Buy or wait, that is the option: the buyer's option in sequential laboratory auctions

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We report results from an experiment on two-unit sequential auctions with and without a buyer's option (which allows the winner of the first auction to buy the second unit). The 4 main auction institutions are studied. Observed bidding behavior is close to Nash equilibrium bidding in the auctions for the second unit, but not in the auctions for the first unit. Despite these deviations, the buyer's option is correctly used in most cases. The revenue-ranking of the 4 auctions is the same as in single-unit experiments. Successive prices are declining when the buyer's option is available.

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1 Introduction

■ In sales of multiple units of a particular good, auction houses often choose to sell the items sequentially, i.e., the items are auctioned separately, one after the other. The advantage of a sequential auction is that it may fit the needs of both small and large buyers, whereas the alternative auction procedure that consists in selling all available units simultaneously, in one shot, may exclude buyers who set low values on the items (which reduces competition at the auction). The main disadvantage of a sequential method is that it can be time-consuming, especially when the total number of units for sale is large. For this reason, auctioneers sometimes provide a so-called buyer's option, which gives the winner of the first auction the right to buy any number of units (1, 2, ..., or all units available). For each unit he/she must pay the winning price established at the first auction. If the winning bidder decides to purchase only part of the total quantity, the remaining items are re-auctioned, in the same manner, through a second auction; and this scheme is repeated until all units are eventually sold.

The buyer's option thus clearly offers the best of both worlds: it allows the auctioneer to speed up sales, while keeping the auction mechanism sufficiently flexible to be of interest for different types of buyers. Not surprisingly therefore, the buyer's option is used in many auctions throughout the world. Cassady (1967) describes how the buyer's option is practiced in fur auctions in Leningrad and London, and fish auctions in English port markets. At the auction market in Aalsmeer, the Netherlands, huge quantities of flowers are sold through sequential descending auctions with a buyer's option (see Van den Berg, Van Ours, and Pradhan (2001)). Well-known auction houses such as Christie's and Sotheby's (see Ashenfelter (1989) and Ginsburgh (1998)) and Drouot (see Février, Roos, and Visser (2005)) systematically use the buyer's option in their sequential ascending auctions of fine wines.

Despite the practical importance of the buyer's option, little attention has been paid to the subject in the literature. The only theoretical article we are aware of is Black and De Meza (1992). They consider the Independent Private Value (IPV) paradigm, and derive equilibrium bidding strategies in two-unit sequential second-price auctions with and without the buyer's option. All buyers in their model have either decreasing demand for the two units (the additional value of the second unit is less than the value of the first unit), or flat demand (both units are valued the same). Empirical studies are also rare. Ashenfelter (1989) and Ginsburgh (1998) report that the option is exercised by many buyers in ascending wine auctions at Christie's and Sotheby's. Van den Berg, Van Ours, and Pradhan (2001) use data on sequential descending auctions of roses to study the declining price phenomenon. Finally, Février, Roos, and Visser (2005), using data on ascending auctions of wine held at Drouot, structurally estimate their bidding model, and find that the seller's revenue in a system where items are auctioned sequentially is the same as in a system based on the buyer's option.

This paper studies two-unit sequential auctions and looks in particular at the role of the buyer's option. We adopt the IPV paradigm and assume that the 2 units are sold to 2 risk-neutral buyers. Buyers desire both units, and their demand for the items is either decreasing, flat, or increasing (implying that the value of the second unit exceeds the value of the first unit). The 4 main auction institutions are considered: first-price, descending (Dutch), second-price (Vickrey), and ascending (English) auctions. Although there are few field examples of first-price and second-price sequential auctions with or without a buyer's option,[1] it is nonetheless of interest to study sealed-bid auctions. As in standard one-unit auction theory, we show that first-price (resp. second-price) and Dutch (resp. English) sequential auctions with or without a buyer's option are theoretically isomorphic. Furthermore the four auction formats generally generate the same expected revenue. By analogy with experimental studies on single-unit auctions (see Kagel (1995)) for a survey), our experimental design thus allows us to test whether bidding behavior is identical and whether there is revenue equivalence.

Other theoretical predictions are confronted with the experimental data as well. We test whether observed bidding behavior corresponds to risk-neutral Nash equilibrium bidding, and whether the buyer's option has the predicted effect on first-auction bidding behavior. We also analyze to what extent the experimental subjects exercise their option (do first-auction winners directly *buy* the second unit, or instead *wait* and attempt to obtain the additional unit in the second auction?), and test if observed frequencies of using the option correspond to predicted frequencies. Predictions on the degree of efficiency of auction outcomes are also tested, and we compare observed price patterns with their predicted counterparts.

Our main empirical findings are the following. Observed bidding behavior matches the predictions of the theory well in the auctions for the second unit. In the auctions for the first unit there are, however, important deviations between the Nash equilibrium strategies and the observed bidding strategies. Although the experimental subjects tend to adjust their bids in the direction that theory predicts, the adjustments are generally too modest. Despite the deviations in the auctions for the first unit, the buyer's option is correctly used in most cases. Our results also indicate that the revenue-ranking of the four canonical auction institutions is the same as that found in single-unit experiments. The ordering of the auctions mechanisms in terms of expected revenue is thus robust to the sequential-two-unit extension considered in this paper. Finally, we find that successive prices in sequential auctions are declining when the buyer's option is available.

Experimental work on multi-unit sequential auctions is rare.[2] Burns (1985) considers sequential English auctions. The experiment is designed to mimic the Australian wool market, and the paper's main objective is to study the effect of market size on auction prices. The observed behavior is not confronted with equilibrium predictions. Keser and Olson (1996) consider sequential first-price auctions and suppose that buyers have single-unit demand functions. Their main objective is to compare observed price-sequences with the predicted patterns derived in Weber (1983), under different design parameters. As in Burns, the paper focuses on one particular auction mechanism, and no attempt is made to examine outcomes under alternative institutions. Robert and Montmarquette (1999) consider several auction institutions, and also provide theoretical foundations for each of them. In their models, the number of items desired by each buyer is a random variable and demand functions are decreasing. They consider sequential Dutch, English and mixed auctions, and compare observed behavior with predicted behavior. None of these experimental papers on sequential auctions analyzes the buyer's option.

[1] An exception is Cassady (1967, p. 197) who describes the electronic auction market in Osaka, Japan, where lots of fruit and vegetables are sold via sequential first-price auctions.

[2] Spurred by the recent FCC auctions, experimental papers on all sorts of *simultaneous* multi-demand auctions are flourishing (see for example Kagel and Levin (2001) and the references therein, and the special issue of the *Journal of Economics & Management Strategy* (1997, Number 3)).

The only experimental paper that looks at the buyer's option is by Katok and Roth (2004). They experimentally study (among other things) two-unit sequential Dutch auctions with a buyer's option. Their model has two small bidders (single-unit demand) and one larger bidder (two-unit increasing demand). This asymmetry between bidders is essential in Katok and Roth's paper as they focus their analysis on the "free rider problem".

The paper proceeds as follows. In the next section the theoretical background is presented. In deriving the risk-neutral Nash equilibrium bidding functions for the different auction institutions, we partly draw on Black and De Meza (1992), Donald, Paarsch, and Robert (1997) and Février (2000). Most results in this section are new. Section 3 describes the experimental design, Section 4 the experimental results, and Section 5 concludes.

2 Theoretical background

■ Suppose that 2 units of a good are auctioned to 2 potential buyers. Each buyer is assumed to be risk-neutral and desires to purchase both units. Adopting the IPV paradigm, let v_i denote the value that buyer i places on the first unit. The value v_i and the value of i's opponent are independently drawn from a uniform distribution on the interval $[0,\overline{v}]$. It is assumed that the value that i places on the second unit is kv_i. The parameter k can take three values: $k \in \left\{\frac{1}{2},1,2\right\}$. The value of k is common knowledge. Note that $k = \frac{1}{2}$ implies that the second unit is valued less than the first unit (decreasing demand), $k = 1$ that both units are valued the same (flat demand), and $k = 2$ that the second unit is valued more than the first (increasing demand).

The 2 units are sold sequentially. The first unit of the good is sold in the first auction. The manner in which it is auctioned depends on the auction institution. Let a indicate the auction institution, $a \in \{D,E,F,S\}$, where D stands for Dutch auction, E for English auction, F for First-price auction, and S for Second-price auction, and let p_1 denote the price the winner of the first auction has to pay for the first unit. When $a \in \{D,E\}$, the unit is auctioned using a clock. When $a = D$, the clock starts very high, and descends until one of the players stops the clock. This player wins the unit and p_1 equals the price at which the clock was stopped. When $a = E$, the clock starts at 0, and increases until one of the players stops the clock. Here the winner of the auction is the player who *did not stop* the clock. The price p_1 he/she has to pay for the first unit is again the amount at which the clock stopped. When $a \in \{F,S\}$, the unit is sold via sealed-bid auctions. Both players submit their sealed bid to the auctioneer who awards the unit to the highest bidder. When $a = F$ the winner pays his/her own bid, i.e., p_1 equals the highest submitted bid. When $a = S$ the winner pays the bid of his opponent, i.e., here p_1 equals the second highest submitted bid. For all institutions a, the price p_1 is revealed to both players once the first auction has ended.

The way in which the second unit is sold depends on whether the buyer's option is available or not. Let o be the indicator for the availability of the buyer's option, $o = N$ if it cannot be used, and $o = Y$ otherwise. For any auction institution a, if $o = N$ the second unit is auctioned under the prevailing rules of institution a. Let p_2 be the price paid for the second unit. If instead $o = Y$ the winner of the first auction has the option to buy 1 or 2 units, at the price of p_1 per unit. When he/she decides to purchase only 1 unit, a second auction is held under the conditions of institution a. When he/she exercises the buyer's option, no second auction is held. Note that in this case we automatically have $p_2 = p_1$.

The theoretical model presented here is essentially based on the framework of Black and De Meza (1992). These authors, however, only considered the second-price auction ($a = S$) and they do not analyze the case of increasing marginal valuation ($k = 2$).

For any given value of a, o, and k, let $G(a,o,k)$ denote the two-stage game described above. We are looking for perfect bayesian equilibria of the game $G(a,o,k)$ in pure and symmetric strategies in the first auction. Let $b_1(v)$ denote the equilibrium strategy of the bidders in the first auction. If $o = Y$, let $bo(p_1) \in \{0,1\}$ indicate whether the winner exercises the buyer's option or not given the auction price p_1, with $bo(p_1) = 1$ meaning that he/she uses his/her option, and $bo(p_1) = 0$ that he/she does not. Finally, let $b_2^w(v, p_1)$ denote the second auction strategy of the winner of the first auction, and $b_2^l(v, p_1)$ the second auction strategy of the loser of the first auction. For practical reasons, these strategies are only confronted with the data when the buyer's option is not available. In the following proposition, the strategies are therefore only given for $o = N$. But in the proof of the proposition (in the Appendix), explicit use is made of the strategies for $o = Y$.

Proposition 1. A perfect bayesian equilibrium (in pure and symmetric strategies in the first auction) of the game $G(a,o,k)$ is:

1. If $a \in \{E,S\}$, $o = N$, and $k \in \left\{\frac{1}{2}, 1, 2\right\}$, then $b_1(v) = kv$, $b_2^l(v, p_1) = v$, $b_2^w(v, p_1) = kv$.
2. If $a \in \{D,F\}$, $o = N$, and $k \in \left\{\frac{1}{2}, 1\right\}$, then no such equilibrium exists.
3. If $a \in \{D,F\}$, $o = N$, and $k = 2$, then $b_1(v) = \frac{1}{2}v$, $b_2^l(v, p_1) = b_2^w(v, p_1) = v$.
4. If $a \in \{E,S\}$, $o = Y$, and $k = \frac{1}{2}$, then $b_1(v)$ is solution of $b_1(v) - \frac{v}{2} = 2\lambda\,(v - b_1(v))\,b_1'(v)$, with $\lambda = 0$ if $b_1(v) \geq \frac{1}{2}\overline{v}$ and $\lambda = 1$ otherwise; $bo(p_1) = 1$ if $p_1 \leq \frac{1}{2}v$ and $bo(p_1) = 0$ if $p_1 > \frac{1}{2}v$.
5. If $a \in \{E,S\}$, $o = Y$, and $k = 1$, then $b_1(v) = v$, $bo(p_1) \in [0,1]$.
6. If $a \in \{E,S\}$, $o = Y$, and $k = 2$, then $b_1(v) = 2v$, $bo(p_1) = 0$.
7. If $a \in \{D,F\}$, $o = Y$, and $k \in \left\{\frac{1}{2}, 1, 2\right\}$, then $b_1(v) = \frac{1+k}{4}v$, $bo(p_1) = 1$.

Let us first comment on the predictions for the English and second-price auctions. As mentioned in the introduction, the behavioral predictions are the same for these two mechanisms. When $o = N$, theory requires bidders to bid kv in the first auction, that is they bid the value for the second unit. While this result is intuitive for flat demand, it is less so when demand is decreasing or increasing. With decreasing demand, bid shading is required because losing the first auction is not necessarily bad news, as it implies a weaker rival in the second auction. With increasing demand, over-bidding is required as the winner of the first auction is also going to be the winner of the second auction. In the second auction (still when $o = N$), it is a dominant strategy for each player to bid the value of the unit for which he/she is bidding. That is, the loser of the first auction should bid v, and the winner of the first auction kv.

When $o = Y$ and $k \in \{1,2\}$, first-auction bidding is the same as in the absence of the buyer's option. In other words, the buyer's option has no effect on first-auction bidding behavior. However, when $k = \frac{1}{2}$, first-auction bidding should be more aggressive than in the absence of the option. The optimal use of the buyer's option is simple when $k = \frac{1}{2}$ or $k = 1$. In the former case it should be used if the first-auction price is lower than the second unit value, and in the latter case the first-auction winner is indifferent between exercising the option or not, which is the meaning of $bo(p_1) \in [0,1]$. When $k = 2$ it is not optimal to use the option because the loser of the first auction is expected to bid less aggressively in the second auction, so the first-auction winner has a higher expected gain by waiting for the second auction.

Let us next comment on the predictions for the Dutch and first-price auctions. Again theory predicts that behavior is identical under the two institutions. When $o = N$, there does not exist a symmetric pure strategy equilibrium in the first auction for $k \in \left\{\frac{1}{2}, 1\right\}$. If such an equilibrium were to exist, the loser of the first auction would learn the valuation of the winner (since p_1 is revealed at the end of the first auction). The first-auction winner would then be in an uncomfortable situation in the second auction. The equilibrium in the second auction would take the following form: the winner of the first auction would play a mixed strategy and the loser a pure strategy. However, this second-auction equilibrium is not compatible with a first-auction pure strategy, since we can show that there always exists a profitable deviation. This means that both players should hide their valuation by playing a mixed strategy in the first auction.

When $o = N$ and $k = 2$, a symmetric pure strategy equilibrium does exist for the Dutch and the first-price auctions. This equilibrium is neither simple to compute and nor intuitive as it implies a relatively low first-auction bid. One might think that player 1 should deviate by bidding $\frac{x}{2}$ (with $x > v_1$) in the first auction in order to increase the probability to win the first unit, and thereby enter the second auction with a stronger valuation $2v_1$. The Appendix shows however that this deviation is not profitable. Indeed, this deviation decreases the expected gain in the first auction (since bidding half of one's valuation is optimal in a single-unit auction), and it does not affect the expected gain in the second auction. Note that in equilibrium the winner of the first auction, say bidder 1, automatically wins the second auction: his/her valuation for the second unit is $2v_1$ while his/her opponent's valuation for the first unit is $v_2 \leq v_1$ (since the first-auction strategy is symmetric), so by bidding v_1 he/she wins the second auction with probability one. Therefore, in equilibrium it is as if both bidders only compete for the first unit.

When $o = Y$, a symmetric pure strategy equilibrium exists for all values of k. Note that in equilibrium, bidders behave exactly as in standard single-unit Dutch or first-price auctions. Indeed, in equilibrium each player bids $\frac{1+k}{4}v$ in the first auction and the winner *always* exercises his/her option. It is thus as if players submit a single bid equal to $\frac{1+k}{2}v$, for a "single good" with a value $(1+k)v$.[3] Note finally that for $k = 2$, first-auction bidding should be more aggressive when the option is available than when it is not available.

3 Experimental design

■ The experiment was conducted on 28 and 29 March 2001 at the *Ecole Nationale de Statistique et de l'Administration Economique* (ENSAE).[4] Students were recruited through personal

[3] Recall that, given our model assumptions, the optimal single-unit bid (in first-price and Dutch auctions) for a good valued at v is $\frac{1}{2}v$.

[4] The ENSAE is one of the leading French institutions of higher learning in the fields of statistics, economics, finance, and actuarial sciences.

emails, and fliers that we dispatched in their mailboxes. Seventy four students (out of roughly 360 students that studied at the time at ENSAE) participated in the experiment. We organized a total of 10 experimental sessions in the computer rooms at ENSAE, and each student took part in only one session. Only one type of auction mechanism was used per session. Table 1 lists for each session the type of auction mechanism that was studied and the number of participants. From Table 1 it can be seen that 22 students participated in the Dutch auctions, 20 in the English auctions, 16 in the first-price auctions, and 16 in the second-price auctions. All sessions were made up of two parts. In each session the first part was devoted to sequential auctions without a buyer's option, and the second part to sequential auctions with a buyer's option.

We start by describing the first part of a session. We began by reading aloud the instructions about the auction rules without a buyer's option. Written versions of the instructions were distributed to the participants and could be consulted at any time during the experimental session.[5] The first part had 12 periods. Since we focus in this paper on auctions with 2 buyers, participants were told that they were in competition with a single person. We also told them that in each period the computer randomly matched each student to another student present in the room (all sessions had an even number of participants). Participants were thus aware of the fact that their opponent differed from period to period. We informed the participants that in each period 2 units of a fictitious good were sold at auction to each couple.

At the start of each period, valuations were independently drawn from a uniform distribution on $[0;\overline{v}]$=[0;FFr50.00]. The computer screen of participant i displayed his/her valuation for the first unit of the good v_i, the prevailing value of k, and his/her valuation for the second unit kv_i. The value of k changed every 4 periods ($k = \frac{1}{2}$ in periods 1-4, $k = 1$ in periods 5-8, and $k = 2$ in periods 9-12). Participants could observe this information for 30 seconds, after which the first auction started (but the information remained on the screen during the auction). The manner in which participants could bid depended on the auction mechanism used during the session. The auction-specific bidding devices are described below.

Once the first auction was over, information concerning the first auction was added to the screen of each subject i. It indicated whether i was the winner or not, his/her own bid (if any), the winning price p_1, i.e., the price he/she or his opponent had to pay for the first unit, and his/her gain associated with the auction ($v_i - p_1$ if i was the winner, 0 otherwise). Since the identity of the winner of the first auction is crucial knowledge in our experiment, we emphasized this by coloring the box marked "Winning bid" blue if i had won the first auction, and red otherwise. The information released between the two auctions differed slightly with the type of auction mechanism. For instance, for the winner of an English auction the box marked "Your bid" remained empty, while for the winner of a Dutch auction this box indicated the price at which he had stopped the clock.

Before the start of the second auction, participants again had a thirty-seconds reflection period during which they could, if they wished, consult all information on their screen (again, all information remained on display during the second auction). The second auction functioned in the same way as the first auction. We stressed the fact that the gain associated with the second auction depended on the outcome of the first auction. Thus, winner i of the second auction would have a gain of $kv_i - p_2$ if he also won the first auction, and a gain equal to $v_i - p_2$ if he

[5] The instructions can be obtained from the authors.

lost the first auction. Once the second auction was terminated for all couples in the room, we proceeded with the next period.

The 12 periods of the first part of each experimental session were preceded by 6 "dry" periods (2 for each value of k). This gave participants the opportunity to familiarize themselves with the bidding method, determine their strategy for the different values of k, and ask questions to the experimenter.

Next we describe the second part of the session, the one that was designed to study the buyer's option. We began by reading aloud the instructions for this part of the experiment. Like the first part it consisted of 12 periods. Each period started exactly like the first part of the experiment: the valuations and the value of k (the values of k alternated as in the first part) were shown on the screen, the first auction started after 30 seconds, and once the first auction was over for player i and his/her rival, their screens updated them on the relevant auction results. Unlike the first part of the session, subjects were told that the winner of the first auction could, if he/she desired, use the buyer's option. If winner i chose to execute his/her option, the period ended, and his/her total gain in the period was $(v_i - p_1) + (kv_i - p_1) = (1+k)v_i - 2p_1$. If he/she chose not to do so, his/her gain associated with the first auction was $v_i - p_1$, and a second auction was held after the thirty-seconds pause. The second auction was in all respects identical to the second auction conducted in the first part of the experiment.

The 12 "wet" periods of the second part of each experimental session were again preceded by dry periods, but now just 3 of them (1 for each value of k) since, at least from a practical point of view, the second part differed little from the first.

As mentioned above, the way in which participants had to submit their bids depended on the auction format. In the first-price and second-price auctions participants could submit their bid by entering a number in a box marked "Submit your bid here". The number could be any positive real integer, i.e., we did not prevent subjects from bidding more than their valuations.

In the Dutch and English auctions bidding took place via numerical clocks. After the 30-second reflection period, the clock appeared on the screens of the participants. In the English auctions the clock started at FFr0.00, augmented continuously at a rate of FFr0.83 per second (FFr50.00 per minute), and stopped automatically at FFr120.00. The clock started and operated simultaneously on the screens of participant i and his/her rival. They could stop the clock at any time by pressing the "Enter" key or "Space bar", or click on a window marked "Stop the clock". If neither i nor his rival had stopped the clock before it reached FFr120.00, the computer randomly selected i or his/her rival as the winner (this never happened during our experiments). In the Dutch auctions the clock started at FFr60.00 (if $k = \frac{1}{2}$ or $k = 1$) or FFr120.00 (if $k = 2$), descended continuously at the speed of FFr0.83 per second, and stopped automatically at FFr0.00. The Dutch clock started and operated simultaneously for subject i and his/her opponent and they could stop it, at any time, like the English clock. If neither i nor his/her rival had stopped the clock before it reached FFr0.00, there was no auction winner (again, this never occurred). Note that as in the sealed-bid auctions, subjects could bid above their valuations (up to a limit) in the clock auctions.

At the start of an experimental session, i.e., at the beginning of the first period, all participants were given a capital balance of FFr50.00. At the end of each period, the gains made during the period were added to the balance, and losses were subtracted from it. We informed the experimental subjects that if the end-of-period balance of a participant was negative (as a result of his/her bidding behavior in the period), the balance would immediately be readjusted to 0. We stressed that balances would only be readjusted at *the end* of a period, in view of the end-of-period balance, and not at some point *during* a period. The reason for censoring the start-of-period balances at 0 is to incite subjects to play well all along the experiment.[6] As it turned out, no experimental subject's capital balance went negative, and it was not necessary to implement the readjustment procedure.

At the end of the session participants were paid in cash their final capital balance divided by two. From a theoretical point of view this 50% cut does not affect bidding behavior. On average we paid FFr229 to the students, the minimum payment was FFr60, and the maximum payment FFr360. Experimental sessions lasted between 1.5 and 2 hours (including about 20 minutes for the instructions).

Before turning to the results, we wish to make a comment about the experimental design, concerning the treatment conditions. As explained above, in each session the subjects went through 6 different treatments. Furthermore, the ordering of the treatments was the same in every session. These aspects of the design may have undesirable consequences. Indeed, having subjects participate in more than one treatment makes them susceptible to hysteresis effects (behavior carrying over from the previous treatment to the current one). If the behavior of subjects in a given treatment influences behavior in later treatments, the data from all but the first treatment may differ from what they would be were the treatments conducted separately, in isolated and independent sessions. It may therefore be difficult to measure certain treatment effects. For instance, since the periods with the buyer's option always occurred after the periods without the buyer's option, the differences may not measure the effects of the option, but the combined effect of the strategic impact of the buyer's option and a carry-over effect. For a similar reason, it may not be possible to obtain estimates of changes in the demand structure, as it may be difficult to disentangle the strategic impact of modifications in the form of the demand function with the carry-over effect. There may be an additional problem if one believes that subjects need to learn how to bid, and consequently make better decisions with more experience. In this case, comparisons of treatments towards the end of the sessions with those at earlier stages may be contaminated by learning effects.

Although we acknowledge these drawbacks in our experimental design, the consequences are perhaps not that important. Indeed, many results in this paper (summarized in the facts listed below) rely on comparisons across auctions mechanisms or on comparisons between first-unit auction behavior and second-unit auction behavior, and these comparisons are unaffected by hysteresis or learning effects (under some standard assumptions). Other results do not rely on treatment comparisons, so these results cannot be biased either. There are some results (that we do not consider important, and therefore not presented as facts) that can be affected by hysteresis or learning effects. The bias here may be small. First, learning is not a crucial issue in our data.[7] Second, hysteresis effects may be small in our data as we put much effort to present the treatments as fully independent and separate games.

[6] Had we not done this, a subject with a very negative balance at the beginning of period 24, would clearly not have been incited to behave optimally in this last period.

[7] An examination of our data at the individual level reveals that subject-specific behavior is generally stable throughout each treatment, i.e., subjects mostly stuck to the same strategy during the four periods of each treatment.

4 Experimental results

■ This section presents our experimental results. First we confront the observed bidding behavior with the Nash equilibrium bidding strategies given in Proposition 1. We then study the impact of the buyer's option on the behavior of the experimental subjects, and analyze how well the option is used in the experiment. Finally, we study revenue, price patterns, and the efficiency of the four auction mechanisms.

□ **Bidding behavior.** Figures 1-24 (see the web Appendix at www.rje.org/main/sup-mat.html) show all first-auction bids for the different values of k for the four auction formats without buyer's option (Figures 1-12) and with buyer's option (13-24).[8] They depict the losing bids for the English auctions, the winning bids for the Dutch auctions, and both winning and losing bids for the sealed-bid auctions. Whenever there is a theoretical prediction (see Proposition 1), the equilibrium bid function $b_1(.)$ is drawn in dashes. For instance, in Figure 2 ($a = S$, $k = \frac{1}{2}$ and $o = N$) the dashed line is the function $b_1(v) = \frac{1}{2}v$, but in Figure 1 ($a = F$, $k = \frac{1}{2}$ and $o = N$) no dashed line is drawn since no prediction is available. For second-price and English auctions with decreasing demand the solution (approximated using simulations) of the differential equation given in Proposition 1 is $b_1(v) = 0.99v - 0.009v^2$. This explains why the dashed line in Figures 14 and 16 is curved.

In each figure we also draw the estimated bid function in solid: the solid line in Figures 14 and 16 is the estimated bid function $\hat{\beta}_0 + \hat{\beta}_1 v + \hat{\beta}_2 v^2$; in all other figures the solid line represents the estimated bid function $\hat{\beta}_0 + \hat{\beta}_1 v$. The estimation method used to obtain the estimated parameters (either $\hat{\beta}_0$ and $\hat{\beta}_1$, or $\hat{\beta}_0, \hat{\beta}_1$ and $\hat{\beta}_2$) differs with the type of auction mechanism. In the Dutch auctions, the estimates $\hat{\beta}_0$ and $\hat{\beta}_1$ are obtained by applying censored regression methods to the random effects model $b_{1it} = \beta_0 + \beta_1 v_{it} + \mu_i + \varepsilon_{it}$. Here v_{it} is i's valuation in period t, μ_i is a subject-specific error term, and ε_{it} is a period-specific error term. The usual assumptions are made regarding these error terms.[9] The bid b_{it} is only observed if player i wins the auction in period t; if i does not win the auction, his/her bid is censored from below at the winning price paid by i's competitor, p_{1t}, so the only thing known in this case is that b_{it} belongs to the interval $[0; p_{1t}]$. In the English auctions, the estimates $\hat{\beta}_0$ and $\hat{\beta}_1$ ($\hat{\beta}_0, \hat{\beta}_1$ and $\hat{\beta}_2$ when $a = E, k = \frac{1}{2}, o = Y$) are again obtained by performing a censored regression analysis. The random effects model is the same as in the Dutch auctions (except when $a = E, k = \frac{1}{2}, o = Y$, in which case the bid equation is $b_{1it} = \beta_0 + \beta_1 v_{it} + \beta_2 v_{it}^2 + \mu_i + \varepsilon_{it}$). Unlike the Dutch auctions, the bid b_{it} is only observed if player i does not win the auction in period t; if i wins the auction, his/her bid is censored from above at the winning price paid by i's competitor, so in this case we only know that b_{it} exceeds p_{1t}. Given that the clock in the English auction was designed to stop at FFr120.00, and given that under decreasing or flat demand it does not seem likely that agents bid far above the upper bound of the support of the distribution of values (FFr50.00), we assume that when i is the winner of the auction, b_{it} belongs to $[p_{1t}$;FFr120.00$]$ if $k = 2$ and to $[p_{1t}$;FFr60.00$]$ if $k = \frac{1}{2}$ or $k = 1$. In the remaining mechanisms, the first-price and second-price auctions, all observations are uncensored. For these sealed-bid auctions the estimates $\hat{\beta}_0$ and $\hat{\beta}_1$ ($\hat{\beta}_0, \hat{\beta}_1$ and $\hat{\beta}_2$ when

[8] The figures corresponding to the second-unit bids can be obtained from the authors.

[9] Both error terms are assumed to be i.i.d. normal random variables: $\mu_i \sim N(0, \sigma_\mu)$ and $\varepsilon_{it} \sim N(0, \sigma_\varepsilon)$. In addition, μ_i is assumed independent of ε_{it}, and v_{it} is independent of μ_i and ε_{it}.

$a = S, k = \frac{1}{2}, o = Y$) follow from maximum likelihood estimation of the random effects model $b_{1it} = \beta_0 + \beta_1 v_{it} + \mu_i + \varepsilon_{it}$ ($b_{1it} = \beta_0 + \beta_1 v_{it} + \beta_2 v_{it}^2 + \mu_i + \varepsilon_{it}$ when $a = S, k = \frac{1}{2}, o = Y$). The assumptions on the error terms are the same as in the oral auctions.[10]

Finally we draw in the figures the 45-degree line in dots. Whenever there is an equilibrium bidding function, a comparison of the dashed and solid lines in the corresponding figure is a quick eyeball test of the theoretical predictions.

Table 2 reports the estimation results of all the first-auction bid equations. The left panel of this table gives the results for the auctions without buyer's option, and the right panel those for the auctions with buyer's option. Table 3 reports the estimations results of the second-auction bid equations without buyer's option (using the same methods as for the first-auction bid equations).[11] The results in the left panel of this table are based on the second-auction bids submitted by the first-auction *winners*, and those in the right panel are based on the second-auction bids of the first-auction *losers*. We do this because winners and losers of the first auction should generally behave differently in equilibrium (see Proposition 1). The estimation results based on the second-auction bids with buyer's option are not reported. The reason is that the buyer's option is frequently used by our experimental subjects, and relatively few second auctions were held during periods 13-24 of the experiment, leaving us, in most cases, with too few data to estimate the second-auction bidding equations reliably when the option is available.

Both tables give the estimated parameters and standard errors, the theoretical predictions of the parameters in the equilibrium bidding strategies, the number of observations used in the regressions, and test results of the hypothesis that observed behavior is in line with predicted behavior.[12] For example, consider the first-auction results for $a = E$, $k = \frac{1}{2}$, $o = N$. The estimate $\hat{\beta_1}$ equals 0.62 and the estimated standard error is 0.04 ($\hat{\beta_0}$ is 3.62, and standard error is 2.08).[13] These estimates are obtained using 80 observations (40 uncensored observations plus 40 censored observations).[14] The predicted slope is $\frac{1}{2}$, and the null hypothesis that $\beta_0 = 0$ and $\beta_1 = \frac{1}{2}$ is rejected.[15]

For two cases in Table 3 ($a = E$, $k = 1$ and $k = 2$) we cannot estimate the regression parameters. The censoring rate is high in these two treatment conditions,[16] and the information in the data is not rich enough to identify the intercept and slope coefficient. Test results are therefore not reported for these two cases.

[10]The underlying bid equation thus has the same stochastic structure in all 4 auction mechanisms. The results for the sealed-bid auctions are similar when using the random effects GLS estimator.

[11]All estimation results in Table 2 and Table 3 are obtained using the xtintreg procedure in Stata (version 9).

[12]In environments where no symmetric pure strategy equilibrium exists, we are not able to characterize any of the mixed strategy equilibria, and comparisons can therefore not be made.

[13]Estimated intercepts are not reported in Tables 2 and 3. Using standard T-tests, the hypothesis that the intercept equals zero is accepted in the large majority of cases. Thus in most cases we can accept the hypothesis that the bidding equations pass through the origin.

[14]As Table 1 indicates, a total of 20 subjects participated in the English auctions. Since each subject played the treatment condition $a = E$, $k = \frac{1}{2}$, $o = N$ four times and since each English auction had one loser and one winner, there are 40 uncensored observations and 40 censored observations.

[15]A limitation of this kind of test procedure is that the null hypothesis can be accepted, not because the null is really true, but because the bid equation is imprecisely estimated.

[16]The number of uncensored observations is just two when $a = E, k = 1$ and zero when $a = E, k = 2$. This reflects the fact that most first-auction winners in these treatment conditions also win the second auctions, which implies that (practically) all corresponding second-auction bids are censored.

Fact 1. In sequential auctions, subjects only partially conform to the strategic implications of the Nash equilibrium in the auction for the first unit. Their observed bidding behavior is close to equilibrium bidding behavior in the auction for the second unit.

As Table 3 shows, second-auction bidding in the English and second-price auctions is almost always in line with theory: 8 out of 10 times we can accept the hypothesis that subjects have played, on average, as the equilibrium strategy predicts. Recall that in all English and second-price auctions for the second unit, it is optimal for players to bid their own valuation (which is in fact a dominant strategy). Thus, like in standard single-unit English and second-price auctions, it is optimal for bidders to reveal their valuation. It is therefore of interest to compare our results with those obtained in the experimental literature on single-unit English and single-unit second-price auctions. The results of Coppinger, Smith, and Titus (1980), Kagel, Harstad, and Levin (1987), and Kagel and Levin (1993), indicate that subjects bid according to equilibrium behavior in single-unit English auctions. Most experimental studies show, however, that in single-unit second-price auctions subjects tend to bid above their value (see for instance Kagel, Harstad, and Levin (1987), and Kagel and Levin (1993)).[17] Compared to the single-unit second-price literature, our findings appear somewhat more in accordance with theory. It should be stressed however that bidding in our experiment and bidding in the single-unit experiments took place in different contexts.[18] These contextual differences can explain the small differences between our results and those obtained in the earlier literature.

Table 3 indicates that observed bidding in the Dutch and first-price auctions with increasing demand also accords with predicted behavior: subjects have played, on average, according to theory 3 times out of 4.

Next we study the results of the auctions for the first unit. In the tests of the theory for all auction mechanisms, Table 2 shows that the null hypothesis is accepted just 7 out of 20 times. The theory is mostly accepted in cases where the equilibrium strategies are relatively transparent, for the English and second-price auction mechanism with flat demand. The equilibrium strategies are relatively transparent in these cases because the outcome of the first auction has no impact on the second auction. Our experimental subjects understood that in this relatively simple setup it is optimal to bid their valuation in the first auction.

The equilibrium predictions are almost always rejected when the demand function is either decreasing or increasing, that is precisely in the situations where our sequential auctions are inherently more complex. (In determining the equilibrium bidding strategies, agents should anticipate that the valuation of the first-auction winner is modified at the start of the second auction.) However, subjects adjust their bids in the direction predicted by theory, thereby acknowledging, in part, the strategic implications of non-flat demand functions. Thus in all second-price and English auctions with decreasing (resp. increasing) demand, subjects understood that equilibrium behavior calls for bid shading (resp. bidding above value), but the extent to which they

[17] Cox, Roberson, and Smith (1982) find that average bidding is below value. Kagel, Harstad, and Levin (1987) and Kagel and Levin (1993) point out that this may result from the fact that the design of Cox, Roberson, and Smith (1982) *did not allow* subjects to bid in excess of their valuation.

[18] Experimental subjects were more informed about their opponents (when they submitted their second-auction bid, they knew p_1). Also, when $k = \frac{1}{2}$ and $k = 2$, bidders in the second auction are no longer symmetric as in the single-unit experiments.

do this is too modest to fit the theory.[19] Similarly, although observed bidding is mostly above equilibrium bidding in the Dutch and first-price auctions with buyer's option, our experimental subjects understood the impact of k on the bidding strategies. As k increases, they bid more aggressively as predicted by the theory.

In summary, observed bidding behavior in the second auction is mostly in line with equilibrium predictions, but bidding in the first auction is generally off-equilibrium. The subjects thus exhibit bounded rationality in the sense that they solve correctly the final stage of the auction games but are apparently unable to apply correctly the backward-induction-arguments necessary to solve the first stage of the game. [20]

Fact 2. In the majority of cases, bidding behavior in English (resp. Dutch) auctions is identical to bidding behavior in second-price (resp. first-price) auctions.

Evidence for Fact 2 follows from comparisons of the estimated bid-function parameters between Dutch (resp. English) and first-price (resp. second-price) auctions. A total of 24 comparisons are made (there are 12 Dutch/first-price couples and 12 English/second-price couples; see Table 2 and Table 3): we thus test for bidding equivalence not only in cases where theory explicitly predicts two auction mechanisms to be isomorphic, but also in cases where there are no theoretic predictions. Wald tests on the equality of parameters show that the null is accepted 10 out of 12 times for the D/F comparisons, and 9 out of 12 times for the E/S comparisons.

Fact 3. In the English auction for the second unit under increasing demand and without buyer's option, the losers of the first auction bid above their valuation. Subjects appear to use a weakly dominated strategy.

Table 3 shows that first-auction losers bid almost 30% above their valuation (and dominant strategy) v in the English auction. A possible explanation for this over-bidding phenomenon is that first-auction losers wish to punish their competitors, out of feelings of envy or because of fairness considerations. Such punitive behavior is also observed in other types of experiments (see Roth (1995) and Zizzo and Oswald (2001)).

There is therefore some evidence that our experimental subjects use a dominated strategy in a game where a dominant strategy is available. This finding is contrary to Kreps (1990) (p. 698), for instance, who argues that mechanism designers may safely expect players to settle on the dominant strategy even when other equilibria (in weakly dominated strategies) exist. Our finding accords with the results obtained by Kagel, Kinross, and Levin (2001).

□ **The buyer's option.** The results regarding the use of the buyer's option can be found in Table 4. The second column reports the number of times the buyer's option is exercised for each auction institution (and in parentheses the relative number of times), and the third column gives the theoretical predictions. Proposition 1 describes the use of the buyer's option only along the

[19] When $a \in \{E,S\}, k = 2, o = N$, the equilibrium strategy exposes players to a financial risk – the possibility of winning only one unit and earning losses on it. This exposure problem has been suggested as an issue in other auctions with synergies (see Kagel and Levin (2005)), and may explain behavior in our study.

[20] Individual-specific investigations of the bids tend to show however that some subjects behaved closer to equilibrium than others. This type of result is also found in the literature on the beauty contest game (see Nagel (1995)).

equilibrium path. Since, however, subjects have deviated from theory in the auctions for the first unit, the third column reports the predictions regarding the use of the option conditional on the first auction price.

For example, in the first-price auction under decreasing demand, the equilibrium strategy is 0.375 v (and the winner should exercise the option). Yet, in the experiment many players bid above $0.375v$. In this case the out-of-equilibrium-path prediction stipulates that the option should be used when the winning price is lower than $\frac{1}{2}v$, and that it should not be used otherwise. In the data the option was used 10 times (83% of the cases) when $p_1 \leq \frac{1}{2}v$, and 3 times (15%) when $p_1 > \frac{1}{2}v$.[21]

Fact 4. The buyer's option is correctly used in most cases.

As Table 4 shows, when $k = \frac{1}{2}$ or $k = 1$, it is optimal to use the buyer's option whenever the value of the second unit exceeds the first-auction winning price.[22] Our experimental subjects follow this strategy 7 times out of 8. The only substantial deviation from theory occurs in the Dutch auction under decreasing demand.

When $k = 2$ the observed frequencies of exercising the option are close to the predicted frequencies for the Dutch and first-price auctions. However, for the English and second-price auctions the observed frequencies are much higher than those predicted by the theory. In the case of the English auction, this deviation may result from the fact that losers of the first auction bid above their valuation (see Fact 3). To avoid paying a higher price, it makes sense to buy the second unit rather than wait for a second auction.

The impact of the buyer's option on first-auction bidding behavior can be studied by comparing the left and right panels of Table 2. To formally check if there is an impact, we perform Wald tests on the equality of the parameters.[23] It turns out that bidding behavior in the first auction is not affected by the presence of a buyer's option in 9 cases out of 12.

□ **Revenue comparisons.** Table 5 reports, for each auction mechanism and value of k, the mean observed revenue and its standard error, the mean revenue as predicted by the theory and its standard error, and the test result of the hypothesis that the observed revenue has the same mean as the predicted revenue. The left panel of the table gives the results for the auctions without buyer's option, and the right panel those for the auctions with the option. Each mean observed revenue is determined by first defining, for each couple and for each period, the seller's revenue $REV_{obs} = p_1 + p_2$, and then by calculating the empirical average of REV_{obs} over all relevant observations (for instance, when $a = E, k = 2$, and $o = N$ the reported mean revenue of 40.95 is the average over 40 observations). Similarly, each mean predicted revenue is determined by first defining, for each realized couple of valuations (v_1, v_2) and for each period, the predicted revenue $REV_{theo}(v_1, v_2)$,[24] and then by calculating the empirical mean over all relevant observations. The null hypothesis that the observed revenue has the same mean as the predicted

[21] In these 3 cases where the option was wrongly used the loss on the second unit was negligible.

[22] When $a \in \{E, S\}$ and $k = 1$, the first-auction winner is in fact indifferent between using the option or not.

[23] When $a \in \{E, S\}$ and $k = \frac{1}{2}$, we test the joint hypothesis that the intercepts and slope coefficients are equal, and that the coefficient β_2 in the right panel equals zero.

[24] Thus $REV_{theo}(v_1, v_2)$ is the revenue level had the couple of subjects with valuations v_1 and v_2 played according to theory.

revenue is tested using two-sample T-tests. As the table shows, the null is accepted just 7 out of 20 times. This finding is consistent with our earlier result that bidding in the auctions for the first unit is often off-equilibrium (see Fact 1).

Next we wish to study whether there is revenue equivalence between auctions, as predicted by the theory.[25] To check whether auctions are equivalent (for given k and o), we simply regress the variable $REV_{obs} - REV_{theo}$ on a constant and indicator variables for the auction types,[26] and test if the coefficients are equal to zero. By subtracting the predicted revenue from the observed revenue, we account for differences (across treatment conditions) in the drawing of valuations. Revenue equivalence is thus tested after controlling for different valuation realizations.

Fact 5. In sequential auctions with or without buyer's option, the first-price auction generates the highest revenue, followed by the Dutch auction, then the second-price auction, and finally the English auction.

The estimation results (not reported here) show that in general the 4 auction mechanisms are indeed ranked as claimed in Fact 5. Standard T-tests indicate that revenue in first-price auctions is statistically larger than revenue in Dutch auctions (2 out of 4 times), or there is equality (2 out of 4 times); revenue in Dutch auctions is statistically larger than revenue in second-price auctions (2 out of 4 times), or there is equality (2 out of 4 times); finally, revenue in second price auctions is statistically larger than revenue in English auctions (2 out of 6 times), or there is equality (4 out of 6 times). Our revenue-ranking of auction formats is identical to that found by Cox, Roberson, and Smith (1982) in their experimental study on one-unit auctions.[27] The revenue-ranking of the 4 main auction institutions thus appears to be robust to the sequential-two-unit extension considered in this paper.

□ **Price patterns.** Table 6 reports for each value of k and each auction mechanism, the mean and standard error of the difference in observed prices $p_2 - p_1$, the mean and standard error of the difference in predicted prices, and whether these predictions are rejected by the data or not. The predicted prices are calculated under the assumption that, given the realized valuations, subjects play according to theory (see Proposition 1). The left panel gives the results for the auctions without buyer's option and the right panel those for the auctions with option. As the table shows, the theoretical predictions are accepted 7 out of 20 times. We will not comment on these individual rejections/acceptations but instead we discuss the following fact.

Fact 6. In sequential auctions, prices are declining when there is a buyer's option.

Evidence for Fact 6 comes from the results in the right panel of Table 6. The empirical average of the difference in observed prices is always negative when the option is available,

[25] It follows from Proposition 1 that $E\left[REV_{theo}(v_1, v_2)\right]$ is the same for all a (given k and o). This revenue equivalence does not hold, however, for $k = \frac{1}{2}, o = Y$ (the auctions $a \in \{D, F\}$ generate a higher expected revenue compared to the auctions $a \in \{E, S\}$).

[26] When $k = \frac{1}{2}, o = N$ (resp. $k = 1, o = N$), we pool all observations from the auctions $a \in \{E, S\}$, and regress $REV_{obs} - REV_{theo}$ on the indicator variable $1(a = S)$. In each of the four other situations (for example $k = 2, o = N$), we pool all auction observations, and regress $REV_{obs} - REV_{theo}$ on $1(a = S)$, $1(a = D)$ and $1(a = F)$.

[27] Cox et al. study first-price, second-price, and Dutch auctions, but not English auctions.

and the difference is significant 7 out of 12 times. The successive prices are thus (significantly) decreasing when the buyer's option is available.

The declining price phenomenon that we find is also observed in the field-data studies mentioned in the introduction. These results seem compatible with theory in the case of the wine auction studies. Indeed, in English auctions under decreasing demand, successive prices should in theory be decreasing (see the right panel of Table 6 and Février, Roos, and Visser (2005) who show that these predictions hold in a model with an arbitrary number of bidders). Furthermore, there is empirical evidence that the demand function for wine is decreasing (see Février, Roos, and Visser (2005)). In the case of the Dutch flower auctions, it seems more difficult to reconcile the observed declining price patterns with theory. Indeed, the theoretical prediction here is that successive prices should remain constant regardless of the form of the demand function (right panel of Table 6). Admittedly, these predictions are only derived in the two-buyer-two-unit model. But if they were also valid in a more general model, then the declining price phenomenon described in Van den Berg, Van Ours, and Pradhan (2001) would truly be an anomaly.[28]

□ **Efficiency.** In this subsection we study auction efficiency. The results can be found in Table 7. For each k and auction mechanism, the table reports the mean and standard deviation of the observed relative efficiency RE, where RE is the ratio between the realized surplus and the maximum surplus available. Thus if bidder 1 wins the first unit and bidder 2 the second unit, then this measure is defined as $RE = \frac{v_1+v_2}{\max\{(1+k)v_1, v_1+v_2, (1+k)v_2\}}$, and if bidder i wins both units, then $RE = \frac{(1+k)v_i}{\max\{(1+k)v_1, v_1+v_2, (1+k)v_2\}}$.

We also report the mean and standard error of the predicted values of RE (calculated under the assumption that subjects play according to Proposition 1).[29] The left panel gives the results for the auctions without a buyer's option and the right panel those for the auctions with the option.[30]

Fact 7. The allocation of the units in the experimental auctions is almost as efficient as predicted by theory.

As Table 7 reports, almost all auction institutions are, in theory, efficient mechanisms. The only exceptions are auctions with a buyer's option and decreasing demand. There is a slight inefficiency in these cases since the buyer's option allows the first-auction winner to buy the second unit while having a lower valuation than his opponent.

As the table indicates, the observed level of efficiency is generally close to the predicted efficiency. In spite of the out-of-equilibrium behavior observed in the data, all auctions turn out to be highly efficient in our experiments. This follows because most players bid more or less

[28]Ashenfelter (1989) observed declining price patterns in sequential (wine) auctions, and called the phenomenon the declining price anomaly. Although later studies have provided theoretical explanations for declining price patterns (Black and De Meza (1992), McAfee and Vincent (1993), and Branco (1997)), the term is used whenever theory is in contradiction with empirical findings.

[29]Note that the standard errors of the predicted means are necessarily equal to zero except when $k = \frac{1}{2}, o = Y$.

[30]Another efficiency measure can be obtained by calculating for each treatment how many times an individual ends up with 0 unit, 1 unit, 2 units, and comparing the frequencies with what should happen in equilibrium. A table summarizing these calculations can be obtained from the authors.

according to the same increasing function of the valuation. Therefore, even if observed bidding is not in line with equilibrium bidding, auction winners are typically the players with the highest valuation for a given unit, thereby guaranteeing high efficiency.

5 Conclusion

This paper experimentally studies two-unit sequential auctions with and without the buyer's option. The 2 identical units are sold to 2 potential buyers. Each buyer desires both units, and their demand function is either decreasing, flat, or increasing. The four best known auction mechanisms are considered: Dutch, English, first-price and second-price auctions.

Some of our model assumptions are restrictive. For instance, in real world auctions the number of participants is typically larger than two, and generally there are more than just two units on sale. Similarly, our assumption that bidders are symmetric and risk neutral buyers is restrictive. However, this is the first paper on the buyer's option, and one of our objectives is to confront subject behavior with theoretical predictions. In future work we plan to refine and extend our model and study the effects of an increase in the number of buyers and the number of units auctioned. We also plan to investigate the role of the buyer's option in a common value setting.

Some of the empirical results obtained in this paper suggest other topics for further research. For example, it would be interesting to test whether subjects manage to adjust and improve their behavior (see Fact 1) by adding more rounds to the design or by giving more information (on the observed strategies and/or the equilibrium strategies) between the rounds. It would also be interesting to study in more detail the causes of the overbidding by first auction losers in the second auction (see Fact 3), and to investigate conditions under which overbidding is reduced.

Appendix

■ Proof of proposition 1

□ **The case** $\mathbf{a} \in \{\mathbf{E},\mathbf{S}\}$, $\mathbf{o} = \mathbf{N}$, $\mathbf{k} \in \left\{\frac{\mathbf{1}}{\mathbf{2}},\mathbf{1},\mathbf{2}\right\}$. The second auction strategies are obtained by the standard dominated strategies argument. Therefore, both the loser and the winner of the first auction bid their valuation: $b_2^l(v,p_1) = v$ and $b_2^w(v,p_1) = kv$. To derive the first auction equilibrium strategies, we have to distinguish the English auction from the second-price auction as the available information is not the same in these two auction institutions.

The case $a = E, k \in \left\{\frac{1}{2},1\right\}$. See Donald, Paarsch, and Robert (1997). *Q.E.D.*

The case $a = E, k = 2$. Let $b_1(v)$ be the first auction equilibrium strategy and v_1 the value of player 1. Suppose the clock has reached p (close to $b_1(v_1)$) and player 1 has to decide to continue or to stop bidding. Let $G(\varepsilon, p)$ denote the expected total gain (for the first and second auctions) for player 1 if he decides to continue with bidding until $p+\varepsilon$:

$$G(\varepsilon,p) = \int_{b_1^{-1}(p)}^{b_1^{-1}(p+\varepsilon)} (v_1 - b_1(w) + 2v_1 - w)\frac{dw}{\overline{v} - b_1^{-1}(p)}.$$

The above expression follows because player 1 can only win the first auction if $p \leq b_1(v_2) \leq p+\varepsilon$, with v_2 being the valuation of player 2. If he wins the first auction, he also wins the second auction because, since p is close to $b_1(v_1)$, we have that $2v_1$ is larger than v_2. On the contrary, if he loses the first auction he also loses the second one. Note that the density in the integral is the conditional density of v_2 given $\overline{v} \geq v_2 \geq b_1^{-1}(p)$. Derivation with respect to ε gives:

$$\frac{\partial G}{\partial \varepsilon}(\varepsilon,p) = \left(b_1^{-1}(p+\varepsilon)\right)' \frac{v_1 - (p+\varepsilon) + 2v_1 - b_1^{-1}(p+\varepsilon)}{\overline{v} - b_1^{-1}(p)}.$$

The equilibrium condition is:

$$\frac{\partial G}{\partial \varepsilon}\left(\varepsilon = 0, p = b_1\left(v_1\right)\right) = 0, \text{ which leads to } b_1\left(v_1\right) = 2v_1.$$

To prove that it is indeed a Nash equilibrium, assume that player 2 follows the strategy $b_1(v_2) = 2v_2$ and assume that player 1 deviates from this strategy and stops at $p < v_1$. In that case he loses the first auction and can only win the second if $v_2 \in \left[\frac{p}{2}, \frac{v_1}{2}\right]$ which would lead to a gain of $v_1 - 2v_2$. But in that case, by bidding until $p = v_1$ player 1 would win both the first and the second auction (because $v_2 < v_1$), which leads to a larger profit of $v_1 - 2v_2 + 2v_1 - v_2$. Assume, now, that player 1 stops at p with $v_1 < p < 2v_1$. In that case losing the first auction also means losing the second because $v_1 < p < 2v_2$ (here in the second auction player 2 bids $2v_2$), therefore it is optimal to bid until $2v_1$. Finally, a deviation $p > 2v_1$ is weakly dominated: it does not improve the gain when $v_2 < v_1$ while it implies a loss when $v_1 < v_2 < \frac{p}{2}$ as the revenue of player 1 is then $3v_1 - 3v_2 < 0$. *Q.E.D.*

The case $a = S, k \in \left\{\frac{1}{2}, 1\right\}$. See Black and De Meza (1992). *Q.E.D.*

The case $a = S, k = 2$. In order to characterize the equilibrium strategy $b_1(.)$, assume that player 1 deviates from $b_1(v_1)$ by bidding $b_1(x)$, with x close to v_1. If he loses the first auction while bidding $b_1(x)$, he is sure to lose the second auction as well. On the contrary, if he wins the first auction he is also sure to win the second auction. That is, the expected total gain of player 1 playing $b_1(x)$ is:

$$G(x) = \int_0^x \left[v_1 - b_1(w) + 2v_1 - w\right] dw.$$

In equilibrium such a deviation cannot be profitable which means that:

$$G'(x = v_1) = 0, \text{ which leads to } b_1(v_1) = 2v_1.$$

To prove that it is a Nash equilibrium, assume that player 2 bids $2v_2$. It is then obvious that a bid equal to $2x$, $x < v_1$, gives player 1 a lower expected gain than a bid equal to $2v_1$ as it does not increase the gain when player 1 wins both auctions but it reduces the probability of winning. Next, a bid equal to $2x$, $v_1 < x$, also reduces the expected gain of player 1 because when player 1 wins the first auction with $2x$ but not with $2v_1$ he has a negative total gain. *Q.E.D.*

□ **The case $\mathbf{a} \in \{\mathbf{D}, \mathbf{F}\}$, $\mathbf{o} = \mathbf{N}$, $\mathbf{k} \in \left\{\frac{1}{2}, 1\right\}$.** The non-existence of a Nash equilibrium with symmetrical pure strategies in the first auction is proved in Février (2000). *Q.E.D.*

□ **The case $\mathbf{a} \in \{\mathbf{D}, \mathbf{F}\}$, $\mathbf{o} = \mathbf{N}$, $\mathbf{k} = 2$.** We first study the second auction assuming that both players bid according to $b_1(.)$ in the first auction. Suppose that player 1 with valuation v_1 won the first auction and let v_2 denote the valuation of player 2. Therefore the value of $b_1(v_1)$ is revealed before the second auction starts and both players know that $v_2 < v_1 < 2v_1$. In equilibrium the second player knows that he cannot win the second auction and therefore a best reply is to bid v_2 in the second auction. By bidding x in the second auction the expected gain of player 1 in the second auction is

$$\text{Prob}\left(x > v_2 | v_2 < v_1\right)\left(2v_1 - x\right) = \min\left\{\frac{x}{v_1}\ ;\ 1\right\}\left(2v_1 - x\right),$$

which is maximized for $x = v_1$. Consequently, both players bid their first-unit valuation in the second auction, i.e. $b_2^l(v, p_1) = b_2^w(v, p_1) = v$.

We now study the first auction. Assume that player 2 bids $b_1(v_2)$ in the first auction and v_2 in the second auction. Suppose first that player 1 bids $b_1(x) > b_1(v_1)$. If player 1 wins the first auction he learns that $v_2 < x$. When player 1 bids y in the second auction, his expected gain in the second auction is therefore $\min\left\{\frac{y}{x}; 1\right\}(2v_1 - y)$. This gain is maximized for $y = v_1$ and equals v_1^2/x. His expected total gain in this case is $(v_1 - b_1(x) + v_1^2/x)$. On the other hand, if player 1 loses the first auction then he learns that $v_2 > x > v_1$ and he loses the second auction as well and his expected gain is zero. His expected total gain is zero. Therefore, when bidding $b_1(x) > b_1(v_1)$, player 1's expected total gain is

$$G(x) = x\left(v_1 - b_1(x) + v_1^2/x\right) + (1 - x)\,0.$$

This expected gain must be maximized in equilibrium for $x = v_1$. The first order condition leads to $v_1 - b_1(v_1) - v_1 b_1'(v_1) \leq 0$. Suppose, now, that player 1 bids $b_1(x) < b_1(v_1)$. If he wins the first auction he learns that $v_2 < x < v_1$ and when he bids y his expected second auction gain is $\min\left\{\frac{y}{x}\ ;\ 1\right\}(2v_1 - y)$, which is maximized at $y = x$. On the other hand, if player 1 loses the first auction he learns the value of v_2. If $v_2 > v_1$, he also loses the second auction as the equilibrium strategy of player 2 is to bid v_2 in the second auction. If $v_2 < v_1$, he wins the second auction by bidding just above v_2. Therefore, when bidding $b_1(x) < b_1(v_1)$, player 1's expected total gain is

$$G(x) = x[v_1 - b_1(x) + 2v_1 - x] + \int_x^{v_1} (v_1 - w)\,dw + (1 - v_1)\,0.$$

The first order condition leads to $v_1 - b_1(v_1) - v_1 b_1'(v_1) \geq 0$. Therefore, combining both first order conditions, the equilibrium first auction strategy satisfies:

$$v_1 - b_1(v_1) - v_1 b_1'(v_1) = 0 \text{ which leads to } b_1(v_1) = \frac{v_1}{2}. \textit{ Q.E.D.}$$

□ **The case $\mathbf{a} \in \{\mathbf{E},\mathbf{S}\}$, $\mathbf{o} = \mathbf{Y}$, $\mathbf{k} \in \left\{\frac{1}{2}, 1, 2\right\}$.** The second auction strategies are obtained by the standard dominated strategies argument: each player bids his valuation $b_2^l(v, p_1) = v$ and $b_2^w(v, p_1) = kv$.

The case $a = E, k = \frac{1}{2}$. We start with the buyer's option. If $\frac{v_1}{2} \geq p_1 = b_1(v_2)$, it is profitable to use the option because if the winner does not execute the option his gain in the second auction is $\max\left\{0, \frac{v_1}{2} - v_2\right\}$, which is lower than $\frac{v_1}{2} - b_1(v_2)$. On the contrary, if $\frac{v_1}{2} < p_1 = b_1(v_2)$, it is clear that the winner must not use the option.

We next study the first auction. As it will become clear later we can restrict ourselves to the search of a first auction equilibrium $b_1(v) \geq \frac{v}{2}$. Suppose the clock has reached p and player 1 has to decide to continue or to stop bidding. It is important to remark that as player 2 is still active at p, his valuation is greater than $b_1^{-1}(p)$. To derive the equilibrium necessary conditions, we assume that p is close to $b_1(v_1)$. Let $G(\varepsilon, p)$ denote the expected total gain if player 1 decides to continue with bidding until $p + \varepsilon$. If player 2 withdraws between p and $p + \varepsilon$, player 1 wins the first auction. As we have assumed that $b_1(v_1) \geq \frac{v_1}{2}$, and that p is close to $b_1(v_1)$, it is not profitable to use the buyer's option. Furthermore, player 1 loses the second auction (indeed, his valuation is divided by two, while the valuation of player 2 remains around v_1). The expected gain in this case is: $\int_{b_1^{-1}(p)}^{b_1^{-1}(p+\varepsilon)} (v_1 - b_1(w)) \frac{dw}{\overline{v} - b_1^{-1}(p)}$.

If player 2 remains active at $p + \varepsilon$, player 1 loses the first auction. As we have shown above, player 2 uses his option if and only if $p + \varepsilon \leq \frac{v_2}{2}$. In case player 2 does not use the option, we have $\frac{v_2}{2} < p + \varepsilon \simeq b_1(v_1) < v_1$ which means that player 1 wins the second auction. The expected gain in this case is: $\int_{b^{-1}(p+\varepsilon)}^{\min(2(p+\varepsilon),\overline{v})} (v_1 - \frac{w}{2}) \frac{dw}{\overline{v} - b_1^{-1}(p)}$. The expected gain is therefore

$$G(\varepsilon, p) = \int_{b_1^{-1}(p)}^{b_1^{-1}(p+\varepsilon)} (v_1 - b_1(w)) \frac{dw}{\overline{v} - b_1^{-1}(p)} + \int_{b^{-1}(p+\varepsilon)}^{\min(2(p+\varepsilon),\overline{v})} (v_1 - \frac{w}{2}) \frac{dw}{\overline{v} - b_1^{-1}(p)}.$$

The equilibrium condition is $\frac{\partial G}{\partial \varepsilon}(\varepsilon = 0, p = b_1(v_1)) = 0$. When $b_1(v_1) \leq \frac{\overline{v}}{2}$ this leads to:
$\frac{1}{b_1'(v_1)}(\frac{v_1}{2} - b_1(v_1)) + 2(v_1 - b_1(v_1)) = 0$. On the contrary, if $b_1(v_1) \geq \frac{\overline{v}}{2}$ we obtain: $\frac{1}{b_1'(v_1)}(\frac{v_1}{2} - b_1(v_1)) = 0$.

This second differential equation combined with the assumption $b_1(v) \geq \frac{v}{2}$ implies that $b_1(\overline{v}) = \frac{\overline{v}}{2}$. The first differential equation and this terminal condition define a unique bidding function which verifies $b_1(v) \geq \frac{v}{2}$.
To end the proof, it is necessary to show that this function constitutes indeed a Nash equilibrium of the game by checking that there is no profitable deviation, which is straightforward. *Q.E.D.*

The case $a = S, k \in \left\{\frac{1}{2}, 1\right\}$. See Black and De Meza (1992). *Q.E.D.*

The case $a = S, k = 2$ or $a = E, k \in \{1, 2\}$. For $a \in \{E, S\}$, $k = 2$, the proof is identical to the proof without buyer's option because it is optimal not to use the buyer's option. Indeed, assume that both players bid $2v$ and that $v_1 > v_2$. Player 1 wins the first auction and the price $p_1 = 2v_2$. If player 1 uses the option he pays the second unit $2v_2$, while if he waits he only has to pay v_2. For $a = E$, $k = 1$, the proof is also identical to the one without buyer's option since the first-auction winner is indifferent between exercising the option or not. *Q.E.D.*

□ **The case $\mathbf{a} \in \{\mathbf{D},\mathbf{F}\}$, $\mathbf{o} = \mathbf{Y}$, $\mathbf{k} \in \left\{\frac{1}{2}, 1, 2\right\}$.** The second auction strategies are obtained from Février (2000) (proposition 4.5). The second auction strategy for the loser of the first auction is

$$b_2^l(v, p_1) = \begin{cases} v & \text{if } v \leq \frac{2k}{1+k} p_1 \\ \frac{kp_1}{1+k}\left(1 - \frac{4kp_1}{(1+k)v}\right) & \text{if } v \geq \frac{2k}{1+k} p_1 \end{cases}$$

The winner of the first auction plays the following strategy: If $p_1 = b_1(v)$ (that is if he played in the first auction according to the equilibrium strategy but he did not use the option), he plays a mixed strategy, such that he bids x, $x \in \left[\frac{kv}{2}, \frac{k-\frac{k^2}{4}}{v}\right]$, with x having the distribution function

$$F(x) = \frac{1-k+\frac{k^2}{4}}{1-\frac{k}{2}} \frac{kv}{2x-kv} \exp\left[\frac{4x-2\left(2k-\frac{k^2}{2}\right)v}{(2x-kv)(2-k)}\right].$$

If $p_1 > b_1(v)$ (that is he played in the first auction a bid above the equilibrium strategy, won the auction and did not use the option) then $b_2^w(v, p_1) = \frac{k}{2}v$. If $p_1 < b_1(v)$ (that is he played in the first auction a bid below the equilibrium strategy, won the auction and did not use the option) then $b_2^w(v, p_1) = \frac{4k-k^2}{1+k} p_1$.

Consider now the first auction. Assume that player 2 bids $b_1(v_2)$. If player 1 bids $b_1(x)$ and uses his option, then his expected gain is:

$$G(x) = x[(1+k)v - 2b_1(x)].$$

The first-order condition is

$$G'(v) = 0 \Leftrightarrow \frac{1+k}{4}v^2 = (vb_1(v))', \text{ which leads to } b_1(v) = \frac{1+k}{4}v.$$

See Février (2000) for the proof that given the strategies described above, it is not profitable to deviate in the first auction and to abstain from using the buyer's option. *Q.E.D.*

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Table 1: Sessions

Session	Type of Auction	Number of Subjects
1	First-price	8
2	Second-price	8
3	Dutch	6
4	English	6
5	Dutch	10
6	English	8
7	First-price	8
8	Second-price	8
9	Dutch	6
10	English	6

Février-Linnemer-Visser, Table 1 of 7

Table 2: First Auction

Type	#Obs.	Without Buyer's Option: Estim. (Std. Err.)	Without Buyer's Option: Prediction	Without Buyer's Option: Accepted	With Buyer's Option: Estim. (Std. Err.)	With Buyer's Option: Prediction	With Buyer's Option: Accepted
			$k = 0.5$			$k = 0.5$	
F	64	0.39 (0.04)	∅	∅	0.46 (0.03)	0.375	No
S	64	0.76 (0.05)	0.5	No	1.05 (.15) -.004 (.003)	$0.99 - 0.009$	No
D	88 (44)	0.42 (0.03)	∅	∅	0.38 (0.03)	0.375	Yes
E	80 (40)	0.62 (0.04)	0.5	No	1.07 (.10) -.008 (.002)	$0.99 - 0.009$	No
			$k = 1$			$k = 1$	
F	64	0.45 (0.06)	∅	∅	0.56 (0.02)	0.5	No
S	64	1.00 (0.03)	1	Yes	1.05 (0.04)	1	Yes
D	88 (44)	0.42 (0.03)	∅	∅	0.52 (0.02)	0.5	Yes*
E	80 (40)	0.94 (0.03)	1	Yes	0.98 (0.02)	1	Yes
			$k = 2$			$k = 2$	
F	64	0.96 (0.06)	0.5	No	0.80 (0.04)	0.75	Yes
S	64	1.31 (0.05)	2	No	1.56 (0.05)	2	No
D	88 (44)	0.70 (0.04)	0.5	No	0.67 (0.05)	0.75	No
E	80 (40)	1.61 (0.07)	2	No	1.42 (0.07)	2	No

Notes: Column headed "#Obs." gives the number of regression observations (when $a \in \{D,E\}$ the number of uncensored observations are in parentheses). Column headed "Estim. (Std. Err.)" gives $\hat{\beta_1}$ ($\hat{\beta_1}$ and $\hat{\beta_2}$ when $a \in \{E,S\}, k = \frac{1}{2}, o = Y$), and standard errors in parentheses. Column headed "Prediction" gives the predicted values of β_1 (β_1 and β_2 when $a \in \{E,S\}, k = \frac{1}{2}, o = Y$). Column headed "Accepted" indicates "Yes" if the theoretical prediction is accepted at the 5% significance level, "Yes*" if it is accepted at the 1% level (but not at the 5% level), and "No" otherwise. The null hypothesis, $\beta_0 = 0$ and $\beta_1 =$ theoretical prediction, is tested using the Wald test (when $a \in \{E,S\}, k = \frac{1}{2}, o = Y$, the null hypothesis is $\beta_0 = 0, \beta_1 = 0.99$ and $\beta_2 = -0.009$).

Février-Linnemer-Visser, Table 2 of 7

Table 3: Second Auction Without Buyer's Option

	Winners of the First Unit				Losers of the First Unit			
Type	#Obs.	Estim. (Std. Err.)	Prediction	Accepted	#Obs.	Estim. (Std. Err.)	Prediction	Accepted
			$k = 0.5$				$k = 0.5$	
F	32	0.42 (0.02)	∅	∅	32	0.50 (0.05)	∅	∅
S	32	0.52 (0.04)	0.5	Yes	32	0.97 (0.05)	1	Yes
D	44 (26)	0.42 (0.04)	∅	∅	44 (18)	0.37 (0.05)	∅	∅
E	40 (14)	0.42 (0.07)	0.5	Yes	40 (26)	0.99 (0.04)	1	Yes
			$k = 1$				$k = 1$	
F	32	0.55 (0.05)	∅	∅	32	0.66 (0.04)	∅	∅
S	32	0.99 (0.04)	1	Yes	32	0.99 (0.05)	1	No
D	44 (29)	0.45 (0.05)	∅	∅	44 (15)	0.40 (0.06)	∅	∅
E	40 (2)	- (-)	1	-	40 (38)	1.00 (0.08)	1	Yes
			$k = 2$				$k = 2$	
F	32	0.92 (0.05)	1	Yes*	32	0.96 (0.07)	1	Yes
S	32	2.29 (0.36)	2	Yes	32	0.97 (0.08)	1	Yes*
D	44 (39)	0.84 (0.05)	1	No	44 (5)	0.86 (0.08)	1	Yes
E	40 (0)	- (-)	2	-	40 (40)	1.27 (0.08)	1	No

Notes: Column headed "#Obs." gives the number of regression observations (when $a \in \{D, E\}$ the number of uncensored observations are in parentheses). Column headed "Estim. (Std. Err.)" gives $\hat{\beta}_1$, and standard errors in parentheses. Column headed "Prediction" gives the predicted values of β_1. Column headed "Accepted" indicates "Yes" if the theoretical prediction is accepted at the 5% significance level, "Yes*" if it is accepted at the 1% level (but not at the 5% level), and "No" otherwise. The null hypothesis, $\beta_0 = 0$ and $\beta_1 =$ theoretical prediction, is tested using the Wald test.

Février-Linnemer-Visser, Table 3 of 7

Table 4: The Buyer's Option

Type	Absolute (Relative) Frequency	Prediction
	$k = 0.5$	
F	10 out of 12 (83%) if $p_1 \leq 0.5v$, else 3 out of 20 (15%)	$bo(p_1) = 1$ if $p_1 \leq 0.5v$, else 0
S	13 out of 14 (93%) if $p_1 \leq 0.5v$, else 0 out of 18 (0%)	$bo(p_1) = 1$ if $p_1 \leq 0.5v$, else 0
D	14 out of 26 (54%) if $p_1 \leq 0.5v$, else 0 out of 18 (0%)	$bo(p_1) = 1$ if $p_1 \leq 0.5v$, else 0
E	27 out of 28 (96%) if $p_1 \leq 0.5v$, else 0 out of 12 (0%)	$bo(p_1) = 1$ if $p_1 \leq 0.5v$, else 0
	$k = 1$	
F	29 out of 32 (91%) if $p_1 \leq v$, else 0 out of 0 (0%)	$bo(p_1) = 1$ if $p_1 \leq v$, else 0
S	25 out of 31 (81%) if $p_1 \leq v$, else 0 out of 1 (0%)	$bo(p_1) \in [0,1]$ if $p_1 \leq v$, else 0
D	37 out of 44 (84%) if $p_1 \leq v$, else 0 out of 0 (0%)	$bo(p_1) = 1$ if $p_1 \leq v$, else 0
E	27 out of 40 (68%) if $p_1 \leq v$, else 0 out of 0 (0%)	$bo(p_1) \in [0,1]$ if $p_1 \leq v$, else 0
	$k = 2$	
F	26 out of 32 (81%)	$bo(p_1) = 1$
S	22 out of 32 (69%)	$bo(p_1) = 0$
D	43 out of 44 (98%)	$bo(p_1) = 1$
E	28 out of 40 (70%)	$bo(p_1) = 0$

Février-Linnemer-Visser, Table 4 of 7

Table 5: Seller's Revenue

Type	Without Buyer's Option			With Buyer's Option		
	Observed	Predicted	Accepted	Observed	Predicted	Accepted
		$k = 0.5$			$k = 0.5$	
F	31.41 (2.00)	∅	∅	33.98 (2.09)	26.77 (1.24)	No
S	27.52 (2.45)	22.43 (2.26)	No	29.15 (3.14)	24.14 (2.52)	No
D	29.81 (1.75)	∅	∅	27.07 (1.93)	24.69 (1.33)	Yes
E	20.12 (2.46)	16.46 (1.85)	No	24.56 (2.70)	22.62 (2.19)	Yes
		$k = 1$			$k = 1$	
F	46.46 (2.80)	∅	∅	39.85 (2.69)	33.24 (2.33)	No
S	33.22 (3.75)	31.65 (3.75)	Yes	37.48 (4.93)	35.71 (5.11)	No
D	37.82 (1.74)	∅	∅	37.86 (2.51)	33.78 (2.03)	No
E	31.10 (3.33)	30.53 (3.40)	Yes	30.27 (3.69)	30.95 (3.72)	Yes
		$k = 2$			$k = 2$	
F	70.91 (4.79)	50.50 (3.16)	No	59.49 (4.48)	47.95 (3.72)	No
S	38.91 (4.13)	42.89 (5.42)	Yes	45.49 (5.94)	49.04 (6.63)	Yes*
D	64.43 (3.81)	52.59 (2.81)	No	63.87 (2.63)	53.11 (2.51)	No
E	40.95 (5.45)	48.80 (5.58)	No	39.84 (4.19)	47.23 (5.00)	No

Notes: Column headed "Observed" gives the mean of observed revenues (standard errors in parentheses). Column headed "Predicted" gives the mean of predicted revenues conditional on realized valuations (standard errors in parentheses). Column headed "Accepted" indicates "Yes" if the theoretical prediction is accepted at the 5% significance level, "Yes*" if it is accepted at the 1% level (but not at the 5% level), and "No" otherwise.

Février-Linnemer-Visser, Table 5 of 7

Table 6: Price Variation $p_2 - p_1$

Type	Without Buyer's Option			With Buyer's Option		
	Observed	Predicted	Accepted	Observed	Predicted	Accepted
		$k = 0.5$			$k = 0.5$	
F	-1.09 (1.17)	∅	∅	-3.74 (1.00)	0.00 (0.00)	No
S	-1.18 (1.28)	4.38 (0.55)	No	-5.29 (1.11)	-1.72 (0.42)	No
D	-3.53 (0.81)	∅	∅	-3.31 (0.64)	0.00 (0.00)	No
E	0.57 (0.75)	3.51 (0.51)	No	-1.00 (0.45)	-1.02 (0.35)	Yes
		$k = 1$			$k = 1$	
F	0.99 (1.49)	∅	∅	-0.18 (0.14)	0.00 (0.00)	Yes
S	0.75 (0.83)	0.00 (0.00)	Yes	-0.07 (0.13)	0.00 (0.00)	Yes
D	1.40 (0.43)	∅	∅	-0.94 (0.40)	0.00 (0.00)	No
E	-0.39 (0.49)	0.00 (0.00)	Yes	-0.72 (0.77)	0.00 (0.00)	Yes
		$k = 2$			$k = 2$	
F	2.02 (2.24)	16.83 (1.05)	No	-0.71 (0.34)	0.00 (0.00)	No
S	0.41 (1.28)	-13.97 (1.73)	No	-0.30 (0.89)	-16.34 (2.21)	No
D	1.95 (1.03)	17.53 (0.94)	No	-0.07 (0.07)	0.00 (0.00)	Yes
E	0.11 (1.50)	-16.03 (1.87)	No	-3.77 (1.73)	-15.74 (1.67)	No

Notes: Column headed "Observed" gives the mean of the difference in observed prices (standard errors in parentheses). Column headed "Predicted" gives the mean of the difference in predicted prices conditional on realized valuations (standard errors in parentheses). Column headed "Accepted" indicates "Yes" if the theoretical prediction is accepted at the 5% significance level, and "No" otherwise.

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Table 7: Relative Efficiency

	Without Buyer's Option		With Buyer's Option	
Type	Observed	Predicted	Observed	Predicted
		$k = 0.5$		$k = 0.5$
F	0.98 (0.02)	∅	0.96 (0.03)	0.96 (0.01)
S	0.99 (0.01)	1.00 (0.00)	0.99 (0.01)	0.99 (0.00)
D	0.98 (0.01)	∅	0.94 (0.03)	0.95 (0.01)
E	0.99 (0.01)	1.00 (0.00)	0.98 (0.02)	0.99 (0.00)
		$k = 1$		$k = 1$
F	0.95 (0.02)	∅	0.98 (0.01)	1.00 (0.00)
S	0.99 (0.01)	1.00 (0.00)	0.99 (0.01)	1.00 (0.00)
D	0.94 (0.01)	∅	0.97 (0.01)	1.00 (0.00)
E	0.99 (0.01)	1.00 (0.00)	1.00 (0.00)	1.00 (0.00)
		$k = 2$		$k = 2$
F	0.96 (0.02)	1.00 (0.00)	0.98 (0.01)	1.00 (0.00)
S	0.99 (0.01)	1.00 (0.00)	0.98 (0.02)	1.00 (0.00)
D	0.95 (0.02)	1.00 (0.00)	1.00 (0.00)	1.00 (0.00)
E	0.99 (0.01)	1.00 (0.00)	1.00 (0.00)	1.00 (0.00)

Notes: Column headed "Observed" gives the mean of the observed relative efficiency (standard errors in parentheses). Column headed "Predicted" gives the mean of the predicted relative efficiency conditional on realized valuations (standard errors in parentheses).

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