole, raising the minimum wage from \( \frac{a_k^2}{2} \) to \( w \in \left[ \frac{a_k^2}{2}, \frac{a_k+1}{2}a_{k-1} \right] \) may improve the social welfare of the unemployed. In other words, if the social planer maximizes a Benthamian welfare function, raising the minimum wage can also be a good policy measure.

To summarize, raising the minimum wage from \( \frac{a_k^2}{2} \) to \( w \in \left[ \frac{a_k^2}{2}, \frac{a_k+1}{2}a_{k-1} \right] \) is slightly detrimental to the welfare of the more educated unemployed, while the less educated unemployed (between \( y \) and \( a_k \)) benefit greatly from this. On the whole, the social welfare of the unemployed may improve.

Some remarks:

- It must be noted that, since the minimum wage is welfare-improving for the less educated only when it reduces the extent of voluntary segmentation, a necessary (but not sufficient) condition for an increase in the minimum wage (beyond \( \frac{a_k^2}{2} \)) to be welfare-improving is that the highest-educated agents of segment \( k \) are not directly constrained. In other words, raising the minimum wage from \( \frac{a_k^2}{2} \) to \( w > \frac{a_k}{2}a_{k-1} \) has clearly a negative effect on the welfare of all types of agents.

\(^{31}\) For example, if the social planer maximizes the welfare of the poorest (a maximin function).
- The unemployed just below and just above the initial minimum wage are affected by an increase in the minimum wage. The agents just below the initial minimum wage earn more, those above earn less. Thus there is a higher fraction of workers whose wage is close to the minimum wage. But the effect does not propagate within the wage distribution, which is consistent with the empirical evidence.

- The more interesting case is the case where \( k = N - 1 \).

5. Conclusion

This paper shows that, in a search framework, raising the minimum wage may reduce the degree of labor market segmentation. More precisely, the high-skilled may become less selective in their search behavior and decide to accept the low-skilled that they were refusing before. When such a phenomenon occurs, the average exit rate from unemployment rises and the welfare of the less skilled unemployed improves.

However, when raising the minimum wage does not change matching strategies and does not reduce the extent of segmentation, this policy is detrimental to the welfare of all the unemployed. In other words, the minimum wage is welfare-improving only when it reduces the degree of voluntary segmentation. Therefore, one necessary (but not sufficient condition) for an increase in the minimum wage to be welfare-improving is that the labor market is initially partially segmented. Another more restrictive condition states that the low-skilled must be sufficiently educated with respect to the other types of agents, and/or, that the number of the low-skilled among the unemployed must be sufficiently high. Note also that raising the minimum wage is never pareto-improving: some agents lose and some others gain. But the interesting point is that the (potential) winners are the less-skilled unemployed.

This paradoxical result can be understood thanks to the «second best» theory proposed by Lipsey et Lancaster [1956] : «It is not true that a situation in which more, but not all, of the optimum conditions are fulfilled is necessarily, or even is likely to be, superior to a situation in which fewer are fulfilled» (p12). Once a minimum wage does exist, raising this existing minimum wage may improve the social welfare of the unemployed. But we can go further. Once search is costly, the unemployed may benefit in average from the minimum wage.

6. Appendix

Proof of proposition 3.1.
Let $w_0, w_1, w_2$, and $w_3$ denote any value of the minimum wage respectively strictly lower than $h_2^2/2$, between $h_2^2$ and $b_2b_2$, between $b_2b_2$ and $h_2^2$, and between $h_2^2$ and $b_2b_2$.

Let $U_{i_k}(d_i)$ denote agent $i$’s utility when agent $i$ chooses the strategy $d_i$ and $w = w_k$.

Let $V_{i_k}(d_i) = \frac{2}{h_i} U_{i_k}(d_i)$.

For $w \in \{w_0, w_1, w_2\}$, a type $a$ agent accepts a type $b$ agent as a partner
\[ \iff V_a(a) < V_a(ab) \iff m_a(a) < m_a(ab) \]
\[ \iff -r/3f_a + \frac{1}{2} \left( \frac{r}{f_a} \right)^2 + 3h_a < -r/3(f_a+f_b) + \left( \frac{1}{2} \right) \left( \frac{r}{f_a+f_b} \right)^2 + 3 \left( \frac{f_ab_a+f_bb_b}{f_a+f_b} \right) \]
\[ \iff 3 \left[ f_a \left( \frac{h_a-h_b}{2f} \right) \right]^2 < h_b \iff C_a < 0 \]

Then, under the conditions $C_a > 0$ et $w = w_0$, a type $b$ agent accepts a type $c$ agent as a partner \( \iff V_{b_0}(b) < V_{b_0}(bc) \iff 3 \left[ f_b \left( \frac{h_b-h_c}{2f} \right) \right]^2 < h_c \iff C_b < 0 \)

We want to show that this condition $C_b < 0$ stays valid for $w = w_1$. More precisely, we want to show that under the hypothesis $C_b < 0$, $V_{b_1}(b) = V_{b_0}(b) < V_{b_1}(bc)$. We will proceed in two steps.

Let $C_b < 0$ and $w = w_1$.

First step

Maximizing equation (3.1) with respect to search intensity, for a given matching strategy, gives the following utilities at the equilibrium:
\[ V_{b_1}(b) = V_{b_0}(b) = \frac{f_bb_b-2mb_0(b)}{f_b} \quad \text{and} \quad V_{c_1}(b) = \frac{f_bb_2-2mc_1(b)}{f_b} \]

It follows that $V_{b_0}(b) < V_{c_1}(b)$ if and only if $m_{b_0}(b) > m_{c_1}(b)$.

Now, when $d_b = \{b\}$, the intensity of search chosen by a type $b$ agent is given by:
\[ [m_{b_0}(b)]^2 + 2m_{b_0}(b) \left[ \frac{r}{f_b} + m_{b_0}(b) \right] - h_b = 0 \quad (6.1) \]

In the same way, when $d_c = \{b\}$, the intensity of search chosen by a type $c$ agent is given by:
\[ [m_{c_1}(b)]^2 + 2m_{c_1}(b) \left[ \frac{r}{f_b} + m_{b_1}(bc) \right] - h_b = 0 \quad (6.2) \]

It follows that $m_{b_0}(b) > m_{c_1}(b) \iff m_{b_1}(bc) > m_{b_0}(b)$.

We need to show that $m_{b_1}(bc) > m_{b_0}(b)$. Now, we know that $m_{b_0}(bc) > m_{b_0}(b)$\footnote{For an unconstrained agent, the optimal search intensity is the maximal one.}. It remains to show that $m_{b_1}(bc) > m_{b_0}(bc)$. At the equilibrium, $m_{b_1}(bc)$ and $m_{b_0}(bc)$ verify respectively:
3 \left( \sum_{j=b,c} f_j \right) \left[ m_{b_0} (bc) \right]^2 + 2r m_{b_0} (bc) = \sum_{j=b,c} f_j h_j \quad (6.3)

By subtracting these equations, we obtain:

\[
sign \left\{ m_{b_0} (bc) - m_{b_0} (bc) \right\} = sign \left\{ (m_{b_1} (bc) - m_{c_1} (b)) \times \left[ \frac{f_b (h_b - h_c)}{r} - 2m_{b_1} (bc) \right] \right\}.
\]

By subtracting equations (6.2) and (6.4), we show that:

\[
sign \left\{ m_{b_1} (bc) - m_{c_1} (b) \right\} = -sign \left\{ \frac{f_b (h_b - h_c)}{r} - 2m_{b_1} (bc) \right\}.
\]

Thus \( m_{b_0} (b) < m_{b_0} (bc) < m_{b_1} (bc) \).

The middle skilled search more intensively when \( w \in \left[ \frac{h^2}{2}, \frac{h_b h_c}{2} \right] \), that is when they are indirectly affected by the minimum wage.

It follows that \( m_{c_1} (b) < m_{b_0} (b) \) and then \( V_{c_1} (b) > V_{b_0} (b) \), which ends the first step.

**Second step**

When agents of type \( b \) accept the \( c \) for \( w = w_1 \), agents \( b \) and \( c \)'s utilities are given by:

\[
V_{b_1} (bc) = 3m_{b_1}^2 (bc) + 2f_c \left[ m_{b_1} (bc) - m_{c_1} (b) \right] \left[ \frac{f_b (h_b - h_c)}{2r} - m_{b_1} (bc) \right] \quad (6.5)
\]

\[
V_{c_1} (b) = 3m_{c_1}^2 (b) + 2m_{c_1} (b) \left[ m_{b_1} (bc) - m_{c_1} (b) \right] \quad (6.6)
\]

By substracting equations (6.5) and (6.6), we obtain:

\[
V_{b_1} (bc) - V_{c_1} (b) = \left( m_{b_1} (bc) - m_{c_1} (b) \right) \times \left[ \frac{3f_c \left( m_{b_1} (bc) + m_{c_1} (b) \right) + f_c \frac{f_b (h_b - h_c)}{r}}{f_c + f_c m_{b_1} (bc) + m_{c_1} (b)} \right]
\]

Thus, \( sign \left\{ V_{b_1} (bc) - V_{c_1} (b) \right\} = sign \left\{ m_{b_1} (bc) - m_{c_1} (b) \right\}. \)

We know on the one hand that \( m_{b_0} (b) > m_{c_1} (b) \) and on the other hand that \( m_{b_1} (bc) > m_{b_0} (bc) > m_{b_0} (b) \).

It follows that \( m_{b_1} (bc) > m_{c_1} (b) \) and thus \( V_{b_1} (bc) > V_{c_1} (b) \).

We showed that \( V_{b_1} (bc) > V_{c_1} (b) > V_{b_0} (b) \) under the condition \( C_b < 0 \). In other words, when \( C_b < 0 \), agents of type \( b \) accept the low-skilled as partners when \( w < \frac{h^2}{2} \) or when \( w \in \left[ \frac{h^2}{2}, \frac{h_b h_c}{2} \right] \).

We showed also that the middle-skilled search more intensively and the low-skilled less intensively when \( w \in \left[ \frac{h^2}{2}, \frac{h_b h_c}{2} \right] \) than when \( w < \frac{h^2}{2} \).
We proceed in the same way for the condition $C_a < 0$. We want to show that if the high-skilled accept the middle-skilled for $w = w_0$, this stays true for $w = w_1, w_2$ or $w_3$. It is obviously true for $w = w_1$ or $w_2$, because neither the high-skilled nor the middle-skilled are concerned by the minimum wage\(^{33}\). For $w = w_3$, we can use the same proceeding than for the condition $C_b < 0$, where $c$ must be replaced by $b$ and $b$ by $a$. Therefore, the hypothesis $C_a < 0$ implies that $V_{a_3} (ab) > V_{a_0} (a)$.

In the same way, we show that if the condition $C_{ab} < 0$ is verified for $w = w_0$, then agents of type $a$ and $b$ still accept the low-skilled as partners for $w = w_1$ or $w_2$.

For $w = w_0$, agents of type $a$ and $b$ accept the $c$ as partners,

\[ \iff V_a (ab) < V_a (abc) \iff m_a (ab) < m_a (abc) \iff \]

\[ - 3 (f_a + f_b) + \left( \frac{r}{f_a + f_b} \right)^2 + 3 \left( \frac{f_a h_a + f_b h_b}{f_a + f_b} \right) > - \frac{r}{3 (f_a + f_b + f_c)} + \left( \frac{r}{f_a + f_b + f_c} \right)^2 + 3 \left( \frac{f_a h_a + f_b h_b + f_c h_c}{f_a + f_b + f_c} \right) \]

\[ \iff 3 \left[ f_a \left( \frac{h_a - h_b}{2r} \right) + f_b \left( \frac{h_b - h_c}{2r} \right) \right] < h_c \iff C_{ab} < 0. \]

Then the hypothesis $C_{ab} < 0$ implies that $V_{a_0} (abc) > V_{a_1} (abc) > V_{a_2} (abc) > V_{a_0} (ab)$.

It only remains to prove that under the conditions $C < 0$ and $w = w_3$, type $a$ agents gain by accepting both the middle and low-skilled as partners. Agents of type $b$ and $c$ are directly constrained when $w = w_3$ and can only match with type $a$ agents.

When the high-skilled accept the middle-skilled but refuse the low-skilled, search intensities verify the following equations:

\[ 3 (f_a + f_b) [m_{a3} (ab)]^2 + f_b (m_{a3} (ab) - m_{b3} (a)) \left[ \frac{f_a (h_a - h_b)}{r} - 2 m_{a3} (ab) \right] + 2 r m_{a3} (ab) - (f_a h_a + f_b h_b) = 0 \]

(6.7)

\[ [m_{b3} (a)]^2 + 2 m_{b3} (a) \left[ \frac{r}{f_a} + m_{a3} (ab) \right] - h_a = 0 \]

(6.8)

Inversely, when the high-skilled accept both the middle and the low-skilled, search intensities are given by:

\[ 3 [m_{a3} (abc)]^2 + 2 r m_{a3} (abc) + f_b (m_{a3} (abc) - m_{b3} (a)) \left[ \frac{f_a (h_a - h_b)}{r} - 2 m_{a3} (abc) \right] \]

\[ + f_c (m_{a3} (abc) - m_{b3} (a)) \left[ \frac{f_a (h_a - h_c)}{r} - 2 m_{a3} (abc) \right] - (f_a h_a + f_b h_b + f_c h_c) = 0 \]

(6.9)

\[ [m_{b3} (a)]^2 + 2 m_{b3} (a) \left[ \frac{r}{f_a} + m_{a3} (abc) \right] - h_a = 0 \]

(6.10)

\(^{33}\)We assume that $C_{ab} > 0$, which means that both type $a$ and type $b$ agents refuse the low-skilled as partners.
\[ m_{b3} (a) = m_{c3} (a) \]  

(6.11)

We proceed in two steps: first, we show that \( m_{a3} (abc) > m_{a3} (ab) \) when \( C < 0 \) is verified; next we show that the welfare can be rewritten as an increasing function of search intensity. This proves that under the conditions \( C < 0 \) and \( w = w_3 \), \( V_{a3} (abc) > V_{a3} (ab) \).

**First step**

If \( m_{a3} (abc) = m_{a3} (ab) \), it follows that \( m_{a3} (abc) = \frac{f_b(h_a-h_c) + f_c(h_b-h_c)}{2r} = \frac{3|m_{a3} (abc)|^2}{4r} + [m_{a3} (abc) - m_{b3} (a)] \left[ \frac{f_b(h_a-h_c)}{r} - 2m_{a3} (abc) \right] - h_c = 0 \). Moreover, \( m_{a3} (abc) - m_{b3} (a) = 2m_{a3} (abc) + \frac{r_f}{r_a} - \sqrt{\left(m_{a3} (abc) + \frac{r_f}{r_a}\right)^2 + h_a} \).

Consequently, \( m_{a3} (abc) > m_{a3} (ab) \) if:

\[
3 \left[ \frac{f_b(h_a-h_c)+f_c(h_b-h_c)}{2r} \right]^2 - f_b \left( \frac{h_b-h_c}{r} \right) \left[ \frac{r_f}{r_a} + f_b(h_a-h_c)+f_c(h_b-h_c) \right] + f_b \left( \frac{h_b-h_c}{r} \right) \times \sqrt{\left[ \frac{r_f}{r_a} + f_b(h_a-h_c)+f_c(h_b-h_c) \right]^2 + h_a - h_c} < 0
\]

That is, \( m_{a3} (abc) > m_{a3} (ab) \) if \( C < 0 \).

**Second step**

Next we show that \( V_{a3} (abc) \) and \( V_{a3} (ab) \) can be rewritten as the same increasing function of search intensity.

By substracting (6.7) and (6.8), we obtain:

\[
\frac{2f_b}{f_a + f_b} \left( \frac{f_b(h_a-h_b)}{2r} - m_{a3} (ab) \right) = - \frac{(m_{a3} (ab) - m_{b3} (a))(3m_{a3} (ab)+m_{b3} (a)+\frac{r_f}{r_a})}{m_{a3} (ab) - m_{b3} (a) + \frac{r_f}{r_a}}
\]

By substracting (6.9) and (6.10), we obtain:

\[
f_b f_a(h_a-h_b) - f_c f_a(h_b-h_c) - 2(f_b + f_c) m_{a3} (abc) = - \frac{(m_{a3} (abc) - m_{b3} (a))(3m_{a3} (abc)+m_{b3} (a)+\frac{r_f}{r_a})}{m_{a3} (abc) - m_{b3} (a) + \frac{r_f}{r_a}}
\]

Thus \( V_{a3} (abc) \) and \( V_{a3} (ab) \) can be rewritten as the same increasing function of search intensity: \( V_{a3} = f (m_{a3}) \), where \( f (m_a) = 3m_a^2 - \frac{(m_a - m_b)^2(3m_a + m_b + \frac{r_f}{r_a})}{(m_a - m_b + \frac{r_f}{r_a})} \).

Since \( m_b = - \left( m_a + \frac{r_f}{r_a^2} \right) + \sqrt{\left( m_a + \frac{r_f}{r_a^2} \right)^2 + h_a } \), function \( f (.) \) can be rewritten:

\[
f (m_a) = 3m_a^2 - \frac{2m_a + \frac{r_f}{r_a}}{2 \left( m_a + \frac{r_f}{r_a^2} \right) - \sqrt{\left( m_a + \frac{r_f}{r_a^2} \right)^2 + h_a}} \left[ \left( m_a + \frac{r_f}{r_a^2} \right)^2 + h_a \right]
\]

\[
2 \left( m_a + \frac{r_f}{r_a^2} \right) - \sqrt{\left( m_a + \frac{r_f}{r_a^2} \right)^2 + h_a}
\]
Now \( f(.) \) is an increasing function of \( m_a \).

Consequently, under the condition \( C < 0 \), it follows that \( V_{aa} (abc) > V_{aa} (ab) \), which means that a type \( a \) agent gains by accepting both the middle and low-skilled as partners.

This ends the proof of proposition 3.1.

**Proof. of proposition 4.1**

The search program faced by a type \( i \) agent is given by equation (4.1). Taking first order conditions with respect to, respectively, \( h_{ri} \) and \( m_i \) gives:

- \( m_i \frac{(h_{ui} - h_{ri})^2}{2} + \int_{h_{ri}}^{h_{ui}} m(h_j) dh_j - 2\Delta r m_i^2 - \left[ 2\Delta r + \int_{h_{ri}}^{h_{ui}} m(h_j) dh_j \right] h_{ri} = 0 \)
- \( \left[ \left( \frac{h_{ui} + h_{ri}}{2} \right) - 2\Delta r \left( \frac{2m_i}{h_{ui} - h_{ri}} \right) \right] \left( 2\Delta r + \int_{h_{ri}}^{h_{ui}} m(h_j) dh_j \right) = 2\Delta r m_i^2 + \int_{h_{ri}}^{h_{ui}} m(h_j) h_j dh_j \)

Adding these two expressions gives agent \( i \)'s search intensity as a function of her reservation endowment of human capital \( h_{ri} \):

\[
m_i = \left( \frac{1}{2\Delta r} \right) \left( \frac{h_{ui} - h_{ri}}{2} \right)^2 \quad (6.12)
\]

Next, replacing \( m_i \) by its expression in the first order condition gives \( h_{ri} \):

\[
2\Delta r + \int_{h_{ri}}^{h_{ui}} m(h_j) dh_j \bigg|_{h_{ri}} = \left( \frac{1}{2\Delta r} \right) \left( \frac{h_{ui} - h_{ri}}{2} \right)^4 + \int_{h_{ri}}^{h_{ui}} m(h_j) h_j dh_j \quad (6.13)
\]

Agent \( i \)'s education does not appear directly in this expression. Therefore, this equation is also valid for any agent \( j \), with \( h_j \in [h_{ri}, h_{ui}] \). In other words, any agent \( j \), with \( h_j \in [h_{ri}, h_{ui}] \) chooses the reservation strategy \( h_{ri} \) and the search efficiency \( m_i \). Thus \( \int_{h_{ri}}^{h_{ui}} m(h_j) dh_j = m_i (h_{ui} - h_{ri}) \) and \( \int_{h_{ri}}^{h_{ui}} m(h_j) h_j dh_j = m_i \left( \frac{h_{ui}^2 - h_{ri}^2}{2} \right) \).

Replacing these expressions in equation (6.13) gives a first relation between \( m_i \) and \( h_{ri} \):

\[
[2\Delta r + m_i (h_{ui} - h_{ri})] h_{ri} = 2\Delta r m_i^2 + m_i \left( \frac{h_{ui}^2 - h_{ri}^2}{2} \right).
\]

Moreover, equation (6.12) gives a second relation between \( m_i \) and \( h_{ri} \): \( 2\Delta r m_i = \left( \frac{h_{ui} - h_{ri}}{2} \right)^2 \). It follows that \( h_{ri} = 3m_i^2 \) and \( 2\Delta r \sqrt{\frac{h_{ui} - h_{ri}}{3}} = \left( \frac{h_{ui} - h_{ri}}{2} \right)^2 \).

For this solution to be unique, the expected value of unemployment \( U_i \) must be concave. \( U_i \) is concave if the Hessian matrix, \( D^2 U_i(h_{ri}, m_i) = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \), where \( \frac{\partial U_i}{\partial m_i} = a \), \( \frac{\partial^2 U_i}{\partial m_i^2} = b \), \( \frac{\partial U_i}{\partial m_i / h_{ri}} = c \) and \( \frac{\partial^2 U_i}{\partial m_i / m_i^2} = d \), is negative.

Now, \( \text{sign} [a] = \text{sign} \left[ - m_i (h_{ui} - h_{ri}) + \int_{h_{ri}}^{h_{ui}} m(h_j) dh_j - 2\Delta r m_i^2 \right] < 0 \) and \( \text{sign} [ad - bc] \geq 0 \). Thus the Hessian matrix is negative and the solution is unique.
To solve for the equilibrium, it is suitable to begin by the case of the highest-educated agent. Her endowment of human capital is \( a_0 = \overline{h} + \Delta \). Since this agent is accepted by everyone, \( h_u(a_0) = a_0 \). Thus her reservation strategy, \( h_r(a_0) \), is given by the following equation:

\[
\left( \frac{h_u(a_0) - h_r(a_0)}{2} \right)^2 = 2r \Delta \sqrt{\frac{h_r(a_0)}{3}}
\]

This means that \( h_r(a_0) < h_u(a_0) = a_0 \). Moreover, any agent whose human capital lies between \( h_r(a_0) \) and \( a_0 \) will choose the same reservation strategy than \( a_0 \).

Now let \( a_1 = h_r(a_0) \). This agent cannot form a partnership with someone whose human capital is higher than \( a_1 \): thus \( h_u(a_1) = a_1 \). Her reservation strategy, \( h_r(a_1) = a_2 \), is given by:

\[
\left( \frac{h_u(a_1) - h_r(a_1)}{2} \right)^2 = 2r \Delta \sqrt{\frac{h_r(a_1)}{3}}
\]

Then, any agent whose human capital lies between \( h_r(a_1) = a_2 \) and \( a_1 \) will choose the same reservation strategy than \( a_1 \).

This downward process can be repeated successively for any segment number \( n \) (\( n = 1, \ldots \)), as long as the lower bound of this segment, \( a_n = h_r(a_{n-1}) \), is higher than \( \overline{h} - \Delta \), the lower bound of the distribution of human capital. The last segment (let this segment be called segment number \( N \)) is truncated: the upper bound of this segment, \( a_{N-1} \), is such that \( h_r(a_{N-1}) < \overline{h} - \Delta \). Now, by definition, the lower bound of the last segment must coincide with the lower bound of the distribution of human capital. Thus \( a_N = \overline{h} - \Delta > h_r(a_{N-1}) \).

**Proof of proposition 4.2** Assume that \( y \geq a_k \). We show that this hypothesis leads to a contradiction.

For \( w = w_0 \), the reservation level chosen by a non-constrained agent of the \( k \)-th segment verifies:

\[
2 \Delta r a_k - \left( \frac{1}{2 \Delta r} \right) \left( \frac{a_k - a_0}{2} \right)^4 = \int_{a_k}^{a_{k-1}} m_k(h) \, dh - a_k \int_{a_k}^{a_{k-1}} m_k(h) \, dh
\]

These agents search for a partner with the intensity: \( m_k = \left( \frac{1}{2 \Delta r} \right) \left( \frac{a_k - a_0}{2} \right)^2 \).

In the same way, for \( w \in \left[ \frac{a_k^2}{2}, \frac{a_{k+1} - a_{k+1}}{2} \right] \), the reservation level chosen by a non-constrained agent of the \( k \)-th segment verifies:

\[
2 \Delta r y - \left( \frac{1}{2 \Delta r} \right) \left( \frac{a_k - y}{2} \right)^4 = \int_{y}^{a_{k-1}} m(h) \, dh - y \int_{y}^{a_{k-1}} m(h) \, dh
\]

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These agents search for a partner with the intensity: \( m_x = \left( \frac{1}{2\pi \sigma^2} \right) \left( \frac{a_x - y}{-\beta} \right)^2 \).

Any agent \( i \) between \( y \) and \( a \) is constrained and chooses a search intensity \( m_i \) such that \( \frac{\partial m_i}{\partial h_i} < 0 \). Therefore, \( m_i < m_x, \forall \ h_i \in [y, x[ \), since agent \( x \) is the first to be constrained.

Let \( A (z) = 2\Delta z - \left( \frac{1}{2\pi \sigma^2} \right) \left( \frac{a_k - z}{-\beta} \right)^4 \), \( \forall \ z \in \{y, a_k\} \). Assuming \( y \geq a_k \) implies that \( A (y) > A (a_k) \).

At the equilibrium, \( A (z) \) is equal to: \( \int_y^{a_k} m_k (h) dh + \int_z^{a_k} m (h) dh \).

Under the hypothesis \( y \geq a_k \), \( A (a_k) = \int_y^{a_k} m_k (h) dh - a_k \int_y^{a_k} m (h) dh \)

\[ = \left[ \int_y^{a_k} m_k (h) dh - a_k \int_y^{a_k} m_k (h) dh \right] + \left[ \int_y^{a_k} m_k (h) dh - a_k \int_y^{a_k} m (h) dh \right]. \]

Now, the second term, \( \int_y^{a_k} m_k (h) dh - a_k \int_y^{a_k} m_k (h) dh \), is strictly positive.

Thus it follows that \( A (a_k) > \int_y^{a_k} m_k (h) dh - a_k \int_y^{a_k} m_k (h) dh \).

We want to compare \( \int_y^{a_k} m_k (h) [h - a_k] dh \) and \( A (y) = \int_y^{a_k} m (h) [h - y] dh \).

Assuming \( y \geq a_k \) implies that \( m_k > m_x > m_i, \forall \ h_i \in [y, x[ \).

As a result, \( m_k (h) [h - a_k] > m (h) [h - y], \forall \ h \in [y, a_k - 1[ \).

This means that \( A (a_k) > \int_y^{a_k} m_k (h) [h - a_k] dh > \int_y^{a_k} m (h) [h - y] dh = A (y) \).

Thus the hypothesis \( y \geq a_k \) leads to a contradiction. It follows that \( y < a_k \).

References


