Optimal Financial Structure and Asset Prices

Johan Hombert*  

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Abstract

I study the welfare properties of competitive equilibria in an economy with financial frictions. In the model, entrepreneurs raise funds to set up a firm, then they exert effort, and finally they trade assets. Private financial contracts do not internalize their impact on asset prices. On the one hand, lower prices depress firms' liquidation value, which reduces the pledgeable income – the collateral effect. On the other hand, lower prices boost entrepreneurs' incentives to achieve good performance, which alleviates the moral hazard problem and raises the pledgeable income – the incentive effect. I show that the latter effect outweighs the former, implying that a decrease in asset prices improves welfare. Even though, from a first-best perspective the competitive equilibrium displays too low asset prices, from a second-best point of view prices are too high.

1 Introduction

Both developed and emerging countries have experienced severe financial and banking crises since at least the 1980s, as illustrated by the ongoing turmoil. These episodes are characterized by a collapse in asset prices, credit and investment. The propagation mechanisms through which plummeting asset prices weaken firms’ balance sheet and worsen credit rationing, have received extensive attention.\(^1\) The perceived harm of these episodes has raised the issue of policy intervention, either preventive (e.g., prudential regulation, monetary policy) or curative (e.g., public bailouts, lender of last resort).\(^2\) However, policy actions that reduce the losses of poor performers are criticized on the ground that they create moral hazard. Any intervention on financial markets should therefore tradeoff the benefits of sustaining the value of collateral against the adverse impact on incentives. Besides, any rationale for public intervention has to include a theory of why private decisions can lead to socially inefficient outcomes.

This paper investigates the tradeoff between collateral and incentives, and exhibit a pecuniary externality that operates through the price of assets. I analyze constrained efficiency by considering a social planner who faces the same constraints as the private economy, and asking whether a modification of financial contracts can lead to a welfare improvement. My

*ENSAE and CREST

\(^1\)The idea of focusing on the general equilibrium feed-back between financial distress and asset prices goes back to Shleifer and Vishny (1992) and Kiyotaki and Moore (1997). See also the subsequent literature that I discuss later on.

main result is that asset prices are too high in the equilibrium. Even though, from a first-best perspective the competitive equilibrium displays too low asset prices, from a second-best point of view prices are too high.

My first contribution is to extend Innes (1990)' model of financing under moral hazard with continuous action and continuous payoff, to the case of a continuous and variable size of the firm, and, more importantly, to relax the assumption that the density of the payoff conditional on the level of effort has the monotonic likelihood ratio property (MLRP). This assumption means that more effort increases the probability of high payoffs, and all the more so for the highest payoffs. Innes (1990) establishes that a debt contract is optimal in that case. I show that this remains true when the scale of investment can be chosen before effort is put in, and the firm can be resized once the quality of the firm is known.

When the MLRP is satisfied, the moral hazard problem is an effort problem. This rules out, for instance, the possibility of risk-shifting. I relax the MLRP assumption and show that the optimal financial contract specifies an investors’ payoff function that is piecewise linear with a slope switching between 0 and 1. The debt contract is one special case of this optimal contract, which arises under a slightly weaker assumption than required by Innes (1990). When there is risk-shifting, the optimal contract can be implemented with a combination of outside senior debt and equity, and inside junior debt. As suggested by Jensen and Meckling (1976) and documented by Kaplan and Strömbärg (2003) in the venture capital industry, this can also be done with convertible securities.

My second contribution is to plug this optimal contracting setting in a general equilibrium framework, and to show that the competitive economy is not constrained efficient. There is a continuum of ex ante identical firms, which are subject to idiosyncratic shocks. Once firms learn their quality, they can adjust their scale of operation by trading assets between each others on a spot market. Since firms are atomistic, they do not take into account the general equilibrium effect of their financial structure on prices, which opens the door to a pecuniary externality.

Lower prices reduce firms’ liquidation value. The flip side of the coin is that they make the expansion of successful firms cheaper. Generally speaking, the optimal financial contract allocates cash flow rights to investors in case of bad performance – and liquidation, and to managers in case of good performance – and firm expansion. Therefore, a low price depresses the investors’ revenue in case of liquidation. This collateral effect tends to reduce the pledgeable income. On the other hand, a low price boosts the manager’s incentives to achieve good performance and purchase additional assets. This incentive effect relaxes the moral hazard problem inside the firm. I show that, in most cases, the incentive effect dominates the collateral effect. Therefore, a social planner can improve welfare with a decrease of asset prices. Since firms are ex ante identical, this also a Pareto improvement from an ex ante point of view.

This paper is related to several strands of literature. First, it contributes to the literature on optimal capital structure. Innes (1990) shows that when the probability density function of the payoff conditional on effort has the MLRP, the optimal contract is a debt contract. Biais and Casamatta (1999) build a double moral hazard model including effort and risk-shifting, with discrete actions and payoff, and show that, the optimal contract can be implemented with a combination of debt and equity when risk-shifting is main source of moral hazard.

This paper also contributes to the literature that focuses on the interactions between asset prices and financial frictions. Kiyotaki and Moore (1997) exhibit mechanisms through which
price drops weaken firms’ balance sheet when they need collateral the most, leading to the amplification of macroeconomic shocks. Shleifer and Vishny (1997) make a similar point in the money management industry to account for limits of arbitrage. Krishnamurthy (2003) shows that these results are not robust to the use of state-contingent contracts, but can be restored if hedging is limited by the limited liability of financial intermediaries.

This work is closely related to recent papers that study welfare from a second-best perspective and identify an externality of firms’ financial decisions. Caballero and Krishnamurthy (2001, 2003) start from the insight that the interest rate is lower than the marginal product of capital in the presence of financial frictions. They show that this provides firms with incentives to have a weak balance sheet, which results in the underprovision of liquidity, or, equivalently in their model, to excessive foreign debt. Gromb and Vayanos (2002) develop a model with financially constrained arbitrageurs and unconstrained investors, making it welfare improving to transfer wealth from the latter towards the former. They show that, if arbitrageurs are expected to reduce their positions, then they are hurt by price drops and the asset price is socially too low, while the opposite holds if they are expected to increase their positions. In their model, the pecuniary externality through the asset price can thus work in either direction, implying that arbitrageurs may too much or not enough risk. In Lorenzoni (2008) and Hombert (2007), firms in a financially constrained sector are expected to sell assets to firms in an unconstrained sector during fire sale episodes. They show that the competitive equilibrium displays too low asset prices and that firms take on too much debt. This category of models includes aggregate uncertainty. However, it can be shown that the pecuniary externalities they exhibit would persist with idiosyncratic risk only. To clarify this point, I write my model with no aggregate uncertainty.

The novelty of my paper is that I endogeneize the fundamental value of assets, whereas the previous papers focus on the incentives to build up liquidity reserves, and take as given the fundamental value of assets. In terms of modelling strategy, I assume that moral hazard takes place before the shock, instead of after the shock. Besides, I neutralize liquidity issues by assuming that credit markets are frictionless once uncertainty is resolved. Endogenous riskiness of assets can also be found in the banking literature, as in Freixas, Parigi and Rochet (2004). However, they assume that banks are run by a their owners and not by managers, which rules out agency issues. In Rochet and Tirole (1996) and Holmström and Tirole (2000), there is moral hazard inside the firm that determines the quality of assets, and thus the decision to liquidate the bank’s portfolio. However, there is no asset market and liquidated assets are lost.

Section 2 presents the basic model. The optimal financial structure is presented in Section 3 and the asset market equilibrium in Section 4. In Section 5, I discuss the main assumptions of the model and propose several extensions.

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3The idea that the competitive equilibrium in economies with financial frictions can be constrained inefficient goes back to Kehoe and Levine (1993).

4It would nevertheless introduce one subtle difference in Gromb and Vayanos (2003) and Lorenzoni (2008). With aggregate uncertainty, ex ante Pareto improvements are possible through transfers across states of nature, whereas only utilitarian welfare improvements exist in a deterministic economy. This distinction does not arise in Caballero and Krishnamurthy (2001, 2003) nor in the present paper, since firms are ex ante identical.
2 Model

There is a mass one continuum of identical risk-neutral entrepreneurs. Each entrepreneur has a project and has initial wealth $A$. There are also competitive risk-neutral investors with a sufficiently large amount of wealth. There are three periods $t = 1, 2, 3$.

At date 1, each entrepreneur raises funds from investors to set up his project (financial contracts are described later on). The scale of the project, $k > 0$, is a continuous variable that can be freely selected. The technology has constant returns to scale with respect to the initial size of the project. The entrepreneur then exerts an effort, $e$, that is also a continuous real variable with support $[\underline{e}, \bar{e}]$. The cost of providing effort $e$ is proportional to the firm size: $ek$.

At date 2, the quality of the project, $\theta$, is revealed. $\theta$ is distributed on $[\underline{\theta}, \bar{\theta}]$, $\underline{\theta} > 0$, with cumulative distribution $F(\theta|e)$ and twice continuously differentiable density $f(\theta|e)$. I assume that, for any $e \in [\underline{e}, \bar{e}]$, $F_{ee}(\theta|e) > 0$ for all $\theta \in (\underline{\theta}, \bar{\theta})$, and $\{\theta : (-F/e/(1 - F))(\theta|e) = h\}$ is of measure zero for all $h > 0$. A higher level of effort improves the quality of the firm in a sense that I shall precise at the end of the Section. Besides, $\theta$ is independent across firms, therefore there is idiosyncratic risk only.

Then, the firm can be rescaled to size $k' \geq 0$ by trading assets with the other firms at the (endogenous) asset price $p$. Returns to scale with respect to the size expansion between date 1 and date 2 are decreasing. Formally, the project pays off $\theta v(k'/k)k$ at date 3, with $v(0) = 0$, $v' > 0$, $v'' < 0$, $v'(0) = +\infty$ and $v'(+\infty) = 0$. Normalizing the discount rate to zero, the net present value of the project, net of the effort cost, is equal to

$$[-1 + p + \theta v(s) - ps - e]k,$$

where

$$s = \frac{k'}{k}$$

is the resizing factor.

The contract between the entrepreneur and the investor specifies the initial investment, $k$, as well as the resizing strategy, $s(\theta)$, and the entrepreneur’s wage, $w(\theta)$, as functions of the quality of the project. The entrepreneur is limited liable, thus

$$w(\theta) \geq 0 \quad \forall \theta. \quad (1)$$

Then, I assume two key contractual frictions (Innes, 1990). First, the entrepreneur’s effort is not verifiable. Second, the actual quality of the firm $\theta$ is not verifiable and the entrepreneur can manipulate the final payoff $\theta v(s(\tilde{\theta}))k$. Precisely, he can (i) secretly borrow to increase the final payoff or (ii) sabotage the firm to decrease the final payoff. By the revelation principle, $s$ and $w$ can nevertheless be written as functions of $\theta$. They must satisfy the incentive compatibility constraints that the entrepreneur does not want to borrow $(\tilde{\theta} - \theta)v(s(\tilde{\theta}))k$ to announce $\tilde{\theta} > \theta$,

$$w(\theta) \geq w(\tilde{\theta}) - (\tilde{\theta} - \theta)v(s(\tilde{\theta}))k \quad \forall \tilde{\theta} > \theta, \quad (2)$$

nor does he want to sabotage the firm and announce $\tilde{\theta} < \theta$,

$$w(\theta) \geq w(\tilde{\theta}) \quad \forall \tilde{\theta} < \theta. \quad (3)$$

Finally, I look for renegotiation-proof contracts. Therefore, for all $\theta$, $s(\theta)$ must maximize $[\theta v(s) - ps]k$. The Inada conditions on $v(.)$ ensure that the solution, which I shall also denote by $s(\theta)$ to save notation, is interior and satisfies the first order condition

$$v'(s(\theta)) = \frac{p}{\theta} \quad \forall \theta.$$
To conclude that Section, I specify in which sense the effort improves \( \theta \). The net present value of continuing a firm of quality \( \theta \) is linear in \( k \). I denote it by \( \pi(\theta)k \), where

\[
\pi(\theta) = [\theta v(s(\theta)) - ps(\theta)].
\]

The expected net present value, net of the effort cost, of a project implemented with effort \( e \) is also linear in \( k \). I denote it by \( V(e)k \), where

\[
V(e) = -1 + p + E[\pi(\theta)|e] - e.
\]

Noticing that \( \pi(\cdot) \) and \( s(\cdot) \) are increasing functions of \( \theta \), the following assumption states that a higher effort improves the firm’s quality.

**Assumption 1.** (a) \( V(e) \) is strictly quasiconcave with an interior maximum \( e^{FB} \), and there exists \( e_{\text{inf}} < e^{FB} \) such that \( V(e_{\text{inf}}) = 0 \). (b) \( E[s(\theta)|e] \) is strictly increasing in \( e \).

Assumption 1-(a) means that a higher effort increases the expected NPV from 0 when the level of effort is equal to \( e_{\text{inf}} \), to its maximum level when the first-best level of effort \( e^{FB} \) is attained. When the level of effort increases above \( e^{FB} \), the marginal cost of effort outweighs the marginal improvement in the quality of the project. Assumption 1-(b) states that effort increases the expected date 2-resizing factor.

### 3 Optimal Financial Structure

#### 3.1 The Problem

The optimal contract between the entrepreneur and the investor consists in an initial investment, a level of effort, and a compensation scheme for the entrepreneur. Since firms are atomistic, they take the asset price as given. The optimal contract solves the following program, denoted by \( \mathcal{P} \):

\[
\max_{k, e, w(\cdot)} \left[ -1 + p + \int_{\theta}^{\bar{\theta}} \pi(\theta)f(\theta|e)d\theta - e \right] k, \quad (4)
\]

s.t.

\[
-1 + p + \int_{\theta}^{\bar{\theta}} \pi(\theta)f(\theta|e)d\theta \left[ k - \int_{\theta}^{\bar{\theta}} w(\theta)f(\theta|e)d\theta \right] \geq -A, \quad (5)
\]

\[
e = \arg \max_{\tilde{e}} \int_{\theta}^{\bar{\theta}} w(\theta)f(\theta|\tilde{e})d\theta - \tilde{e}k, \quad (6)
\]

The objective function is the expected NPV, \( V(e)k \). The participation constraint (5) imposes that the investor’s profit is nonnegative. The incentive compatibility constraint (6) states that the entrepreneur’s effort is determined by its compensation scheme.

Program \( \mathcal{P} \) can be simplified after some manipulations. First, there are constant returns to scale with respect to initial investment \( k \). Formally, the program is homogeneous of the
first order in \( k \) and \( w(.) \) (and \( A \)). It then proves useful to define the compensation of the manager by unit of investment

\[
z(\theta) = \frac{w(\theta)}{k} \quad \forall \theta.
\]

The set of wage functions (by unit of investment) that satisfy the limited liability and manipulability constraints (1), (2) and (3), is

\[
\mathcal{Z} = \left\{ z : [\bar{\theta}, \bar{\theta}] \mapsto \mathbb{R} \text{ such that } z(\theta) \geq 0 \text{ and } 0 \leq \frac{z(\theta_2) - z(\theta_1)}{\theta_2 - \theta_1} \leq v(s(\theta_2)), \forall \theta_1 < \theta_2 \right\}.
\]

Second, the incentive compatibility constraint (6) can be replaced by the corresponding first order condition. Indeed, as formally shown in Appendix A.1, the expected revenue of the entrepreneur, net of the effort cost, is twice continuously differentiable in \( e \) and the second derivative is strictly negative by \( F'_{ee} > 0 \). Problem \( P \) then rewrites as

\[
\max_{k,e,z(.) \in \mathcal{Z}} \left\{ -1 + p + \int_{\bar{\theta}}^{\bar{\theta}} \pi(\theta)f(\theta|e)d\theta - e \right\} k,
\]

s.t.

\[
-1 + p + \int_{\bar{\theta}}^{\bar{\theta}} \pi(\theta)f(\theta|e)d\theta - \int_{\bar{\theta}}^{\bar{\theta}} z(\theta)f(\theta|e)d\theta \geq -A,
\]

\[
\int_{\bar{\theta}}^{\bar{\theta}} z(\theta)f_e(\theta|e)d\theta = 0.
\]

3.2 The Optimal Contract

The power of incentives given to the manager are limited by the manipulability constraint that the entrepreneur does not borrow to announce a higher \( \theta \). This imposes a maximum level of effort that can be implemented.

**Lemma 1.** There exists \( e_{\text{sup}} \in [e^{FB}, \bar{e}] \) such that the set of efforts that can be implemented is \([e, e_{\text{sup}}]\).

**Proof.** See Appendix A.1. \( \square \)

We can restrict our attention to contracts that generate a positive NPV, that is, to \( e \geq e_{\text{inf}} \). For any \( e \in [e_{\text{inf}}, e_{\text{sup}}] \) and \( k > 0 \), I denote by \( z^e(.) \in \mathcal{Z} \) the wage function (by unit of investment) that implements the level of effort \( e \) while maximizing the investor’s profit. I defer the determination of \( z^e(.) \), which is the main contribution of this Section, to the next subsection to streamline the exposition. The term in brackets in equation (8) is the investor’s revenue by unit of investment, which I assume to be strictly negative. Therefore, when the effort is \( e \), the highest level of investment that is compatible with the investor’s rationality and the entrepreneur’s incentives, is

\[
K(e) = \frac{A}{1 - p - \int_{\bar{\theta}}^{\bar{\theta}} (\pi(\theta) - z^e(\theta))f(\theta|e)d\theta}.
\]

I also assume that \( K(.) \) is a strictly decreasing function, which is the case provided that \( F \) is sufficiently convex in \( e \).
Assumption 2. (a) $K(e)$ is strictly decreasing on $e \in [e_{\text{inf}}, e_{\text{sup}}]$, with $K(e_{\text{inf}}) > 0$. (b) $V(e)K(e)$ is strictly concave on $e \in [e_{\text{inf}}, e_{\text{sup}}]$.

The problem consists in maximizing $V(e)K(e)$ on $e \in [e_{\text{inf}}, e_{\text{sup}}]$. On $[e_{\text{inf}}, e^{FB}]$, $V(.)$ is strictly increasing and $K(.)$ is strictly decreasing. There is a tradeoff between quality and size. On the one hand, a higher level of effort increases the average quality of the project. On the other hand, it requires a higher compensation to the entrepreneur, which lowers the investor’s profit by unit of investment, and reduces the scale of the project. The optimal level of effort never rises above $e^{FB}$, since it would reduce the NPV and consume additional pledgeable income.

Proposition 1. $P$ has a unique solution, with effort $e^*(p) \in (e_{\text{inf}}, e^{FB})$ and investment $k^*(p) = K(e^*(p))$.

Proof. Immediate. \hfill \qed

The optimal contract is parameterized by the asset price $p$, since firms take the asset price as given. Note that the participation constraint is always binding. I denote by $\lambda$ the Lagrangian multiplier of the participation constraint (8), and by $\mu$ the Lagrangian multiplier of the incentive compatibility constraint (9).

3.3 Financial Structure

I now turn to the main result of this Section, namely the determination of the optimal wage function. In the following, $e$ denotes the optimal level of effort $e^*(p)$. I keep the notation $e$ to signal that the determination of the wage function that implements a given level of effort while minimizing the expected wage, is valid for any $e \in [e_{\text{inf}}, e_{\text{sup}}]$, not only for $e^*(p)$. Although the program (7)-(8)-(9) is written in terms of wage by unit of investment $z(.)$, it is equivalent to work with the total wage function $w(.) = z(.)k$.

First, $w(\theta)$ is optimally set to 0, since only the slope of the wage function matters for incentives. Indeed, if $w(\theta) > 0$, lowering all the $w(\theta)$ by $w(\theta)$ relaxes the participation constraint without affecting the incentive compatibility constraint.

Then, let me inspect the optimal slope of $w(.)$ for a given $\theta$: What is the effect of increasing $w(t)$ by an infinitesimal amount $\delta w$ for all the $t \in [\theta, \bar{\theta}]$? On the one hand, it lowers the expected revenue of the investor by

$$(1 - F(\theta|e))\delta w.$$ 

On the other hand, it increases the marginal return of effort for the entrepreneur by

$$(-F_e(\theta|e))\delta w.$$ 

Starting from the optimal contract, given the values of the Lagrangian multipliers, increasing $w(t)$ by a small amount for all the $t \in [\theta, \bar{\theta}]$ improves the objective function if

$$-\lambda(1 - F(\theta|e)) - \mu F_e(\theta|e) > 0.$$ (10)

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6Proving the existence of the Lagrangian multipliers in the problem (7)-(8)-(9) requires mathematical results in infinite dimension. Actually, the following analysis shows that the dimensionality of the problem can be reduced to three, which makes it straightforward to see that the Lagrangian multipliers exist.
Conversely, decreasing $w(t)$ by a small amount for all the $t \in [\theta, \bar{\theta}]$ improves the objective function if the inequality is reversed. Besides, the manipulability constraints (2) and (3) imply that the slope of $w(.)$ in $\theta$ is comprised between 0 and the slope of the firm’s value, $v(s(\theta))k$. The optimality of the wage function implies that one cannot increase its slope when inequality (10) holds, nor decrease its slope when the reverse inequality holds. Therefore, in the optimal contract, the slope of $w(.)$ in $\theta$ is equal to $v(s(\theta))k$ when $(-F_e/(1-F))(t|e) > \lambda/\mu$, and to 0 when $(-F_e/(1-F))(t|e) < \lambda/\mu$.$^7$ The interpretation of the ratio $(-F_e/(1-F))(\theta|e)$ is the (percentage) increase of the likelihood of a type better than $\theta$ when the level of effort increases.

**Proposition 2.** The wage function in the optimal contract is given by

$$w(\theta) = \int_{\theta}^{\bar{\theta}} \frac{1 - F_e(t|e^*(\mu))}{t^{|e|}} v(s(t))k dt \quad \forall \theta \in [\theta, \bar{\theta}].$$

(11)

**Proof.** See Appendix A.2.

The analysis leading to Proposition 2 can be extended to any level of effort $e \in [e_l, e_{sup}]$ to show that the wage function that implements $e$ while minimizing the expected wage, is of the form

$$z^e(\theta)k = \int_{\theta}^{\bar{\theta}} \frac{1 - F_e(t|e)}{t^{|e|}} v(s(t))k dt \quad \forall \theta \in [\theta, \bar{\theta}],$$

with $h > 0$. Plugging this expression into the program (7)-(8)-(9) implies that the dimensionality of the problem can be reduced to three, with choice variables $k$, $e$ and $h$, ensuring the existence of the Lagrangian multipliers $\lambda$ and $\mu$. Proposition 2 can be restated in terms of the delta of the entrepreneur, defined as the derivative of the wage $w(\theta)$ with respect to the firm’s value $R = \pi(\theta)k$:

$$\Delta(R) = \frac{dw}{dR} = \begin{cases} 1 & \text{if } (-F_e/(1-F))(\pi^{-1}(R/k)|e) \geq \lambda/\mu, \\ 0 & \text{if } (-F_e/(1-F))(\pi^{-1}(R/k)|e) < \lambda/\mu. \end{cases}$$

(12)

The delta of the entrepreneur is equal to 1 when the costs of increasing the expected wage are outweighed by the benefits of higher effort, which is the case when the ratio $-F_e/(1-F)$ is large.

Proposition 2 generalizes Biais and Casamatta (1999)’s results to continuous levels of effort and payoff. Their model has three possible outcomes, from which they have to extrapolate the (continuous) capital structure.$^8$ I now turn to the implementation of the optimal contract with standard financial instruments. This depends on the shape of the function $(-F_e/(1-F))(.|e)$. Consider first that the ratio $-F_e/(1-F)$ is increasing in $\theta$, as on Figure 1. This means that a higher effort increases (multiplicatively) the probability of a type better than $\theta$ by more

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$^7$Remind that equation (10) holds with an equality for a set of $\theta$s of measure zero. This implies that the slope of $w(.)$ in these points does not matter.

$^8$There are two additional differences between my model and theirs. First, their effort is two-dimensional. I show in Section 5.3 that all my results extend to the case of a multi-dimensional effort. Second, they do not impose any manipulability constraints, which becomes an unreasonable assumption as one switches to continuous payoffs. Indeed, since the entrepreneur is risk-neutral, the optimal contract would be to pay him an infinite wage for $\theta \in \arg \max(.-F_e/(1-F))(t|e)$ and nothing otherwise. I propose in Section 5.6 a modelling strategy to weaken the assumption of profit manipulation.
when $\theta$ is higher. Effort improves the distribution of $\theta$ in the sense of first order stochastic dominance. In that case, the optimal wage function is flat as long as the firm’s value is below a threshold $\theta_0$, and steep with the same slope than the firm’s value above that threshold. This a debt contract with face value $\pi(\theta_0)k$. Therefore, a sufficient condition for the optimal contract to be a debt contract is that $(1 - F)(\theta|e)$ has the monotone likelihood ratio property (MLRP), where a family of functions $\{\varphi(\theta|e)\}$ is said to have the MLRP if $\varphi_e/\varphi$ is increasing in $\theta$ (Milgrom, 1981).

![Figure 1: When $1 - F$ has the MLRP, debt is optimal.](image)

**Proposition 3.** When $(1 - F)(\theta|e)$ has the MLRP, the optimal financial contract is implemented with outside debt.

Innes (1990) establishes that $f$ having the MLRP is a sufficient condition for the optimality of debt. This is a stronger condition than $1 - F$ satisfying the MLRP, since

$$\frac{\partial}{\partial \theta} \frac{-F_e(\theta|e)}{1 - F(\theta|e)} = \frac{f(\theta|e)}{(1 - F(\theta|e))^2} \int_0^{\theta} \left( \frac{f_e(t|e)}{f(t|e)} - \frac{f_e(\theta|e)}{f(\theta|e)} \right) f(t|e) dt.$$ 

The reasoning leading to Proposition 3 uncovers that the economic force that makes debt optimal, is that effort increases (multiplicatively) the likelihood of a quality better than $\theta$ by more when $\theta$ is larger.

The sufficient condition in Proposition 3 can actually be weakened. Indeed, it suffices that $-F_e/(1 - F)$ be increasing in $\theta$ in the neighborhood of the $\theta$s such that $-F_e/(1 - F)(\theta|e) > 0$. For instance, if $-F_e/(1 - F)$ is negative for $\theta$ below a threshold, and positive and increasing above that threshold, as on Figure 2, then a debt contract is still always optimal. This comes from the fact that the wage function is always flat when $F_e > 0$, otherwise it would consume pledgeable income and reduce incentives, hence the monotonicity of $-F_e/(1 - F)$ matters only when it is positive. An interpretation of the ratio $-F_e/(1 - F)$ represented on Figure 2 is that exerting effort consists in taking on more risk. In that case, a debt contract is still appropriate since it provides the manager with a convex payoff function.

Consider now that the ratio $-F_e/(1 - F)$ is hump-shaped in $\theta$, as represented on Figure 3. The economic interpretation is that a low effort consists in risk-shifting. In that case, the optimal wage function is flat for $\theta$ below a threshold $\theta_1$ and above another threshold $\theta_2 > \theta_1$,
when effort involves risk-taking, debt is optimal. Such a financial structure can be implemented with outside debt with face value \( \pi(\theta_1)k \) and outside equity, together with inside junior debt with face value \( (\pi(\theta_2) - \pi(\theta_1))k \). A common form of inside debt are pensions. Yermack and Sundaram (2007) document that pensions represent a significant component of CEOs’ compensation in large U.S. companies. As suggested by Jensen and Meckling (1976), the optimal contract can also be implemented by giving convertible debt to the investor, with face value \( \pi(\theta_1)k \) and a conversion price of \( \pi(\theta_2)k \) for the totality of the shares. Kaplan and Strömberg (2003) document that convertible preferred stock is extensively used in venture capital contracts.

Proposition 4. When \(( -F_e/(1 - F))(\theta|e) \) is hump-shaped, the optimal financial contract
is implemented with a combination of outside debt and equity and inside junior debt, or equivalently with convertible securities.

Note that if $-F_e/(1 - F)$ is hump-shaped, but $(-F_e/(1 - F))(\bar{\theta} | e) > \lambda/\mu$, as on Figure 4, then a debt contract is still optimal. Such a distribution function does not, however, exactly fits a risk-shifting problem. Indeed, effort increases the probability of a type higher than $\theta$ for high $\theta$s, although less than for intermediate $\theta$s.

Figure 4: When $-\frac{F_e}{1 - F}$ is hump-shaped and $-\frac{F_e}{1 - F}(\bar{\theta} | e) > 0$, (a) debt or (b) convertible securities can be optimal.

Finally, one can complexify the shape of the ratio $(-F_e/(1 - F))(\theta | e)$. In the general case, the optimal payoff of the entrepreneur (or, by complementarity, of the investor) is continuous piecewise linear with a slope switching between 0 and 1, as illustrated in Figure 5.

Figure 5: In the general case, the wage function is piecewise linear with a slope switching between 0 and 1.
4 Capital Market Equilibrium

4.1 Competitive Equilibrium

I now solve for the competitive equilibrium of the economy. I define a competitive equilibrium as an asset price and firms’ financial contracts, such that financial contracts are optimal and the asset market clears.

In Section 3, I have determined the optimal financial contract for any given asset price. The optimal initial investment $k^*(p)$ and level of effort $e^*(p)$ depend on the asset price in an ambiguous way. The direct effect of a higher price is to reduce the benefits of effort and to increase the resale value of assets, which tends to decrease $e^*$ and increase $k^*$. However, as shown later on, the asset price affects the firm’s pledgeable value, which modifies the optimal contract in an ambiguous way. These indirect effects potentially make $e^*(p)$ locally increasing in $p$. As will appear shortly, this opens the door to multiple equilibria on the asset market. Although multiple equilibria would not modify the essence of my results, I assume that the direct effect outweighs the indirect ones to streamline the analysis.

Assumption 3. $e^*(p)$ is continuous and strictly decreasing, unless $e^*(p) \in \{e, \pi\}$.

Conversely, for a given financial contract that implements investment $k$ and effort $e$, I can aggregate individual behaviors on the asset market at date 2. Firms with a low $\theta$ liquidate assets, while firms with a high $\theta$ purchase assets. The asset market clears at the price $p^*(e)$, such that

$$\int_{\underline{\theta}}^{\overline{\theta}} (s(\theta, p^*(e)) - 1) f(\theta|e)d\theta = 0. \quad (13)$$

Notice that the equilibrium asset price does not depend on $k$. This comes from the fact that the technology has constant returns to scale with respect to $k$. Therefore, investment at date 1 generates as much demand as supply on the asset market at date 2. Then, equation (13) indicates that the equilibrium price $p^*(e)$ increases with effort. Indeed, more effort increases the number of firms that purchase assets, by Assumption 1-(b), which requires a higher asset price to restore the equilibrium, since $\partial s/\partial p < 0$. Besides, $p^*(e)$ is continuous in $e$ and comprised between $\underline{\theta} v'(1)$ and $\overline{\theta} v'(1)$, since all firms are on the buy side when $s(\underline{\theta}, p) \geq 1$ and on the sell side when $s(\overline{\theta}, p) < 1$.

A competitive equilibrium of the economy is a triplet $(k^{CE}, e^{CE}, p^{CE})$ which satisfies $k^{CE} = k^*(p^{CE})$, $e^{CE} = e^*(p^{CE})$ and $p^{CE} = p^*(e^*(p^{CE}))$. Assumption 3 ensures that the equilibrium is unique. The existence comes from the intermediate value theorem. Indeed, $p^*(e^*(.))$ is continuous, strictly positive and bounded, therefore it has a fixed point.

4.2 Welfare

I now analyze the optimality of the competitive equilibrium. I define welfare as the sum of all the entrepreneurs’ and investors’ profits. In the competitive equilibrium, since investors earn

\[ ^9 \text{Precisely, a higher price reduces the firms’ pledgeable value, which has two effects on the optimal contract. First, it tends to reduce both } k^* \text{ and } e^* \text{ to restore the investor’s participation constraint. Second, since the participation constraint is more binding (the Lagrangian multiplier increases), it becomes more interesting to relax that constraint, which is done by reducing } k^* \text{ and increasing } e^*. \]

\[ ^{10} \text{I now add } p \text{ as an argument in the functions } s(\theta, p), \pi(\theta, p), V(e, p) \text{ and } K(e, p). \]

\[ ^{11} \text{In Assumption 3, it is actually sufficient to have } e^*(p) \text{ strictly increasing in the neighborhood of } p \text{ such that } p = p^*(e^*(p)). \]
zero profit and there is a mass one of identical firms, welfare is equal to a firm’s NPV.

Consider a social planner who, at date 1, chooses the financial contracts \((k, e, w(.))\). She faces the same constraints as the private economy: investors break even, entrepreneurs choose the effort that maximizes their revenue, they are limited liable, they can manipulate \(\theta\), the resizing strategy must be renegotiation-proof and is thus given by \(s(\theta, p)\). The only difference with the individual entrepreneur’s problem is that the social planner takes into account the impact of financial contracts on the equilibrium price on the asset market. Noticing that the strict concavity of \(V(e)K(e)\) implies identical \(k\) and \(e\) for every firm in the optimal allocation, the problem of the social planner writes as

\[
\max_{k, e, w(.), p} \left[ -1 + p + \int_\theta^\theta \pi(\theta)f(\theta|e)d\theta - e \right] k,
\]

subject to the participation constraint of the investor, with Lagrangian multiplier \(\lambda\), and the incentive compatibility of the entrepreneur, with Lagrangian multiplier \(\mu\). After integrating the expected wage by parts, the participation constraint rewrites as

\[
\left[ -1 + p + \int_\theta^\theta \pi(\theta)f(\theta|e)d\theta \right] k - \int_\theta^\theta w(\theta)f(\theta|e)d\theta \geq -A,
\]

limited liability (1) and manipulability constraints (2) and (3),

\[
p = p^*(e).
\]

The maximization program of the social planner makes clear that asset prices are endogenous. By contrast, in the competitive equilibrium, since each firm neglects the impact of its individual behavior on the asset price, there is potentially a pecuniary externality that operates through the asset price. Before stating formally the difference between the competition outcome and the second-best, let me show heuristically how the asset prices in these two allocations compare.

Starting from the competitive equilibrium, what can the social planner do to increase welfare, or equivalently, firms’ NPV? She can change slightly the asset price by modifying slightly financial contracts. Since we started from the competitive equilibrium, financial contracts are optimal. The envelope theorem implies that an infinitesimal variation of the financial contract has no first order effect on a firm’s NPV. Only the variation of the asset price itself has a first order effect. The social planner should therefore modify slightly financial contracts in order to increase the asset price if the (total) derivative of firm’s NPV with respect to \(p\) is strictly positive, and decrease \(p\) if it is strictly negative. To determine the sign of that derivative, remind that the NPV is the maximum of

\[
-1 + p + \int_\theta^\theta \pi(\theta)f(\theta|e)d\theta - e
\]

subject to the participation constraint of the investor, with Lagrangian multiplier \(\lambda\), and the incentive compatibility of the entrepreneur, with Lagrangian multiplier \(\mu\). After integrating the expected wage by parts, the participation constraint rewrites as

\[
\left[ -1 + p + \int_\theta^\theta \pi(\theta)f(\theta|e)d\theta \right] k - \int_\theta^\theta \frac{\pi(\theta)f(\theta|e)}{1 - F(\theta|e)} \geq \frac{\lambda}{\mu} \int_\theta^\theta v(s(\theta, p))k(1 - F(\theta|e))d\theta \geq -A.
\]
Similarly, an integration by parts of the incentive compatibility constraint gives
\[
\int_{\theta}^{\overline{\theta}} 1 - \frac{F_e(\theta|e)}{1 - F(\theta|e)} \geq \frac{\lambda}{\mu} v(s(\theta, p)) k(-F_e(\theta|e)) d\theta = k.
\] (16)

The total derivative of the firm’s NPV with respect to the asset price is therefore equal to
\[
\frac{dNPV}{dp} = \frac{\partial(14)}{\partial p} + \lambda \frac{\partial(15)}{\partial p} + \mu \frac{\partial(16)}{\partial p} = (1 + \lambda) \int_{\theta}^{\overline{\theta}} (1 - s(\theta, p)) k f(\theta|e) d\theta
\]
\[
+ \lambda \int_{\theta}^{\overline{\theta}} 1 - \frac{F_e(\theta|e)}{1 - F(\theta|e)} \geq \frac{\lambda}{\mu} \theta|v''(s(\theta, p))| k(1 - F(\theta|e)) d\theta
\]
\[
- \mu \int_{\theta}^{\overline{\theta}} 1 - \frac{F_e(\theta|e)}{1 - F(\theta|e)} \geq \frac{\lambda}{\mu} \theta|v''(s(\theta, p))| k(-F_e(\theta|e)) d\theta.
\]

The first term is the direct effect of the asset price on the firm’s NPV. Since firms are identical and the asset market clears, each firm sells on average as many assets as it buys, implying that this term is equal to zero. The second term is the effect of the asset price on the participation constraint. The entrepreneur is mostly rewarded when \(\theta\) is high (not exactly in case of risk-shifting). Symmetrically, the investor essentially gets the liquidation value of the firm, therefore a higher price relaxes the participation constraint. This is the collateral effect. The third term is the effect of the asset price on the incentive compatibility constraint. When the asset price increases, the firm’s value for high \(\theta\)s is lower, which makes it more difficult to incentivize the entrepreneur. Therefore, a higher price strengthens the incentive compatibility constraint. When the asset price increases, the firm’s value for high \(\theta\)s is lower, which makes it more difficult to incentivize the entrepreneur. Therefore, a higher price strengthens the incentive compatibility constraint through an incentive effect. Although the collateral effect and incentive effect work in opposite directions, the overall effect can always be signed:
\[
\frac{dNPV}{dp} = \int_{\theta}^{\overline{\theta}} 1 - \frac{F_e(\theta|e)}{1 - F(\theta|e)} \geq \frac{\lambda}{\mu} \theta|v''(s(\theta, p))| k(1 - F(\theta|e)) - \mu(-F_e(\theta|e)) d\theta < 0,
\]
since the integrand is strictly negative (except on a set of measure zero on which it is equal to zero). The incentive effect always dominates the collateral effect. This implies that a small decrease of the asset price below its competitive level improves welfare.

Besides, to ensure that the social planner’s problem has a unique solution, which I denote by \((k_{SB}, e_{SB}, p^*_{SB})\), I assume that

**Assumption 4.** \(V(e, p^*(e)) K(e, p^*(e))\) is strictly quasiconcave in \(e\).

To complete the analysis, I compare the competitive equilibrium and the second-best with the first-best allocation. The first-best corresponds to an economy with no financial constraint. For given price \(p\) and investment \(k\), the first-best level of effort is \(e^{FB}(p)\), as defined in Assumption 1-(a). However, with no participation constraint, the optimal level of investment is \(k \rightarrow +\infty\). In order to have a well-defined benchmark, I put aside the determination of \(k\) and define the first-best asset price by \(p^{FB} = p^*(e^{FB}(p^{FB}))\). It is necessarily unique.
Proposition 5. The asset price in the competitive equilibrium is strictly lower than in the first-best, but strictly larger than in the second-best:
\[ p^{SB} < p^{CE} < p^{FB}. \]

Proof. See Appendix A.3.

In the competitive equilibrium, firms do not take into account the impact of their effort on the asset price. They implement a socially too high level of effort, which increases the asset price. This makes more difficult for the other firms to incentivize the entrepreneur. Even though it also raises the value of their collateral, the incentive effect dominates. Therefore, the decentralized level of effort is too high. Equivalently, the decentralized level of investment is too low.

Proposition 6. In the competitive equilibrium, firms do not invest enough and provide too much effort:
\[ k^{SB} > k^{CE}, \]
\[ e^{SB} < e^{CE} < e^{FB}. \]

Proof. Immediate given that \( p^*(e) \) is strictly increasing in \( e \) and \( K(e,p) \) is strictly decreasing in \( e \) and \( p \).

The second-best allocation improves on the competitive allocation in the sense of utilitarian welfare. Since firms are ex ante identical, this is also a Pareto improvement from an ex ante point of view.

4.3 Example: Debt Contract

The intuition of the results is best understood with an example. Consider that \( 1 - F \) has the strict MLRP, i.e., \(( -F_e / (1 - F)) (\theta | e) \) is strictly increasing in \( \theta \), so that a debt contract is optimal. I denote the face value by \( \pi(\theta_0) k \). Increasing the face value by 1 raises the expected revenue of the investor by \( 1 - F(\theta_0 | e) \) and reduces the marginal return of effort for the entrepreneur by \( -F_e(\theta_0 | e) \). The optimal face value satisfies
\[ \lambda(1 - F(\theta_0 | e)) = \mu(-F_e(\theta_0 | e)), \]
and the MLRP implies that \( \lambda(1 - F(\theta | e)) > \mu(-F_e(\theta | e)) \) for \( \theta < \theta_0 \) and \( \lambda(1 - F(\theta | e)) < \mu(-F_e(\theta | e)) \) for \( \theta > \theta_0 \).

The resizing technology is given by \( v(0) = 0, v'(s) = 1 \) if \( s < 1 \), \( v'(s) = 0.5 \) is \( s \in (1, 2) \) and \( v'(s) = 0 \) if \( s > 2 \). Therefore, the optimal date 2-investment satisfies
\[ s(\theta, p) = \begin{cases} 
0 & \text{if } \theta < p, \\
1 & \text{if } p < \theta < 2p, \\
2 & \text{if } \theta > 2p. 
\end{cases} \]
The firm’s value, in terms of unit of initial investment, is then given by
\[ \pi(\theta, p) = \begin{cases} 
p & \text{if } \theta < p, \\
\theta & \text{if } p < \theta < 2p, \\
1.5\theta - p & \text{if } \theta > 2p. 
\end{cases} \]
The sharing rule between the entrepreneur and the investor is represented by the solid lines on Figure 6. Finally, the equilibrium asset price satisfies $F(p|e) = 1 - F(2p|e)$. Assume that $\theta_0 \in (p, 2p)$.

![Figure 6: Debt contract and piecewise linear technology.](image)

When the asset price increases by 1, the expected revenue of the entrepreneur increases by $F(p|e)k = (1 - F(2p|e))k$, and the marginal return of effort is reduced by $(-F_e(2p|e))k$. The new payoff functions are represented by the dotted lines on Figure 6. The effect on the NPV is

$$\frac{dNPV}{dp} = \lambda(1 - F(2p|e))k - \mu(-F_e(2p|e))k < 0.$$ 

5 Discussions and Extensions

5.1 Returns to Scale

Let me clarify the role of returns to scale in the model. There are decreasing returns to scale with respect to date 2-investment, $k'$. This assumption is essential, because it prevents that the single firm with the highest $\theta$ acquires all the assets in the economy. I have formalized this assumption with a payoff function $\theta_v(k'/k)$ that satisfies the Inada condition as regards final size $k'$. Alternatively, I could have used an adjustment cost by positing a payoff function $\theta_v(k'/k) = \theta k'/k - c(k'/k - 1)$, with $c(0) = 0$, $c'(0) = 0$ and $c'' > 0$. In that case, the Inada condition $v'(0) = +\infty$ is not satisfied any more and low $\theta$-firms can be completely shut down. That would not modify the analysis. All what is needed is that $v'(\cdot)$ is decreasing in the neighborhood of $+\infty$, to prevent the $\tilde{\theta}$-firms to absorb all the assets.

By contrast, the assumption of constant returns to scale with respect to initial investment, $k$, can be relaxed. If the final payoff, $\theta_v(s)k$, is replaced by $\theta v(s)u(k)$, with $u(0) = 0$, $u' > 0$ and $u'' < 0$, and the cost of effort $ek$ becomes $e c(k)$, with $c(0) = 0$, $c' > 0$ and $c'' > 0$, it is straightforward to check that all the results still hold. Moreover, if $u'(+\infty) = 0$ or
$c'(\infty) = +\infty$, then the first-best level of investment $k^{FB}$ is well-defined. In that case, the model would also admit an initial wealth of entrepreneurs $A$ equal to 0.

5.2 Aggregate Shock

Shocks $\theta$ are idiosyncratic so that there is no aggregate uncertainty. The model and the results are easily extended to the case of aggregate shocks, as long as the asset price $p$ is contractible. This is true because contracting on $p$ boils down to contracting on the aggregate state of nature. In that case, there is one compensation scheme by state of nature, and each one satisfies Proposition 2, where the distribution function is now conditional on the aggregate state of nature. The other results then follow. In particular, the price of the asset is too high in every state of nature.

5.3 Multidimensional Effort

The model can be extended to the case of a multidimensional effort $e = (e_1, \ldots, e_N)$. Consider for simplicity a separable cost of effort, $\sum_{i=1}^{N} e_i k$. I still assume that the $N$ first order conditions corresponding to the incentive compatibility constraint,

$$\int_{\theta}^{\bar{\theta}} w(\theta) \frac{\partial f}{\partial e_i}(\theta | e) d\theta - k = 0 \quad \forall \ i = 1, \ldots, N,$$

are sufficient. I denote by $\mu_i$ their respective Lagrangian multiplier. The optimal wage function is derived as in the basic model. Increasing the slope of $w(.)$ in $\theta$ reduces the investor’s profit by $1 - F(\theta | e)$ and increases the marginal return of effort $e_i$ by $-F_e(\theta | e)$. Thus, the optimal wage function is steep if $\sum_i \mu_i(-F_e(\theta | e)) \geq \lambda(1 - F(\theta | e))$, and flat otherwise:

$$w(\theta) = \int_{\theta}^{\bar{\theta}} \sum_{i=1}^{N} \mu_i \left[ \frac{1}{1 - F(\theta | e)} \right] v(s(t, p)) k dt \quad \forall \ \theta \in [\theta, \bar{\theta}].$$

The competitive asset price is still socially too high, since

$$\frac{dNPV}{dp} = \int_{\theta}^{\bar{\theta}} \sum_{i=1}^{N} \mu_i \left[ \frac{1}{1 - F(t | e)} \right] v'(s(\theta, p)) k \lambda(1 - F(\theta | e)) - \sum_{i=1}^{N} \mu_i(-F_i(\theta | e)) \right] dt < 0.$$

5.4 Risk-Averse Entrepreneur

If we introduce risk-aversion in the model, the entrepreneur and investors contract not only to fund the project, but also to share risk. I show that if the main motive for contracting remains to raise funds, then the asset price is still too low. By contrast, if risk-aversion is so strong that risk-sharing becomes the main motive for contracting, then the competitive equilibrium is socially optimal.

Assuming that the entrepreneur is risk-averse and its preferences are additively separable
in revenue and effort, the problem writes as

\[
\max_{k, e, w(\cdot)} \int_{\underline{\theta}}^{\bar{\theta}} u(w(\theta)) f(\theta|e) d\theta - ek,
\]

subject to

\[
-1 + p + \int_{\underline{\theta}}^{\bar{\theta}} \pi(\theta) f(\theta|e) d\theta \geq A,
\]

\[
e = \arg \max_{\tilde{e}} \int_{\underline{\theta}}^{\bar{\theta}} u(w(\theta)) f(\theta|\tilde{e}) d\theta - \tilde{e}k,
\]

limited liability (1) and manipulability constraints (2) and (3),

where \( u' > 0 \) and \( u'' < 0 \). If the manipulability constraint (2) is binding on a set of strictly positive measure, then a lower \( p \) increases \( v(s(\theta)) \) for the corresponding \( \theta \), which relaxes the non-borrowing manipulability constraint. In that case, the competitive level of the asset price is still socially too high.

If, instead, the non-borrowing constraint is never binding, the competitive equilibrium achieves constrained efficiency. This is the case if, for all \( \theta \in [\underline{\theta}, \bar{\theta}] \), increasing or decreasing \( w(\theta) \) does not improve the objective function, which provides the first order condition:

\[
u'(w(\theta)) f(\theta|e) - \lambda f(\theta|e) + \mu u'(w(\theta)) f_e(\theta|e) = 0.
\]

Differentiating this expression with respect to \( \theta \) provides

\[
w'(\theta) = \frac{u'(w(\theta))(f'(\theta|e) + \mu f'_e(\theta|e)) - \lambda f'_e(\theta|e)}{|u''(w(\theta))|(f(\theta|e) + \mu f_e(\theta|e))},
\]

which must be comprised between 0 and \( v(s(\theta))k \) for all \( \theta \). This is the case if \( u(\cdot) \) is sufficiently concave, that is, if the entrepreneur is very risk-averse.

### 5.5 Distribution of Bargaining Power

So far I have used the standard assumption that investors are competitive so that entrepreneurs extract all the surplus of the project. The results are robust to a modification of the distribution of bargaining power. Assume that entrepreneurs are competitive and investors extract all the surplus. The optimal contract then solves

\[
\max_{k, e, w(\cdot)} \left[ -1 + p + \int_{\underline{\theta}}^{\bar{\theta}} \pi(\theta) f(\theta|e) d\theta \right] k - \int_{\underline{\theta}}^{\bar{\theta}} w(\theta) f(\theta|e) d\theta,
\]

subject to

\[
\int_{\underline{\theta}}^{\bar{\theta}} w(\theta) f(\theta|e) d\theta - ek - A \geq R,
\]

\[
e = \arg \max_{\tilde{e}} \int_{\underline{\theta}}^{\bar{\theta}} u(w(\theta)) f(\theta|\tilde{e}) d\theta - \tilde{e}k,
\]

limited liability (1) and manipulability constraints (2) and (3),

where \( R \geq 0 \) is the reservation utility of the entrepreneur.

The participation constraint of the entrepreneur is necessarily binding, otherwise the objective function can be improved by lowering \( k \) and \( w(\cdot) \) by a same factor. The incentive compatibility constraint is also binding, otherwise one should increase \( e \). Increasing the slope of \( w(\cdot) \) in \( \theta \) reduces the objective function by \( 1 - F(\theta|e) \), relaxes the participation constraint
(with Lagrangian multiplier $\lambda$) by $1 - F(\theta|e)$ and the incentive compatibility constraint (with Lagrangian multiplier $\mu$) by $-F_e(\theta|e)$. The optimal wage function is steep if, and only if, the weighted sum of these three terms is positive, hence

$$w(\theta) = \int_\theta^\theta 1 - \frac{F_e(t|e)}{1 - F(t|e)} \geq 1 - \lambda \frac{\mu}{\theta} v(s(t))k dt \quad \forall \theta \in [\underline{\theta}, \bar{\theta}]$$

The objective function is the investor’s profit, but since the entrepreneur’s profit (net of the effort cost) is fixed, the derivative of the NPV is the same as the derivative of investor’s profit, and it is strictly negative:

$$\frac{dNPV}{dp} = \int_\theta^\theta 1 - \frac{F_e(t|e)}{1 - F(t|e)} \geq 1 - \lambda \frac{\mu}{\theta} v''(s(\theta,p))k[(1 - \lambda)(1 - F(\theta|e)) - \mu(-F_e(\theta|e))]d\theta < 0.$$  

### 5.6 Manipulability Constraints

The manipulability constraints can be weaken by imposing a cost to manipulate the payoff. Assume that the entrepreneur incurs a cost proportional to the magnitude of the manipulation. Precisely, secrete loans are extended with an interest rate $\alpha \geq 0$, and it costs $\beta \geq 0$ to reduce the firm’s value by 1. The manipulability constraints then become

$$w(\theta) \geq w(\tilde{\theta}) - (1 + \alpha)(\tilde{\theta} - \theta)v(s(\tilde{\theta}))k \quad \forall \tilde{\theta} > \theta,$$

and

$$w(\theta) \geq w(\tilde{\theta}) - \beta(\tilde{\theta} - \theta)k \quad \forall \tilde{\theta} < \theta.$$  

In the optimal contract, the slope of the wage function is now $1 + \alpha$ times the slope of the firm’s value function in the incentive parts, and $-\beta$ times the firm size in the non-incentive parts if the wage is not already equal to zero:

$$w(\theta) = \int_\theta^\theta \left(-1 - \frac{F_e(t|e)}{1 - F(t|e)} \geq 1 - \lambda \frac{\mu}{\theta} v(s(t))k dt \quad \forall \theta \in [\underline{\theta}, \bar{\theta}].$$

The total derivative of the expected NPV with respect to the asset price is still negative:

$$\frac{dNPV}{dp} = \int_\theta^\theta \left(-1 - \frac{F_e(t|e)}{1 - F(t|e)} \geq 1 - \lambda \frac{\mu}{\theta} v''(s(\theta,p))k[(1 - \lambda)(1 - F(\theta|e)) - \mu(-F_e(\theta|e))]d\theta < 0.$$  

The manipulability constraints can also be tighten, by assuming that the entrepreneur can divert a part of the firm’s value. This amounts to taking $\beta \in [-1, 0]$. In that case, the sign of the above expression becomes indeterminate. This indicates that the result can be overturn. Consider for instance the case $\alpha = -\beta = 1$, which implies that the contract is constrained to be a debt contract with a face value no smaller than $\pi(\theta)k$. Since it remains optimal to set $w(\theta) = 0$, then the optimal face value is exactly $\pi(\theta)k$. The participation constraint becomes

$$[-1 + p + \pi(\theta,p)]k \geq -A.$$
and the incentive compatibility constraint,
\[
\int_\theta^\theta (\pi(\theta, p) - \pi(\theta, \hat{p})) k f_e(\theta|e) d\theta = k.
\]
Finally, the derivative of the NPV with respect to the asset price is given by
\[
\frac{dNPV}{dp} = \lambda(1 - s(\theta, p)) k - \mu \int_\theta^\theta s(\theta, p) k f_e(\theta|e) d\theta,
\]
which may be either positive or negative. If, for instance, date 2-returns to scale are strongly
increasing, then \( s(\theta, p) \) is close to 1 and \( s(\theta, p) \) is only slightly increasing, thus an increase of
the asset price improves welfare.

This suggests that restricting the set of contracts can overturn the result that the decentralized asset price is too high. The point is made more clearly in the following extension.

5.7 Non-Optimal Financial Structure

The result that the incentive effect dominates the collateral effect hinges on the optimality of
the financial contract. To illustrate that point, consider that there is a risk-shifting problem,
but only a (non-optimal) debt contract can be used. \(-F_e/(1 - F)\) is humped-shape and
negative in \( \theta \), as on Figure 7, and the wage function is constrained to be of the form
\[
w(\theta) = \int_0^\theta 1_{t \leq \theta_0} v(t) k d\theta \quad \forall \, \theta \in [\theta, \bar{\theta}] .
\]
The optimal face value, \( \pi(\theta_0) k \), of this debt contract is given by the first order condition
\[
\lambda(1 - F(\theta_0)) - \mu(-F_e(\theta_0)) = 0.
\]
Since \(-F_e/(1 - F)\) is humped-shape, there exists two values \( \theta_0 < \bar{\theta}_0 \) that satisfy the first
order condition. However, using that \(-F_e/(1 - F)\) is increasing in \( \theta_0 \) and decreasing in \( \bar{\theta}_0 \), it
is straightforward to show that the second order condition \( \lambda f(\theta_0) + \mu f_e(\theta_0) < 0 \) is satisfied
for \( \theta_0 \) only.

The derivative of the NPV with respect to the asset price is equal to
\[
\frac{dNPV}{dp} = \int_\theta^{\bar{\theta}} \frac{v'(s(\theta, p))}{\theta v''(s(\theta, p))} k[\lambda(1 - F(\theta|e)) - \mu(-F_e(\theta|e))] d\theta.
\]
The integrand is positive for \( \theta \in [\theta_0, \bar{\theta}_0] \) and negative for \( \theta \in [\bar{\theta}_0, \bar{\theta}] \). The integral may be
positive or negative. It tends to be positive if \(-F_e/(1 - F)\) is strongly negative for high \( \theta s \),
which occurs if the risk-shifting problem is large, implying that the competitive asset price is
socially too low.

The conclusion of this example is that the result that the incentive effect dominates the
collateral effect comes from the optimality of the financial structure. When the contract is
far from the optimal one, as a debt contract in the presence of a severe risk-shifting problem,
the collateral effect can dominate the incentive effect.
Figure 7: A debt contract is used although $1 - F$ does not have the MLRP.

References


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A Appendix

A.1 Proof of Lemma 1

The proof proceeds in several steps:

1. The incentive compatibility constraint is equivalent to the first order condition.

2. For $e \geq e_{\inf}$, there exists a real $x$, such that the wage function (by unit of investment) that implements $e \geq e_{\inf}$ and minimizes the expected wage, is of the form

$$z^e(\theta) = \int_{\theta}^{\Theta} \frac{1}{1 - F_e(\theta|e)} x v(s(t))dt \quad \forall \theta.$$ 

3. Existence of $e_{\sup}$.

**Step 1** I work by unit of investment to allege notations. Let $z(.) \in \mathcal{Z}$. The manipulability constraints (2) and (3) imply that $z(.)$ is Lipschitz-continuous. Therefore, $z(.)$ is absolutely continuous and there exists a collection $\{\delta(t), t \in [\theta, \Theta]\}$ such that

$$z(\theta) = z(\bar{\theta}) + \int_{\theta}^{\bar{\theta}} \delta(t) v(s(t))dt \quad \forall \theta.$$ 

Besides, limited liability implies that $z(\bar{\theta}) \geq 0$, non-sabotage that $\delta(\theta) \geq 0$, and non-borrowing that $\delta(\theta) \leq 1$, except possibly on a set of measure zero.

The second derivative of the expected wage,

$$\int_{\theta}^{\bar{\theta}} z(\theta)f_{ee}(\theta|e)d\theta = -\int_{\theta}^{\bar{\theta}} \delta(\theta) v(s(\theta))F_{ee}(\theta|e)d\theta,$$

is strictly negative, implying that the first order condition is necessary and sufficient.\(^\text{12}\)

**Step 2** Let $e \geq e_{\inf}$ such that there exists a wage function in $\mathcal{Z}$ that implements effort $e$, and let $z(.)$ the one that minimizes the expected wage. Notice first that $z(\bar{\theta}) = 0$, otherwise lowering the wage by the same constant for all $\theta$ relaxes the participation constraint without affecting the incentive compatibility constraint. Writing $z(.)$ as in Step 1, let me show that there exists a real $x$ such that

$$\delta(\theta) = 1 - \frac{F_e}{1 - F_e}(\theta|e) \geq x \quad \forall \theta.$$ 

By contradiction, assume that there exists $y$ such that the sets $\Theta_0 = \{\theta : \delta(\theta) > 0 \text{ and } (-F_e/1 - F_e)\theta|e < y\}$ and $\Theta_1 = \{\theta : \delta(\theta) < 1 \text{ and } (-F_e/1 - F_e)\theta|e > y\}$ are of strictly positive measure. I can build another wage function $\tilde{z}(.)$ that generates the same level of effort than $z(.)$, with a strictly lower expected wage:

$$\tilde{z}(\theta) = \int_{\theta}^{\bar{\theta}} \tilde{\delta}(t)dt \quad \tilde{\delta}(\theta) = \begin{cases} 0 & \text{if } \theta \in \tilde{\Theta}_0, \\ 1 & \text{if } \theta \in \tilde{\Theta}_1, \\ \delta(\theta) & \text{otherwise}, \end{cases}$$

\(^{12}\)If the wage function is constant, then the second derivative is equal to zero, but the first order condition cannot be satisfied.
where the subsets $\tilde{\Theta}_0 \subset \Theta_0$ and $\tilde{\Theta}_1 \subset \Theta_1$ are such that
\[
\int_{\tilde{\Theta}_0} \delta(\theta)(-F_e(\theta|e))d\theta = \int_{\tilde{\Theta}_1} (1 - \delta(\theta))(-F_e(\theta|e))d\theta > 0.
\]
The new wage function $\tilde{z}(.)$ generates the same level of effort than $z(.)$, since
\[
\int_{\Theta} (\tilde{z}(\theta) - z(\theta)) f_e(\theta|e)d\theta = \int_{\Theta} (\tilde{\delta}(\theta) - \delta(\theta))(-F_e(\theta|e))d\theta
\]
\[
= -\int_{\tilde{\Theta}_0} \delta(\theta)(-F_e(\theta|e))d\theta + \int_{\tilde{\Theta}_1} (1 - \delta(\theta))(-F_e(\theta|e))d\theta
\]
\[
= 0,
\]
and it lowers the expected wage, since
\[
\int_{\Theta} (\tilde{z}(\theta) - z(\theta)) f(\theta|e)d\theta = \int_{\Theta} (\tilde{\delta}(\theta) - \delta(\theta))(1 - F(\theta|e))d\theta
\]
\[
= -\int_{\tilde{\Theta}_0} \delta(\theta)(1 - F(\theta|e))d\theta + \int_{\tilde{\Theta}_1} (1 - \delta(\theta))(1 - F(\theta|e))d\theta
\]
\[
< -\int_{\tilde{\Theta}_0} \delta(\theta)\frac{-F_e(\theta|e)}{y}d\theta + \int_{\tilde{\Theta}_1} (1 - \delta(\theta))\frac{-F_e(\theta|e)}{y}d\theta = 0,
\]
which contradicts the optimality of the wage function $z(.)$.

**Step 3** To establish the existence of $e_{sup}$, I first show that $x \geq 0$ in the definition of $z^e(.)$. For any effort $e$ and real $y$, I define $z(.)$ by
\[
z(\theta; e, y) = \int_{\Theta} 1_{\frac{1}{1-F_e(\theta|e)} \geq y} v(s(t))dt.
\]
The level of effort implemented by $z(\cdot; e, y)$, which I denote by $E(e, y)$, satisfies
\[
0 = \int_{\Theta} z(\theta; e, y)f_e(\theta|E(e, y))d\theta - E(e, y) = \int_{\Theta} 1_{\frac{1}{1-F_e(\theta|e)} \geq y} v(s(\theta))(-F_e(\theta|E(e, y)))d\theta - E(e, y).
\]
Differentiating with respect to $y$ gives
\[
\frac{\partial E(e, y)}{\partial y} = \frac{\partial}{\partial y} |_{E(e, y)} \int_{\Theta} 1_{\frac{1}{1-F_e(\theta|e)} \geq y} v(s(\theta))(-F_e(\theta|E(e, y)))d\theta
\]
\[
= \frac{1}{1 + \int_{\Theta} 1_{\frac{1}{1-F_e(\theta|e)} \geq y} v(s(\theta))F(e)\theta|E(e, y))d\theta},
\]
where the denominator is strictly positive since $F_{ee} > 0$. The numerator is equal to $0$ if $y$ is outside the image of $(-F_e/(1-F))(\cdot|e)$. Otherwise, it has the opposite sign than $y$. Therefore, denoting the image of $(-F_e/(1-F))(\cdot|e)$ by $[y_{min}, y_{max}]$, where $y_{min} \leq 0$ and $y_{max} > 0$, $E(e, y)$ is constant on $(-\infty, y_{min}]$, strictly increasing on $[y_{min}, 0]$ (unless $E(e, y) = e$), strictly decreasing on $[0, y_{max})$ (unless $E(e, y) = \bar{e}$), and constant on $[y_{max}, +\infty)$. Besides, $E(e, 0)$ is equal to $e^{FB}$ if $y_{min} = 0$, and strictly larger than $e^{FB}$ if $y_{min} < 0$, and $E(e, y_{max}) = \bar{e}$. 24
Therefore, there exists a unique \( x \geq 0 \), and possibly another \( x' < 0 \), such that \( E(e,x) = e \). The expected wage is strictly smaller with \( x \geq 0 \), which concludes this step of the proof.

This analysis shows that a level of effort \( e \) can be implemented if, and only if, \( E(e,0) \geq e \), or
\[
\int _\Theta ^1 - F_e(\theta|e) - \frac{d}{de}E[s(\theta)|e]d\theta - e \geq 0.
\]
The LHS of this expression is decreasing in \( e \), and positive in \( e^{FB} \), therefore there exists \( e_{sup} \geq e^{FB} \) such that \( e \) is implementable if, and only if, \( e \in [\xi, e_{sup}] \).

A.2 Proof of Propositions 2

Show that \( x = \lambda/\mu \). The derivative of the Lagrangian with respect to \( x \),
\[
\frac{\partial}{\partial x} \int _\Theta ^1 - \frac{e}{\frac{d}{de}F_e(\theta|e)} - x^2 v(s(\theta)) (\lambda(1 - F(\theta|e))) - \mu(-F_e(\theta|e))) k d\theta,
\]
has the same sign than \( (\lambda - \mu x)(1 - F(\theta|e)) \). The first order condition therefore implies \( x = \lambda/\mu \).

A.3 Proof of Proposition 5

I first show that the first-best is unique. I then solve for the second-best.

First-best Since \( p^*(e) \) is strictly decreasing, it suffices to show that \( e^{FB}(p) \) is strictly decreasing to obtain the uniqueness of the first-best allocation.

For a given \( p \), the first-best level of effort is defined by \( (\partial V/\partial e)(e^{FB}(p), p) = 0 \) and the second order condition is satisfied by Assumption 1-(a). Then
\[
\frac{\partial e^{FB}}{\partial p} = \frac{\frac{\partial^2 V}{\partial e \partial p}}{\frac{\partial^2 V}{\partial e^2}} = -\frac{\frac{d}{de}E[s(\theta)|e]}{\frac{\partial^2 V}{\partial e^2}} < 0,
\]
by Assumption 1-(b).

Second-best The problem of the social planner consists in maximizing \( V(e, p^*(e))K(e, p^*(e)) \) on \( e \). The partial derivative of \( V(e, p) \) with respect to \( p \),
\[
\frac{\partial V}{\partial p}(e, p) = \int _\Theta ^1 (1 - s(\theta, p)) f(\theta|e)d\theta,
\]
is equal to zero in \((e, p^*(e))\), for any \( e \). To compute the partial derivative of \( K(e, p) \) with respect to \( p \), remind that \( K(e, p) = -A/\Pi_I(e, p) \), where \( \Pi_I(e, p) \) is the maximum level of the investor’s profit compatible with a level of effort \( e \) and a price \( p \):
\[
\Pi_I(e, p) = \max _{z(\cdot) \in \mathcal{Z}} -1 + p + \int _\Theta ^1 \pi(\theta, p)f(\theta|e)d\theta - \int _\Theta ^1 z(\theta)) f(\theta|e)d\theta,
\]
s.t. \( \int _\Theta ^1 z(\theta)f(\theta|e)d\theta - 1 = 0 \).

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An argument along the line of the proof of Lemma 1 shows that the solution is reached for

\[ z^{(e,p)}(\theta) = \int_\theta^\theta 1 - \frac{F_1(t|e) \geq x(e,p)}{x(e,p)} v(s(t,p)) dt \quad \forall \theta, \]

where \(1/x(e,p)\) is the Lagrangian multiplier of the incentive compatibility constraint. An integration by part gives

\[ \Pi_I(e,p) = -1 + p + \int_\theta^\theta \pi(\theta,p)f(\theta|e)d\theta - \int_\theta^\theta \frac{1-F_1(x(e,p))}{x(e,p)}(1-F(\theta|e))d\theta, \]

then

\[ \frac{\partial \Pi_I}{\partial p}(e,p) = \int_\theta^\theta (1-s(\theta,p))f(\theta|e)d\theta \]

\[ + \int_\theta^\theta \frac{1-F_1(\theta|e) \geq x(e,p)}{x(e,p)} \frac{v'(s(\theta,p))}{\theta |v''(s(\theta,p))|} (1-F(\theta|e))d\theta \]

\[ - \frac{1}{x(e,p)} \int_\theta^\theta \frac{1-F_1(\theta|e) \geq x(e,p)}{x(e,p)} \frac{v'(s(\theta,p))}{\theta |v''(s(\theta,p))|} (-F_e(\theta|e))d\theta. \]

The first term is equal to zero in \((e, p^*(e))\). The sum of the second and the third terms rewrites as

\[ \int_\theta^\theta \frac{1-F_1(\theta|e) \geq x(e,p)}{x(e,p)} \frac{v'(s(\theta,p))}{\theta |v''(s(\theta,p))|} \left[ (1-F(\theta|e)) - \frac{1}{x(e,p)}(-F_e(\theta|e)) \right] d\theta, \]

which is strictly negative given the dummy function. Since \(\Pi_I\) is strictly negative, \(\partial K/\partial p\) is strictly negative in \((e, p^*(e))\) for any \(e\).

Finally, \((VK)(e, p^*(e))\) is strictly quasiconcave in \(e\) by Assumption 4, and

\[ \frac{d(VK)}{de}(e, p^*(e)) = \frac{\partial(VK)}{\partial e} + \frac{\partial p^*}{\partial e}(e, e^*(p)) \frac{\partial K}{\partial p}(e, p^*(e)) \]

is strictly positive in \(e = e_{inf}(p^{CE})\), since \(V = 0\) and \(\partial(VK)/\partial e > 0\), and strictly negative \(e^{CE}\), since \(V > 0\) and \(\partial(VK)/\partial e > 0\). Therefore, the problem of the social planner has a unique maximum in \(e^{SB} \in (e_{inf}(p^{CE}), e^{CE})\).