REAL-TIME PRICING OF ELECTRICITY FOR RESIDENTIAL CUSTOMERS: ECONOMETRIC ANALYSIS OF AN EXPERIMENT

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SUMMARY

Under real-time pricing, a network operator sets the price level for a period according to a predefined scheme which depends on the state of demand and costs, and announces this price shortly before the period begins. The French state-owned electric utility experimented with a six-rate real-time tariff, which divides the year into three types of days and each day into two periods. The number of days of each type is known in advance to the consumer, but the type of any particular day is announced only at the end of the preceding day. In order to evaluate the responsiveness of customers to this pricing option, we estimate the Frisch demand functions for daily electricity consumption, derived from a simple dynamic model based on an additively separable intertemporal utility function. As the marginal utility of expenditure which enters the Frisch demands follows a known stochastic process, the econometric model has a state-space representation. We can then apply the Kalman filter to compute the log-likelihood function associated with each consumer’s time series of electricity consumption. The main result of the analysis is that the real-time tariff improves the welfare of a majority of consumers participating in the experiment.

1. INTRODUCTION

Real-time pricing is advocated by professional economists and consultants as a method for recovering costs in response to short-term conditions of demand and supply in network industries. When demand is highly uncertain, it could exceed capacities with some probability. Instead of using some rationing scheme, the theory suggests adjusting prices continuously and instantaneously depending on the rate of network utilization and the duration of congestion. This pricing mechanism was first proposed by Vickrey in 1971. In practice, it is currently feasible in the electric power industry where supplementary capacity can meet additional demand of any end-user customer linked to the network, as long as total capacity is not reached. (Failure costs are incurred when outages happen.) More precisely, it is now technically possible to send through the network a limited number of signals at a specific hour or time informing the customers about the energy price levels for the next period. The French state-owned electric

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utility, Electricité de France (known as EDF), which has a long experience in peakload pricing, has recently experimented such a real-time tariff for its residential customers.\textsuperscript{1} We report on this unique experiment, with the objective of providing a measure of its welfare effects for consumers and to estimate the degree of substitution in consumption under flexible electricity prices. By the very nature of this tariff, which introduces uncertainty in day-to-day prices of electricity, the investigation must rely on a methodology different from the approaches followed in numerous cost-benefit studies of residential time-of-use rates published in the literature.\textsuperscript{2}

The electricity demand of French households has shown three general trends over the past decade: steady increase in global consumption, smoothing of daily consumption and growing unpredictable winter peak demands. The combination of the last two trends explains the flattening of load curves and the variability of the peaks' location. Two explanations exist for these changes. First, the increasing flattening of daily demands results mainly from the application, since 1965, of a tariff based on two daily fixed periods. This induces residential customers to switch consumption from the peak period to the off-peak period.\textsuperscript{3} The load factor (the ratio of average to peak load) is now very high, reaching about 90\% on winter weekdays. Second, the development of electric space heating has caused a concentration of peak demands in winter, especially on the coldest days. Winter consumption peaks are therefore much more linked to the outside temperature and, as a result, cannot easily be forecasted. Hence, while the peak demand used to occur during a well-known number of hours per day during many winter weekdays, it now covers the most costly hours in winter (about 396 hours during a year on average). The system demand has then moved from a largely predictable peak during specific weekdays to a random peak occurring on a few winter days. The exact dates are unforeseeable since they depend on outside temperatures.

Obviously, these changes affect electricity supply decisions. EDF is able to meet demand in off-peak periods by relying on baseload generating sources, mainly nuclear power plants with low marginal cost.\textsuperscript{4} Winter consumption peaks, however, require the use of power plants mainly burning fuel oil, with a significantly higher variable (fuel) cost. The availability of thermal and hydraulic generating facilities further accentuates these cost contrasts. Because plants with a high variable cost and a low capital cost are required to satisfy a peak of short duration, the differential between costs at the peak and at other hours of the year is quite large.

In order to reflect cost and demand variation in their pricing policies, several electric power producers have developed real-time tariffs, based on flexible rates according to the state of generation equipment and network utilization. The advantage of flexible tariffs is to allow price to reflect current marginal costs, instead of only expected marginal costs as is the case with fixed-price schedules. The problem is to define a set of flexible periods. It is impractical to change prices continuously and instantaneously, because this would require a very expensive and elaborate information system for notifying customers. Moreover, spot prices could complicate equipment choice decisions of households in an inefficient way. However, because

\textsuperscript{1}The new tariff will be offered to all French households by the end of 1996. In practice, given the technical support it requires, it will be gradually extended to different rural and urban areas of France. Wilson (1994) offers a complete description of electricity tariffs at EDF and, in particular, of real-time tariffs.

\textsuperscript{2}There are two Annals of the \textit{Journal of Econometrics} on peak-load pricing, one edited by Lawrence and Aigner in 1979 and one edited by Aigner in 1984. For recent work on this subject, see Mountain and Lawson (1992) and Aigner \textit{et al.} (1994).

\textsuperscript{3}For a long time, French households have had the choice between a standard tariff with a single rate and a tariff with two different rates for off-peak and peak periods.

\textsuperscript{4}The short-run marginal cost is mainly determined by the cost of nuclear fuel, which is low. Computations of marginal cost at EDF is a complex matter which cannot be detailed in this study. They stem from original work by M. Boiteux.
the cost of electronic monitoring devices has considerably decreased, the transaction costs of implementing real-time tariffs are now reasonable when the number of price signals to be transmitted is limited. The basic principle of real-time tariffs is then simple. Users who have chosen such a tariff are informed at short notice that next period will be or will not be a peak and that, as specified by the contract, particular clauses, such as quantity rationing and/or very high prices, will apply. The idea is to provide incentives to consumers to substitute between periods and/or (if it is possible) among different types of energy. The consumers then determine their consumption levels by comparing their willingness-to-pay with the predetermined marginal cost.

There are, however, obstacles to real-time pricing. Assuming that the producer is able to announce a peak and its associated cost, the transmission of sophisticated price signals must be economically justified. The crucial factor is the capacity of consumers to use these signals, that is, their capacity to reduce peak consumption and to defer consumption from peaks to other periods. This is why real-time tariffs cannot be made mandatory, and are offered to customers on an optional basis. Evaluating the responsiveness of consumers to real-time signals, and measuring the welfare gains or losses due to this option are therefore crucial steps in optimal tariff design.

We perform an econometric investigation of these two questions. Our study is based on data collected during an experiment of a real-time tariff for residential customers conducted by EDF over 1989 to 1992. The experimental tariff includes six rates. The year is divided into three types of days and each day is separated into two periods. The number of days of each type is known by the consumer in advance; but the type of any particular day is only announced at the end of the preceding day. Clearly, the tariff introduces uncertainty concerning future prices of electricity and requires modelling the dynamics of consumer decisions. The solution we adopt is to estimate the Frisch demands of daily electricity consumption derived from a simple dynamic model based on a separable intertemporal utility function. These demands correspond to the quantity bundles that maximize the expected value of discounted utility. They depend on current prices and on the marginal utility of electricity expenditure, which in any period summarizes unobservable temporal variables and captures both past behaviour and forward-looking expectations. From one day to the next, the marginal utility of expenditure can be viewed as an individual-specific fixed effect and can be estimated from repeated observations on each individual. It turns out that the system of Frisch demand functions, and the equation governing the temporal pattern of the marginal utility of expenditure, form a state-space model. We can then apply the Kalman filter to compute the log-likelihood function associated with our panel data set. The goodness-of-fit we obtain seems to justify the use of this methodology. Alternative approaches, however, are conceivable but wait on our agenda for future research. The main result of the analysis is that the real-time tariff improves economic welfare for a majority of consumers.

The study is organized as follows. Section 2 offers a detailed account of the experiment. Section 3 is devoted to the presentation of the theoretical model and to the econometric specification. The empirical results are discussed in Section 4.

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1 In fact the electronic meter no longer imposes a constraint for the implementation of a tariff. In particular, the number of seasonal time-of-day periods is now mainly conditioned by the size of an electronic memory, which is easy to build and not costly. This new generation of meters therefore features higher efficiency and less cost than the former technology of electromechanical meters.

2 One could think of a system where the consumer informs the producer of its willingness-to-pay, and then the producer determines its production level. This scheme could be applicable for large business customers. Notice that it resembles the method proposed by MacKie-Mason and varian (1994) for pricing the Internet.
2. THE EXPERIMENTAL CONTEXT

The experimental pricing scheme results from the evolution of the temporal structure of electricity demand by French households as well as from changes in the cost structure. In the standard peakload pricing, households have no strong incentives to adjust the daily levels and the relative shares of peak and off-peak consumption: Indeed each day they face two fixed prices, one for each of the two predetermined time-of-use periods. In contrast, the real-time tariff introduces uncertainty concerning future prices of electricity and induces households to optimize their electricity needs over a very short period.

2.1. Rules

Under the experimental tariff, the year is divided into three types of days and each day into two periods. The three groups of days, marked by colours (which, for mnemonic or cultural reasons, are blue, white and red) are defined according to three levels of long-run marginal costs,\(^7\) from the least expensive to the most expensive one. The number of days of each type is fixed \textit{ex ante}: EDF has allocated 22 reds, 43 whites, and 300 blues in a year.\(^8\) Red and white colours approximatively correspond to colder periods, which, given the temperate climate of France, happen mainly during winter (from the beginning of November to the end of April).\(^9\) However, climatic variations are not the unique factors determining the types of days: in particular, EDF must take into account the availability of different power generating facilities. Thus the three different types of days correspond to marginal costs computed on an average basis, according to these various factors considered by the electric utility. In addition, as in the standard peakload price setting, each day consists of two fixed time-of-day periods. Their length, however, changes with the type of day. For blue and white days, the peak period lasts for 16 hours, from 7:00 a.m. to 11:00 p.m., and then the off-peak period begins; for red days, the ending time of the peak period is 1:00 a.m. Again, each period is associated with a predetermined level of prices.

Households do not know the dates at which red and white days occur. By choosing a high price ratio between the most expensive time period (peak hours of red days) and the least expensive time period (off-peak hours of blue days), the structure of relative prices should provide strong incentives for households to modify their electricity consumption as soon as a red or a white day is announced.\(^10\) This colour is chosen by the national dispatching centre of EDF at the end of the day for the next day, when it has tangible information about the next day's marginal costs of electricity supply, given predictions on the climatic factors and demand. The signal is then transmitted to the customer through the network by 8:00 p.m. and displayed on their meters.\(^11\)

While it introduces uncertainty in the consumption decision, the tariff should give the customer a stable and simple signal which is in principle, usable and comprehensible. Unlike a

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\(^7\) Long and short-run marginal costs are defined with respect to optimally installed capacity of each type of generating plants for a long-term time horizon. Any additional electricity demand must be satisfied by starting up installed power generating facilities or by building one new unit of production. Doing so, the marginal production cost with a given capacity (short-run marginal cost) and the marginal production cost of the new capacity (long-run marginal cost) must be equal.

\(^8\) The 300 blue days correspond approximatively to the exclusive utilization of baseload sources.

\(^9\) In the tariff contract, red days must be chosen from 1 November to 31 March while white days can be placed any time during the year. In reality, we observed no white days between 1 May and 31 October during the experiment.

\(^10\) The subscription fee for this new tariff option is fixed over a year. It should not affect daily decisions.

\(^11\) Note that weather forecasts are broadcast by the main French TV channels around 8:30 p.m. for the next day during the week and for the next week on each Sunday.
spot price reflecting day-to-day cost variations, the rule of a fixed yearly number of white and red days and of fixed levels of energy prices should somewhat help customers to form their expectations, and hence more easily calculate their yearly electricity expenditure and choose their stock of capital goods. As indicated earlier, this pricing pattern is built on long-run marginal costs calculated on average, according to the availability of thermal power-generating equipment, the state of the hydraulic system, and the level of outside temperatures. For this reason, real-time pricing based on long-run marginal costs may favour efficient long-term investment in multi-energy heating systems for households.

2.2. Sample and Data

Before launching this new pricing scheme in September 1993, EDF experimented with 800 households, over approximately a three-year period beginning in September 1989. In fact, data on hourly electricity consumptions are available for only 60 households. Among them, 26 households have been subject to a six-price real-time scheme from November 1991 to April 1992; the 34 other participants experienced a four-price tariff from September 1989 to December 1991, because, at the beginning of the experiment, the meters could not record more than four signals. Table I details the different price levels used during the experiment. The

<table>
<thead>
<tr>
<th>Table I. Structure of electricity prices during the experiment</th>
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<tr>
<td><strong>Four-price tariff</strong> (cents per kWh)</td>
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<td></td>
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<tr>
<td>From 18.4.1990</td>
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<td>From 01.3.1991</td>
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<td>From 25.2.1992</td>
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</tbody>
</table>

<table>
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<tr>
<th><strong>Six-price tariff</strong> (cents per kWh)</th>
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<tbody>
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<td></td>
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<tr>
<td></td>
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<tr>
<td>From 25.2.1992</td>
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<tr>
<td>From 20.2.1993</td>
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</tbody>
</table>

<table>
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<tr>
<th>Period duration in hours per year</th>
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<tbody>
<tr>
<td>2400h</td>
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<tr>
<td>4800h</td>
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<tr>
<td>344h</td>
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<tr>
<td>688h</td>
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<tr>
<td>132h</td>
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<tr>
<td>396h</td>
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</table>

12 At this point one may ask why a real-time rate system rather than an insurance system is chosen. It is well known that insurance against climatic hazard or natural catastrophes (earthquakes, for instance) is not easy to set up. Although there are examples of private insurances against hailstorms (especially for farmers), usually these risks are socially insured by some public funds. Indeed, risk pooling is difficult when risks are correlated, i.e., are affecting all individuals. In this sense climatic variations differ from car accidents. When risks are correlated, prices should reflect changes in the aggregate demand, which is the rationale of peak load pricing. Prices should reflect capacity shortages. By insuring against climate variations, one would end up with a uniform price system, which can be far from marginal cost pricing. One could think of an insurance system only if transportation costs of electricity are negligible and a great number electric systems are interconnected. Note, however, that, because marginal costs are computed on average, the structure of the real-time tariff system proposed by EDF introduces some insurance.

13 All the prices given here and in what follows do not include taxes.
experimental tariff with four rates was formed by equalizing the rates of adjacent time-of-use periods, i.e. on the one hand, the blue peak and white off-peak periods, and on the other, the white peak and red off-peak periods. Since 1992, the electronic meters are compatible with the six-price tariff. Note that the price of the most expensive time period (peak hours of red days) is about seventeen times higher during the experiment than the price of the least expensive time period (off-peak hours of blue days). Table I also gives the total number of hours per period and per year in order to balance the price ratios.

Participation in the experiment was voluntary. No compensation for bills over the past average amount was offered ex ante to customers. EDF’s commercial agencies spent time on convincing potential participants that they had a very high probability of saving money from the tariff. Very few complaints were recorded ex post and were compensated partially. In fact, the characteristics of customers who could benefit from the real-time tariff were precisely defined by looking at the load curves of different residential users of electricity. This fact, combined with the small number of participants for which data were recorded, produced a rather homogeneous sample. All households, except one, live in a detached house. In a companion paper (written in French and available upon request) we show that many variates, such as the type of housing, the ownership of different appliances, have no variability over the sample. Households are located in areas of distinct climatic conditions. Indeed, the experiment concerned six areas, four in the east and the centre of France (with a colder climate), one near Paris and one on the Atlantic coast (with a milder climate).

Note also another important point related to the data collected on individuals participating in the experiment. For sociological and cultural reasons, it is very difficult to ask a French person for his or here income or even class of income. Therefore only the professional group to which each household belongs is known. But this is rather loosely related to income. Moreover, expenditures on other goods are not surveyed during the experiment. This restricts the type of utility function which can be estimated.

The experiment duration was rather long and produced a substantial amount of data. Indeed, during the experiment, the participants were equipped with a specific metering system (different from the main meter) capable of recording average electricity consumption in kilowatts every ten minutes. We judged that these data were not useful for our purpose and we have based the analysis on the hourly peak and off-peak consumption obtained by aggregating these data. However, sometimes consumptions for several hours were missing due to technical problems related to the specific meter. In this case, we simply extrapolated the average hourly consumption, computed over the hours for which data were recorded, to the missing observations in the same period. The dependent variables used in the empirical analysis are the

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14 Given the structure of EDFs tariffs, customers who, under the real-time tariff, do not change their electricity consumption patterns over a year, must spend on average the same amount as under the standard tariff.
15 Doing so, EDF implements some kind of third-degree price discrimination, that is, uses observable characteristics to offer adequate prices. EDF knows the consumption patterns of customers and can, on this basis, characterize a priori a group that could benefit from the real-time tariff. Note that this pricing scheme is offered to French customers as an option. Customers will have to choose basically between two pricing systems: the real-time tariff and the standard peak/off-peak tariff. This allows for some second-degree price discrimination, and should improve efficiency. The extent to which these pricing rules ensure equity is a more complex story. It seems reasonable to expect that low-income and low-electricity consumption households would not benefit from the real-time pricing system. Note that, for a vacation or a country house only occupied during weekends or summer, a household has an interest to apply for this tariff. In general, a high-income household that has a two-energy heating systems should save on its electricity expenditure by correctly using the real-time pricing system.
16 Aggregating consumption over time avoids the problem of zeros. It seems that zero consumption is very rare. In fact, except for a few cases, it was impossible to distinguish between true zeros and missing data (i.e., zeros due to a defective metering system).
average daily consumptions in kilowatt-hours during the peak and off-peak periods.\footnote{The main meter records the total consumption in each of the four or six time-of-use periods between two billing dates. These records were not available to us. They could be useful to check the consistency of our data treatment, but not for studying the day-to-day substitution.} We ended up with a sample of 29,323 observations, by cumulating all daily records for the 60 customers.

2.3. Experimental Conditions and Overall Appraisal

The setting of a real-time tariff system on an electric power network makes it possible, for any consumer who desires the service, to join it with the automatic control of certain appliances so that certain uses of electricity automatically depend on the signals launched by the network operator. However, at the time of the experiment, sophisticated energy management systems were not yet available to households. Thus, customers participating in the experiment could be fitted out with only a rather crude control system for heating and appliances. Only a few households had management systems for certain domestic usages (heating and hot water).

Households were not regularly informed about the stock of red and white days left. They had to record this information if they wanted it. All of these technical constraints limited the capacity of households to substitute consumption across periods to the extent currently possible. The experiment gives a conservative view of effects of the tariff. One could expect a higher level of responsiveness in the currently improved technical environment. Table II indicates that, for our sample, the average hourly electricity consumption over peak periods for red days is nevertheless 27% less than for blue days. The reduction is more than one third over off-peak periods between red and blue days. Hence the real-time tariff has a significant effect on average.

Given the experimental conditions mentioned above, the success of the tariff was very depends on the behaviour of households. In this respect, the level of uncertainty about the types of days is a crucial factor when households are making their plans and expectations. Table III presents the empirical frequency of colours over the week, as it occurred during the experiment. Note that there are no red days and only a few white days during the weekend.\footnote{At the beginning of the experiment, the stock of red days had been spent at the very end of winter. Since then, EDF has managed the stock of days left more efficiently.}

<table>
<thead>
<tr>
<th>Kilo watt</th>
<th>Blue days</th>
<th>White days</th>
<th>Red days</th>
</tr>
</thead>
<tbody>
<tr>
<td>Off-peak periods</td>
<td>2.088</td>
<td>1.759</td>
<td>1.326</td>
</tr>
<tr>
<td>Peak periods</td>
<td>2.180</td>
<td>1.782</td>
<td>1.601</td>
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<table>
<thead>
<tr>
<th>Percentage</th>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
<th>Saturday</th>
<th>Sunday</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blue days</td>
<td>82.8</td>
<td>78.4</td>
<td>75.1</td>
<td>76.5</td>
<td>75.9</td>
<td>93.1</td>
<td>97.1</td>
</tr>
<tr>
<td>White days</td>
<td>13.7</td>
<td>11.5</td>
<td>15.4</td>
<td>16.1</td>
<td>18.9</td>
<td>6.9</td>
<td>2.9</td>
</tr>
<tr>
<td>Red days</td>
<td>3.5</td>
<td>10.1</td>
<td>9.5</td>
<td>7.4</td>
<td>5.2</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>
substitution. Clearly there is an incentive to postpone certain activities (like washing) until the weekend: electricity is not easily stored. Whether households are using the signals received on their meters and are taking care of the number of red and white days left at each date is an open issue.

The model we adopt to study the behaviour of participants in the experiment is influenced by this experimental context. Two assumptions underlie our approach. First, customers are not able to implement complex strategies based on the state of the stock of remaining red and white days. Thus we favour a model emphasizing a day-to-day optimization. Second, customers decide more or less on an annual budget for their energy consumption and try to keep their daily energy expenditure consistent with this objective. The plausibility of these assumptions must be assessed by taking up alternative models in future research.

As the sample built from the experiment is not representative of the French population, we cannot generalize conclusions drawn from the experiment or the following econometric analysis. We can expect to derive only an estimate of welfare effects for the individuals participating in the experiment, and to assess why there are losses or gains.

3. THE ECONOMETRIC MODELLING

This section presents a theoretical and econometric framework for analysing an individual consumer’s demand for electricity during the real-time pricing experiment. We start from a simple dynamic model based on an intertemporally separable utility function and follow the line of the literature on the life-cycle allocation of household expenditures by estimating a system of Frisch demand functions. (See among others, Heckman and MaCurdy, 1980; Browning et al., 1985, Blundell et al. 1990, 1994.) One feature of our approach is to estimate this system by using the Kalman filter.

3.1. The Theoretical Framework

Consider an individual consumer who has to choose electricity consumption over a time horizon of \( T \) time units \((t = 1, \ldots, T)\). The time unit here is one day. Assume that individual preferences are additively separable between days and that the within-day utility only depends on the quantities of electricity consumed during two distinct periods, namely the peak and off-peak pricing periods of the time unit. These quantities are denoted \( x_1 \) and \( x_2 \), respectively. In the experimental tariff scheme, electricity prices have to be considered as random variables from the consumer viewpoint. Indeed, the colour of a day, and so the electricity prices, depends roughly on the average daily outside temperature, which is a random variable. Moreover, the outside temperature may also directly influence individual preferences. Hence the individual utility function should depend on the time index. This question is again discussed later.

The total (annual) expenditure on electricity \( \omega_0 \) is fixed by the consumer as the result of a two-stage budgeting procedure. (See Deaton and Muellbauer, 1980, on the validity of this assumption and Parks and Weitzel, 1984, on the application of this assumption to time-of-use experiments.) In turn, it requires the separability of preferences between electricity consumption and non-electricity goods and services. As in a tariff experiment, data are not usually available to permit complete modelling of the households’ choices, separability is a convenient but non-testable hypothesis. The consumer has to allocate \( \omega_0 \) across the different period.

If \( \omega_t \), denotes total expenditure for electricity goods left over at the beginning of period \( t \) \((t = 1, \ldots, T)\) then the consumer’s problem is to choose, for any date \( t \), the feasible quantities
\( x_{1t}, x_{2t}, \) and \( \omega_{t+1} \) that will maximize his or her intertemporal value function:

\[
V_t = V_t(\omega_t) = \max_{\omega_{t+1}, x_{1t}, x_{2t}} u_t(x_{1t}, x_{2t}) + \frac{1}{1 + \rho} E_t[V_{t+1}(\omega_{t+1})]
\]

(1)

subject to the total expenditure constraint for period \( t \),

\[
\omega_{t+1} = \omega_t - p_{1t} x_{1t} - p_{2t} x_{2t} = \omega_t - y_t
\]

(2)

with \( \omega_0 \) unknown constant and \( \omega_T = 0 \). In these equations, \( y_t \) is the daily expenditure on electricity, \( u_t = u_t(x_{1t}, x_{2t}) \) is the within-day utility function, \( \rho \) is the fixed individual discount rate, \( E_t(\cdot) \) denotes the conditional expectation given the available information up to time \( t \), and \( p_i \) \((i = 1, 2)\) is the electricity price for period \( i \) \((i = 1 \text{ for peak periods, } i = 2 \text{ for off-peak periods})\) during day \( t \) \((t = 1, \ldots, T)\). Note that the within-day utility is assumed to be independent of past consumptions.

We assume that \( \omega_0 \) is large enough. It is senseless to envision that the consumer could be able to spend in one period all the budget left from the preceding period. So we assume \( \omega_t > 0 \) for any \( t \), which, in any case, can be checked \textit{ex post}.

First-order conditions for programme (1) are:

\[
E_t \left( \frac{\partial u_t}{\partial x_{1t}} - \lambda_t p_{1t} \right) = 0, \quad i = 1, 2
\]

(3a)

\[
E_t \left( \frac{1}{1 + \rho} \frac{\partial u_t}{\partial \omega_t} \right) = \lambda_t
\]

(3b)

where \( \lambda_t \) is the marginal utility of past expenditure \( \omega_t \) at time \( t \), or the inverse of the price associated with the expected future utility calculated at time \( t \). At the optimal solution, we have

\[
\lambda_t = \frac{\partial V_t}{\partial \omega_t} = \frac{\partial u_t}{\partial y_t}
\]

(3c)

The expectation operator in equation (3a) is superfluous as it bears on current variables. Note that the intertemporal separability assumption and the hypothesis of a within-day utility function independent of past consumptions lead to a solution which does not require us to posit a model for the formation of expectations on the colour of the next day. These assumptions also prevent us from considering a process for habit or stock formation. Relaxing these hypotheses is left to further research.

Equation (3b) provides an orthogonality condition which we exploit in the econometric model below. From equations (3a), we deduce the Frisch demand functions

\[
x_{it}^* = \xi_t(p_{1t}, p_{2t}, \lambda_t), \quad i = 1, 2
\]

(4)

---

19 Here the real interest rate is set to zero, which seems a reasonable assumption in this context of daily decisions.

20 Implicitly the utility function has all the good assumptions in order to avoid corner solutions.

21 In particular, this assumption means that the model is based on the idea that storage capacities on several days are very limited. The consumption of lighting cannot be substituted between days. For hot water and heating, the storage can only be realized for several hours. One can postpone the use of a washing-machine or a dryer but is less likely to do so with the dishwasher.
which depend on observable electricity prices at time $t$, but also on past behaviour and expectations on prices and other variables through the only variate $\lambda_r$.

In this framework of a stochastic intertemporal model, the assumption of additive separability produces a demand system which is analytically tractable, and can be estimated once the difficulty introduced by the non-observability of $\lambda_r$'s is overcome. This is done in two steps. First, we derive the parametric expression of Frisch demand functions from the specification of an indirect utility function. Second, we observe that the system of Frisch demand functions has a state-space representation which allows us to estimate this system by means of the Kalman filter, nested in the maximum likelihood estimation procedure.

### 3.2. The Econometric Specification

Let

$$\Phi(p_t, y_t) = \Phi(p_{1t}, p_{2t}, y_t) = \max_{x_{1t}, x_{2t}} \{ u(x_{1t}, x_{2t}); p_{1t}x_{1t} + p_{2t}x_{2t} = y_t \}, \quad \forall t = 1, \ldots, T$$  \hspace{1cm} (5)

be the within-day indirect utility function of the consumer. The preceding dynamic intertemporal program can be easily written in terms of this indirect utility function. One may indeed solve programme (1) in two steps. First, one chooses the electricity consumptions for each day; then one optimizes the sequence of residual expenditures.

In the remaining part of this section the time index $t$ will be omitted when no confusion can arise. Consider the following quasi-homothetic indirect utility function (also known as the Gorman polar form):

$$\Phi(p, y) = -\exp \left[ -\frac{y - a(p) - \epsilon(p)}{b(p)} \right]$$  \hspace{1cm} (6)

where the functions $a(p), \epsilon(p), \text{ and } b(p)$ are homogeneous of degree one.\footnote{This specification of the indirect utility function contains an implicit normalization which can be estimated. See Alessie et al. (1989). One may in particular multiply the right-hand side of equation (6) as well as the function by the same parameter. At the estimation stage, we did not reject the null hypothesis that this parameter is equal to one. We decided to report only on results with this parameter set to one.}

Roy's identity provides the expression of the Marshallian demand for good $i$, which is

$$x_i = a_i(p) + \frac{y - a(p) - \epsilon(p)}{b(p)} b_i(p) + \epsilon_i$$  \hspace{1cm} (7)

where $a_i(p) \equiv \partial a(p)/\partial p_i$, $\epsilon_i(p) \equiv \partial \epsilon(p)/\partial p_i$ and $b_i(p) \equiv \partial \beta(p)/\partial p_i$. We may deduce the marginal utility of expenditure given by

$$\lambda = \frac{\partial \Phi}{\partial y} = b(p)^{-1} \exp \left[ -\frac{y - a(p) - \epsilon(p)}{b(p)} \right]$$  \hspace{1cm} (8)

or, equivalently, by

$$\ln \lambda = -\ln b(p) - \frac{y - a(p) - \epsilon(p)}{b(p)}$$  \hspace{1cm} (9)
Then, from equations (8) and (9), it is easy to deduce the demand functions with a constant $\lambda$, or Frisch demand functions, as

$$x_i = \xi_i(p, \lambda) + \epsilon_i = a_i(p) - b_i(p)(\ln b(p) + \ln \lambda) + \epsilon_i \tag{10}$$

We now choose some parametric specifications for functions $a(p)$ and $b(p)$. The price index function $a(p)$ is approximated through a generalized Leontief form, i.e.

$$a(p) = a_{11}p_1 + a_{22}p_2 + 2a_{12}(p_1p_2)^{1/2} \tag{11}$$

For $b(p)$ we consider a translog form, which may be written as

$$\ln b(p) = \beta_1 \ln p_1 + (1 - \beta_1)\ln p_2 + \frac{1}{2} \beta_{11}\left(\ln \frac{p_1}{p_2}\right)^2 \tag{12}$$

To account for a possible effect of temperature and of individual characteristics on preferences, the parameters of $a(p)$ and $b(p)$ could be made functions of these variables.

In addition, we must specify the error term $\epsilon(p)$. Since prices are perfectly known, measurement errors can only happen on consumptions. Assume that these errors are additive, that is to say,

$$x_i = x_i^* + \epsilon_i, \quad i = 1, 2 \tag{13}$$

where $x_i$ and $x_i^*$ are the observed and true levels of consumption, respectively. Then the observed expenditure is

$$y \equiv p'x = p'x^* + p'\epsilon \tag{14}$$

with obvious notations. So we specify $\epsilon(p)$ as

$$\epsilon(p) \equiv p'\epsilon = p_1\epsilon_1 + p_2\epsilon_2 \tag{15}$$

We now complete the stochastic specification of the model. First, we assume that $\epsilon_n$ is normally distributed, with a zero mean and with the following covariance structure:

$$E(\epsilon_i^2) = \sigma_i^2, \quad i = 1, 2 \forall t$$

$$E(\epsilon_i \epsilon_j) = 0, \quad i \neq j, \forall t, s \tag{16}$$

Second, we can take advantage of the orthogonality condition (3b) by assuming that the logarithm of the marginal utility of expenditure follows a martingale with drift, i.e.

$$\ln \lambda_t = \ln \lambda_{t-1} + \ln(1 + \rho) + \eta_t \tag{17}$$

The error term $\eta_t$ is assumed to be Gaussian, with a zero mean and a finite variance $\sigma_{\eta}^2$.

All equations introduced so far should be indexed by an individual index. The preceding stochastic assumptions apply to all daily observations for each individual in the sample. A final hypothesis is that observations among individuals are uncorrelated.

Summarizing, we propose to approximate the first-order conditions (3a) and (3b) by estimating the system formed by equation (10) together with equation (17). The demand functions defined by equation (10) are explained by the observed prices and two latent variables: the marginal utility of daily expenditure and the measurement errors. While daily prices are identical for all individuals and are time-varying, the marginal utility of daily expenditure appears as a individual-specific time-varying effect, which evolves according to a non-stationary
process. Note that, because of the simultaneous presence of $\lambda$ in the peak and off-peak demand functions, they are correlated. When panel data are available, the 'constant' specification of demands can be estimated from the repeated observations on cross-sectional units. Note that estimating Frisch demands instead of Marshallian demands avoids the use of the total expenditure as an explanatory variable, which is the usual practice.

### 3.3. The State-Space Representation

Equations (10) and (17) may be viewed as the measurement equation and the transition equation of a state-space model, respectively. This justifies the use of the Kalman filter to build up the likelihood function and to estimate parameters. Following the notation used by Harvey (1990) in his presentation of the Kalman filter, we define

$$
x_t = \begin{pmatrix} x_{1t} \\ x_{2t} \end{pmatrix}, \quad Z_t = \begin{pmatrix} b_{1t}(p_t) \\ b_{2t}(p_t) \end{pmatrix}, \quad d_t = \begin{pmatrix} a_{1t}(p_t) - b_{1t}(p_t)\ln b_{1t}(p_t) \\ a_{2t}(p_t) - b_{2t}(p_t)\ln b_{2t}(p_t) \end{pmatrix}, \quad \varepsilon = \begin{pmatrix} \epsilon_{1t} \\ \epsilon_{2t} \end{pmatrix}
$$

$$
\mu_t = \ln \lambda_t, \quad c = \ln(1 + \rho)
$$

$$
H = \begin{pmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{pmatrix}, \quad Q = \sigma_q^2
$$

With this notation, the system consisting of equations (10) and (17) may be written

$$
x_t = Z_t \mu_t + d_t + \varepsilon_t, \quad E(\varepsilon_t) = 0, \quad \text{Var}(\varepsilon_t) = H
$$

$$
\mu_t = \mu_{t-1} + c + \eta_t, \quad E(\eta_t) = 0, \quad \text{Var}(\eta_t) = Q
$$

(18a) (18b)

Moreover, we assume that $E(\varepsilon, \eta_t) = 0$, $E(\varepsilon, \mu_0) = 0$, and $E(\eta_t, \mu_0) = 0$ for any $t \geq 1$, where $\mu_0$ is a value of the initial state.

When successive observations for each individual in the sample are not independent, which is obviously the case here, the Kalman filter produces a calculable form for the individual log-likelihood function. The individual likelihood function is given by

$$
\ln L(x) = \sum_{i=1}^{T} p(x_i | X_{i-1})
$$

(19)

where $p(x_i | X_{i-1})$ is the conditional distribution of $x_i$ given the past history $X_{i-1}$ (from 1 up to $t - 1$). Recall that, as the error terms of the model are normally distributed, this conditional distribution is also normal. By adding all individual log-likelihood functions, we derive the function to be optimized.

When estimates are obtained, the Kalman filter can be used to assess the fit of the model as well as to compute the estimated values of the marginal utility of daily expenditure.

### 4. THE EMPIRICAL RESULTS

In this section we report on the empirical analysis. The results indicate that the chosen method offers a good representation of the data-generating process, which allows us to provide an evaluation of the real-time pricing experiment.
4.1. Estimation Results

The technique proposed in the previous section is applied to data provided by the real-time pricing experiment. As the daily discount rate cannot be identified, the optimization of the log-likelihood function is performed at different values of this parameter taken on a very large interval.\(^{23}\) Results are reported for only three values of \(\rho\) covering a very large range, specifically, \(\rho = 0.00\%,\ 2.56\%\) and \(5.26\%.\) The optimization procedure is based on routines of the GAUSS statistical language. The Kalman filter converges very rapidly for each estimation of the model. When choosing initial values for the equations of the Kalman filter, we have simply followed the recommendations given by Harvey (1990) when the transition equation is a random walk. In the following we use parameter estimates obtained when \(\rho = 2.56\%\).

Results are reported in Table IV. The value of the discount rate has only a very slight impact on the estimates. All parameters are always highly significant. The goodness-of-fit for the two demand equations (measured on Table IV by \(R^2_1\) and \(R^2_2\) for peak and off-peak demand functions, respectively) is quite high, even though individual observable heterogeneity is not incorporated into the model. Figures 1 and 2 display, for a particular customer,\(^{24}\) the observed and estimated demands over peak and off-peak periods during 100 days in winter time (from December 1991 to February 1992). These graphs can be reproduced for other periods and for other individuals. Clearly, the effect of the tariff is striking. Note first, that peak and off-peak demands are highly correlated, their temporal patterns being quite similar. Second, observe that there is either a catching-up effect after red or white days, which pleads for a model with stock effects, or the effect of more or less correct expectations. We favour here the interpretation in terms of expectations, but we recognize that a model that would be able to discriminate between the two effects would be required. Nonetheless, note that this customer always returns to his average consumption when he experiences a sequence of several blue days, and often increases his consumption just before a red day.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>(\rho = 0.00%)</th>
<th>(\rho = 2.56%)</th>
<th>(\rho = 5.26%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\alpha_{11})</td>
<td>8.835 ± 1.154</td>
<td>8.835 ± 1.154</td>
<td>8.834 ± 1.152</td>
</tr>
<tr>
<td>(\alpha_{22})</td>
<td>11.913 ± 2.126</td>
<td>11.913 ± 2.126</td>
<td>11.914 ± 2.124</td>
</tr>
<tr>
<td>(\beta_1)</td>
<td>0.717 ± 0.004</td>
<td>0.717 ± 0.004</td>
<td>0.718 ± 0.004</td>
</tr>
<tr>
<td>(\beta_{11})</td>
<td>0.080 ± 0.003</td>
<td>0.080 ± 0.003</td>
<td>0.080 ± 0.003</td>
</tr>
<tr>
<td>(\sigma_{1}^2)</td>
<td>14.823 ± 0.350</td>
<td>14.823 ± 0.350</td>
<td>14.822 ± 0.350</td>
</tr>
<tr>
<td>(\sigma_{2}^2)</td>
<td>1.889 ± 0.311</td>
<td>1.888 ± 0.311</td>
<td>1.890 ± 0.312</td>
</tr>
<tr>
<td>(\sigma_{3}^2)</td>
<td>8.337 ± 0.516</td>
<td>8.335 ± 0.516</td>
<td>8.343 ± 0.516</td>
</tr>
<tr>
<td>(R^2_1)</td>
<td>0.836</td>
<td>0.835</td>
<td>0.835</td>
</tr>
<tr>
<td>(R^2_2)</td>
<td>0.936</td>
<td>0.936</td>
<td>0.934</td>
</tr>
<tr>
<td>(\ln L)</td>
<td>−6.91973</td>
<td>−6.91941</td>
<td>−6.92040</td>
</tr>
</tbody>
</table>

\(^{23}\) Note that the values of the discount rate we consider are small compared to price variations implied by the real-time tariff.

\(^{24}\) This customer is a clerical employee, he has 3 children, he lives in the east of France, he is the lucky owner of a four-bedroom house fitted out with an electric heating (completed by an additional system), he may use a crude control system for his heating, and he owns standard household equipments (but no dryer and no air conditioning). He uses the six-price tariff and he is one of the most responsive customers to the experimental tariff.
Figure 1. Peak demand of a customer during winter

Figure 2. Off-peak demand of a customer during winter
As mentioned earlier, some information is available on individual characteristics, on the type of heating system, on the ownership of different appliances and on the energy management system. Moreover, the average daily outside temperature at the different locations of the experiment is also known. One way to introduce these variables into the model is through the parameters of functions \( a(p) \) and \( b(p) \) specified by equations (11) and (12). Notice that the homogeneity constraint on the demand functions restricts the ways for introducing exogenous variables. Basically, one can consider linear relationships between the structural parameters and these variables. So far, we could not obtain convergence of the optimization procedure for various specifications and choices of variables. One possible reason is that the exogenous variables specific to individuals do not vary much across individuals. Moreover, these variables are probably correlated with the latent individual-specific effect \( \lambda \), i.e., the marginal utility of daily expenditure. Recall that, in the context of our stochastic intertemporal model, \( \lambda \) should stay constant as long as no new information arises. Clearly, most of individual characteristics and variables describing the stock of electric appliances do not change during the experiment. This is not the case for temperature. However, in a real-time pricing scheme, electricity prices of every day are in part varying with the weather conditions. Therefore prices are conveying the new information while the marginal utility of daily expenditure keeps track of the invariant information. When we introduce individual characteristics, we fall into a problem of exogeneity since the marginal utility of expenditure plays the role of an error term in the demand system while it is related to these characteristics.

As a support of this conjecture, we fitted, for each day, the estimated values of the marginal utility of daily expenditure (obtained from the estimates of the dynamic model using the Kalman filter) on some individual dummy variables characterizing the types of heating and energy management system owned by households and on the average outside temperature. We report the OLS estimates in Table V. All parameters are highly significant. In addition, we present the GLS estimate of a random effect model where temperature is the single explanatory variable. Again the fit is good and the parameter is significant. Note that the value of the parameter associated with the temperature does not change much between these two estimations.

<table>
<thead>
<tr>
<th>Dependents variable: ( \lambda )</th>
<th>OLS estimates</th>
<th>GLS estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>77.090</td>
<td></td>
</tr>
<tr>
<td>Electric heating</td>
<td>-15.804</td>
<td></td>
</tr>
<tr>
<td>Electric heating with additional system</td>
<td>-22.664</td>
<td></td>
</tr>
<tr>
<td>Combined oil—electric boiler</td>
<td>-12.954</td>
<td></td>
</tr>
<tr>
<td>Heating by accumulation system</td>
<td>-27.636</td>
<td></td>
</tr>
<tr>
<td>Electric pump heating</td>
<td>-15.323</td>
<td></td>
</tr>
<tr>
<td>Heating is on for blue off-peak only</td>
<td>-19.399</td>
<td></td>
</tr>
<tr>
<td>Heating is off for red peak only</td>
<td>7.474</td>
<td></td>
</tr>
<tr>
<td>Heating is off for red days/white peaks</td>
<td>-2.629</td>
<td></td>
</tr>
<tr>
<td>Temperature</td>
<td>-1.431</td>
<td>-1.473</td>
</tr>
</tbody>
</table>

\( R^2 \)  

<table>
<thead>
<tr>
<th></th>
<th>OLS estimates</th>
<th>GLS estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.129</td>
<td>0.535</td>
</tr>
</tbody>
</table>
4.2. Analysis of Individual Behaviour

Let $\epsilon_{ij}^p$, $\epsilon_{ij}^h$, and $\epsilon_{ij}^f$ denote the price elasticities of the Marshallian, Hicksian, and Frisch demands for good $i$ with respect to the price of good $j$, respectively. If $e_i$ represents the elasticity of the demand for good $i$ with respect to the daily electricity expenditure and if $w_i$ denotes the share of the electricity expenditure allocated to good $i$, then

$$\epsilon_{ij}^p = \epsilon_{ij}^h + e_i e_j w_i \sigma + e_i \sigma$$

(20)

as Houthakker (1960) has shown. We define $\sigma$, the intertemporal elasticity of substitution, as

$$\sigma = \left( \sum_i \frac{\partial \chi}{\partial p_i} \frac{p_i}{y} \right) - 1 \quad i = 1, 2$$

(21)

where $\chi(p, \lambda)$ represents the expenditure amount for which the consumer's marginal utility attains the level of marginal utility of daily expenditure $\lambda$ given that prices are equal to $p$. Moreover, following Browning (1985), the intertemporal effect can be defined as $e_i \sigma$, while the intratemporal effect $\sigma_{ii}$ is the expression between brackets in the equation

$$\epsilon_{ij}^f = [\epsilon_{ij}^h + e_i e_j (e_i w_i - 1)] + e_i \sigma = \sigma_{ii} + e_i \sigma$$

(22)

which results from equation (20). Goods are specific substitutes (respectively, complements) if the intratemporal effect is negative (resp. positive).25

Table VI shows the sample means and standard deviations (given in parentheses) of Marshallian price elasticities for peak and off-peak demands. This table also gives the elasticities of these demands with respect to the daily electricity expenditure, the intratemporal effects and the intertemporal elasticity of substitution. Moreover, these statistics are calculated according to the day type (i.e. its colour). Because distributions of these elasticities contain large outliers, these summary statistics are computed after having trimmed about 5% of all observations.

Table VI suggests the following conclusions. First, own-price elasticities are high but significantly less than unity. Second, cross-price elasticities are negative and quite high. However, the off-peak demand elasticity with respect to peak prices is much higher in absolute value than the peak demand elasticity with respect to off-peak rates. Note that, from a blue to a red day, the most noticeable change is the increase in the off-peak demand elasticity with respect to peak prices. Moreover, when the peak rate increases, the customer decreases more the off-peak consumption than the peak demand during white or red days, while the customer does the opposite on blue days. In some sense, electricity needs appear to be more incompressible during the peak hours of white and red days. Third, the peak and off-peak electricity goods are substitutes (compensated cross-elasticities, computed on average overall, types of days are positive but small). They are also specific substitutes, because the intratemporal effects are slightly negative. Given that French households have been accustomed to time-of-use pricing for a long time, they probably have already adapted their consumption habits. Clearly one cannot expect a quite high level of substitutability between off-peak and peak demands of electricity. Fourth, the expenditure elasticities are very close to unity. Finally, the intertemporal elasticity of substitution is significantly different from zero, but takes very low values.26 However, these values vary greatly with day type as shown in Figure 3. Again, this diagram indicates that consumers respond in the expected way to price variations.

25 In a two-good system, the two goods cannot be Hicksian complements but they can be specific complements.

26 Since expenditure elasticities are close to one, the intertemporal effect (see equation (22)) is of the same order of magnitude as the intertemporal elasticity of substitution.
<table>
<thead>
<tr>
<th></th>
<th>All types of days</th>
<th>Blue days</th>
<th>White days</th>
<th>Red days</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Peak</td>
<td>Off-peak</td>
<td>Peak</td>
<td>Off-peak</td>
</tr>
<tr>
<td>$\xi_f$</td>
<td>-0.79 (0.1)</td>
<td>-0.18 (0.2)</td>
<td>-0.78 (0.2)</td>
<td>-0.19 (0.2)</td>
</tr>
<tr>
<td>$\xi_f$</td>
<td>-0.93 (0.5)</td>
<td>-0.28 (0.3)</td>
<td>-0.74 (0.2)</td>
<td>-0.37 (0.5)</td>
</tr>
<tr>
<td>$\epsilon_i$</td>
<td>0.99 (0.1)</td>
<td>1.44 (1.2)</td>
<td>1.01 (0.2)</td>
<td>1.18 (0.6)</td>
</tr>
<tr>
<td>$\sigma_{11}$</td>
<td>-0.018</td>
<td>(0.12)</td>
<td>-0.028</td>
<td>(0.14)</td>
</tr>
<tr>
<td>$\sigma_{22}$</td>
<td>-0.019</td>
<td>(0.30)</td>
<td>-0.049</td>
<td>(0.32)</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>-0.035</td>
<td>(0.04)</td>
<td>-0.034</td>
<td>(0.04)</td>
</tr>
</tbody>
</table>
4.3. Welfare Effects

In this dynamic setting, a welfare evaluation of the real-time pricing scheme can be obtained by comparing, for each individual, the present discounted value of his or her daily electricity expenditure incurred during the experiment and the present discounted value of the expenditure that allows the consumer to achieve the same level of indirect utility under the standard two-price tariff.\textsuperscript{27} This second expenditure is not known for the participants in the experiment. However, given the within-day indirect utility specified in equation (6), this expenditure is obtained as

\[ \bar{y}_t = \frac{b(\bar{p}_t)}{b(p_t)} (y_t - a(p_t)) + a(\bar{p}_t) \]  \hspace{1cm} (23)

where \( p_t \), and \( y_t \) are, respectively, electricity prices and daily expenditure for day \( t \) during the experiment, and \( \bar{p}_t \) is the vector of electricity prices for day \( t \) under the standard rate pricing schedule. In fact one has \( \bar{p}_t = (\bar{p}_{1t}, \bar{p}_{2t}) = (\bar{p}_1, \bar{p}_2) \), \( \forall t \), where \( \bar{p}_1 \) and \( \bar{p}_2 \) are known constant prices. Let

\[ G = \sum_{t=0}^{T} \left( \frac{1}{1 + \rho} \right) \left( \frac{y_t - \bar{y}_t}{\bar{y}_t} \right) \]  \hspace{1cm} (24)

\textsuperscript{27}In the standard peak/off-peak pricing option in 1992, the peak and off-peak rates are 56.89 and 32.29 French cents per kWh, respectively.
be a measure of change in welfare (i.e. a measure of average savings to each consumer). If $G$ is negative, the ex ante consumer welfare increases with the real-time pricing option. The distribution of estimated individual gains or losses is given in Figure 4. This shows that a majority of consumers involved in the experiment have benefited from the new tariff option. The mean of $G$ is 7.96%. (For $\rho = 0.00\%$, $G = 4.53\%$.) Moreover, if we take into account the fixed fees that customers have to pay for consumers energy, the effect is slightly higher, because, on average, the fee for the real-time tariff is lower than for the standard option (according to the individual power subscribed).

This evaluation also shows that consumers with the experimental real-time tariff option for a longer period (almost all the consumers subject to the four-price tariff) have increased their welfare with the new pricing scheme. Should we conclude that customers need time to get accustomed to the real-time tariff rules and that customers subject to the six-price tariff will obtain positive welfare gains in the future? Probably, as the analysis is terminated at a given date, the process behind the experiment is in some sense censored, which introduces a bias. In any case, we may point out these two following facts.

First, customers who benefited most from the real-time tariff are not necessarily located in colder areas. Except one customer, all the others living in temperate regions of France (west) gained from the experimental tariff. Indeed this is not surprising. As the tariff is the same all over the electrical network for residential customers (due to the existence of a regulatory constraint), a customer living in colder areas receives the same price signal as the one enjoying mild temperatures, but the former cannot restrain consumption as much as the latter. If this is true, once could expect to observe a larger fraction of households living in colder areas that would be worse off. However, about 50% of customers (in the sample) living in colder areas of France (east and centre) increased their welfare.

![Figure 4. Distribution of welfare effects, customer by customer](image)

---

28 As prices are based on long-run marginal costs (see footnote 7), any saving on the electricity bill roughly corresponds to the same amount of cost saving (from not having maintained higher capacity for instance). In theory, this pricing policy is socially efficient.
Second, as a related issue, one may look at the distribution of individual welfare losses and gains according to the type of heating system installed in the house. Whatever the heating system, their owners are equally distributed between losers and winners during the experiment. In particular, more than 50% of households owing an electric space heating system have benefited from the tariff. This is noticeable as these households have a priori a lower capacity to adapt their consumptions when a red or white day is announced. However, a customer who must only rely on electricity may be more concerned by the optimization of consumption and more responsive to price signals than a customer endowed with a dual-energy heating system, but reluctant to switch from one energy to the other when the management system of the heating is rather crude. Moreover, during the experiment, the price of fuel was low, which reduces the effectiveness of incentives conveyed by the electricity tariff. This raises the question of the competition between producers of energy.

5. CONCLUSION

The main result of this study is that the real-time tariff experiment conducted by EDF on residential customers improved the welfare of the majority of participants. However, the sample built during this experiment has a small size that does not allow us to generalize this result further. It is also quite homogeneous which, in part, prevents us from measuring the influence of individual characteristics or other environmental variables on electricity consumption. Moreover, we are not able to treat the problem of selection bias, induced by the optional nature of the tariff. Finally, it is too early to observe the effect of this type of tariff on the equipment choices of households.

Our method consists of using the Kalman filter to estimate the system of Frisch demands of daily electricity consumption, derived from a classical dynamic model of household behaviour. Although our method performs quite well, it is based on several assumptions. In particular, the utility function is additively separable; there is no effect of stocks; there is no process of habit formation; the household uses a two-stage budgeting procedure; and the model requires that the consumer has some knowledge of annual expenditure on electricity. These hypotheses avoid the problem of specifying a model of expectation formation on the stock of red and white days left. We cannot infer from our approach whether the customer makes use of the information on the stock of days. To take up this problem, one should consider the type of approach proposed by Rust (1994) and Pakes (1994).

The analysis is restricted to a two-good system. Leisure (or time) is absent. Clearly the opportunity cost of time could be a crucial variable here since, without sophisticated energy management systems, a real-time tariff requires an effort from the consumer. However his/her attitude, or the time he or she is ready to spend to achieve an efficient use of electric equipment, is unobservable. This raises two questions. First, how do we measure this unobservable variable recover it when data are collected by a firm and cannot bear on all the aspects of consumer behaviour? Second, does the real-time tariff give the right incentive to the customer?

While the investigation is performed by means of a simple model which provides a fruitful specification, this paper opens a vast field for further research.

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