Is Economic Activity in the G7 Synchronised? Identifying the Determinants of Common Movements across Countries*

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Abstract

This paper analyses the co-movement in activity, measured by GDP and industrial production, between the G7 countries for the period 1972 to 2002. For that purpose, a dynamic factor model is estimated using Kalman filtering techniques. In addition to seperating common and country-specific - idiosyncratic - developments of output, we try to identify the causes underlying the observed co-movement: To what extent is it driven by common shocks and to what extent can cross-country/cross-area spill-over effects account for the observed co-movement? We find that the ouput developments in G7 countries are driven to a substantial extent by common dynamics. A significant part of the co-movement, especially in the first half of the sample, can be explained by developments in the price of oil, an important and easily identifiable common shock. The anlysis suggests that, in addition, area-specific common factors play an important role, separating the sample into a North-American (US, Canada) and a Continental-European (France, Germany, Italy) area, with the UK and Japan being somewhat separate from these areas. We find that developments in the North-American factor have a strong lagged impact on the Continental-European factor, while the reverse is not true. Furthermore, the strength of the cross-area spill-overs from America to Europe appear to have become stronger over the sample period, suggesting that international linkages have increased in the process of globalisation.

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1 Introduction

The issue of international business cycle linkages, or more generally, of possible co-movements in economic activity across countries and regions has recently resurfaced to the attention of analysts, policymakers and academic scholars. The simultaneous slowdown experienced to different degrees by all the G7 economies starting in the final part of 2000 underlines that the developments even in relatively large economies cannot always be analysed in isolation. For policymakers, the ability to gauge the nature and the magnitude of cross-country co-movements in economic activity may be crucial in order to assess the developments in their own domestic economy. This paper aims primarily at characterizing the common fluctuations amonst G7 economies by analysing the properties of real GDP growth in those countries. In particular, the paper first isolates and measures the historical evolution of the degree of output growth co-movements in the different countries and then assesses the relative importance of common shocks, affecting the different economies at the same time versus direct linkages between countries ("spill overs") in generating such co-movements. In addition, the analysis explores the role of oil price developments, which are considered a classical example of a common shock, in determining the joint fluctuation of real GDP growth in the G7 economies.

The methodology employed consists in the estimation of a dynamic common factor model for the G7 countries' GDP (and industrial production) growth using Kalman filtering techniques. The general model specification assumes the process for real GDP growth in each of the G7 countries to be driven by a country-specific autoregressive component and a latent component, which is common to all series. This latent component - or common factor - is also assumed to follow a univariate autoregressive process. In a subsequent step, "area-wide" common factors are introduced by allowing two separate univariate autoregressive common factors which are each shared by only a subset of countries. Finally, we try to disentangle "area-wide" common factors and "area-wide" spill-over effects by allowing the two "area-wide" common factors to interact. This is achieved by modelling the "area-wide" common factors as a multivariate autoregressive process, with each of the two common factors explaining its own and the other common factor's next period value.

The analysis shows that the G7 countries share common output dynamics with clearly identifiable common swings in activity across the G7. The paper ranks G7 countries according to two measures of syncronisation: the share of each country's total variance of real GDP growth explained by the variance of the common factor and, alternatively, the correlation between the common factor and the real GDP growth series. Furthermore, the paper finds statistical support for a (time-varying) effect of oil price developments on the common factor, confirming that oil prices, indeed, constitute one important variable partly responsible for the observed co-movement among economic activity in different countries. More concretely, while oil price developments are important in explaining the common factor during the 1970s, their relevance decreases in the last two decades. However, during the most recent downturn the role of oil prices appears to have played a somewhat more important role again.

Using the general model specification with one global common factor, the paper tries to disentangle common shocks from spill-over effects by exploiting the feature of the Kalman filter that it distributes the overall variance between the idiosyncratic and the common part of the system shocks. As a consequence, an initially idiosyncratic shock, which gets transmitted to other countries, will be identified as a common factor shock by the filter. From the analysis of the evolution of the variance of the idiosyncratic and the common factor disturbances, it appears that spill-over effects have gained in importance in explaining the co-movements among G7 countries' growth developments, especially from the mid-1980s to the mid-1990s.

The "area-wide" specification of common factors identifies significant "area-wide" common effects on real GDP growth developments in the G7 countries, with the "areas" being a North-American one (comprising the United States and Canada) and a Continental-European one (comprising Germany, France and Italy). Japan and the UK appear to be somewhat separated from these areas. The analysis with cross-area linkages indicates strong spill-over effects from the North-American area affecting the European area. At the same time, spill-over effects in the opposite direction - from Europe to North America - prove to be small and statistically insignificant. Interestingly, the model specification with inter-related "area-wide" common factors results in the loss of significance of the global common factor. This suggests that what was originally modelled as a pure global common factor was instead to a large extent capturing the mixture of joint fluctuations in a subset of countries' real GDP growth and cross-area spill over effects.

Modelling common fluctuation in economic variables by using the dynamic factor approach presents clear advantage as compared to simpler and more direct approaches like the one that analyses the evolution of pure bi-variate correlation[1]. First, the analysis of simple correlation cannot allow for the separation of the idiosyncratic component from the purely common source of joint co-movements. Second, static correlation analysis, by definition, misses the possible persistence of common fluctuations.

Within the family of the dynamic factor models there currently exist two main approaches: the linear state space model and the generalised dynamic factor (GDF) model.. The former approach, which this paper adopts, uses a parametic specification of the underlying dynamics, thereby allowing precise statistical inference. Unlike the present paper, none of the existing studies based on this approach attempts to disentangle the possible underlying causes of the measured common dynamics. For example, Gregory, Head and Raynauld (1997) use dynamic factor analysis similar to that developed in this paper to study fluctuations in GDP, consumption and investment in the G7 countries for the period 1970 to 1993. The authors find significant world and country-specific components for all countries without any clear lead or lag relationship between the world component and individual country series. In a related paper, Gregory and Head (1999) analyse common factors in the behaviour

[1] Seee among the others Baxter and Stockman (1989), Gerlach (1988), Stockman (1988) and more recently Doyle and Faust (2002).

of productivity, investment and the current account for the G7 countries, with the main objective of assessing the validity of the intertemporal approach to the current account. The authors find that productivity and investment are strongly affected by a common world component, which in turn is very similar to US movements in investment[2].

The generalized dynamic factor model, the alternative approach, was pioneered by Forni and Reichlin (1998)[3]. These models extend the dynamic factor models introduced by Geweke (1977), Sargent and Sims (1977) and Geweke and Singleton (1981), where the observed process Y_t is stationary and in the simplest version of the model, is determined by a single common factor F_t and n idiosyncratic components $(u_{1t}, \cdots, u_{nt}) = u_t'$. In the model specification, F_t is an unobservable stationary process, u_t is made of n independent stationary processes and the processes F_t and u_t are independent. Using the moving average representation of F_t, this model can be written as $Y_t = a(L)\xi_t + u_t$ where ξ_t is a scalar white noise (with unit variance). Similarly, u_t can be written $u_t = B(L)\varepsilon_t$ where $B(L)$ is a diagonal matrix and $\varepsilon_t = (\varepsilon_{1t}, \cdots, \varepsilon_{nt})$, the ε_{it} being independent white noises with unit variance. Hence, it is possible to decompose and estimate the spectral matrix of Y_t in order to compute the share of the variance explained by the common factor at various frequencies. The GDF approach allows for a limited amount of cross correlations between the components of ε_t. Identifiability is obtained by assuming that n goes to infinity and is based on assumptions about the behavior of the eigenvalues of the spectral density matrix of Y_t , when n goes to infinity. The common components are recovered asymptotically using dynamic principal components theory (see Brillinger (1981). The main disadvantage of the GDF approach is that these models necessitates a very large number of cross sections of idiosyncratic components in order to be valid. Furthermore, no asymptotic distributional theory is available, thus making it impossible to perform any kind of statistical inference[4].

The rest of the paper is organised as follows: Section 2 sets the general model employed in the paper to estimate the common factor and briefly decribes the dataset to which the model is applied. Section 3 contains the analysis of the model specification for the case of one common factor. In this section some evidence on the role of oil prices in determining the developments in the common factor is also presented. Section 4 presents the analysis involving "area-wide" common factors, while Section 5 introduces interdependencies between the "area-wide" common factors in order to identfiy possible cross-area spill-over effects. Section 6 concludes.

[2] In a more recent paper, Kose et al. (2002) employ a Bayesian dynamic factor model to estimate common components in the main macroeconomic aggregates in a sixty-country sample covering seven regions of the world. They find significant evidence of an important effect of the world common factor and of less important regional factor.

[3] In an application close to the spirit of the analysis performed in this paper, Forni and Reichlin (2001) analyse output growth for 138 regions of nine European countries and 3075 counties for 48 US states for the period 1970-1993. They find that for most European countries, the common European component explains the main part of the output variance.

[4] See Reichlin (2002).

2 The linear state space model

The general idea underlying dynamic factor models is that the cross correlations between the components of a n-dimensional stochastic process Y_t can be captured by k unobservable factors $Z_{1,t}, ..., Z_{k,t}$, where k is considerably smaller than n. As a special case, the dynamic factor model can be formulated as a linear state space model by assuming that Y_t depends linearly on the matrix of factors Z_t, which in turn follows a autoregressive process. Concretely, the linear state space model can be written as follows:

$$\begin{aligned} Y_t &= \mu_t + B_t Z_t + M_t \varepsilon_t \\ Z_t &= \nu_t + D_t Z_{t-1} + N_t \xi_t \end{aligned} \tag{1}$$

$$\begin{aligned} V(M_t \varepsilon_t) &= R_t \\ V(N_t \xi_t) &= Q_t \end{aligned}$$

where ε_t and ξ_t are independent Gaussian white noise terms. Y_t is the n-vector of observed variables, Z_t is an unobserved state vector, and $\mu_t, B_t, M_t, \nu_t, D_t, N_t$ are functions of an unknown vector of parameters θ and of the past values of Y_t. θ is finite dimensional and, therefore, the model is parametric and can be estimated by the Kalman filter, which consists of a recursive procedure which computes the best approximations of the states Z_t, given the information through time t. As the Kalman filter provides at each step k the likelihood function for $k+1$ conditional on information given at k, the log-likelihood function for the entire sample can be constructed as a by-product of the Kalman filtering.[5] Therefore it is possible to maximise this function with respect to the vector θ of unknown parameters appearing in $B_t, D_t, R_t, Q_t, M_t, N_t$, μ_t or ν_t. The first set of n equations - the measurement equations - explain the relation between the unobserved vector Z_t and a n vector of measurements Y_t. Moreover, exogenous or lagged dependent variables μ_t can also enter the measurement equations, as well as a n vector of disturbances $M_t \varepsilon_t$. The second set of r equations - the transition equations - describe the evolution of an $(r \times 1)$ unobservable vector Z_t in response to a $(r \times 1)$ vector ν_t of exogenous or lagged dependent variables and a $(r \times 1)$ vector of disturbances $N_t \xi_t$.

In this paper the general model specification assumes that the process for Y_t can be separated into three components:

$$Y_t = AY_{t-1} + BZ_t + \varepsilon_t \tag{2}$$

$$V[\varepsilon] = \begin{bmatrix} \sigma_1^2 & & 0 \\ & \ddots & \\ 0 & & \sigma_n^2 \end{bmatrix}$$

[5] The expression of the log likelihood of the unknown paramters is given in Appendix A.1.1 (Eq.(7)).

where σ_i^2 denotes $V[\varepsilon_i]$.

The B matrix measures the instantaneous impact of the common factors on each series Y_i, its entries are called factor loadings or sensitivities.

A is assumed to be diagonal in most specifications, thus capturing the core notion of the dynamic factor model that the co-movements of the multiple time series arise from the single source Z.

Furthermore, the dynamic process of the state vector Z_t is assumed to follow:

$$Z_t = DZ_{t-1} + \eta_t \tag{3}$$

The dynamics of the unobservable factors is univariate as long as D is diagonal. Moreover, an identification constraint is imposed on the variance-covariance matrix of the disturbances η :

$$V[\eta] = Id(r)$$

With reference to the general notation introduced in equation 1 the actual general model implemented in the paper results in:

$$\mu_t = AY_{t-1} \ , \ B_t = B \ , \ \nu_t = 0 \ , \ D_t = D \ , \ R_t = V[\varepsilon] \ , \ Q_t = Id(r)$$

As the dimension of θ is, in some of our model specifications, quite large and each stage of the Kalman filter requires a large number of calculations, we use a two-step procedure for the purpose of maximisation: First, we apply the Expectations Maximisation (EM) algorithm[6], which allows to find estimates in the region of the maximum reasonably quickly even from poor starting values. However, the EM algorithm does not have quadratic convergence properties and thus converges only slowly in the vicinity of the maximum. Moreover, the EM algorithm does not provide an estimate of the information matrix. In a second step, we therefore apply the numerical BFGS maximisation algorithm provided by GAUSS. We calculate the information matrix based on the results by Engle and Watson (1981).[7]

2.1 Data

We use (seasonally adjusted) quarterly real GDP figures for the G7 countries from 1970.2 to 2002.2. In general, the series are taken from the International Monetary Fund database[8]. In subsection 5.1, (seasonally adjusted) monthly industrial production series of the G7 countries are considered. These data are taken from the IMF database, covering the period 1970.M2-2002.M2 at monthly frequency.

An augmented Dickey-Fuller test was used to check for non-stationarity in the level data and the null hypothesis of non-stationarity was accepted for all series.

[6] The general algorithm and its application to our models are presented in the appendix.

[7] The two ways of calculating the information matrix are desribed in more detail in the appendix

[8] However, in the case of Japanese GDP data the OECD database was used, as the IMF series exhibits a spike in 1980.1, reflecting a change in statistical methodology for calculating Japanese GDP. In addition, the IMF series is not seasonally adjusted prior to that date.

Therefore, in order to render the data stationary, we use log differenced data. Furthermore the differenced data are demeaned by removing the sample mean and the variance is standardized to one.

Before considering the estimates of the dynamic factor models, it is useful to take a look at the correlation pattern present in the data. Table 2.1 contains the cross-correlation coefficients for the GDP growth rates of the G7 countries. All the correlation coefficients are positive, with the highest correlation in the case of the USA-Canada and the France-Italy GDP growth rate pairs. With regard to the first order autocorrelation (see Table 2.2), all GDP growth rates except those for Germany exhibit positive autocorrelation, with Italy having the largest autocorrelation, followed by Canada and France.

Table 2.1 Cross-correlations among real GDP growth rates

	U.S.A.	Japan	U.K.	Canada	Germany	France	Italy
U.S.A.	1						
Japan	0.223	1					
U.K.	0.260	0.292	1				
Canada	0.529	0.193	0.200	1			
Germany	0.122	0.122	0.349	0.012	1		
France	0.248	0.205	0.246	0.223	0.398	1	
Italy	0.176	0.104	0.116	0.206	0.293	0.531	1

Table 2.2 First order autocorrelations among treal GDP growth rates.

	U.S.A.	Japan	U.K.	Canada	Germany	France	Italy
$corr(X_t, X_{t-1})$	0.279	0.097	0.057	0.374	-0.087	0.310	0.449

3 The case of one common factor

This first application of the general model depicted in Section 2 involves a single common factor Z. We can then rewrite Eq.(2) and (3) as:

$$\begin{aligned} Y_{i,t} &= a_i Y_{i,t-1} + b_i Z_t + \varepsilon_{i,t} \\ Z_t &= d Z_{t-1} + \eta_t \end{aligned} \tag{4}$$

In this case, since the $\varepsilon_{i,t}$ are serially and contemporaneously uncorrelated, comovements among the $Y_{i,t}$ are explained only by the single common factor.

(Insert Figure 1 here)

Figure 1 plots the estimated common factor. According to this factor the G7 economies experienced four major downturns since 1970: In the mid-1970s, early

1980s and early 1990s, and most recently in 2000. The downturns in the mid-1970s and early 1980s were both very sharp, while the downturn in the early 1990s was preceded by a generally delining trend starting in the second half of the 1980s. Similarly with respect to the upturns, some were very sharp (1974-1975, 1993) while the downturn in the early 1980s was followed by long period of gradual expansion. These activity patterns and their timing conform reasonably well with common wisdom on global economic developments over recent decades. In particular, the first and second downturns correspond to the timing of the two oil price shocks, although the impact of the latter on economic activity appears to be much less pronounced.

The parameter estimates are given in Table 3.1. The lagged dependent variable is significant only in the case of Canada, Germany and Italy, while the impact coefficients of the global component on the GDPs (b_i) are all significant and similar in magnitude across the different countries. The global factor exhibits a relatively high degree of autocorrelation, as indicated by the value of almost 0.7 for the coefficient d, which suggests high persistence of common output developments on real GDP growth in each of the G7 economies.

Table 3.1 Parameter estimates (Model with one common factor)

	U.S.A.	Japan	U.K.	Canada	Germany	France	Italy
a_i	0.115	-0.055	-0.097	0.248	-0.277	0.001	0.284
	(0.086)	(0.087)	(0.088)	(0.084)	(0.086)	(0.094)	(0.084)
b_i	0.299	0.309	0.331	0.238	0.408	0.496	0.336
	(0.075)	(0.077)	(0.078)	(0.071)	(0.081)	(0.088)	(0.074)
σ_i	0.881	0.910	0.898	0.875	0.839	0.721	0.778
	(0.058)	(0.060)	(0.060)	(0.057)	(0.060)	(0.064)	(0.054)
d	0.693						
	(0.087)						

$Y_{i,t} = a_i Y_{i,t-1} + b_i Z_t + \varepsilon_{i,t}$
$Z_t = d Z_{t-1} + \eta_t$

The estimation of model (4) as a simple univariate first order autoregressive process[9] validates the inclusion into the model of a dynamic common factor[10].

(Insert Figure 2 here)

Figure 2 plots the common factor - weighted by its country-specific impact coefficient b_i[11] - together with the actual GDP series for the US, Japan and Germany. It

[9] It can be shown that the marginal dynamics of each $Y_{i,t}$ implied by the common factor model is an ARMA(2,1) process and thus allows for significantly more complex dynamics than the simple AR(1) in this reduced specification.

[10] The value of the AIC in the case of the simple AR(1) specification is -1249.8 compared to -1190.9 in the case of the dynamic common factor model.

[11] In Figure 2 the impact of the common factor on real GDP growth appears to be similar in the case of the US, Japan and Germany. However, the series actually differs by a factor given by the b_is in Table 3.1.

appears that the slowdown in the mid 1970s is indeed well-explained by the common factor[12]. Only in Japan did GDP growth in one quarter deviate significantly from the level suggested by the common factor. During no other downturn does the common factor appear to explain country-specific growth performance as well as in this first one, as growth rates in the various countries fluctuate more strongly around the path suggested by the common factor. Nonetheless the common factor appears to be able to explain some important general cyclical trends in GDP growth in the individual countries. In addition, it helps to identify a few episodes as more or less country specific. In the case of the US, the strong growth performance in 1983/1984 appears to be very much a country specific phenomenon, similar to the recession experienced in 1990/1991. In the case of Germany, the "post-reunification boom" from 1989 to 1991 stands out as a country-specific event. Regarding Japan, it is interesting to note that the slow growth performance in the 1990s - and, especially, in the second half of the decade - is largely a country-specific phenomenon, reflecting the delayed effects of the bursting of the asset price bubble in the early 1990s. Moreover, the difference in volatility between the common factor and the quarterly GDP growth figures increases substantially in the latter half of the 1990s. This may indicate that the policy measures intended to revive the Japanese economy have introduced a substantial amount of idiosyncratic volatility into the economy[13].

Based on the model estimates, it is possible to derive some measures of syncronisation in the G7 economic developments, by looking both at the amount of volatility of each single real GDP growth series that is explained by the volatility of the common factor and to the impact effect of the evolution of the common factor on each of these series. Table 3.2, which reports the shares, S_i, of the total real GDP growth variance accounted for by the common factor[14], documents the particularly large role played by the common factor in the case of France and Italy, where its variance explains around one half to one third of these countries' real GDP growth variance. For the other countries the share is somewhat lower but still accounts for one sixth to one fourth of these countries' growth variations.

Table 3.2 Shares of variance of real GDP growth accounted for by the common factor.

	U.S.A.	Japan	U.K.	Canada	Germany	France	Italy
S_i	0.206	0.170	0.186	0.168	0.235	0.477	0.349

In addition, we compute the correlations between the global factor and the GDP series. Unlike in the case of the share of variance explained by the common factor, here the emphasis is more on the contemporaneous impact of the common factor on a country's GDP growth rate, rather than on the entire effect (including lagged responses to the common factor), which is captured by the b_is in Table 3.1. Table 3.3 shows that this correlation is relatively high for all countries, especially for

[12]In subsection 3.2 the paper provides empirical evidence supporting the significant role of oil price developments in determining the evolution of the common factor.

[13]Alternatively it may reflect shortcomings in the GDP statistics, although it seems difficult to explain why such statistical problems may have become more prevalent during the 1990s.

[14]The method of computation, which is based on the infinite moving average representation of the dynamic process, is described in the appendix.

the continental European countries. In addition, with the exception of Canada (whose series is strongly correlated with the one of the U.S.A.), all series of the G7 countries have higher correlations with the global factor than with the series of any other countries (see Table 2.1 for reference). It is interesting to notice that the ranking of countries' degree of syncronisation emerging from Tables 3.2 and 3.3 is rather different from the ranking according the values of the b_is in Table 3.1. The Continental-European block of the G7 countries remains the most affected by the common factor, but the US position is higher in the ranking by degree of syncronisation as it is in terms of total effect of the common factor. Japan appears to be the country whose real GDP growth is least synchronised with the common factor.

Table 3.3 Correlation of the GDP series with the global factor.

	U.S.A.	Japan	U.K.	Canada	Germany	France	Italy
$Corr(Z_t, Y_{i,t})$	0.517	0.463	0.485	0.476	0.542	0.781	0.640

Figure 3 shows the evolution of the correlation between Z_t and $Y_{i,t}$ over time for the G7 countries using a 4-year rolling window It is apparent that the correlation of individual countries' growth rates with the common factor has overall been fairly high (up to 0.7 in the early 70s), although a moderately declining trend can be identified. The only country, which saw a slight increase in the correlation, is France, while Canada's correlation trend was basically flat. On the other side, strong declines in the correlation trend can be observed especially for the US and Japan. Thus overall it appears that the synchronisation of business cycles has declined over the past three decades.Against that background, the most recent pick-up in correlation is particularly noteworthy, as it underlines the synchronised nature of the most recent downturn.

(Insert Figure 3 here)

3.1 The syncronisation of G7 real GDP growth: common shocks vs. spill-over effects.

Based on the correlation evidence alone it is, however, not possible to say anything about the reasons for this decline or, more generally, about the reason for the overall level of synchronisation of business cycles among the G7 countries. In principle, synchronisation can be attributed to three different causes: all countries are affected by a common shock, to which they react in similar ways. Second, a subgroup of the countries - possibly only a single country - experiences a shock, which is transmitted to the other countries through the various international transmission channels. And third, the countries happen to experience similar country-specific shocks. All these cases would be captured as a shock to the common factor in the current estimation framework, without being able to distinguish between the different cases.

Although a quantification of the different explanations to the observed level of overall synchronisation is not possible in the current framework, some insights into the causes behind changes in the degree of synchronisation documented above may be, nonetheless, possible. An increase in the variance of idiosyncratic shocks ε_{it} in the measurement equation, which one might consider a decrease in international synchronisation, would result in a decline of the correlation between a country's growth rates and the common factor, while leaving the covariance unchanged. In contrast, an increase in the variance of the common factor, as captured by an increase in the error term η_t in the transition equation, would be reflected in an increae of both the correlation and the covariance between the growth rate and the common factor[15]. Thus, an increase in the synchronisation of a country's growth performance with the international business cycle would be reflected in an increase in the correlation between the two series, either due to a decline in the idiosyncratic growth fluctuations or due to an increase in the variance of the common shock, whose impact on the correlation is more than offset by the associated increase in the covariance between the two series.

In addition, it is even possible to make inferences about the changing relative importance of common shocks and spillover effects. If merely the variance of η_t changes, this would indicate a change in the relevance of common shocks for international growth fluctuations. If, however, the change in the variance of η_t is accompanied by a change of the variance of ε_{it} in opposite direction this would indicate a change in the importance of spillover effects. The case of strengthening international transmission channels may help to illustrate this case. Let us assume for the sake of argument that initially there are no common shocks and no spillover effects. In that case the growth of the various countries would not be correlated except as the result of sampling errors. If due to increasing global integration the fluctuations in a certain country start to affect other countries in a systematic (and linear) way, this introduces a certain degree of comovement into the system, which in our modeling framework would be picked up by the common factor. Concretely, the part of the idiosyncratic shock which causes spillover effects, will no longer be counted as a idiosyncratic shock but will instead be categorized as a common shock. As a consequence, the variance of the idiosyncratic shock in the country from which the spillover effects originate will decrease, while the variance of the common shock will increase. In principle, therefore, even the countries from which the spillovers originate could be identified. Of course, such a clear pattern would apply only in the theoretical ceteris paribus case. in reality other factors may play a role as well and may dominate the development of the two variances. For example, an increase in international spillovers may be masked by an autonomous increase in idiosyncratic volatility or a simultaneous decline in the variability of common shocks. Nonetheless, it is useful to take a close look at the evolution of the variance of the two shocks over time.

(Insert Figure 4 here)

[15] This follows directly from the expression for the covariance between y_{it} and z_t:

$$Cov(Y_{i,t}, Z_t) = \frac{b_i}{(1-a_i b_i)(1-d^2)}\sigma_\eta^2$$

Figure 4 depicts the evolution of the variance of η_t and ε_{it} using again a 4-year rolling window. The variance of η_t declines more or less continuously until the mid-1980s. After that, it remains relatively constant, with a moderate gradual increase up to the mid-1990s, followed by a relatively sharp drop in 1997. In 2001, this drop is partly reversed again. Regarding the variance of the ε_{it}, no uniform pattern exists. For a number of countries, the variance of ε_{it} has seen a noticeable decline in the second half of the sample similar to the variance of η_t. This is particularly the case for the US, Canada, the UK and Italy. Unlike the other countries, Japan experienced a marked increase in the variance of ε_{it} in the second half of the 1990s. The timing of the increase in Q2 1997 suggests that it might be related to the fiscal tightening implemented at that time.

It seems reasonable to assume that the decline in the variance of η_t, and thus of Z_t, over the first half of the sample period was the result of the declining importance of oil price disturbances and therefore of truly common shocks, rather than the result of declining global integration and reduced spill-over effects. As this effect has dominated this period it would be difficult to identify more gradual shifts due, for example, to increasing international spill-over effects. The decline in the variance of ε_{it} observed for a number of countries over the same period may be the result of lower idiosyncratic policy volatility related to the consequences of the oil price shocks to which countries reacted in very different ways. The results do unfortunately not provide any clear evidence as to whether international spill-over effects have become more important over the sample period, as there is very little evidence of a general decline in idiosyncratic volatility associated with increased common volatility. However, for some subperiods such a pattern can be identified for individual countries.

The strong increase of the variance of the Japanese idiosyncratic shock in Q2 1997 is particularly striking, as it is accompanied by a noticeable contemporaneous decline of the variance of the common shock. This movement in both series is partly reversed in Q3 2001. Although this finding would be consistent with a temporary isolation of Japan from the other G7 countries, in the sense that spill-overs from shocks to the Japanese economy were reduced during that period, it appears difficult to explain why such a sudden and temporary disconnect should have occured. A possible explanation might be that the increase in the consumption tax in Japan in early 1997, which contributed to a slowdown in economic activity, marked the beginning of a period of domestic policy volatility characterised by limited international ramifications.

Possibly more interestingly, if one focuses on the average idiosyncratic shock variance for all countries, the gradual increase in the variance of the common shock from the mid-1980s until the mid-1990s coincides with a declining trend in the idiosyncratic shock variance. Thus, during this decade the effects of increasing international integration may indeed have dominated the evolution of the error variances. Of course, this finding is merely suggestive of such an interpretation, as the same pattern could be the result of two completely independent developments in the two variance series. However, as the pattern can be observed for a relatively extended period of time, it seems to be driven by a gradual process, such as globalisation, rather than a one-time event, such as a change in the policy framework.

3.2 The common factor and oil prices

To the extent that the observed comovement in G7 activity is driven by common shocks rather than spillover effects, fluctuations in oil prices may be one specific and easily identifiable underlying factor driving the common factor This is, for example, suggested by the coincidence of the first two G7 recessions identified by the common factor with the two oil price shocks in the early and late 1970s. In order to test for the role of oil prices more formally, we regressed the common factor on various lags of the log changes of the oil price[16], refining the specification from general to specific. The low values of the Durbin-Watson coefficient indicate a substantial degree of autocorrelation in the error term, which we take explicitly into account in the estimation. Table 3.4 contains the estimation results.

Table 3.4 Regression of Z on log differenced oil prices (1970.2 to 2002.2)

α_1	α_3	α_4	α_5	AR_ε
-0.015	-0.035	-0.028	-0.019	0.806
(0.009)	(0.010)	(0.010)	(0.010)	(0.054)

R^2
0.70

$Z_{1,t} = \alpha_1 oil_{t-1} + \alpha_3 oil_{t-3} + \alpha_4 oil_{t-4} + \alpha_5 oil_{t-5} + \varepsilon_t$

In general, the high autocorrelation in the error terms indicates that oil prices by themselves are not able to explain the persistence in the common factor and that therefore other factors are likely to play an important role in explaining the movement of the common factor. Nonetheless oil prices appear to be an important part of the explanation for the comovement of the G7 economies. Figure 5 depicts the actual common factor and the estimated values (excluding the autoregressive error component) obtained in the oil regression . In line with perceived wisdom, oil prices were particularly important during the two recessions in the mid-1970s and the early 1980s, although the renewed slowdown around 1980-1981 seems to have been relatively unrelated to oil prices. Likewise the G7 recession in the early 1990s appears to be due largely to other factors than oil prices. In contrast, during the latest downturn in 2000-2001 oil prices again seem to play a non-negligible role. In general, it appears that during the second half of the sample period a phase shift has occurred, with output reacting more slowly to changes in oil prices.

(Insert Figure 5 here)

Thus, a uniform lag structure over the entire sample may not be appropriate and we reestimated the model for two seperate subperiods: from 1970:2 to 1986:1

[16] We used the price of Western Texas oil. The original price quotation in US-dollars was converted into a weighted currency price taking exchange rate movements among the currencies of the G7 countries into account.

and from 1986:2 to 2002:2. Figures 6 and 7 show the respective estimated common factors. The estimation for the first subsample (see Table 3.5) shows that oil prices become even more important for the first two recessions when one allows for a change in the structural relationship. Similarly, although clearly less important in the second half of the sample period oil prices maintain some explanatory even in the later subperiod (see Table 3.6), which was characterised by much less pronounced swings in oil prices. The different lag structure in the two sub-samples also underlines the changed response of economic activity to oil price shocks, with the response in the first sub-period being considerably more immediate than in the second.

(Insert Figure 6 and 7 here)

Table 3.5 Regression of Z_1 on log differenced oil prices (1970.2 to 1986.1)

α_3	α_4	AR_ε
-0.074	-0.060	0.817
(0.016)	(0.0.016)	(0.075)

R^2	DW
0.78	1.62

$Z_{1,t} = \alpha_3 oil_{t-3} + \alpha_4 oil_{t-4} + \varepsilon_t$

Table 3.6 Regression of Z_1 on $(oil_t, ..., oil_{t-5})$ from 1986.2 to 2002.2

α_5	α_7	α_9	α_{10}	AR_ε
-0.019	-0.021	-0.020	-0.020	0.817
(1.075)	(1.075)	(0.016)	(0.0.016)	(0.075)

R^2	DW
0.71	1.52

$Z_{1,t} = \alpha_5 oil_{t-5} + \alpha_7 oil_{t-7} + \alpha_9 oil_{t-9} + \alpha_{10} oil_{t-10} + \varepsilon_t$

Overall, these results suggest that a substantial part of the common fluctuations in G7 activity may be explained by oil shocks. The current estimates provide a lower bound for the importance these shocks, as there may exist important other factors influencing economic activity across countries. In general, they can, however, be expected to be quantitatively less important individually and possibly more difficult to measure. Therefore, the dynamic factor model may be a convenient tool to capture their overall effect without having to specify all factors separately. The costs involved in such a strategy is that one captures in addition to comovement introduced by common shocks also comovement resulting from spill-over effects between countries[17].

[17] In this respect is important to note, however, that even if all common shocks were properly measured and introduced into the estimation procedure it would be very difficult to disentangle the direct effect of these shocks on a country's growth rate and the indirect effects from spill-overs of the common shock on other countries.

4 "Areas" Model

In this section, the general model is extended by introducing dynamic factors which are common only to a sub-set of series, in addition to a single factor common to all the G7 real GDP growth series[18]. More precisely, let us consider n areas, and for each area (area i) k_i series. We will refer to these series by using the notation: $Y_{i,t}^j$, where $j = 1, ..., k_i$ indexes the series of the i^{th} considered area.

Let W_t be an unobserved factor affecting all of the series, and $N_{i,t}$ be a factor common to all the series in area i. We will refer to them as the global common and area-specific common factors. Each series $Y_{i,t}^j$ can thus be decomposed into four separate components:

$$Y_{i,t}^j = a_i^j Y_{i,t-1}^j + b_i^j W_t + c_i^j N_{i,t} + \varepsilon_{i,t}^j \quad \forall i \quad \forall j \tag{5}$$

Here, b_i^j measures the impact of W_t on the j^{th} series of area i and c_i^j measures the impact of the area-specific common component on the j^{th} series of area i. As before, we assume that $(\varepsilon_{1,t}...\varepsilon_{n,t,t}\, , W_t, N_{i,t})$ are uncorrelated at all lead and lags, which is achieved by assuming that ε is a white noise and that ε,W and the N_is are independent.

Denoting with $(\sigma_i^j)^2$ the variance of $\varepsilon_{i,t}^j$, we have

$$V[\varepsilon] = \begin{bmatrix} \begin{bmatrix} (\sigma_1^1)^2 & & 0 \\ & \ddots & \\ 0 & & (\sigma_1^{k_1})^2 \end{bmatrix} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \begin{bmatrix} (\sigma_n^1)^2 & & 0 \\ & \ddots & \\ 0 & & (\sigma_n^{k_n})^2 \end{bmatrix} \end{bmatrix}$$

Both, the global and the area-specific common factors are assumed to follow univariate autoregressive processes of order one:

$$W_t = d_w W_{t-1} + \eta_{w,t}$$

$$N_{i,t} = d_i N_{i,t-1} + \eta_{i,t} \tag{6}$$

[18]The estimation of the general model (1) with two common factors shared by all countries (not reported here) suggests that rather than being influenced only by one global factor, subsets of countries may be affected by area-specific common factors. When adding a second global factor to the one factor model of Section 3, one of the global factors resembles very closely the original common factor. The second factor appears to capture asymmetries among the countries, as the factor loadings are positive for some countries and negative for others. In particular, we find that the factor loadings for the US and Canada share the same sign, while the factor loadings for the Continental European countries France, Germany and Italy have the opposite sign. The factor loadings for the second factor in the case of Japan and the UK are insignificant.

where we add the following identification condition:

$$V\begin{bmatrix}\eta_{w,t}\\ \eta_{1,t}\\ \vdots\\ \eta_{n,t}\end{bmatrix}=Id_{n+1}$$

The results of the two-factor model suggest that two distinctive area factors may play a role: a North-American (US and Canada) and a Continental European (France, Germany and Italy) one. As the loading coefficients for Japan and the UK were insignificant it is not clear whether or where these countries should be included. Based on the results from various area specifications, we chose to exclude Japan from the analysis, as it does not appear to belong to any of the areas. For the UK, we estimated the model separately for the two cases of the UK belonging to the North-American and the Continental-European area. Using the notations introduced in 4, $i \in \{1, 2\}$ indexes the area, and $j \in \{I, II, III, IV\}$ indexes the j^{th} country in the considered area.

Figure 8 plots the three common factors estimated. The global factor W_t shares the main feature of the global factor obtained earlier in the case of a single common factor for all countries. Thus although in the present specification area-specif developments are taken into account, the six analysed countries still clearly show a high degree of comovement - even across areas and the common dynamics are more persistent than in the single-factor model as indicated by the autoregressive coefficient of the transition equation. In addition to the globally shared dynamics the estimation results also show that area-specific common dynamics play an important role. Although the two area factors show some similarities in their evolution, they capture important differences in the two areas. In this respect, more pronounced downturns in activity in the North-American (+UK) factor than in the Continental-European factor in the early 1980s can be seen. Furthermore, the effect of the German reunification boom and subsequent decline in activity, which clearly affected neighboring countries as well and thus shows up as common movement of these countries, can be identified in the Continental-European factor, while being absent from the North-American (+UK) factor. As shown in Table 4.1 the persistence of the global factor is considerably higher than that of the two area-specific factors, whose autoregressive coefficients are very similar. For most countries, the area-specific factor is much more important than the truely global factor in explaining growth developments (see Table 4.2) Only in the case of the UK and France does the share of variance explained by the global factor exceed that explained by the respective area factors, with the important difference that both factors together explain only a small share of growth variance in the case of the UK (15%), while in the case of France the two factors play a much more important role (65% explained by the two factors).

(Insert Figure 8 here)

Table 4.1 Parameter estimates (Model with global and two area factors)
UK included in the North-American area

	U.S.A.	U.K.	Canada	Germany	France	Italy
a_i^j	-0.263	-0.203	0.198	-0.284	0.073	0.224
	(0.165)	(0.120)	(0.080)	(0.084)	(0.107)	(0.083)
b_i^j	0.497	0.433	0.255	0.236	0.318	0.204
	(0.151)	(0.141)	(0.085)	(0.084)	(0.096)	(0.078)
c_i^j	0.752	0.108	0.363	0.390	0.561	0.378
	(0.281)	(0.176)	(0.125)	(0.100)	(0.124)	(0.093)
σ_i^j	0.213	0.808	0.742	0.828	0.605	0.740
	(0.889)	(0.110)	(0.058)	(0.062)	(0.010)	(0.058)

d_w	0.726	d_1	0.288	d_2	0.553
	(0.100)		(0.178)		(0.136)

$$Y_{i,t}^j = a_i^j Y_{i,t-1}^j + b_i^j W_t + c_i^j N_{i,t} + \varepsilon_{i,t}^j \quad \forall i \quad \forall j$$
$$W_t = d_w W_{t-1} + \eta_{w,t}$$
$$N_{i,t} = d_i N_{i,t-1} + \eta_{i,t}$$

Table 4.2 Shares of variance accounted for by common factors

	USA	UK	Canada	Germany	France	Italy
S_w	0.381	0.307	0.194	0.083	0.197	0.131
S_i	0.570	0.018	0.170	0.173	0.428	0.283

S_w : share of variance accounted for by the global factor
S_i : share of variance accounted for by the area-specific factor

The small share of the growth variance explained by the area factor in the case of the UK may indicate that it is more appropriately modeled as belonging to the Continental-European factor. Figure 9 and Tables 4.3 and 4.4 report the results of this alternative specification. Overall, it appears that the main features of the area factors and the global factor prove to be relatively robust to this modification. However, also in this specification UK output growth does not appear to be strongly affected by the area specific factor, with the share of variance explained by the area factor actually being slightly lower than in the alternative specification. In contrast to the previous results, however, the global factor explains now almost one fourth of the UK output variance. The two specifications can also be compared more formally: Since the number of unknown parameters is the same in the two specifications, using the Akaike criteria to compare the two models amounts to comparing the two log-likelihoods of the models. According to this test the specification with the UK part of the North-American area performs slightly better than the alternative.[19] A sperate estimation of the models two sub-samples of roughly equal length (1970.2-1986.2 and 1986.3-2001.4) suggests that this slightly better fit mainly reflects the growth dynamics of the first sub-sample, as the log likelihoods in the second sub-sample are basically identically, suggesting that the UK could equally well be seen as belonging

[19] The log likelihood of the model with the UK in the North-American area is -974.518, while that for the alternative model with the UK part of the Continental-European area is -975.141.

to North-American or the Continental-European area.[20] In any case, the share of variance explained by the area factor in the case of the UK is small and the concrete specification choice does not appear to affect the qualitative results of the analysis.

(Insert Figure 9 here)

Table 4.3 Parameter estimates (Model with global and two area factors)
UK included in the Continental-European area

	U.S.A.	Canada	U.K.	Germany	France	Italy
a_i^j	-0.288	0.185	-0.166	-0.291	-0.044	0.229
	(0.171)	(0.080)	(0.108)	(0.085)	(0.101)	(0.083)
b_i^j	0.444	0.251	0.350	0.181	0.289	0.183
	(0.141)	(0.095)	(0.123)	(0.101)	(0.123)	(0.097)
c_i^j	0.816	0.345	0.098	0.456	0.557	0.395
	(0.267)	(0.140)	(0.142)	(0.108)	(0.123)	(0.098)
σ_i^j	0.015	0.778	0.859	0.811	0.625	0.737
	(0.307)	(0.067)	(0.077)	(0.066)	(0.088)	(0.058)

d_w	0.759	d_1	0.333	d_2	0.534
	(0.098)		(0.237)		(0.146)

$Y_{i,t}^j = a_i^j Y_{i,t-1}^j + b_i^j W_t + c_i^j N_{i,t} + \varepsilon_{i,t}^j \quad \forall i \quad \forall j$
$W_t = d_w W_{t-1} + \eta_{w,t}$
$N_{i,t} = d_i N_{i,t-1} + \eta_{i,t}$

Table 4.4 Shares of variance accounted for by common factors

	USA	Canada	UK	Germany	France	Italy
S_w	0.327	0.207	0.231	0.054	0.183	0.121
S_i	0.672	0.159	0.012	0.231	0.421	0.298

S_w : share of variance accounted for by the global factor
S_i : share of variance accounted for by the area-specific factor

5 Spill-over effects

As the above analysis shows, the growth performance in the individual G7 countries contains a common element, which can be described by autoregressive common factors - either global or area-specific. In addition to being able to describe the common component, it would be useful to gain a better understanding of the determinants of the co-movement of growth rates. As shown above, common shocks in the form of oil price movements can explain some part of the observed co-movement. In this section, the issue of spill-over effects will be adressed in more detail: To what extent do developments in one country/area affect growth in other countries/areas?

[20]For the sub-period 1970.2-1986.2 the likelihood for the model with the UK in the North-American area is −487.178, compared to −488.455 in the case of the alternative specification.

Unfortunately, such exercise is complicated by the fact that without clear a priori identifying restrictions, contemporaneous spill-overs, i.e. spill-overs which affect another country within the same period, can not be analysed. Thus we have to rely on the intertemporal pattern of impulse and response to measure spill-over effects. Such a procedure will, of course, only allow insights if a substantial part of cross-country spill-over effects operate relatively slowly, taking, for example, in the present case up to one quarter to materialise.

We tried to identify cross-country spill-over effects through appropriate adjustments of the model specification. In particular, two approaches were initially chosen. First, the A matrix, instead of being diagonal, was specified as a non-diagonal matrix. In this case, the model becomes a mixture of a first-order VAR and a dynamic factor model, where common movements in the variables can be driven by the common factor and through cross-country spill-over effects, captured by the off-diagonal elements of the A matrix. Second, instead of affecting other countries directly through the A matrix, growth developments in the individual countries were specified as affecting each other through its impact on the common factor. To that end, lagged growth rates of each country were added to the transition equation. In this case, growth in each country is thus influenced by an index of lagged growth rates in the different countries. Unfortunately, the estimation results using these specifications were not very encouraging. In particular, many of the coefficients capturing the spill-over effects turn out to be insignificant and/or have the "wrong" sign. As one reason for these findings may be the large number of parameters to be estimated, especially in the case of the first specification using a non-diagonal A matrix, we chose an alternative specification, in which spill-over effects exist between areas rather than countries. More specifically, we model spill-over effects by allowing the D matrix to be non-diagonal, thereby creating lagged links between the area-specific common factors. Initially, we estimated the model with a global factor and two area factors connected through spill-over effects. The results showed that the global factor changes dramatically in this case and no longer lends itself to an easy economic interpretation. Furthermore, we find that dropping the global factor from the model does not affect the results of the analysis and leaves the Akaike criterion basically unchanged.[21] Thus we report here only the results for the model without the global factor.

Concretely, the measurement equations are:

$$Y_{i,t}^{j} = a_i^j Y_{i,t-1}^{j} + c_i^j N_{i,t} + \varepsilon_{i,t}^{j} \quad \forall i \quad \forall j$$

and the transition equations are:

$$\begin{bmatrix} N_{1,t} \\ \vdots \\ N_{n,t} \end{bmatrix} = \begin{bmatrix} d_{1,1} & \cdots & d_{1,n} \\ \vdots & \ddots & \vdots \\ d_{n,1} & \cdots & d_{n,n} \end{bmatrix} \begin{bmatrix} N_{1,t-1} \\ \vdots \\ N_{n,t-1} \end{bmatrix} + \begin{bmatrix} \eta_{1,t} \\ \vdots \\ \eta_{n,t} \end{bmatrix}$$

[21] In fact, the Akaike criterion is slightly higher for the model with the global common factor included for the specification with the UK belonging to the North-American area, it while it is slightly lower in the alternative specification witht the UK in the Continental-European area. In any case, the differences are very small.

where $V[\eta_t] = Id_n$. The state vector N_t is thus assumed to follow a VAR process of order one.

Given the slightly higher value of the Akaike criterion, we report the results for the specification with the UK being part of the North-American area. Figure 10 shows that the area-specific factors are very similar to the ones in specification with global factor and either independent or related area-specific common factors.The Akaike criterion suggests that the present specification is in fact superior to the model with the global factor and two independent area-specific factors, despite the fact that it contains fewer factors to explain the shared growth dynamics.

Regarding the interaction between the two area-specific common factors it is interesting to note that the North-American factor has a strong lagged impact on the Continental-European factor (in Table 5.1 $d_{2,1}$ is large and statistically significant), while the reverse is not true; the coefficient for the lagged Continental-European factor, $d_{1,2}$, in the equation describing the evolution of the North-American factor is small and insignificant. This finding is also illustrated by the impulse response functions in Figure 11, where the impact of an innovation in the area-specific factors on the various countries is depicted.

Table 5.1 Parameter estimates (Model with two interrelated area-specific factors)
UK included in North-American area

	U.S.A.	Canada	U.K.	Germany	France	Italy
a_i^j	-0.173	0.117	-0.047	-0.277	0.008	0.194
	(0.109)	(0.090)	(0.087)	(0.084)	(0.098)	(0.086)
c_i^j	0.661	0.494	0.297	0.432	0.546	0.455
	(0.119)	(0.123)	(0.101)	(0.126)	(0.100)	(0.061)
σ_i^j	0.621	0.715	0.926	0.826	0.666	0.693
	(0.100)	(0.064)	(0.061)	(0.062)	(0.075)	(0.061)

$d_{1,1}$	0.629	$d_{1,2}$	0.018	$d_{2,1}$	0.395	$d_{2,2}$	0.434
	(0.119)		(0.101)		(0.123)		(0.126)

$Y_{i,t}^j = a_i^j Y_{i,t-1}^j + c_i^j N_{i,t} + \varepsilon_{i,t}^j \quad \forall i \quad \forall j$
$N_t = (d_{i,j}) N_{t-1} + \eta_t \quad i \in \{1,2\}, j \in \{1,2\}$

(Insert Figures 10 here)

(Insert Figures 11 here)

The area-specific shocks explain roughly half of the growth variance in the case of the US and Canada, with virtually all the explanatory power coming from innovations in the North-American factor (see Table 5.2). In the case of the UK, the overall share explained by the factors is again considerably lower, but also comes virtually exclusively from the North-American factor. For the Continental-European countries also most of the variance is explained by their "own" area-specific factor. However, for these countries the "other" area-specific factor also accounts for a substantial share of the growth variance. In relative terms, the contribution is around one third (Germany) to one half (Italy) of the Continental-European factor. Overall, Germany and the UK are the countries with the smallest share of the growth variance explained by the common factors.

Table 5.2 Shares of variance accounted for by the innovations of the common factors

	USA	Canada	UK	Germany	France	Italy
S_1	0.604	0.482	0.140	0.071	0.175	0.172
S_2	0.000	0.000	0.000	0.195	0.376	0.321

S_i : share of variance accounted for by the innovations of area-specific factor N_i

Using the estimation results, it is possible to calculate hypothetical area-specific factors, which show the evolution of the factors assuming that the respective areas had not been affected by developments in the other area. For the North-American factor the two series, with and without the effect of the Continental-European factor, are, as expected, virtually identical and therefore omitted. For the Continental-European factor some interesting difference can, however, be identified (see Figure 12). For example, it appears that in the early 1970s Europe's growth performance was lifted up by the positive spill-over effects from North America. In contrast, the slowdowns in the early 1980s would have been less pronounced, if Europe had been shielded from the influence of developments in North America. Likewise in the early 1990s European growth was strongly negatively affected by the recession in the US and also the most recent downturn in Europe was to a considerable part a reflection of adverse developments in North America. At the same time, however, the subsequent growth reversal in North America has also helped to revive European growth, bringing the Continental European factor back to a level where it would have been without the intermittent downward pressure from the North-American factor.

(Insert Figure 12 here)

When the UK is included in the Continental-European area the results are very similar. This can, for example, be seen in the shares of variance explained by the two area-specific factors (see Table 5.3). In the case of the US and Canada, the share of variance explained by both the North-American and the European factor increases slightly, but the effect of the European factor on the North-American factor remains small and insignificant[22]. In the case of the European countries, the share of variance explained by the Continental-European factor decreases slightly for Italy, while it increases for France and Germany. The effect of the North-American factor on the European one is reduced in this specification, except in the case of Germany, suggesting that possibly part of the spill-overs in the previous specification originated in the UK. For the UK itself the overall share of variance explained by the two factors remains virtually unchanged, although, of course the bulk of the explanation now comes from the "own" - European - factor.

[22] The estimated value of the parameter $d_{1,2}$ (see Table 7.1) in the specification with the UK in the Continental European factor is 0.079 and its standard deviation is 0.100.

Table 5.3 Shares of variance accounted for by the innovations of the common factors

	USA	Canada	UK	Germany	France	Italy
S_1	0.632	0.485	0.038	0.070	0.156	0.139
S_2	0.007	0.006	0.102	0.224	0.392	0.301

S_i : share of variance accounted for by the innovations of area-specific factor N_i

5.1 Industrial production

A similar picture arises if one uses industrial production data instead of GDP[23]. The advantage of industrial production figures is that they are available at a monthly frequency. Thus it might be possible to better identify spill-over effects to the extent that these materialise relatively quickly. On the other hand, industrial production data tend to be more volatile than GDP figures and it may therefore be more difficult to extract systematic relationships from them. The results, however, show that some meaningful relationships and co-movements also exist for industrial production. In fact, the global common factor for industrial production is very similar to the factor for GDP, with a correlation coefficient (using a 7 year rolling window) of between 0.5 and 0.9 between the two factors. The correlation declined somewhat during the 1990s but has increased again recently, standing currently at around 0.7.

For the model with two interrelated area-specific factors the results are reported in Table 5.4. As suggested by the Akaike criterion, the UK has been placed in the Continental-European area [24]. The North-American factor is considerably more persistent than the European one, as reflected in the high autoregressive coefficient $d_{1,1}$. Again, the North American is largely unaffected by developments in Europe, with the coefficient $d_{1,2}$ being small and insignificant and the share of Canadian and US industrial production growth variance explained by innovations in the Continental-European factor being close to zero (see Table 5.5). In contrast, the share of growth variance for the European countries explained by innovations in the North-American factor is substantially larger, especially if seen relative to the contribution of the "own" factor innovations.

[23]The dataset used in the estimation of the common factor in industrial production monthly rates of change spans the period 1970.M2 to 2000.M2

[24]The log-likelihoods for this specification is -2853.69, compared to -2855.33 for the case where the UK is included in the North-American area.

Table 5.4 Parameter estimates (Industrial production: Model with two interrelated area-specific factors)

	U.S.A.	Canada	U.K.	Germany	France	Italy
a_i^j	0.051	-0.293	-0.209	-0.442	-0.431	-0.442
	(0.077)	(0.055)	(0.051)	(0.047)	(0.048)	(0.050)
c_i^j	0.432	0.354	0.271	0.251	0.356	0.439
	(0.079)	(0.058)	(0.064)	(0.058)	(0.063)	(0.071)
σ_i^j	0.728	0.851	0.934	0.856	0.830	0.770
	(0.054)	(0.040)	(0.038)	(0.035)	(0.040)	(0.048)

$d_{1,1}$	0.724	$d_{1,2}$	0.082	$d_{2,1}$	0.345	$d_{2,2}$	0.100
	(0.090)		(0.130)		(0.101)		(0.147)

$Y_{i,t}^j = a_i^j Y_{i,t-1}^j + c_i^j N_{i,t} + \varepsilon_{i,t}^j \quad \forall i \quad \forall j$

$N_t = (d_{i,j}) N_{t-1} + \eta_t \quad i \in \{1,2\}, j \in \{1,2\}$

Table 5.5 Shares of variance accounted for by the innovations of the common factors

	USA	Canada	UK	Germany	France	Italy
S_1	0.461	0.201	0.017	0.012	0.023	0.036
S_2	0.003	0.001	0.074	0.073	0.143	0.224

S_i : share of variance accounted for by the innovations of area-specific factor N_i

5.2 Sub-sample analysis

It seems reasonable to assume that the process of globalisation strengthens the cross-country linkages through which developments in one country/area spill over to other countries/areas. As past decades have been characterised by a substantial reduction of trade barriers, innovations in transportation and communication technologies and increasing financial integration, the size of spill-over effects between countries for a given shock should therefore have increased over the sample period.[25] In order to analyse the evolution of spill-over effects over time we use again the GDP series and split the overall sample period into two distinct sub-periods: 1970.2-1986.1 and 1986.2-2002.2. For the second sub-period, the model specification with the UK included in the North-American area yields negative spill-overs from the Continental-European factor to the North-American factor, which are difficult to interpret economically, especially given the sign reversal between the first and second sub-period. Therefore we base our analysis on the results for the alternative specification with the UK being included in the Continental-European area.

The estimation results for the two subsamples are shown in Tables 5.6 to 5.9.

[25] This does not necessarily mean that the co-movement between activity in different countries/areas necessarily has increased. Apart from the possibility of increased idiosyncratic shocks, discussed earlier in the text, globalisation can also be expected to lead to increased regional specialisation, thus reducing the prevalence of common shocks across countries/area.

Table 5.6 Parameter estimates (Model with two interrelated area-specific factors) 1972.2-1986.1, UK included in the Continental-European factor

	U.S.A.	Canada	U.K.	Germany	France	Italy
a_i^j	-0.333	0.150	-0.206	-0.465	0.097	0.428
	(0.185)	(0.111)	(0.117)	(0.113)	(0.112)	(0.099)
c_i^j	0.939	0.451	0.420	0.708	0.496	0.340
	(0.160)	(0.126)	(0.155)	(0.160)	(0.140)	(0.081)
σ_i^j	0.007	0.795	0.862	0.532	0.748	0.716
	(0.702)	(0.081)	(0.083)	(0.121)	(0.080)	(0.069)

$d_{1,1}$	0.458	$d_{1,2}$	0.172	$d_{2,1}$	0.398	$d_{2,2}$	0.350
	(0.160)		(0.126)		(0.155)		(0.159)

$Y_{i,t}^j = a_i^j Y_{i,t-1}^j + c_i^j N_{i,t} + \varepsilon_{i,t}^j \quad \forall i \quad \forall j$

$N_t = (d_{i,j}) N_{t-1} + \eta_t \quad i \in \{1,2\}, j \in \{1,2\}$

Table 5.7 Shares of variance accounted for by the innovations of the common factors 1972.2-1986.1, UK included in the Continental-European factor

	USA	Canada	UK	Germany	France	Italy
S_1	0.961	0.330	0.047	0.107	0.102	0.098
S_2	0.039	0.017	0.185	0.535	0.323	0.250

S_i : share of variance accounted for by the innovations of area-specific factor N_i

Table 5.8 Parameter estimates (Model with two interrelated area-specific factors) 1986.2-2002.2, UK included in the Continental-European factor

	U.S.A.	Canada	U.K.	Germany	France	Italy
a_i^j	-0.064	-0.228	0.494	-0.062	-0.096	-0.355
	(0.115)	(0.185)	(0.113)	(0.119)	(0.144)	(0.117)
c_i^j	0.471	0.758	0.136	0.243	0.587	0.534
	(0.139)	(0.119)	(0.143)	(0.135)	(0.079)	(0.253)
σ_i^j	0.771	0.319	0.792	0.931	0.531	0.698
	(0.079)	(0.253)	(0.071)	(0.085)	(0.116)	(0.091)

$d_{1,1}$	0.723	$d_{1,2}$	-0.056	$d_{2,1}$	0.447	$d_{2,2}$	0.475
	(0.138)		(0.120)		(0.143)		(0.139)

$Y_{i,t}^j = a_i^j Y_{i,t-1}^j + c_i^j N_{i,t} + \varepsilon_{i,t}^j \quad \forall i \quad \forall j$

$N_t = (d_{i,j}) N_{t-1} + \eta_t \quad i \in \{1,2\}, j \in \{1,2\}$

Table 5.9 Shares of variance accounted for by the innovations of the common factors 1986.2-2002.2, UK included in the Continental-European factor

	USA	Canada	UK	Germany	France	Italy
S_1	0.399	0.883	0.062	0.053	0.297	0.163
S_2	0.003	0.006	0.051	0.071	0.413	0.291

S_i : share of variance accounted for by the innovations of area-specific factor N_i

Overall, it appears that the structure of the dynamic relationships between G7 growth developments has changed noticeably over time, as suggested by the differences in the estimated parameters between the two sub-periods. In particular, the dynamics of the common factors appear to have changed The persistence of both, the North-American and the Continental-European factor has increased over time, especially in the case of the former. As to spill-over effects, the size of the $d_{1,2}$ coefficient, measuring the effect of the Continental-European factor on the North-American factor, declined between the first and second sub-period, turning slightly negative in the latter. However, in both sub-periods the coefficient is insignificant. In contrast, the coefficient $d_{2,1}$is significant and positive in both sub-periods. Furthermore the coefficient increased over time. These findings thus suggest that Europe may have become more suceptible to shocks originating in North-America, while North America remains insulated from developments in Europe.

Regarding the shares of variance explained by innovations in the two common factors, the share attributable to the North-American factor has increased over time for most countries, with the increase for Canada, France and Italy being particularly pronounced. In contrast, in the case of the US and Germany the share has declined. The share of variance explained by the Continental-European factor has declined for most countries, with the exception of France nad Italy. Among the European countries the shares for the UK and Germany declined significantly, which in the latter case might partly be explained by idiosyncratic movements associated with the economic ramifications of re-unification. The changes over time are also illustrated by the impulse response function for the respective sub-periods. A shock to the North-American-specific factor has clearly stronger repercussions on the Italian and French growth rates in the second period than in the first one. Moreover, with the exception of Italy, we can observe that the responses are more persistent during the second period.

(Insert Figure 13 and 14 here)

6 Conclusion

The paper identifies a number of stylised facts about the observed common fluctuations among G7 real GDP growth rates and tries to disentangle the contribution of common shocks - in particular, oil price fluctuations - and cross-country/cross-area spill-over effects in determining those joint fluctuation.

The estimation of a dynamic common factor model for the G7 countries' GDP (and industrial production) growth using Kalman filtering techniques allows first to isolate and measure the historical evolution of the degree of output growth co-movements in the different countries and then to assess the relative importance of common shocks, affecting the different economies at the same time versus direct linkages between countries ("spill overs") in generating such co-movements. The general model specification assumes the process for real GDP growth in each of the G7 countries to be driven by a country-specific autoregressive component and a latent component, which is common to all series and follows a univariate autoregressive process.

By extending the model so to include "area-wide" common factors which are each shared by only a subset of countries, the paper also disentangles the "area-wide" common factors and "area-wide" spill-over effects.

An important result of the analysis is to show that the G7 countries share common output dynamics with clearly identifiable common swings in activity across the G7. Furthermore, the paper finds statistical support for a (time-varying) effect of oil price developments on the common factor, confirming that oil prices, indeed, constitute one important variable partly responsible for the observed co-movement among economic activity in different countries. The "area-wide" specification of common factors identifies significant "area-wide" common effects on real GDP growth developments in the G7 countries and strong spill-over effects running from the North-American area to the European area, but not vice versa. Similar results are obtained when a factor common to the G7 industrial production series is estimated. The approach employed in the paper has a number of advantages over other approaches used in analysing the same problems, like the evolution of pure bi-variate correlation or the generalised dynamic factor (GDF) model.

The paper provides thus a number of interesting insights from which a number of interesting areas for future research can be derived. In particular, one line of future research would be to deepen the understanding of the economic determinants of common fluctuations and area linkages, especially looking at the developments in the last two decades. In that respect, the asymmetry between North America and (Continental) Europe as a source of spill-over effects would be worth exploring further. For that purpose a more careful analysis of the characteristics of the channels of international transmission of country-specific shocks would be useful..

A Mathematical Appendices

A.1 Filtering and Smoothing

A.1.1 Filtering

When estimating a state space model, we face two classes of unknowns. There are unknown parameters appearing in $G_t, H_t, R_t, Q_t, M_t, N_t$, μ_t or ν_t, and there are the unobservable vectors ρ_t. The first class of unknowns are inputs of the Kalman filter and conditional on these parameters values, best approximations of the vector ρ_t based on all information through time t are obtained by the filter.

By denoting with $\rho_{t|\tau}$ the estimate of ρ_t upon information $Y^\tau = \begin{pmatrix} Y_1 & \cdots & Y_\tau \end{pmatrix}'$, the output of the Kalman filter is $\rho_{t|t}$.

Let $\Sigma_{t|\tau}$ denote the variance-covariance matrix of ρ_t based upon information $\begin{pmatrix} Y_1 & \cdots & Y_\tau \end{pmatrix}$.

The procedure consists of the prediction and updating equations.

The prediction equations are

$$\begin{aligned}\rho_{t|t-1} &= \nu_t + H_t\,\rho_{t-1|t-1}\\ \Sigma_{t|t-1} &= H_t\Sigma_{t-1|t-1}H_t' + Q_{t-1}\end{aligned}$$

and

$$\begin{aligned}Y_{t|t-1} &= \mu_t + G_t\,\rho_{t|t-1}\\ \Omega_{t|t-1} &= R_t + G_t\Sigma_{t|t-1}G_t'\end{aligned}$$

where $Y_{t|t-1} = E(Y_t|Y^{t-1})$ and $\Omega_{t|t-1} = V(Y_t|Y^{t-1})$.
The updating equations are

$$\begin{aligned}\rho_{t|t} &= \rho_{t|t-1} + K_t(y_t - y_{t|t-1})\\ \Sigma_{t|t} &= (Id - K_tG_t)\Sigma_{t|t-1}\end{aligned}$$

where K_t, which is called the gain of the filter, is given by

$$K_t = \Sigma_{t|t-1}G_t'(R_t + G_t\,\Sigma_{t|t-1}G_t')^{-1}$$

The filter consists in computing recursively these equations, given initial values $\rho_{1|0} = m_0$ and $\Sigma_{1|0} = P_0$. These values can arise naturally. For example if ρ_t follows an stationary autoregressive process, the filter would be initialised with the unconditional mean and variance of ρ_t. In our study, we worked with demeaned series, and that is why the unconditional mean of ρ_t is zero. In order to get the unconditional variance of ρ_t, we use the results of the variance decomposition presented in Appendix **??**.

Finally, the log likelihood function of the unknown parameters (gathered in the vector θ) is then easily formed. Let λ_t denote the innovations in Y_t ($\lambda_t = Y_t - Y_{t|t-1}$), the log likelihood can then be written as

$$L(Y^T;\theta) = -\frac{NT}{2}\log(2\pi) - \frac{1}{2}\sum_{t=1}^{T}\left(\log\left|\Omega_{t|t-1}(\theta)\right| + \lambda_t'(\theta)\Omega_{t|t-1}^{-1}(\theta)\lambda_t(\theta)\right) \tag{7}$$

A.1.2 Smoothing

The smoothing consists in a refinement of the approximations of the unobserved states ρ_t. More precisely, the filtering uses information up to time t whereas the smoothing use information up to time T in order to approximate ρ_t (this approximation is denoted $\rho_{t|t} = E(\rho_t|Y^t)$). The smoothing is a backwards-recursive procedure which computes the following values:

$$\rho_{t|T} = E(\rho_t|Y^T) \qquad \Sigma_{t|T} = V\left[(\rho_t - \rho_{t|T})(\rho_t - \rho_{t|T})'|Y^T\right]$$

The recursive equations are given by

$$\begin{aligned}\rho_{t|T} &= \rho_{t|t} + F_{t+1}(\rho_{t+1|T} - \rho_{t+1|t})\\ \Sigma_{t|T} &= \Sigma_{t|t} + F_{t+1}(\Sigma_{t+1|T} - \Sigma_{t+1|t})F_{t+1}'\\ \text{where } F_{t+1} &= \Sigma_{t|t}H_{t+1}'\Sigma_{t+1|t}^{-1}\end{aligned}$$

A.2 The EM algorithm

A.2.1 Formulation for a Linear State Space Model.

We consider two stochastic processes Y_t and Z_t. Y_t is observed at $t = 1, ..., T$. Z_t is not observed.

We use the following notations:

$Y^T = (Y_1', Y_2', ..., Y_T')'$

$Z^T = (Z_1', Z_2', ..., Z_T')'$

Note that the former vectorρ_t (see) is here denoted by Z_t.

$L(Y^T, Z^T; \theta)$ is the log probability density function of (Y^T, Z^T).

$L(Y^T; \theta)$ is the log probability density function of Y^T (or the log-likelihood function).

$L(Z^T|Y^T; \theta)$ is the log conditional probability density function of Z^T given Y^T.

We can write

$$L(Y^T; \theta) = L(Y^T, Z^T; \theta) - L(Z^T|Y^T; \theta) \tag{8}$$

The (i+1)th iteration of the EM algorithm consists in taking first the expected value of () , given the Y^T vector available

$$L(Y^T; \theta) = E_{\theta^{(i)}}\left[L(Y^T, Z^T; \theta)|Y^T\right] - E_{\theta^{(i)}}\left[L(Z^T|Y^T; \theta)|Y^T\right]$$

where $\theta^{(i)}$ is the estimate of θ obtained at the i-th iteration, and then in maximising $E_{\theta^{(i)}}\left[L(Y^T, Z^T; \theta)|Y^T\right]$ with respect to θ. This maximisation gives $\theta^{(i+1)}$. The basic principle of this algorithm is based on the following lemma.

Lemma 1 *With the notations introduced above:*

$$L(Y^T; \theta^{(i+1)}) geqslant L(Y^T; \theta^{(i)}) \tag{9}$$

Proof. $L(Y^T; \theta^{(i+1)}) - L(Y^T; \theta^{(i)}) =$

$$\underbrace{E_{\theta^{(i)}}\left[L(Y^T, Z^T; \theta^{(i+1)})|Y^T\right] - E_{\theta^{(i)}}\left[L(Y^T, Z^T; \theta^{(i)})|Y^T\right]}$$
$$\underbrace{-E_{\theta^{(i)}}\left[L(Z^T|Y^T; \theta^{(i+1)})|Y^T\right] + E_{\theta^{(i)}}\left[L(Z^T|Y^T; \theta^{(i)})|Y^T\right]}$$

The first underlined term is positive due to the maximisation, and as regards the second, it can be rewritten this way

$$E_{\theta^{(i)}}\left[\log\left(\frac{l(Z^T|Y^T; \theta^{(i)})}{l(Z^T|Y^T; \theta^{(i+1)})}\right)|Y^T\right] \tag{10}$$

This term is positive too since this is a conditional Kullback information of $\theta^{(i)}$ *vs* $\theta^{(i+1)}$. ■

A.2.2 Application

The "Linear" Algorithm

The Linear State Space Model considered here is

$$\begin{aligned} Y_t &= AY_{t-1} + BZ_t + \varepsilon_t \\ Z_t &= dZ_{t-1} + \eta_t \end{aligned}$$

where Y_t is a $n \times 1$ vector, and Z_t is a scalar.
We assume that A and the variance-covariance matrix of ε_t are diagonal:

$$V(\varepsilon_t) = \begin{pmatrix} \sigma_1^2 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sigma_n^2 \end{pmatrix} \; A = \begin{pmatrix} a_1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & a_n \end{pmatrix}$$

We take $V(\eta_t) = 1$. Let θ denote the parameters to estimate, that is to say

$$\theta = (a_1, \cdots, a_n, b_1, \cdots, b_n, d, \sigma_1^2, \cdots, \sigma_n^2)$$

And as

$$L(Y^T, Z^T; \theta) = \sum_{t=1}^{T} \log \left(f(Y_t, Z_t | Y^{t-1}, Z^{t-1}; \theta) \right)$$

we have to maximise

$$E_{\theta^{(i)}} \left(L(Y^T, Z^T; \theta) | Y^T \right) = -\frac{(n+1)\, T}{2} \log (2\pi) +$$

$$+E_{\theta^{(i)}} \left(-\frac{1}{2} \sum_{t=1}^{T} \left\{ \sum_{j=1}^{n} \left[\log \left(\sigma_j^2 \right) + \frac{(Y_{j,t} - a_j\, Y_{j,t-1} - b_j\; Z_t)^2}{\sigma_j^2} \right] + \left[(Z_t - dZ_{t-1})^2 \right] \right\} \, |Y^T \right)$$

where $\theta^{(i)}$ is the value of θ at the ith iteration. The first order conditions are

$$E_{\theta^{(i)}} \left(\frac{\partial L(Y^T, Z^T; \theta)}{\partial \theta} | Y^T \right) = 0$$

which gives

$$E_{\theta^{(i)}} \left(\sum_{t=1}^{T} Y_{j,t-1} \left(Y_{j,t} - \widehat{a}_j\, Y_{j,t-1} - \widehat{b}_j\; Z_t \right) |Y^T \right) = 0 \quad (11)$$

$$E_{\theta^{(i)}} \left(\sum_{t=1}^{T} Z_t \left(Y_{j,t} - \widehat{a}_j\, Y_{j,t-1} - \widehat{b}_j\; Z_t \right) |Y^T \right) = 0 \quad (12)$$

$$\widehat{\sigma}_j^2 - \frac{1}{T} E_{\theta^{(i)}} \left(\sum_{t=1}^{T} \left(Y_{j,t} - \widehat{a}_j\, Y_{j,t-1} - \widehat{b}_j\; Z_t \right)^2 |Y^T \right) = 0 \quad (13)$$

$$E_{\theta^{(i)}} \left(\sum_{t=1}^{T} Z_{t-1} \left[\left(Z_t - \widehat{d} Z_{t-1} \right) \right] |Y^T \right) = 0 \quad (14)$$

This leads to formulae similar to the OLS in linear models. However, some terms need to be replaced by "smoothed" values. As an example, (14) results in:

$$\widehat{d} = \frac{E_{\theta^{(i)}}\left(\sum_{t=1}^{T} Z_{t-1}Z_t|Y^T\right)}{E_{\theta^{(i)}}\left(\sum_{t=1}^{T} Z_{t-1}^2|Y^T\right)}$$

The numerator is then calculated using the following equation:

$$E_{\theta^{(i)}}\left(\sum_{t=1}^{T} Z_{t-1}Z_t|Y^T\right) = \sum_{t=1}^{T} Cov_{\theta^{(i)}}\left(Z_{t-1}, Z_t|Y^T\right)+\sum_{t=1}^{T} E_{\theta^{(i)}}\left(Z_t|Y^T\right) E_{\theta^{(i)}}\left(Z_{t-1}|Y^T\right) \tag{15}$$

The three terms of the right-hand side of equation (15) given by the Kalman Smoother. Therefore, the set of equations (11)(12)(13)(14) leads to a linear system.

Note that this algotithm needs to include Z_{t-1} in the transition equations in order to get $Cov\left(Z_{t-1}Z_t|Y^T\right)$ and $E\left(Z_{t-1}|Y^T\right)$ by smoothing.

The "Bilinear" Algorithm

Let us consider now the following State Space Model, involving the dynamics of the errors:

$$\begin{aligned} Y_t &= BZ_t + \xi_t \\ Z_t &= dZ_{t-1} + \eta_t \\ \xi_t &= R\xi_{t-1} + \varepsilon_t \end{aligned}$$

This model amounts to adding an autoregressive term in the measurement equation:

$$\begin{aligned} Y_t &= RY_{t-1} + BZ_t - RB\ Z_{t-1} + \varepsilon_t \\ Z_t &= dZ_{t-1} + \eta_t \end{aligned}$$

Bilinear constraints on the parameters appear in the RB term. Assuming that the R matrix is diagonal, $L(Y^T, Z^T; \theta)$ is from now onwards given by

$$\begin{aligned} L(Y^T, Z^T; \theta) &= K - \frac{1}{2}\sum_{t=1}^{T}\sum_{i=1}^{n}\left[\log\left(\sigma_i^2\right) + \frac{\left(Y_{i,t} - a_i\, Y_{i,t-1} - b_i\ Z_t + r_i b_i Z_{t-1}\right)^2}{\sigma_i^2}\right] \\ &\quad -\frac{1}{2}\sum_{t=1}^{T}\left[\left(Z_t - dZ_{t-1}\right)^2\right] \end{aligned}$$

which contains a product of parameters $r_i b_i$. Denoting with $\theta^{(i)}$, the vector of estimates at the ith iteration, a way to implement the (i+1)th iteration is the fallowing "zig-zag" procedure:

$$1)\quad \begin{pmatrix} r_1^{(i+1)} \\ \vdots \\ r_n^{(i+1)} \end{pmatrix} \quad = \quad Arg\max_{r} E_{\theta^{(i)}}\left[L(Y^T, Z^T; \begin{pmatrix} r_1 \\ \vdots \\ r_n \end{pmatrix}; \begin{pmatrix} b_1^{(i)} \\ \vdots \\ b_n^{(i)} \\ d^{(i)} \\ \sigma_1^{2\,(i)} \\ \vdots \\ \sigma_n^{2\,(i)} \end{pmatrix})|Y^T \right]$$

$$2)\quad \begin{pmatrix} b_1^{(i+1)} \\ \vdots \\ b_n^{(i+1)} \\ d^{(i+1)} \\ \sigma_1^{2\,(i+1)} \\ \vdots \\ \sigma_n^{2\,(i+1)} \end{pmatrix} \quad = \quad Arg\max_{b,d,\sigma^2} E_{\theta^{*(i)}}\left[L(Y^T, Z^T; \begin{pmatrix} r_1^{(i+1)} \\ \vdots \\ r_n^{(i+1)} \end{pmatrix}; \begin{pmatrix} b_1 \\ \vdots \\ b_n \\ d \\ \sigma_1^2 \\ \vdots \\ \sigma_n^2 \end{pmatrix})|Y^T \right]$$

where $\theta^{*(i)} = \begin{pmatrix} r_1^{(i+1)} & \cdots & r_n^{(i+1)} & b_1^{(i)} & \cdots & b_n^{(i)} & d^{(i)} & \sigma_1^{2\,(i)} & \cdots & \sigma_n^{2\,(i)} \end{pmatrix}$

A.3 Information Matrix

In this section, we will describe two ways of deriving estimates of the Fisher Information Matrix.

The first subsection will use the formula

$$\widehat{I}_F^{(1)}\left(\widehat{\theta}_T\right) = \frac{1}{T}\sum_{t=1}^{T}\frac{\partial \log f_t\left(Y_t/Y^{t-1};\widehat{\theta}_T\right)}{\partial\theta}\frac{\partial \log f_t\left(Y_t/Y^{t-1};\widehat{\theta}_T\right)}{\partial\theta'} \tag{16}$$

The second subsection will use this other formula

$$\widehat{I}_F^{(2)}\left(\widehat{\theta}_T\right) = -\frac{1}{T}\sum_{t=1}^{T}E_{\widehat{\theta}_T}\left(\frac{\partial^2 \log f_t\left(Y_t/Y^{t-1};\widehat{\theta}_T\right)}{\partial\theta\partial\theta'}|Y^{t-1}\right) \tag{17}$$

A.3.1 Computation using a sequence of smoothing

In this subsection, we consider the estimate of the Fisher Information Matrix given by (16).

Using obvious notations, we have

$$\frac{\partial \log f_t\left(Y_t/Y^{t-1};\widehat{\theta}_T\right)}{\partial\theta} = \frac{\partial \log l\left(Y^t;\widehat{\theta}_T\right)}{\partial\theta} - \frac{\partial \log l\left(Y^{t-1};\widehat{\theta}_T\right)}{\partial\theta} \tag{18}$$

Moreover

$$\frac{\partial \log l\left(Y^t;\widehat{\theta}_T\right)}{\partial\theta} = \frac{\partial \log l\left(Y^t,Z^t;\widehat{\theta}_T\right)}{\partial\theta} - \frac{\partial \log l\left(Z^t|Y^t;\widehat{\theta}_T\right)}{\partial\theta} \tag{19}$$

and if we take the conditional expectation of (19), given y^t, with the parameter value $\widehat{\theta}_T$

$$\frac{\partial \log l\left(Y^t;\widehat{\theta}_T\right)}{\partial\theta} = E_{\widehat{\theta}_T}\left(\frac{\partial \log l\left(Y^t,Z^t;\widehat{\theta}_T\right)}{\partial\theta}|Y^t\right) - E_{\widehat{\theta}_T}\left(\frac{\partial \log l\left(Z^t|Y^t;\widehat{\theta}_T\right)}{\partial\theta}|Y^t\right) \tag{20}$$

The last term of the right-hand side of (20) vanishes:

$$\frac{\partial \log l\left(Y^t;\widehat{\theta}_T\right)}{\partial\theta} = E_{\widehat{\theta}_T}\left(\frac{\partial \log l\left(Y^t,Z^t;\widehat{\theta}_T\right)}{\partial\theta}|Y^t\right) \tag{21}$$

Moreover

$$\log l\left(Y^t,Z^t;\widehat{\theta}_T\right) = \sum_{s=1}^{t}\log f\left(Y_s|Y^{s-1},Z^s;\widehat{\theta}_T\right) + \log f\left(Z_s|Y^{s-1},Z^{s-1};\widehat{\theta}_T\right)$$

Thus, by (21),

$$\frac{\partial \log l\left(Y^t;\widehat{\theta}_T\right)}{\partial\theta} = E_{\widehat{\theta}_T}\left(\frac{\partial}{\partial\theta}\left\{\sum_{s=1}^{t}\log f\left(Y_s|Y^{s-1},Z^s;\widehat{\theta}_T\right)+\log f\left(Z_s|Y^{s-1},Z^{s-1};\widehat{\theta}_T\right)\right\}|Y^t\right)$$

With (18) and (16) we finally get

$$\widehat{I}_F^{(1)}\left(\widehat{\theta}_T\right) = \frac{1}{T}\sum_{t=1}^{T}\left\{M_t - M_{t-1}\right\}\left\{M_t - M_{t-1}\right\}$$

Where

$$M_t = E_{\widehat{\theta}_T}\left(\frac{\partial}{\partial\theta}\left\{\sum_{s=1}^{t}\log f\left(Y_s|Y^{s-1},Z^s;\widehat{\theta}_T\right)+\log f\left(Z_s|Y^{s-1},Z^{s-1};\widehat{\theta}_T\right)\right\}|Y^t\right)$$

The computation of each M_t then requires a pass of the Kalman smoother, using only the t first terms of the y series.

A.3.2 Second Computation

This subsection will use the formula of the information matrix given by (17). The main results described here come from the article by Engle and Watson (1981) [8].

Let λ_t denote the difference between Y_t and the best prediction of Y_t, based on information at $t-1$

$$\lambda_t = Y_t - Y_{t|t-1}$$

$$\lambda_t = Y_t - \mu_t - G_t\,\rho_{t|t-1} \tag{22}$$

$\Omega_{t|t-1}$,the variance-covariance matrix of the innovation λ_t, is actually a function of the parameters of the model, and is recursively computed by the Kalman filter. The log-likelihood of the sample is then given by

$$L(Y^T;\theta) = -\frac{NT}{2}\log(2\pi) - \frac{1}{2}\sum_{t=1}^{T}\left(\log\left|\Omega_{t|t-1}(\theta)\right| + \lambda_t'(\theta)\Omega_{t|t-1}^{-1}(\theta)\lambda_t(\theta)\right) \tag{23}$$

We denote by θ the parameter appearing in L.

The following expressions for derivatives of a symmetric matrix will be used

$$\frac{\partial\left|S\right|}{\partial x} = \left|S\right|\ tr(S^{-1}\frac{\partial S}{\partial x}) \tag{24}$$

$$\frac{\partial S^{-1}}{\partial x} = -S^{-1}\frac{\partial S}{\partial x}S^{-1} \tag{25}$$

Let now differentiate L_t in (23) with respect to θ_i, using (24) and (25)

$$\frac{\partial L_t}{\partial\theta_i} = -\frac{1}{2}tr\left(\Omega_{t|t-1}^{-1}\frac{\partial\Omega_{t|t-1}}{\partial\theta_i}\right) - \left(\frac{\partial\lambda_t}{\partial\theta_i}\right)'\Omega_{t|t-1}^{-1}\lambda_t + \frac{1}{2}\lambda_t'\,\Omega_{t|t-1}^{-1}\frac{\partial\Omega_{t|t-1}}{\partial\theta_i}\Omega_{t|t-1}^{-1}\lambda_t$$

Taking the trace of the last term gives

$$\frac{\partial L_t}{\partial \theta_i} = -\frac{1}{2} tr \left[\left(\Omega_{t|t-1}^{-1} \frac{\partial \Omega_{t|t-1}}{\partial \theta_i} \right) \times \left(I - \Omega_{t|t-1}^{-1} \lambda_t \lambda_t' \right) \right] - \left(\frac{\partial \lambda_t}{\partial \theta_i} \right)' \Omega_{t|t-1}^{-1} \lambda_t$$

$$\frac{\partial L_t}{\partial \theta_i} = L_{1,t} + L_{2,t}$$

In order to calculate the second order derivative of the log-likelihood, we have to write

$$\begin{aligned} \frac{\partial L_{1,t}}{\partial \theta_i} &= -\frac{1}{2} tr \left[\frac{\partial}{\partial \theta_j} \left(\Omega_{t|t-1}^{-1} \frac{\partial \Omega_{t|t-1}}{\partial \theta_i} \right) \times \left(I - \Omega_{t|t-1}^{-1} \lambda_t \lambda_t' \right) \right] \\ & -\frac{1}{2} tr \left[\Omega_{t|t-1}^{-1} \frac{\partial \Omega_{t|t-1}}{\partial \theta_i} \Omega_{t|t-1}^{-1} \frac{\partial \Omega_{t|t-1}}{\partial \theta_j} \Omega_{t|t-1}^{-1} \lambda_t \lambda_t' \right] \\ & +\frac{1}{2} tr \left[\Omega_{t|t-1}^{-1} \frac{\partial \Omega_{t|t-1}}{\partial \theta_i} \Omega_{t|t-1}^{-1} \times \left(\frac{\partial \lambda_t}{\partial \theta_j} \lambda_t' + \lambda_t \frac{\partial \lambda_t}{\partial \theta_j}' \right) \right] \end{aligned}$$

Conditionally to Y_t, the only random terms of the latter equation are the λ_t. That is why the first term vanishes when taking the expected value. Moreover, (22) shows that $\frac{\partial \lambda_t}{\partial \theta_i}$ depends only on the information at $t-1$, hence, the expected value of the third term is zero. All this leads to

$$\frac{\partial L_{1,t}}{\partial \theta_i} = -\frac{1}{2} tr \left(\Omega_{t|t-1}^{-1} \frac{\partial \Omega_{t|t-1}}{\partial \theta_i} \Omega_{t|t-1}^{-1} \frac{\partial \Omega_{t|t-1}}{\partial \theta_j} \right) \tag{26}$$

As far as $L_{2,t}$ is concerned

$$\frac{\partial L_{2,t}}{\partial \theta_i} = -\frac{\partial^2 \lambda_t}{\partial \theta_i \partial \theta_j} \Omega_{t|t-1}^{-1} \eta_t - \left(\frac{\partial \lambda_t}{\partial \theta_i} \right)' \frac{\partial \Omega_{t|t-1}^{-1}}{\partial \theta_j} \lambda_t - \left(\frac{\partial \lambda_t}{\partial \theta_i} \right)' \Omega_{t|t-1}^{-1} \frac{\partial \lambda_t}{\partial \theta_j}$$

For the same reasons as above, the first two terms vanishes when taking the conditional expected values. Since the third depends only on the past innovations, its conditional expected value is equal to itself.

$$E_{\widehat{\theta}_T} \left(\frac{\partial L_{2,t}}{\partial \theta_i} | Y^{t-1} \right) = -\left(\frac{\partial \eta_t}{\partial \theta_i} \right)' M_{t|t-1}^{-1} \frac{\partial \eta_t}{\partial \theta_j} \tag{27}$$

Finally, the ij^{th} element of the information matrix is the negative of the sum of (26) and (27) summed over all time period. This gives

$$\widehat{I}_{F\, i,j}^{(2)} = \sum_t \left\{ \frac{1}{2} tr \left(\Omega_{t|t-1}^{-1} \frac{\partial \Omega_{t|t-1}}{\partial \theta_i} \Omega_{t|t-1}^{-1} \frac{\partial \Omega_{t|t-1}}{\partial \theta_j} \right) + \left(\frac{\partial \lambda_t}{\partial \theta_i} \right)' \Omega_{t|t-1}^{-1} \frac{\partial \lambda_t}{\partial \theta_j} \right\} \tag{28}$$

A.4 Variance decomposition

A.4.1 Infinite moving average representation

The notations used in this section are the same as in section **??**.

Let U_t denote the vector $\left[\begin{array}{cc} Y_t & \rho_t \end{array} \right]'$.The transition and measurement equation involved in our models can be written

$$(\Phi_0 - \Phi_1 L)U_t = \left[\begin{array}{c} M_t\,\varepsilon_t \\ N_t\,\xi_t \end{array} \right] \quad or \quad (Id - \Phi_0^{-1}\Phi_1 L)U_t = \Phi_0^{-1} \left[\begin{array}{c} M_t\,\varepsilon_t \\ N_t\,\xi_t \end{array} \right] \tag{29}$$

where

$$\begin{array}{rcl} V(M_t\,\varepsilon_t) & = & R_t \\ V(N_t\,\xi_t) & = & Q_t \end{array}$$

For instance, for model 2, we have

$$\Phi_0 = \left[\begin{array}{cc} Id_n & -B \\ 0 & Id_2 \end{array} \right] \quad \Phi_0^{-1} = \left[\begin{array}{cc} Id_n & B \\ 0 & Id_2 \end{array} \right] \quad \Phi_1 = \left[\begin{array}{cc} A & 0 \\ 0 & D \end{array} \right]$$

Assuming that the eigen values of the matrix $\Phi_0^{-1}\Phi_1$ lies strictly inside the unit circle, Eq.(29) is equivalent to

$$U_t = \sum_{k=0}^{\infty} (\Phi_0^{-1}\Phi_1)^k L^k\, \Phi_0^{-1} \left[\begin{array}{c} M_t\,\varepsilon_t \\ N_t\,\xi_t \end{array} \right] \tag{30}$$

Eq.(30) presents the infinite moving average rpresentation of U_t. Therefore

$$V(U_t) = \sum_{k=0}^{\infty} (\Phi_0^{-1}\Phi_1)^k\, (\Phi_0^{-1}) \left[\begin{array}{cc} R_t & 0 \\ 0 & Q_t \end{array} \right] (\Phi_0^{-1})'(\Phi_0^{-1}\Phi_1)'^k \tag{31}$$

since ε_t and ξ_t are white are independent gaussian white noises.

A.4.2 Computation of the shares of variance in the various models

The computations of the shares of variance accounted for by the common factors is based on Eq.(31).

Let assume that $N_t = Id$, which is the case in all our models. Then, we have also $Q_t = Id$. In addition, the empirical variances of the $Y_{j,t}$s are assumed to be standardized to one.

$\xi_{i,t}$ is the zero-mean unit variance disturbance entering the transition equation describing the dynamics of the i^{th} common factor.

Let us potentially take $V(\xi_{i,t}) = 0$, which adds a zero in the diagonal of Q_t. This changed $V(\xi_t)$ is denoted by $Q_{i,t}$. Let s denote the vector of the n first coefficients of the diagonal of $V(U_t)$ (obtained by computing the right-hand side of (31) with

$Q_{i,t}$ instead of Q_t). The vector s presents the shares of the variances of the $Y_{j,t}$s which are not explained by the innovations $\xi_{i,t}$ of the i^{th} common factor.

If $u = (1, ..., 1)'$, we denote with p the vector $u - s$. This vector p gives the shares of the variances of the $Y_{j,t}$s accounted for by the disturbances ξ_i (since $V(Y_{j,t}) = 1$). Moreover, if the common factors are independent from each others, we will finally get the shares of the variances of the $Y_{j,t}$s accounted for by the i^{th} common factor.

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Figure 1

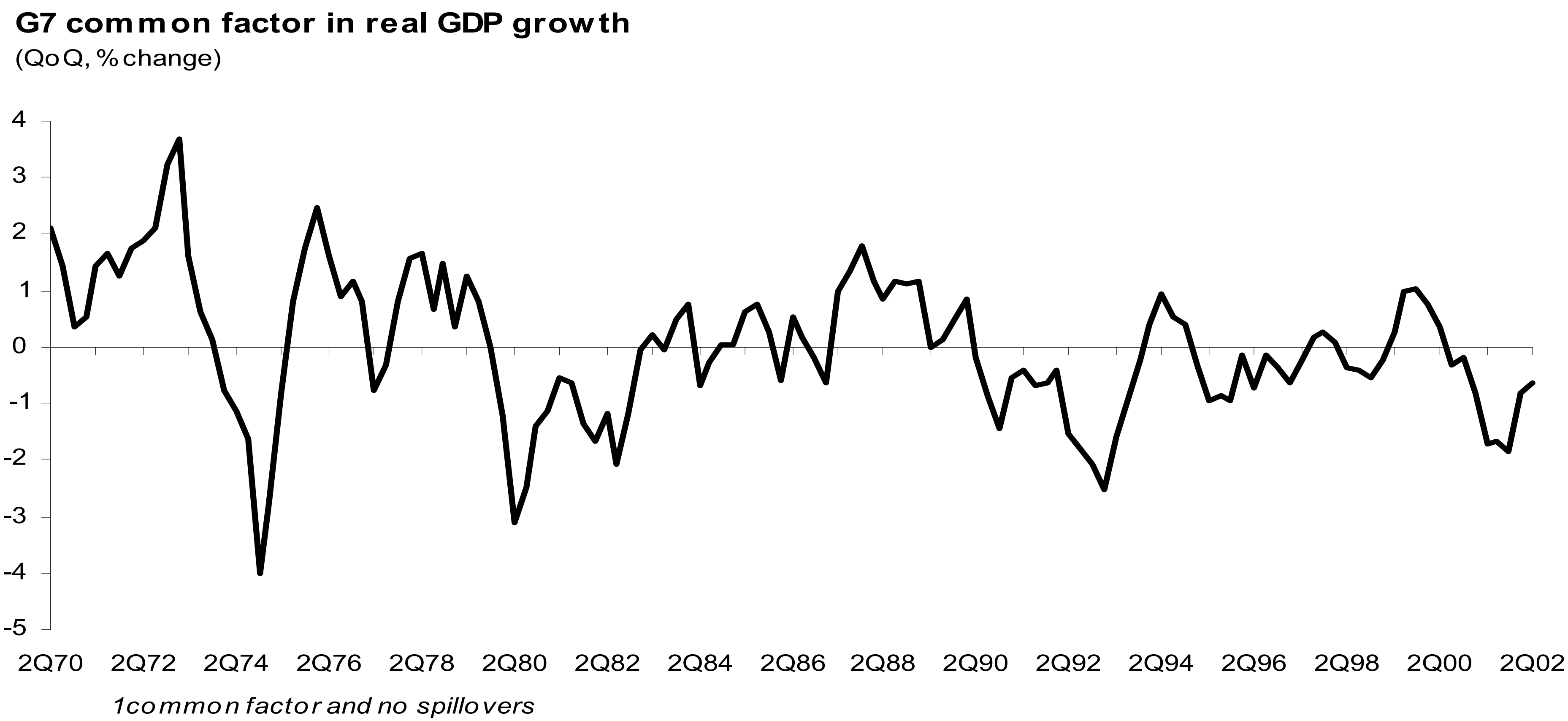
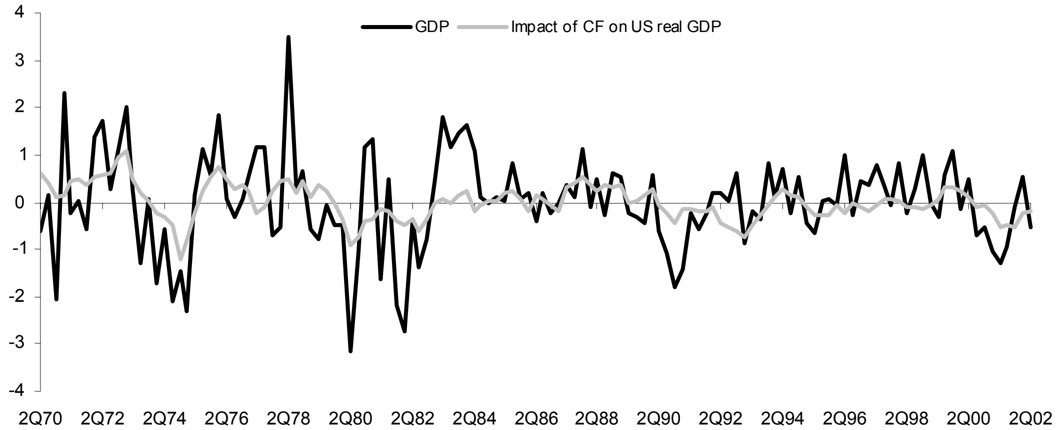

G7 common factor in real GDP growth
(QoQ, % change)
4
3
2
1
0
-1
-2
-3
-4
-5
2Q70
2Q72
2Q74
2Q76
2Q78
2Q80
2Q82
2Q84
2Q86
2Q88
2Q90
2Q92
2Q94
2Q96
2Q98
2Q00
2Q02
1common factor and no spillovers

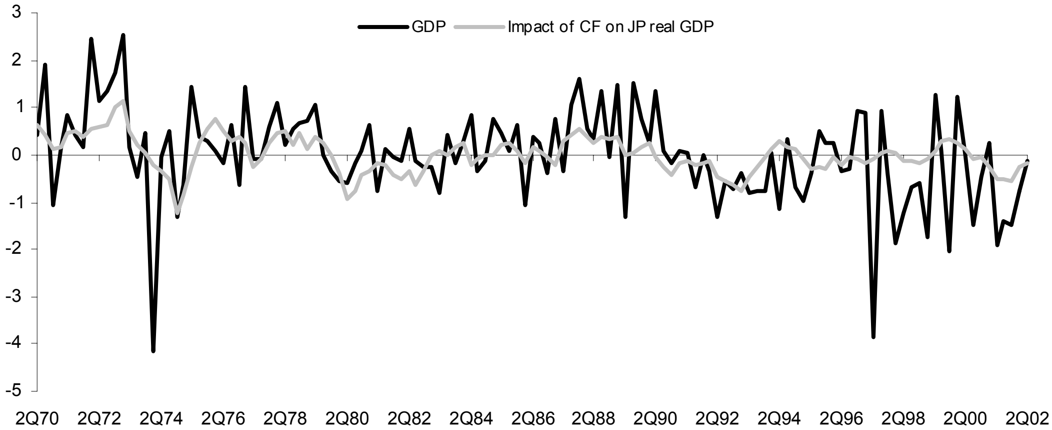
Figure 2

US - Real GDP growth rates: actual and impact contribution of the common factor
(QoQ, % change)



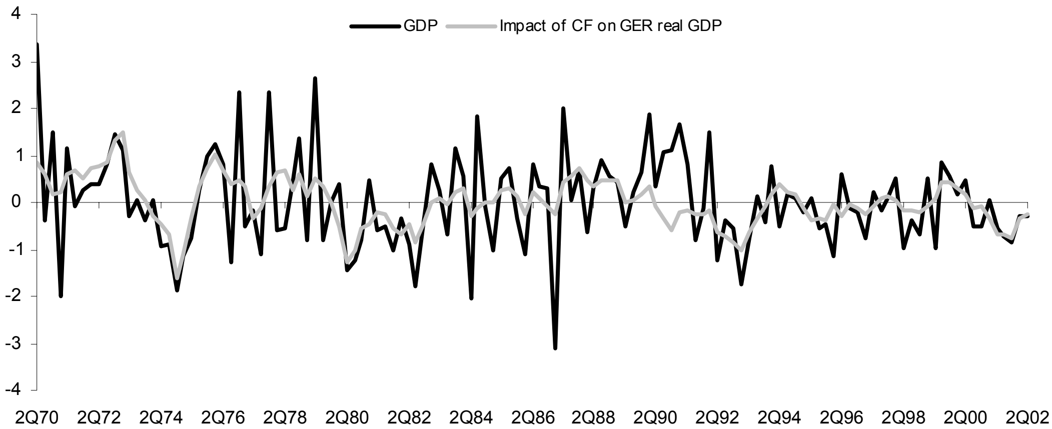
1common factor and no spillovers. Real GDP series are standardised

JAP - Real GDP growth rates: actual and impact contribution of the common factor
(QoQ, % change)



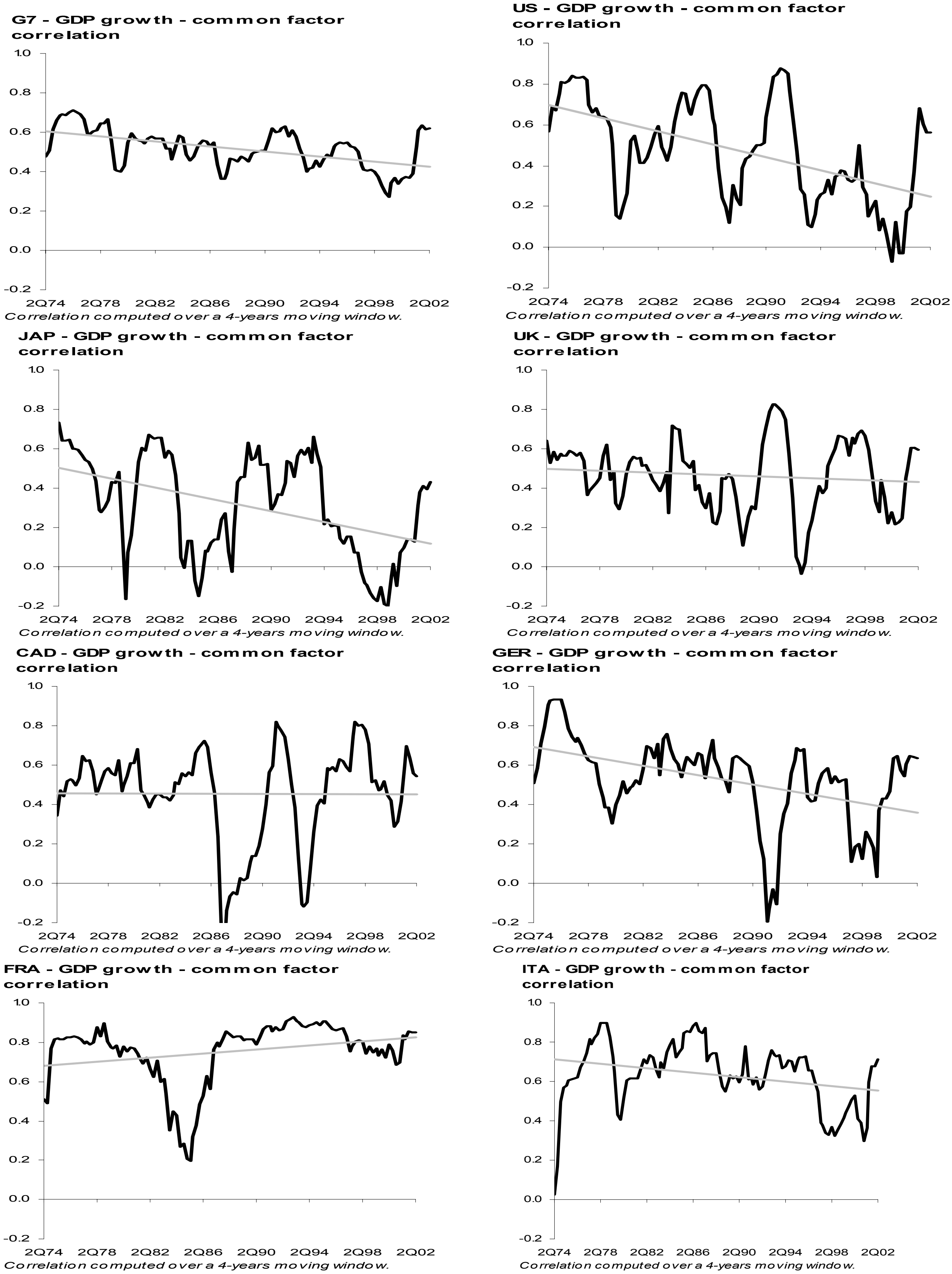
1common factor and no spillovers. Real GDP series are standardised

GER - Real GDP growth rates: actual and impact contribution of the common factor
(QoQ, % change)



1common factor and no spillovers. Real GDP series are standardised

Figure 3



G7 - GDP growth - common factor correlation
Correlation computed over a 4-years moving window.
US - GDP growth - common factor correlation
Correlation computed over a 4-years moving window.
JAP - GDP growth - common factor correlation
Correlation computed over a 4-years moving window.
UK - GDP growth - common factor correlation
Correlation computed over a 4-years moving window.
CAD - GDP growth - common factor correlation
Correlation computed over a 4-years moving window.
GER - GDP growth - common factor correlation
Correlation computed over a 4-years moving window.
FRA - GDP growth - common factor correlation
Correlation computed over a 4-years moving window.
ITA - GDP growth - common factor correlation
Correlation computed over a 4-years moving window.
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
2Q74
2Q78
2Q82
2Q86
2Q90
2Q94
2Q98
2Q02

Figure 4

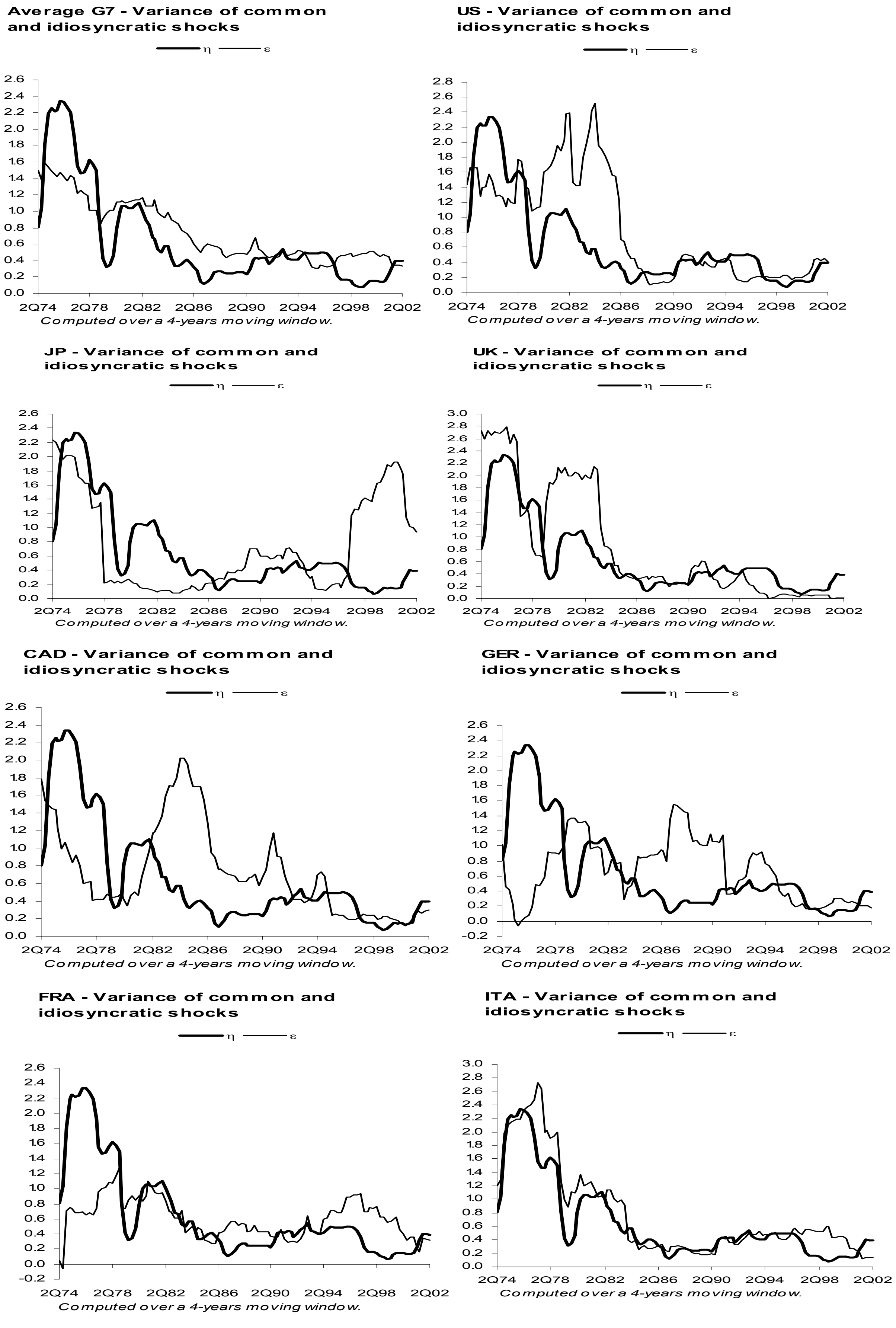
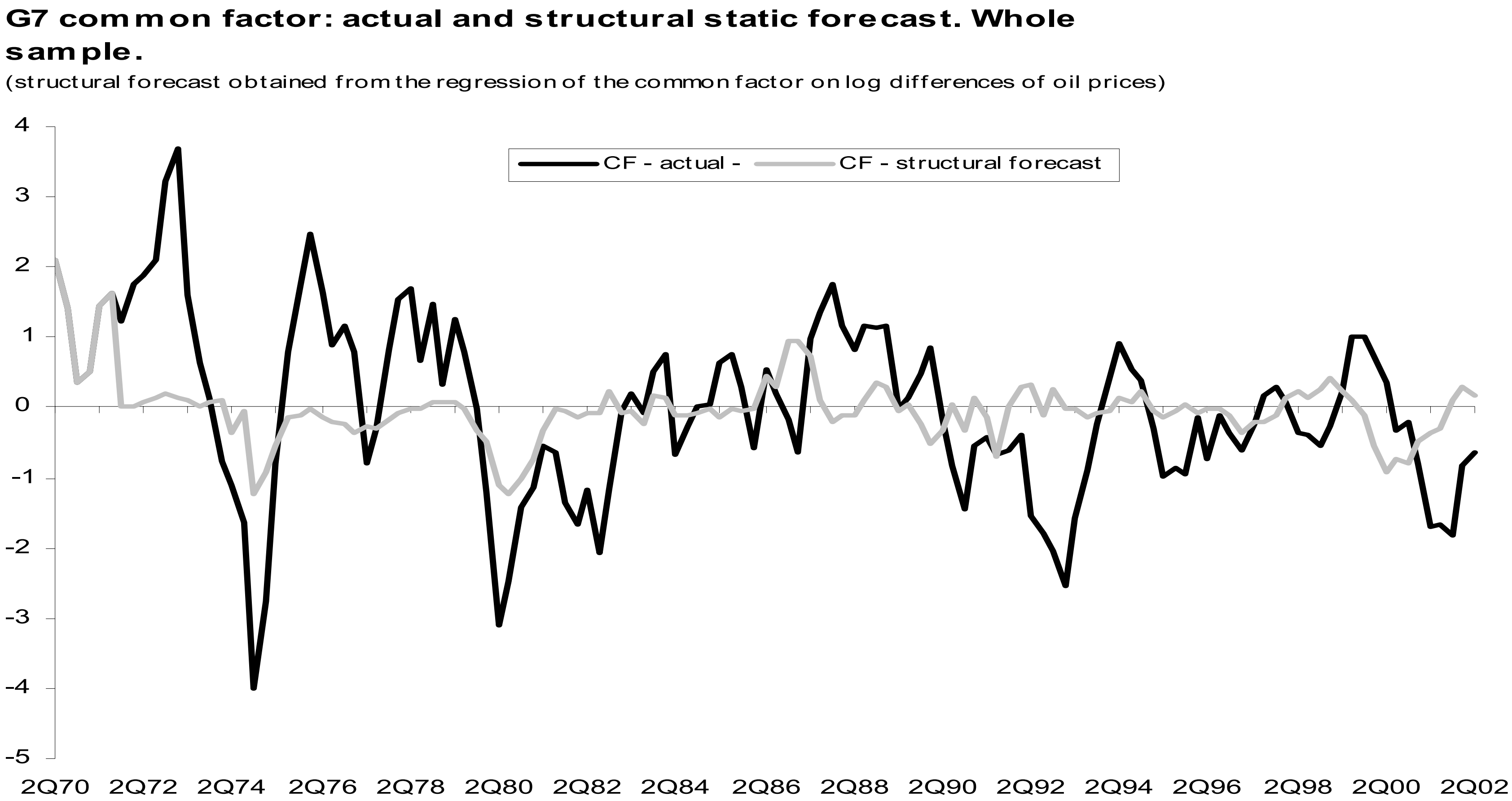

Average G7 - Variance of common and idiosyncratic shocks
η ε
2Q74 2Q78 2Q82 2Q86 2Q90 2Q94 2Q98 2Q02
Computed over a 4-years moving window.
US - Variance of common and idiosyncratic shocks
η ε
2Q74 2Q78 2Q82 2Q86 2Q90 2Q94 2Q98 2Q02
Computed over a 4-years moving window.
JP - Variance of common and idiosyncratic shocks
η ε
2Q74 2Q78 2Q82 2Q86 2Q90 2Q94 2Q98 2Q02
Computed over a 4-years moving window.
UK - Variance of common and idiosyncratic shocks
η ε
2Q74 2Q78 2Q82 2Q86 2Q90 2Q94 2Q98 2Q02
Computed over a 4-years moving window.
CAD - Variance of common and idiosyncratic shocks
η ε
2Q74 2Q78 2Q82 2Q86 2Q90 2Q94 2Q98 2Q02
Computed over a 4-years moving window.
GER - Variance of common and idiosyncratic shocks
η ε
2Q74 2Q78 2Q82 2Q86 2Q90 2Q94 2Q98 2Q02
Computed over a 4-years moving window.
FRA - Variance of common and idiosyncratic shocks
η ε
2Q74 2Q78 2Q82 2Q86 2Q90 2Q94 2Q98 2Q02
Computed over a 4-years moving window.
ITA - Variance of common and idiosyncratic shocks
η ε
2Q74 2Q78 2Q82 2Q86 2Q90 2Q94 2Q98 2Q02
Computed over a 4-years moving window.

Figure 5



G7 common factor: actual and structural static forecast. Whole sample.
(structural forecast obtained from the regression of the common factor on log differences of oil prices)
CF - actual -
CF - structural forecast
4
3
2
1
0
-1
-2
-3
-4
-5
2Q70
2Q72
2Q74
2Q76
2Q78
2Q80
2Q82
2Q84
2Q86
2Q88
2Q90
2Q92
2Q94
2Q96
2Q98
2Q00
2Q02

Figure 6

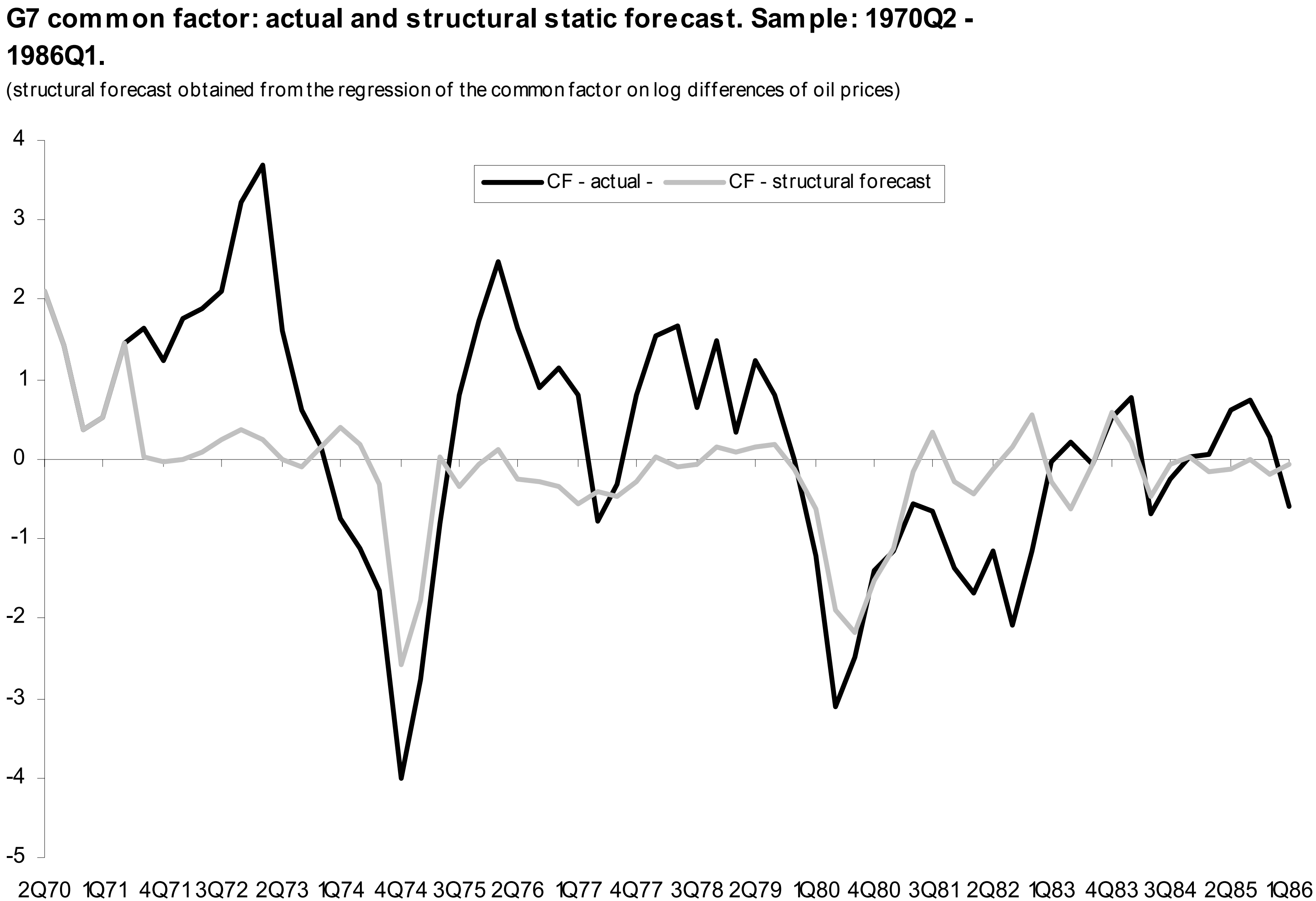

G7 common factor: actual and structural static forecast. Sample: 1970Q2 - 1986Q1.
(structural forecast obtained from the regression of the common factor on log differences of oil prices)
CF - actual -
CF - structural forecast
4
3
2
1
0
-1
-2
-3
-4
-5
2Q70
1Q71
4Q71
3Q72
2Q73
1Q74
4Q74
3Q75
2Q76
1Q77
4Q77
3Q78
2Q79
1Q80
4Q80
3Q81
2Q82
1Q83
4Q83
3Q84
2Q85
1Q86

Figure 7

G7 common factor: actual and structural static forecast. Sample: 1986Q2 - 2002Q2.

(structural forecast obtained from the regression of the common factor on log differences of oil prices)

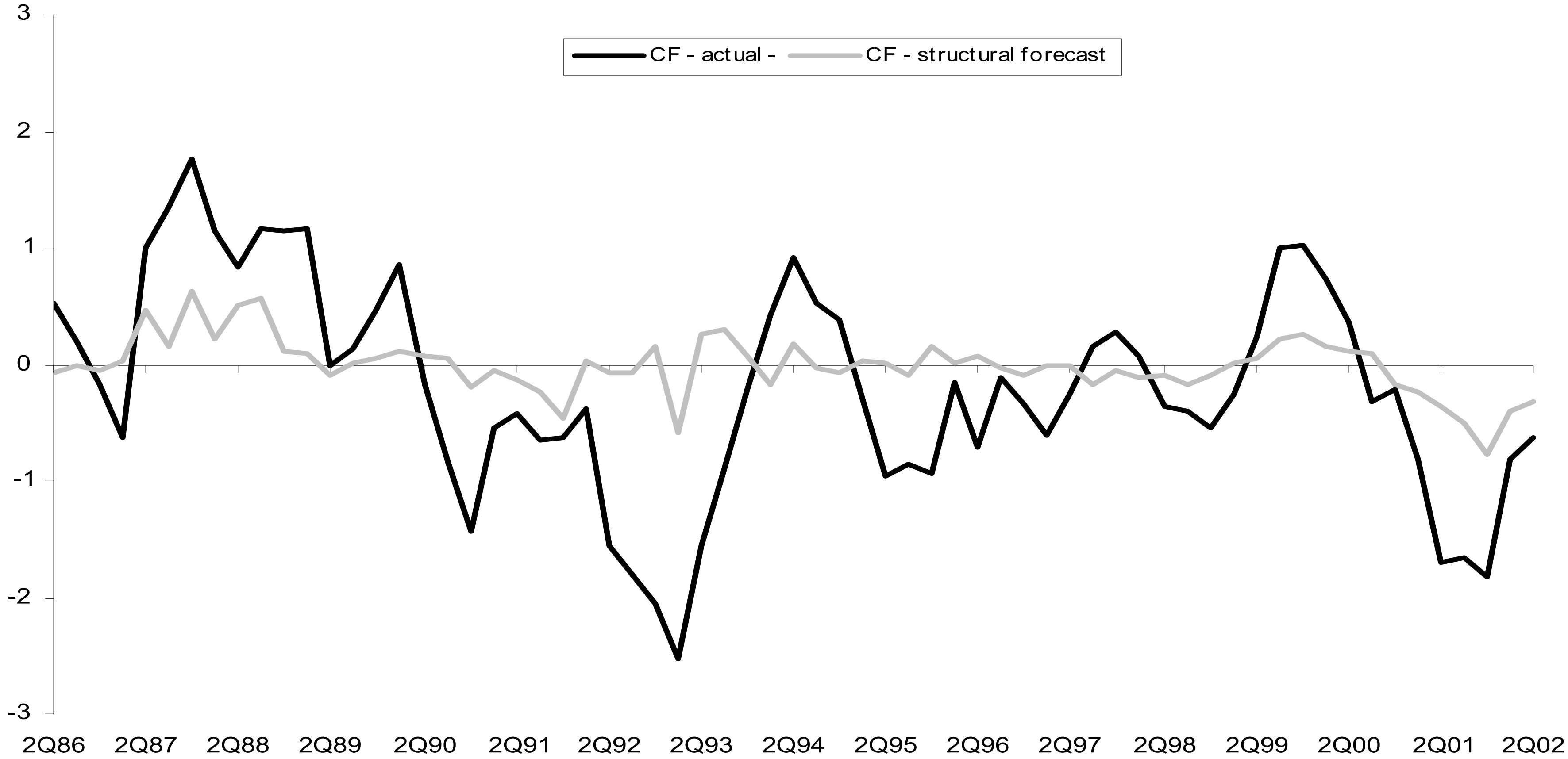
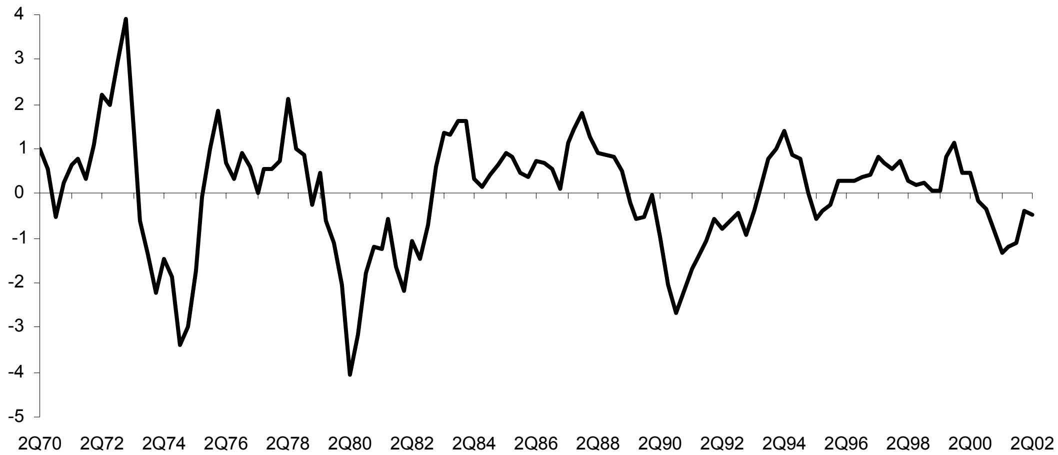


Figure 8

G7 common factor in real GDP growth

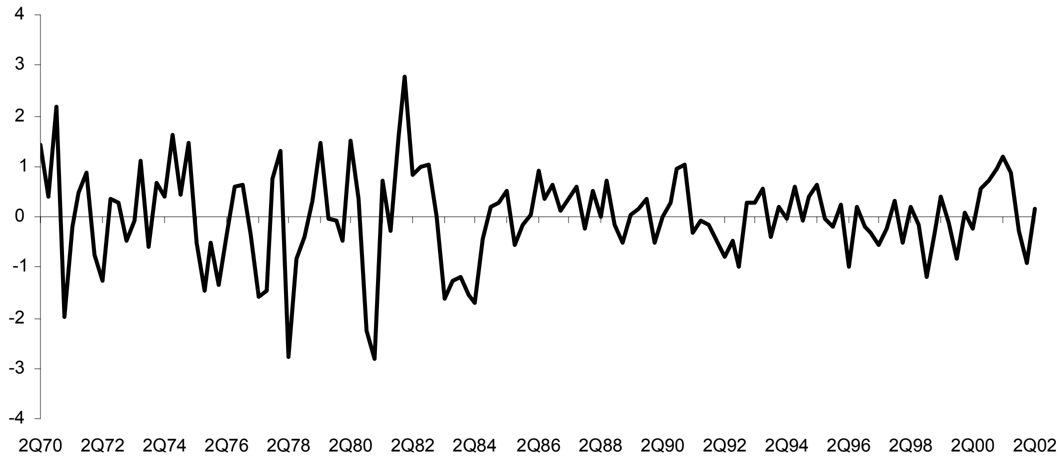
(QoQ, % change)



1common factor, 2 regional factors and no spillovers. UK in the North-American factor

North-American factor in real GDP growth

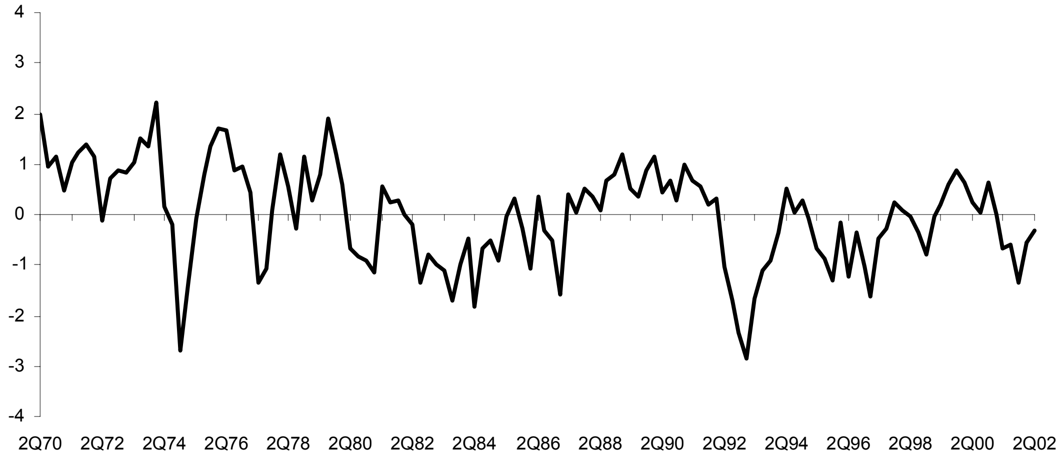
(QoQ, % change)



1common factor, 2 regional factors and no spillovers. UK in the North-American factor

Continental European factor in real GDP growth

(QoQ, % change)

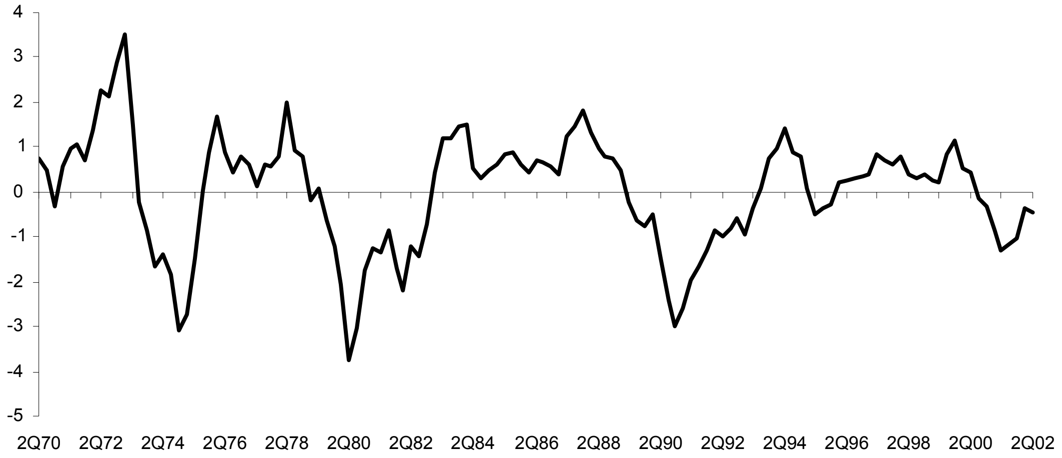


1common factor, 2 regional factors and no spillovers. UK in the North-American factor

Figure 9

G7 common factor in real GDP growth

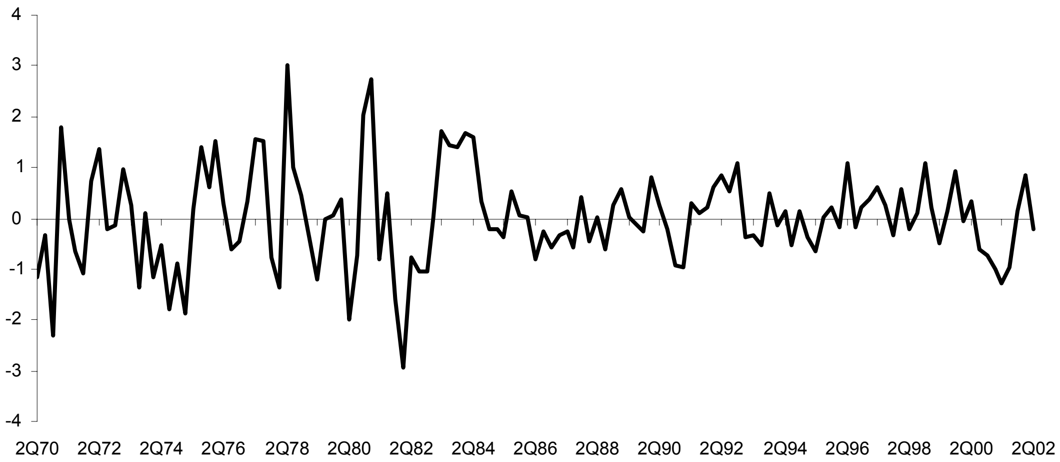
(QoQ, % change)



1common factor, 2 regional factors and no spillovers. UK in the Continental European factor

North-American factor in real GDP growth

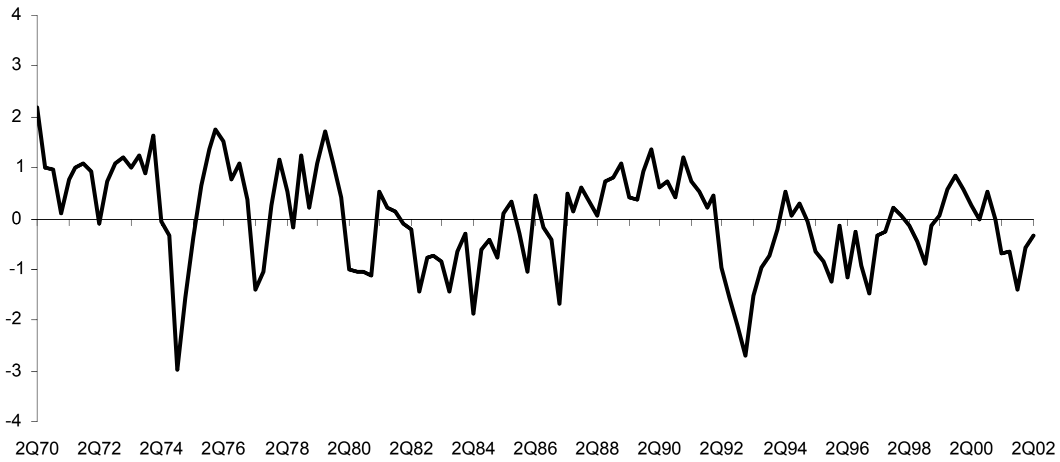
(QoQ, % change)



1common factor, 2 regional factors and no spillovers. UK in the Continental European factor

Continental European factor in real GDP growth

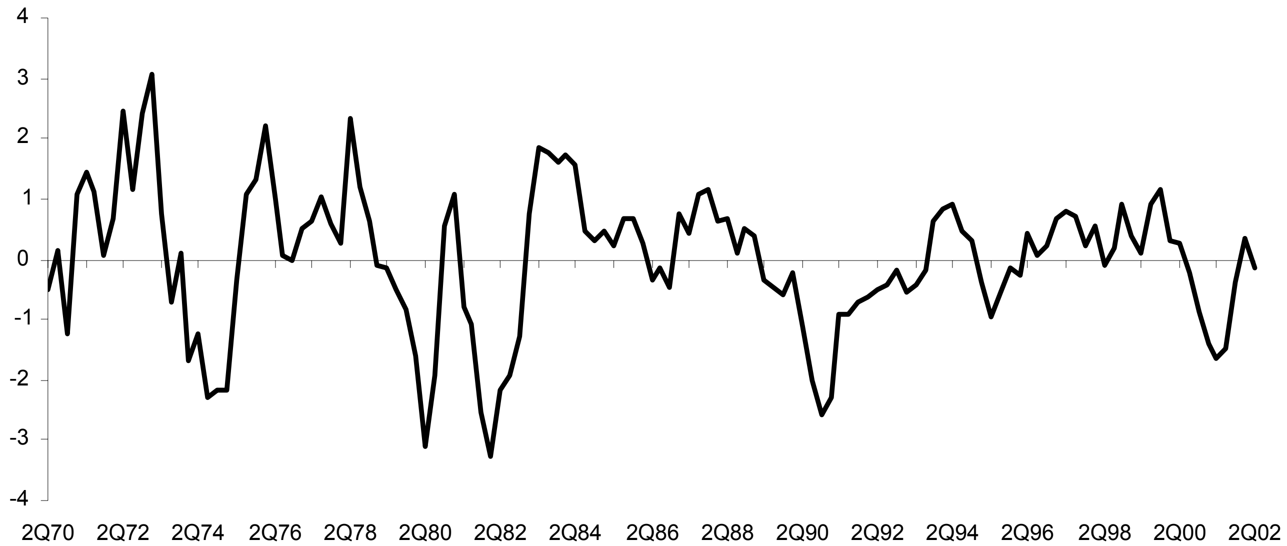
(QoQ, % change)



1common factor, 2 regional factors and no spillovers. UK in the Continental European factor

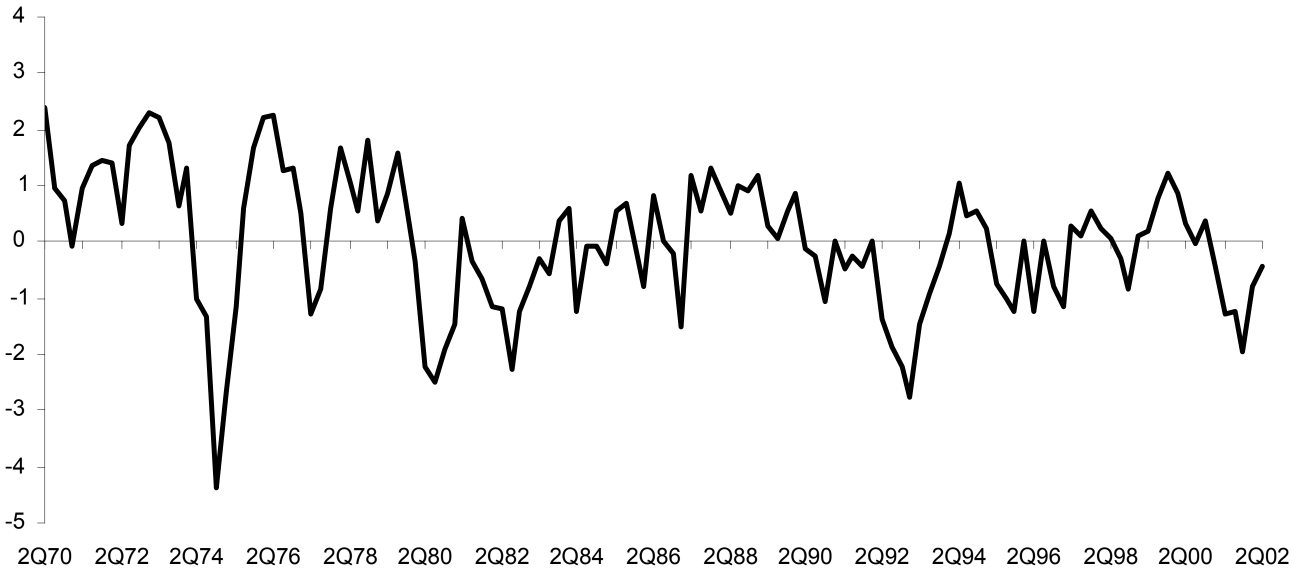
Figure 10

North- American common factor
(QoQ, % change)



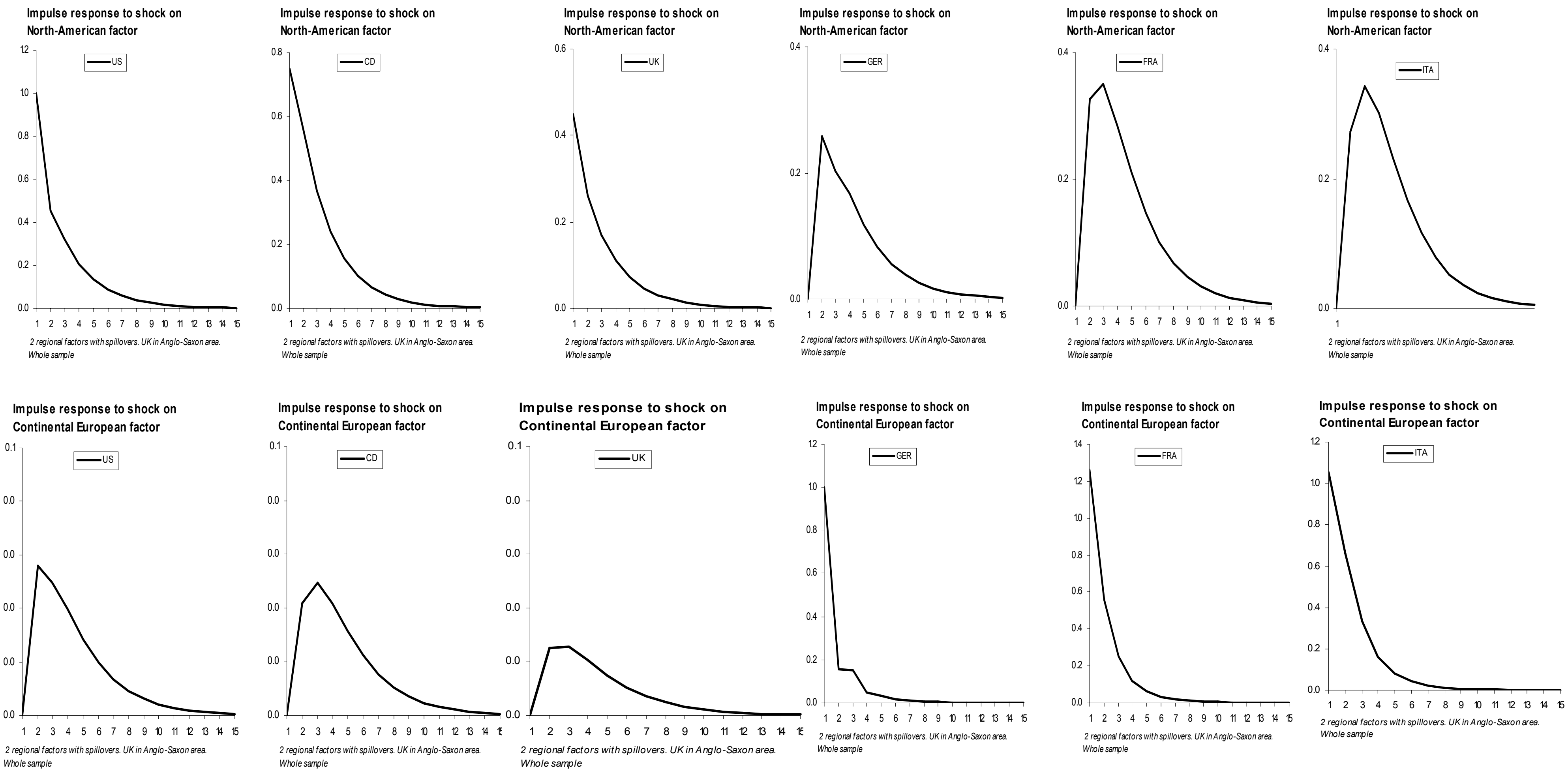
2 regional factors with spillovers. UK in North-American area. Whole sample

Continental European common factor
(QoQ, % change)



2 regional factors with spillovers. UK in North-American area. Whole sample

Figure 11

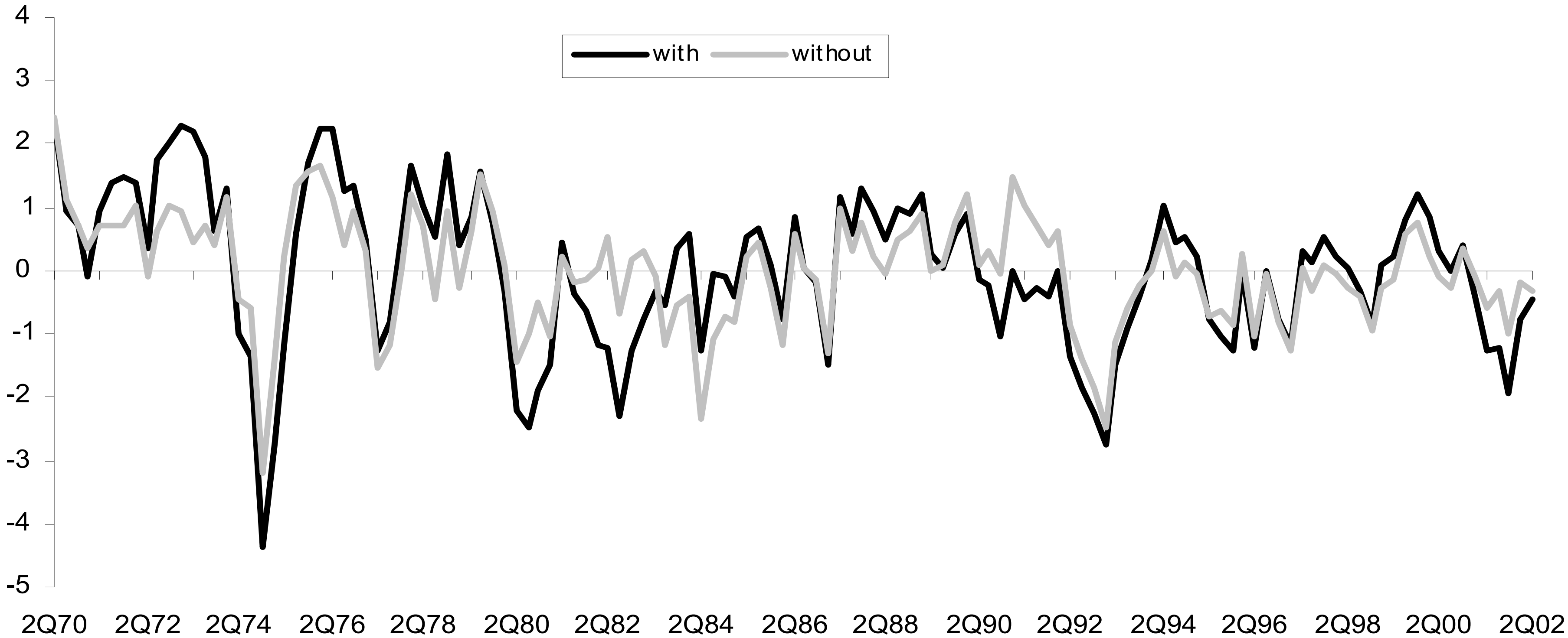


Impulse response to shock on North-American factor
US
2 regional factors with spillovers. UK in Anglo-Saxon area. Whole sample
Impulse response to shock on North-American factor
CD
2 regional factors with spillovers. UK in Anglo-Saxon area. Whole sample
Impulse response to shock on North-American factor
UK
2 regional factors with spillovers. UK in Anglo-Saxon area. Whole sample
Impulse response to shock on North-American factor
GER
2 regional factors with spillovers. UK in Anglo-Saxon area. Whole sample
Impulse response to shock on North-American factor
FRA
2 regional factors with spillovers. UK in Anglo-Saxon area. Whole sample
Impulse response to shock on Norh-American factor
ITA
2 regional factors with spillovers. UK in Anglo-Saxon area. Whole sample
Impulse response to shock on Continental European factor
US
2 regional factors with spillovers. UK in Anglo-Saxon area. Whole sample
Impulse response to shock on Continental European factor
CD
2 regional factors with spillovers. UK in Anglo-Saxon area. Whole sample
Impulse response to shock on Continental European factor
UK
2 regional factors with spillovers. UK in Anglo-Saxon area. Whole sample
Impulse response to shock on Continental European factor
GER
2 regional factors with spillovers. UK in Anglo-Saxon area. Whole sample
Impulse response to shock on Continental European factor
FRA
2 regional factors with spillovers. UK in Anglo-Saxon area. Whole sample
Impulse response to shock on Continental European factor
ITA
2 regional factors with spillovers. UK in Anglo-Saxon area. Whole sample

Figure 12

Continental European (with and without the contribution of North-American specific innovations)

(QoQ, % change)



2 regional factors with spillovers. UK in North-American area. Whole sample

Figure 13

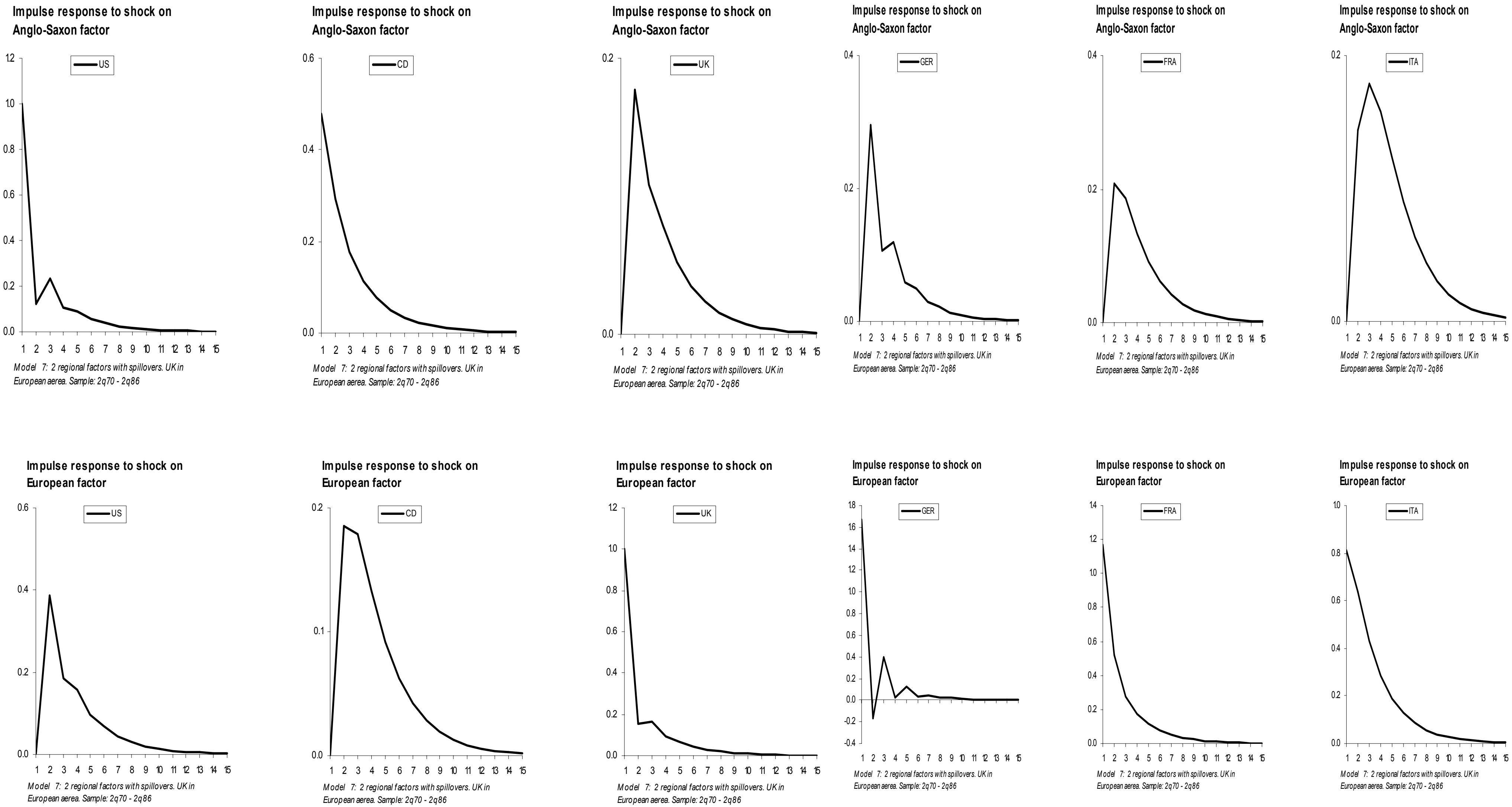
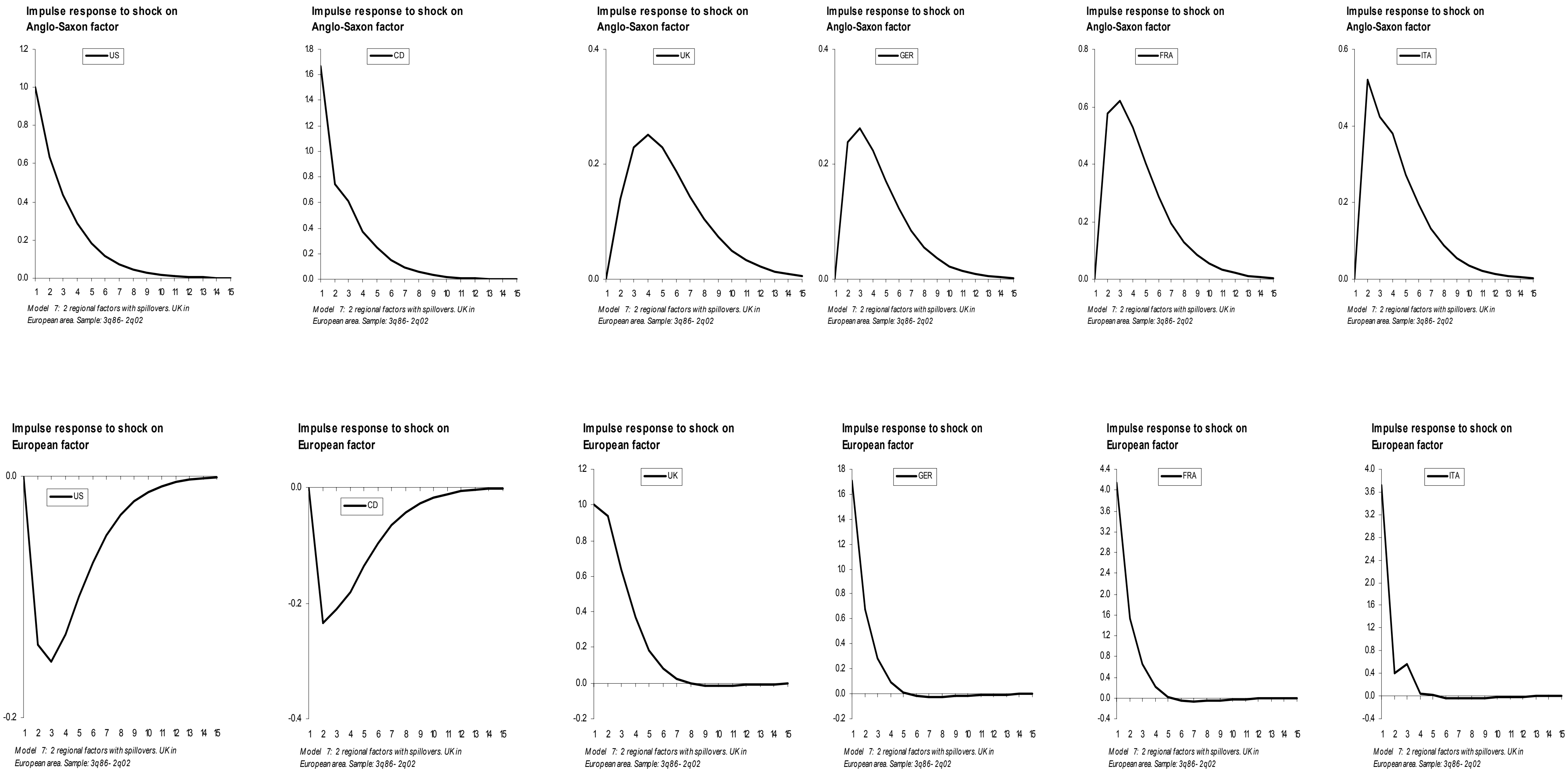

Impulse response to shock on Anglo-Saxon factor
US
Model 7: 2 regional factors with spillovers. UK in European aerea. Sample: 2q70 - 2q86
Impulse response to shock on Anglo-Saxon factor
CD
Model 7: 2 regional factors with spillovers. UK in European aerea. Sample: 2q70 - 2q86
Impulse response to shock on Anglo-Saxon factor
UK
Model 7: 2 regional factors with spillovers. UK in European aerea. Sample: 2q70 - 2q86
Impulse response to shock on Anglo-Saxon factor
GER
Model 7: 2 regional factors with spillovers. UK in European aerea. Sample: 2q70 - 2q86
Impulse response to shock on Anglo-Saxon factor
FRA
Model 7: 2 regional factors with spillovers. UK in European aerea. Sample: 2q70 - 2q86
Impulse response to shock on Anglo-Saxon factor
ITA
Model 7: 2 regional factors with spillovers. UK in European aerea. Sample: 2q70 - 2q86
Impulse response to shock on European factor
US
Model 7: 2 regional factors with spillovers. UK in European aerea. Sample: 2q70 - 2q86
Impulse response to shock on European factor
CD
Model 7: 2 regional factors with spillovers. UK in European aerea. Sample: 2q70 - 2q86
Impulse response to shock on European factor
UK
Model 7: 2 regional factors with spillovers. UK in European aerea. Sample: 2q70 - 2q86
Impulse response to shock on European factor
GER
Model 7: 2 regional factors with spillovers. UK in European aerea. Sample: 2q70 - 2q86
Impulse response to shock on European factor
FRA
Model 7: 2 regional factors with spillovers. UK in European aerea. Sample: 2q70 - 2q86
Impulse response to shock on European factor
ITA
Model 7: 2 regional factors with spillovers. UK in European aerea. Sample: 2q70 - 2q86

Figure 14


Impulse response to shock on Anglo-Saxon factor
US
Model 7: 2 regional factors with spillovers. UK in European area. Sample: 3q86- 2q02
Impulse response to shock on Anglo-Saxon factor
CD
Model 7: 2 regional factors with spillovers. UK in European area. Sample: 3q86- 2q02
Impulse response to shock on Anglo-Saxon factor
UK
Model 7: 2 regional factors with spillovers. UK in European area. Sample: 3q86- 2q02
Impulse response to shock on Anglo-Saxon factor
GER
Model 7: 2 regional factors with spillovers. UK in European area. Sample: 3q86- 2q02
Impulse response to shock on Anglo-Saxon factor
FRA
Model 7: 2 regional factors with spillovers. UK in European area. Sample: 3q86- 2q02
Impulse response to shock on Anglo-Saxon factor
ITA
Model 7: 2 regional factors with spillovers. UK in European area. Sample: 3q86- 2q02
Impulse response to shock on European factor
US
Model 7: 2 regional factors with spillovers. UK in European area. Sample: 3q86- 2q02
Impulse response to shock on European factor
CD
Model 7: 2 regional factors with spillovers. UK in European area. Sample: 3q86- 2q02
Impulse response to shock on European factor
UK
Model 7: 2 regional factors with spillovers. UK in European area. Sample: 3q86- 2q02
Impulse response to shock on European factor
GER
Model 7: 2 regional factors with spillovers. UK in European area. Sample: 3q86- 2q02
Impulse response to shock on European factor
FRA
Model 7: 2 regional factors with spillovers. UK in European area. Sample: 3q86- 2q02
Impulse response to shock on European factor
ITA
Model 7: 2 regional factors with spillovers. UK in European area. Sample: 3q86- 2q02