Heterogeneity in Consumer Price Stickiness: A Microeconometric Investigation

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We examine heterogeneity in price stickiness using a large, original, set of individual price data collected at the retail level for the computation of the French consumer price index. For that purpose, we estimate at a very high level of disaggregation, a piecewise-constant hazard model, as well as competing-risks duration models that distinguish between price increases, price decreases, and product replacements. The main findings are the following: (a) at the product–outlet-type level, the baseline hazard function of a price spell is nondecreasing; (b) cross-product and cross-outlet-type heterogeneity is pervasive, both in the shape and the level of the hazard function as well as in the impact of covariates; (c) there is strong evidence of state dependence, especially for price increases; (d) there is an asymmetry because determinants of price increases differ from those of price decreases.

KEY WORDS: Duration models; Hazard function; Sticky prices.

1. INTRODUCTION

Assessing price rigidity is a notoriously crucial issue from a macroeconomic perspective, in particular for monetary policy. A typical approach to this issue is to investigate time series of aggregate or semiaggregate price indices. There are, however, several motivations for adopting a microeconomic approach. First, many models of price rigidity proposed in the macroeconomic literature are explicitly based on microeconomic behavior (see, for instance, Taylor 1998, for a survey, and Taylor 1980; Calvo 1983; Sheshinski and Weiss 1983; Dotsey, King, and Wolman 1999, for important contributions) so that microdata are a relevant testing ground. In particular, the use of microdata may overcome the problem of observational equivalence of models that emerges at the aggregate level (in the case of the New Keynesian Phillips curve, see, e.g., Rotemberg 1987). Second, such data shed light on the heterogeneous patterns of price-setting behaviors that do coexist in the economy. Heterogeneity in average price durations has recently been documented using individual consumer price data by Bils and Klenow (2004) for the United States, and by Dhyne et al. (2006) for the euro area (see also the references therein).

The present article implements duration models to investigate price stickiness. For that purpose, we use a unique microeconomic dataset consisting of the individual consumer price quotes collected by Institut National de la Statistique et des Etudes Economiques (INSEE) in French outlets for the computation of the Consumer Price Index (CPI hereafter). Though rarely applied so far to price data, the hazard function approach is a relevant framework since it allows to characterize the sign and the magnitude of time- and state-dependence in price-setting. Indeed, econometric duration analysis typically relies on proportional hazard models which specify the hazard function, namely the instantaneous conditional probability of an event, as the product of a baseline hazard function, which captures time-dependence, and of a multiplicative term depending on a set of potentially time-varying covariates, which captures state-dependence. In the context of our investigation, the hazard function is the instantaneous conditional probability of adjusting the price of an item, given the elapsed duration since the last price change. More formally, in continuous time (which is the time scale consistent with the econometric framework developed later), if we denote $P_t$ the price of an item at date $t$, the hazard rate of a price change at date $t$ can be defined as:

$$h_t = \lim_{s \to 0} \Pr(P_{t+s} \neq P_t | P_u = P_t, \forall u \in [0, t]) / s,$$

where the price is assumed to be reset at date 0.

Sticky price models have predictions in terms of the hazard function for a price change. For instance, Calvo’s model (Calvo 1983) relied on a constant hazard function, whereas using a general equilibrium model, Dotsey et al. (1999) showed that the menu costs assumption leads to an increasing hazard function. More generally, models of price adjustment in the literature predict nondecreasing hazard functions. One striking crude result, however, appears to be in contradiction with all price-setting models, since the overall hazard function for a price change is estimated to be decreasing, as illustrated by Figure 1 which reports the nonparametric estimate of the piecewise hazard function for manufactured goods in our sample (see also Alvarez, Burriel, and Hernando 2005; Baudry, Le Bihan, Sevestre, and Tarrieu 2007; Dias, Robalo Marques, and Santos Silva 2005, for similar evidence). One possible route to rationalize this puzzle is the mover–stayer effect. To illustrate this point, let us consider the following example, adapted from the textbook by Cameron and Trivedi (2005, p. 611). Suppose that there are two
types of products in the economy, denoted F (flexible price) and S (sticky price). The price changes of the type-F (respectively, S) product are generated by a time-constant hazard of \( \lambda \) (respectively, \( .1 \)). The economy is a 50/50 mixture of these two types of products. Then for a large number of samples of 100 type-F products, we observe an average of 40 price changes during the first time unit, of 24 changes in the second period, and of 14.4 in the third. For the samples of 100 type-S products, we observe an average of 10, 9, and 8.1 price changes during the first, second, and third time units. However, the aggregate proportions of price changes are \( (40 + 10)/200 = .25 \), \( (24 + 9)/150 = .22 \), and \( (14.4 + 8.1)/117 = .192 \). Here the declining overall hazard rate is a direct consequence of the aggregation of two different subpopulations (products), which have both constant but different hazard rates. Then the inadequate treatment of heterogeneity can produce biased estimates of the hazard function (see Heckman and Singer 1984, or in the context of price changes, Alvarez et al. 2005). To handle heterogeneity and eliminate the mover-stayer effect, we take advantage of the wide coverage and of the very large size of the dataset to estimate duration models at a very high level of disaggregation, namely at the product-outlet-type level. On the whole, duration models are estimated for several hundreds of strata.

Two types of tests are successively considered in this article, each focusing on a specific class of price-stickiness models. In a first step, we concentrate on the main assumption of Calvo’s model (Calvo 1983), namely the constancy of the hazard function of price changes, notwithstanding the fact those price changes may correspond to price increases or price decreases. We find that at a very disaggregated level, the constancy of the hazard function cannot be rejected in 35% of CPI weighted product-outlet strata. The remaining cases can be split roughly equally into increasing and decreasing hazard rates. To investigate these cases, we explore the patterns and the determinants of price changes in the corresponding strata. We distinguish between price increases and decreases because the specific hazard rates corresponding to these two types of events may have different time patterns. This leads us to estimate competing-risks duration models. This strategy, which treats price increases and decreases as separate events, allows us to examine another possible source of heterogeneity, namely “state heterogeneity.” Indeed, the competing-risks duration model allows for differences in baseline hazard functions corresponding to different types of terminating events, but it also allows to test for different effects of the same time-varying covariates on the specific hazard rates of price increases and price decreases. Such an approach is thus in line with theoretical state-dependent models that predict the impact of covariates to be different and asymmetric for price increases and price decreases (see Caballero and Engel 1993a, b). Following the seminal microeconometric analysis of Cecchetti (1986), we test here for the presence of state-dependence by considering time-varying covariates such as the cumulative sectorial rate of inflation.

The outline of the article is as follows. Predictions of theoretical sticky price models in terms of state- and duration-dependence in price-setting are briefly reviewed in the next section. Section 3 describes the dataset. Sections 4 and 5 present the results based on the two aforementioned alternative specifications. Section 6 summarizes the main results and concludes.

2. STICKY PRICES AND THE HAZARD FUNCTION: THEORETICAL BACKGROUND

This section reviews the main models of price-setting behavior used in monetary economics, focusing on their implications in terms of the hazard function. A more detailed survey of these models can be found in, for example, Taylor (1998). Models of price rigidity can be broadly classified into two categories: time-dependent models and state-dependent models (see, e.g., Blanchard and Fisher 1989, pp. 388–389). Discriminating between the alternative forms of price stickiness is an important issue, because the type of nominal rigidity matters for the reaction of the macroeconomy to various shocks, as established by, for example, Kiley (2002) and Dotsey and King (2005).

Time-dependent models assume that price changes take place at fixed or random intervals. A prominent model is Taylor’s staggered contracts model (Taylor 1980), which assumes that prices and wages are negotiated for fixed periods, say one year. Even though formal price contracts do not in general exist for consumer prices, Taylor’s model may reflect the practice of changing the price every year, say in January. As a consequence, the probability of a price change should be zero for the first periods (say, eleven months following a price change) and exhibit one spike at the contract renewal. If contracts of different lengths coexist in the economy, several modes in the hazard function for a sample of price spells may be expected. In the monetary policy literature, a widespread alternative to price or wage contracts is Calvo’s model (Calvo 1983). In this model, each firm has a constant instantaneous probability of changing its price, irrespective of the time elapsed since the last price change, so that the hazard function is flat. Calvo’s constant hazard model is arguably acknowledged by most researchers to be a crude approximation to a fully fledged price-setting policy. Yet this model is widely used in monetary economics, with model calibration often relying on a Calvo-type interpretation of microeconomic data (e.g., Woodford 2003). It is thus important to assess whether such an approximation is reasonable. Yet another time-dependent model, which encompasses both Taylor’s and Calvo’s models, is the truncated Calvo model (see Wolman 1999), in which price spells are assumed to have a maximum duration, so that the hazard function should be flat up to this maximum value.

State-dependent models predict that the probability of a price change varies according to the state of the economy. State-dependence with infrequent price changes typically emerges from menu cost models. These models imply that a firm will not change its price if the foregone profit due to a deviation of its current price from the optimal price is smaller than the menu cost, that is, the fixed cost of changing price. Sheshinski and Weiss (1977, 1983) have proposed such a model, in which the probability of a price change is predicted to decrease with the size of the menu cost, whereas the size of the price change is found to increase with the size of the menu cost. Generally, the probability of a price increase is predicted to be an increasing function of the inflation rate trend (though a counterexample was exhibited by Sheshinski and Weiss (1977)). More recently, Dotsey et al. (1999) have proposed in a general equilibrium context a state-dependent pricing model that generalizes
Calvo’s model by assuming a random menu cost. A steady-state result is that the hazard function increases with the time elapsed since the previous price change, because firms that have set their prices a long time ago are more likely to observe a price gap in excess of the menu cost. The slope of the hazard function depends on several parameters of the model such as the steady-state inflation and the shape of the demand function faced by firms (see Dotsey and King 2005). An alternative approach to state-dependence was proposed by Caballero and Engel (1993a,b) and relies on the definition of an adjustment hazard function. The probability of a price change is postulated to have an “increasing hazard property,” that is, to be an increasing function of the gap between the current price and the optimal price that would be set if nominal rigidities were (transitorily) removed. Caballero and Engel (1993a) argued that Calvo’s model and the $(S, s)$ adjustment rules can be viewed as special cases of the adjustment hazard model, and that asymmetries should be allowed in the adjustment hazard. Caballero and Engel (1999) further motivated the adjustment hazard framework and the “increasing hazard property” using a menu cost model where adjustment costs vary across firms.

We focus hereafter on the predictions of sticky price models in terms of the hazard function for price changes. Although we acknowledge that time- and state-dependent models also have implications in terms of the size of a price change, the estimation of a joint model for frequency and size is left for further research. To motivate the advantage of a hazard function approach, note that Taylor’s and Calvo’s models are observationally equivalent if one considers the aggregate frequency of price changes, that is, the proportion of firms changing their price at a given date. Both models indeed predict the aggregate frequency of price changes to be constant through time, at least if contracts are staggered. The two models have in contrast different predictions with respect to the shape of the hazard function, which is constant in the Calvo case. All price models reviewed above, though, share the testable implication that the unconditional hazard function is a nondecreasing function of the elapsed duration since the previous price change (except for potential spikes in the hazard function).

State-dependent models predict that the probability of a price change depends on the distance of the current price from the optimal price which is a function of covariates. In the general case, covariates may affect in an asymmetric manner the probability of a price increase and that of a price decrease, so that testing for state-dependence requires distinguishing between price increases and price decreases. Note that Calvo’s and state-dependent models cannot be easily nested into a single framework because under Calvo’s pricing rule the probabilities of a price decrease and of a price increase may fluctuate according to the environment, though they sum up to a constant probability of price change at each date. Calvo’s model does not produce straightforward restrictions on the probabilities of price decreases and of price increases.

3. THE DATA

The data used in our analysis are the individual price records collected by INSEE for the computation of the French CPI. This is an original dataset, with regard to both its contents and its size. In this section, we briefly document these data. Further details on the methodology used for data collection are contained in INSEE (1998).

3.1 The Original Dataset

The sample contains monthly CPI records from July 1994 to February 2003. These data cover around 65% of the total weight of the CPI. Individual price data for fresh foods, rents, purchase of cars, and administered prices such as electricity or telephone (when still regulated) are not included in the dataset made available to us. The number of price quotes in the initial database is around 13 million price observations, and around 2.3 million price spells. With each individual record the information recorded includes the price level, an individual product code (outlet and product categories), the year and month of the record, and a “type of record” code (indicating whether the price record is a regular one, a sales price, an “imputed” price due to stockout, and so on).

Some specific data issues have been dealt with prior to estimation. For instance, due to temporary stock-outs or holidays, “missing” prices are not uncommon. Those unobserved prices are most often replaced by INSEE using an imputation procedure. For our purpose, it was found more relevant to replace any unobserved price by the previous price observed for the same item. This avoids creating “artificial” price changes due to the very likely discrepancy between the missing price and the average price over other outlets, as imputed by INSEE in its computation of the CPI. As the observation period goes from 1994:7 to 2003:2 (prices being set in euros from 2002:1 onward), we also take the euro cash change-over into account. Consequently, we divide all prices recorded before 2002:1 by 6.55957, the official French franc/euro exchange rate. We ensure that price spells are unaffected prior to the cash change-over, and that in January 2002 price changes corresponding to a simple rounding up to two digits are not counted as price spell terminations (see Baudry et al. 2007, for details on these issues and other aspects of data treatment).

3.2 Attrition, Censoring, and Trimming

3.2.1 Attrition. Individual price data are affected by attrition, corresponding to statistical units “leaving” the sample before the end of the observation period. Two sources of attrition in price records may be distinguished. First, products have life-cycles: “old” products disappear from the market and “new” ones appear. The time series of price observations for a specific product is very likely to be interrupted at some point during the observation period. Second, outlets or firms may close, which obviously interrupts the time series of price observations for all products sold by the outlet or the firm.

Product replacement is quite common in some sectors (in particular in the clothing sector) and is not uncommon in general. Indeed, replacements represent about 20% of the price spell endings in the subsample used for estimation. They induce attrition and cannot be left out of our analysis, as product replacements indeed provide an opportunity to change prices. However, while the data at hand allow to identify product replacements, we cannot assess whether a given product replacement is associated with a price increase or a price decrease.
3.2.2 Censoring. Censoring is a major issue when analyzing durations in general, and in our context in particular. Indeed, there are at least three reasons that may cause price spells to be censored.

First, the observation period is restricted by the database availability. The first spell in a price trajectory is typically left-censored, and the last one is right-censored. For instance, our dataset starts with price records of July 1994. Presumably, some prices recorded at that date were set before the beginning of the available sample, and we do not observe the starting date of those spells.

Second, the sampling of products and outlets by the statistical institute is also likely to generate some censoring. Indeed, the statistical institute may decide to discard a specific product from the “representative” CPI basket because of a shrinking demand for certain product types, although those products may still be sold in outlets (e.g., video cassette recorders with the advent of DVD players). Then, the last price spell of such a product will be right-censored. Conversely, every year, the INSEE introduces new products in the CPI basket. It is likely that the prices of such products have been set before their first record by INSEE agents. Their price will start to be recorded during the course of one price spell. This will generate left-censoring of the first price spell.

Third, outlets and firms may decide to stop selling a product while its price path is followed up by the statistical institute. In such a case, the procedure most often adopted by statistical agencies consists of replacing the “old product” by either a close substitute in the same outlet or by the same product sold in another outlet. It is then very likely that the price of the “replacing” item is set before the first price observation of this product. Then the price spell of the new product is left-censored.

3.2.3 Trimming. Because our price database covers a 10-year period, price trajectories that are observed for each product/outlet pair (i.e., each statistical unit) are made of several price spells. This mechanically results in an over-representation of units that are characterized by short spells, and it is likely to induce a downward bias in the hazard function estimate (see Dias et al. 2005, for a discussion on this issue). Indeed, whereas clothing articles and services both represent about 18% of price records, they respectively account for 18% and 9% of all price spells. Then, in order to use a sample that is representative of the population of product/outlets, we have randomly selected one non-left-censored price spell by product category for each outlet. This corrects for the over-representation of items/outlets with short spell durations and it also results in a more manageable database without substantial information loss.

Moreover, some trimming of the original dataset proves to be necessary. First, all left-censored spells are discarded. The reason for this exclusion is that dealing with left-censored spells requires making nontestable assumptions about the price-setting behavior before the beginning of the observation period (see Heckman and Singer 1984). It is generally not recommended to discard censored spells from a sample because they may correspond to units having specific characteristics and behavior. When the dataset contains only one spell per unit, ignoring those left-censored spells is equivalent to withdrawing the corresponding units, and this is then likely to induce a selection bias. However, our situation is quite different because we observe in general several spells for each unit. Thus, the risk of creating a selection bias is quite low as all individual price trajectories are sampled, with the only exception of those made of only one spell (being right- and left-censored). Such spells represent only a very small fraction of spells (around 4% of the total number of spells), so that we can reasonably expect such a bias, if it exists, to be of a small magnitude.

Second, price spells corresponding to sales or temporary rebates are also removed. These spells are identified using the “type of observation” code, since the dataset allows to identify whether a price quote corresponds to a sales promotion discount, in the form either of seasonal sales or of temporary discounts. The quite specific behavior of such spells (they last most often for a short period of time, typically less than 3 months) leads us to discard them from the analysis. Indeed, for spells corresponding to such events, the impact of covariates is expected to be weak: “sales price” spells do not end because the cumulated inflation has reached a threshold during the spell but because sales are temporary by nature. Moreover, their baseline hazard is quite different from that of other spells because such price spells last for only a short period of time. Keeping this particular group of spells in our sample would then have added a kind of heterogeneity that we cannot take into account because in our framework, heterogeneity is assumed to affect the level but not the shape of the baseline hazard. In addition, it is worth mentioning that such spells correspond to a rather small fraction of spells in our sample. Altogether, sales and temporary rebates represent 2.68% of price quotes and 11% of price spells. For all these reasons, we discard sales price spells from our estimation sample. We also check whether some temporary sales and promotions could not be appropriately flagged in our dataset by spotting all “temporary price deviations,” that is, spells such that the prices of the preceding and subsequent spells are identical. It appears that such temporary decreases represent only 2% of the spells of the full sample (see table 8 in Baudry et al. 2007). Finally, we also discard price trajectories for which price quotes are collected quarterly, to avoid spurious spikes in the hazard function.

The number of observations left in the subsample is 164,626, out of around 2.3 million in the original dataset. To understand this significant reduction of the sample size, note first that removing left-censored spells typically suppresses one spell out of three. More importantly, selecting one spell per product and outlet amounts to selecting one spell out of 10 to 20, the number typically available in each outlet/product type cell. In some cases with short durations such as oil products, as many as 100 spells are available for one outlet, of which we keep one only.

The distribution of the number of spells according to various criteria (sector, outlet type, destination) is presented in the first column of Table 1. Note that the coverages of the “services” outlet type on the one hand, and of the “services” sector on the other hand, do not match exactly, and are not included in each other. For instance, restaurants fall in the outlet-type category “traditional outlet” while belonging to the “services” sector. Conversely, gasoline sold in gas stations appears in the “services” cell for outlet type, but in the “energy” sector. The adopted sectorial breakdown is here rather detailed, the manufacturing sector being disaggregated into durable goods, clothing, and other manufactured goods, because of the very specific
pattern of price-setting in clothes and durable goods. Given the coverage restriction noted above, the “food” sector in the following tables refers to processed food and meat, while “energy” refers essentially to oil-related energy. In some tables, we weight results using CPI weights. CPI weights are available in our database for products at the six-digit level of the Coicop (Classification of individual consumption by purpose) nomenclature. We have in addition used the number of price records by type of outlet (at the six-digit level) to create a weighting scheme by outlet type within each type of product. The motivation for this choice is that the collection of price records by INSEE aims at reflecting the market share of each outlet type. Whereas food products and large outlets appear to be over-represented in the sample of spells, the sample is representative of the CPI in terms of broad sectors, once weights are taken into account.

3.3 Price Durations: Some Stylized Facts

Table 1 provides some elementary results on the sample of price spells and their duration. The average duration of price spells, a standard indicator of price stickiness, is 7.44 months, and 8.22 months when using CPI weights. This indicator is obviously affected by right-censoring (as indicated in the lower panel). Average duration strongly varies across sectors. The main relevant contrast is between services and other types of goods. The average duration of a price spell is about twice larger in the service sector (11.86 months) than in the manufacturing sector, which includes durable goods, clothing, and other manufactured goods, and in the food sector (6.62 months). Heterogeneity across outlet types is significant as well: the average

![Figure 1. Hazard function for price changes of manufactured goods.](image-url)

NOTE: Average duration is in months. The coverage of the “services” outlet type and of the “services” sector are distinct. The column “Sectoral CPI weights” reports the CPI weight of components for sectors. For the “outlet type” rows, the breakdown of price quotes by type of outlet is reported in this column.

(see Baudry et al. 2007; Dhyne et al. 2006). As noted previously the decreasing pattern of the hazard function stands as a puzzle, and a possible cause of the decrease of the hazard function lies in the heterogeneity of price-setters’ behaviors (see, e.g., Heckman and Singer 1984; Kiefer 1988; and, in a price-setting context, Alvarez et al. 2005).

4. TESTING TIME–DEPENDENCE: A PIECEWISE–CONSTANT HAZARD SPECIFICATION

4.1 Empirical Strategy and Specification

This section describes our empirical strategy to test for time-dependence. In line with the mover–stayer example outlined in the Introduction, our empirical strategy aims at controlling as much as possible for heterogeneity. Price spell durations vary across products (e.g., food, gasoline, clothes, services, etc.), outlets (hypermarts, general stores, traditional “corner shops,” etc.), and over time. Outlets have their specific pricing policy, depending on the type of product they sell, on the characteristics of their customers, and on the competition with other retailers. Differences in the evolution of costs across sectors and in the production and merchandising technologies may also contribute to differences in pricing behavior across different types of goods. To account for these differences, the approach followed hereafter is to stratify the sample by products and outlet types. We stratify the data at the highest available level of disaggregation, simultaneously in terms of the type of good and of the type of outlet. For each price spell, the item type is available through the Coicop nomenclature at the six-digit level. There are 271 Coicop categories of products in our sample. The type of outlet is also available through an indicator variable. To define strata we use the dataset classification that distinguishes between 11 outlet types. For convenience, when reporting the results, we group them into five categories only. Overall there are 1,775 strata with at least one price spell.

For each stratum, we estimate a piecewise-constant hazard model for price changes (see App. A for a definition). Note that in this model product replacements are treated as price changes (see Baudry et al. 2007, for a discussion). To obtain meaningful results, we impose constraints both on the minimum number of observations (spells) in each stratum and on the model itself. More precisely, we require that each stratum contains at least 120 observations (spells) and that at least 30 spells are not right-censored. Under these criteria, the number of strata is equal to 396. The average number of spells per stratum is 321.1. On the whole, 127,145 spells were used in the analysis. Note that we have performed a similar analysis at the Coicop five-digit level (leading to a lower number of strata and to a larger number of spells per model); results were essentially unchanged.

The statistical framework used in this section to characterize the hazard function for a price change is rather standard. The reader is referred to Kiefer (1988) and Lancaster (1997) for comprehensive presentations of the econometric analysis of durations. Estimation is performed by maximizing a likelihood function that is presented in Appendix A. For estimating the piecewise-constant model, we have constrained the baseline hazard to be constant from a duration of 14 months onward (namely, \( h_s = h_{14} \) for \( s \geq 14 \)). Moreover, the hazard is constrained to be zero when there are no observed price changes during a given month. Otherwise, we constrain the hazard function to be positive by setting \( h_l = \exp(b_l), l = 1, \ldots, 14 \), and we optimize the log-likelihood function over the \( b_l \)'s. For each piecewise-constant hazard model, we perform tests on the shape of the hazard function. In particular, testing for a constant baseline hazard function, one of the main predictions of Calvo’s model, is performed by conducting a Wald test of the null hypothesis \( H_0: b_1 = b_2 = b_3 = \cdots = b_{14}. \) Estimation is conducted using the GAUSS software constrained maximum likelihood procedure.

One concern is the presence of unobserved heterogeneity. For instance, consider a hazard function found to be constant by the above procedure. In presence of unobserved heterogeneity, the true hazard function may actually be increasing, which is not consistent with Calvo’s model. To investigate this issue we estimated piecewise-constant hazard models with a Gamma-distributed unobserved heterogeneity term (as in Meyer 1990). The likelihood of this model is detailed in Appendix A.

To illustrate the main features of the statistical models and to provide some typical results, it is worthwhile to present an example. The chosen item is pastry in two different types of outlets, namely supermarkets and traditional outlets (bakeries). The estimated hazard function, obtained under the assumption of a piecewise-constant hazard specification, is reported in Figure 2. As Figure 2 makes clear, the shapes of the hazard function to be constant from a duration of 14 months onward (namely, \( h_s = h_{14} \) for \( s \geq 14 \)). Moreover, the hazard is constrained to be zero when there are no observed price changes during a given month. Otherwise, we constrain the hazard function to be positive by setting \( h_l = \exp(b_l), l = 1, \ldots, 14 \), and we optimize the log-likelihood function over the \( b_l \)'s. For each piecewise-constant hazard model, we perform tests on the shape of the hazard function. In particular, testing for a constant baseline hazard function, one of the main predictions of Calvo’s model, is performed by conducting a Wald test of the null hypothesis \( H_0: b_1 = b_2 = b_3 = \cdots = b_{14}. \) Estimation is conducted using the GAUSS software constrained maximum likelihood procedure.

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To illustrate the main features of the statistical models and to provide some typical results, it is worthwhile to present an example. The chosen item is pastry in two different types of outlets, namely supermarkets and traditional outlets (bakeries). The estimated hazard function, obtained under the assumption of a piecewise-constant hazard specification, is reported in Figure 2. As Figure 2 makes clear, the shapes of the hazard function to be constant from a duration of 14 months onward (namely, \( h_s = h_{14} \) for \( s \geq 14 \)). Moreover, the hazard is constrained to be zero when there are no observed price changes during a given month. Otherwise, we constrain the hazard function to be positive by setting \( h_l = \exp(b_l), l = 1, \ldots, 14 \), and we optimize the log-likelihood function over the \( b_l \)'s. For each piecewise-constant hazard model, we perform tests on the shape of the hazard function. In particular, testing for a constant baseline hazard function, one of the main predictions of Calvo’s model, is performed by conducting a Wald test of the null hypothesis \( H_0: b_1 = b_2 = b_3 = \cdots = b_{14}. \) Estimation is conducted using the GAUSS software constrained maximum likelihood procedure.

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tions sharply differ across the two types of outlets. For bakeries, the slope of the overall hazard function is positive. A striking feature is that, for bakeries, the peak of the hazard function occurs at month 12. For supermarkets, the hazard function is decreasing, which suggests that some individual (unobserved) heterogeneity may still be omitted. This example clearly indicates that a proportional hazard specification (i.e., treating the outlet type as a covariate having a proportional effect on the baseline hazard function) would not be relevant here.

### 4.2 Estimation Results

Results of the tests about the shape of the hazard functions are reported in Tables 2 and 3. Table 2 focuses on a benchmark case, namely the constant hazard function predicted by Calvo’s model. It provides the percentage of strata for which the null hypothesis of a constant baseline hazard is not rejected.

Results can be summarized as follows:

1. The first striking result is the rather important rate of nonrejection of the hypothesis of a time-constant hazard. In more than 55% of the strata, this assumption cannot be rejected, using a 5% level Wald test. Changing the significance level of the test does not alter substantially this result. The overall nonrejection rate using CPI weights is lower than the nonweighted one (35.2%), but it still suggests that Calvo’s model is consistent with the pricing behaviors of one-third of the strata. Allowing for peaks at various locations increases the nonrejection rate up to 74%.

2. There is significant heterogeneity both across sectors and across outlets. The assumption of a constant baseline hazard is relevant in a majority of cases for manufactured goods and for processed food. Using CPI weights, it is not rejected in 46.7% of cases for food, 72.9% for durable goods, and 55.2% for other manufactured goods. It is most often rejected for energy and services, where nonrejection rates are respectively 27.5% and 14.9%. In addition, a constant hazard seems to capture the price change behavior of large outlets, whereas it is rejected for traditional corner shops and service providers. For instance, we do not reject Calvo’s assumption for 56.9% of hypermarkets versus 13.6% of service providers. A possible explanation might be the pricing strategy of large outlets, where price changes and the availability of products are part of the marketing policy.

3. Whenever the assumption of a time-constant hazard is not rejected, there is considerable heterogeneity in the level of the hazard function. This is shown in Figure 3, which plots the distribution of the estimated hazard rates for the restricted set of strata in which the hazard rate is estimated to be constant. In line with descriptive evidence, the hazard is highest for energy products and lowest for services. Moreover, the estimated hazard rates vary substantially across types of outlets, but also within each of those groups. In particular, prices are much more flexible in hyper- and supermarkets than in the other types of outlets.

4. When the null hypothesis of a constant hazard is rejected, it is often the case that the hazard is estimated to be increasing, or that it exhibits peaks at various dates. A first informal assessment is that, in such cases, many of the hazard functions have an overall increasing pattern, as in Figure 2(b). The second and third columns of Table 3 report the percentage of cases for which the slope of an ordinary least squares (OLS) line fit through the point estimates of the piecewise-constant hazard

### Table 2. Wald tests for the time-constancy of the hazard rate (single-spell piecewise-constant hazard models)

<table>
<thead>
<tr>
<th># strata</th>
<th># strata in which a constant hazard is not rejected</th>
<th>% of nonrejection of a constant hazard</th>
<th>% of nonrejection of a constant hazard weighted</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overall</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nonrejection at the 5% level</td>
<td>396</td>
<td>218</td>
<td>.551</td>
</tr>
<tr>
<td>Nonrejection at the 1% level</td>
<td>396</td>
<td>257</td>
<td>.649</td>
</tr>
<tr>
<td>Nonrejection at the 10% level</td>
<td>396</td>
<td>188</td>
<td>.475</td>
</tr>
<tr>
<td>Nonrejection with a peak at months 1 or 2</td>
<td>396</td>
<td>293</td>
<td>.740</td>
</tr>
<tr>
<td>Nonrejection with a peak at months 5, 6, or 7</td>
<td>396</td>
<td>259</td>
<td>.654</td>
</tr>
<tr>
<td>Nonrejection with a peak at months 11, 12, or 13</td>
<td>396</td>
<td>271</td>
<td>.684</td>
</tr>
<tr>
<td>By type of good</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Food</td>
<td>169</td>
<td>104</td>
<td>.615</td>
</tr>
<tr>
<td>Energy</td>
<td>15</td>
<td>8</td>
<td>.533</td>
</tr>
<tr>
<td>Durable goods</td>
<td>28</td>
<td>21</td>
<td>.750</td>
</tr>
<tr>
<td>Clothes</td>
<td>52</td>
<td>9</td>
<td>.173</td>
</tr>
<tr>
<td>Other manuf. goods</td>
<td>78</td>
<td>57</td>
<td>.731</td>
</tr>
<tr>
<td>Services</td>
<td>54</td>
<td>19</td>
<td>.352</td>
</tr>
<tr>
<td>By type of outlet</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hypermarkets</td>
<td>96</td>
<td>57</td>
<td>.594</td>
</tr>
<tr>
<td>Supermarkets</td>
<td>80</td>
<td>54</td>
<td>.675</td>
</tr>
<tr>
<td>Traditional corner shops</td>
<td>98</td>
<td>43</td>
<td>.439</td>
</tr>
<tr>
<td>Services</td>
<td>45</td>
<td>14</td>
<td>.311</td>
</tr>
<tr>
<td>Others</td>
<td>77</td>
<td>50</td>
<td>.649</td>
</tr>
</tbody>
</table>

NOTE: Figures in columns 4 and 5 are percentages of cases for which a Wald test at the 5% level does not reject $H_0: h_1 = \cdots = h_{14}$.

Table 3. Characterizing the hazard shape when the hazard rate is not constant

<table>
<thead>
<tr>
<th></th>
<th># strata</th>
<th>% of increasing hazards when the constant hazard is rejected</th>
<th>% with peaks at months 1, 6, or 12 when the constant hazard is rejected</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>unweighted</td>
<td>weighted</td>
</tr>
<tr>
<td>Overall</td>
<td>178</td>
<td>.500</td>
<td>.578</td>
</tr>
<tr>
<td>By type of good</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Food</td>
<td>65</td>
<td>.123</td>
<td>.154</td>
</tr>
<tr>
<td>Energy</td>
<td>7</td>
<td>.000</td>
<td>.000</td>
</tr>
<tr>
<td>Durable goods</td>
<td>7</td>
<td>.429</td>
<td>.259</td>
</tr>
<tr>
<td>Clothes</td>
<td>43</td>
<td>.860</td>
<td>.872</td>
</tr>
<tr>
<td>Other manuf. goods</td>
<td>21</td>
<td>.571</td>
<td>.757</td>
</tr>
<tr>
<td>Services</td>
<td>35</td>
<td>.829</td>
<td>.834</td>
</tr>
<tr>
<td>By type of outlet</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hypermarkets</td>
<td>39</td>
<td>.179</td>
<td>.116</td>
</tr>
<tr>
<td>Supermarkets</td>
<td>26</td>
<td>.115</td>
<td>.072</td>
</tr>
<tr>
<td>Traditional corner shops</td>
<td>55</td>
<td>.691</td>
<td>.625</td>
</tr>
<tr>
<td>Services</td>
<td>31</td>
<td>.806</td>
<td>.821</td>
</tr>
<tr>
<td>Others</td>
<td>27</td>
<td>.593</td>
<td>.708</td>
</tr>
</tbody>
</table>


function is positive. Results are particularly clear for other manufactured products and services: the hazard rates can be classified as increasing in 87.2% and 83.4% of cases, respectively. There is a clear contrast between food and energy products on the one hand, and manufactured products and services on the other hand. The rejection of a time-constant hazard for the latter group mainly corresponds to an increasing shape, while it corresponds to a decreasing pattern for the former group of products.

To capture the influence of specific peaks on hazard, we have first implemented a test inspired by Taylor’s model by testing for a time-constant hazard, except in some given months (namely, months 1, 6, and 12). Note that testing for strict Taylor contracts (hazard equal to zero outside one peak) leads to systematic rejection, because the estimated values of the hazard function are significantly positive on intervals between the peaks. A possible interpretation of this test is the coexistence of several types of price-setting behaviors, which could correspond to several Taylor models associated with different contract durations. Results of this test are presented in the last two columns of Table 3. Each column gives the percentage of strata for which the assumption of a constant baseline hazard is not rejected, when allowing for one (or several) spikes at specific months (an obvious caveat is that the number of estimated models is rather limited for some subgroups). The main conclusion is that allowing for such specific peaks makes the constancy of the hazard an acceptable assumption. Note that the localization of peaks has sector-specific patterns. The mode in month 1 is dominant in energy due to short spells for gasoline. For services, the occurrence of a peak around month 12 is frequent, suggesting that a mix of Calvo’s units and Taylor’s 12-month contracts is an acceptable characterization of the behavior of price-setters in services.

(5) Unobserved heterogeneity does not seem to be relevant. Results with a Gamma-distributed unobserved heterogeneity (not reported here) are close to those obtained in the case without unobserved heterogeneity, and the variance of the heterogeneity term is close to zero in all strata. The example of pastry is given in Figure 2: the piecewise hazard function with unobserved heterogeneity, which is reported in dotted line, is nearly indistinguishable from that resulting from the specification without unobserved heterogeneity. Using a likelihood ratio (LR) test, unobserved heterogeneity is never significant. This suggests that our stratification strategy captures heterogeneity rather well.

Overall, the above results show that estimating models at a highly disaggregated level allows to solve the decreasing hazard “puzzle” and to recover estimates in better accordance with theoretical models. Indeed, using the results reported in Tables 2 and 3, the proportion of estimated nondecreasing hazard rates equals approximately 78%. This share is computed by adding the fractions of strata in which the hazard rate of price changes is classified as “constant” and as “increasing,” hence $0.551 + (1 - 0.551) \times 0.500$. The corresponding share is equal to 72.7% when CPI weights are used. Thus most strata can be described as having either a constant or an increasing hazard rate, which is consistent with either Calvo’s model or a menu cost model. An alternative approach, which is consistent with Calvo’s and Taylor’s models, consists of counting strata in

![Figure 3. Distribution of the estimated baseline hazard (strata for which hazard-constancy is not rejected).](image-url)
Figure 4. Overall hazard functions for actual and simulated data (pooling strata with constant hazard). (--- actual data; —— simulated data.)

which the hazard rate is either fully constant or constant when allowing for peaks. The percentage of strata with nondecreasing hazard rates is then equal to 82.9% when CPI weights are not used, and to 69.9% when these weights are taken into account.

To conclude this section, we illustrate how disaggregation solves the decreasing hazard paradox through a mover–stayer phenomenon. For this purpose, we perform the following exercise. We consider the set of 218 strata for which the assumption of a constant hazard is not rejected. For each of these strata, we use the estimated hazard parameter to simulate a sample of spells whose number is equal to twice the actual number of spells in each strata, which gives 620 spells on average per strata and 135,000 spells overall. We then pool all simulated spells, and estimate a piecewise hazard function for the whole sample of simulated durations. Results are presented in Figure 4, which also plots (in solid line) the hazard function estimated for the pooled observations in the same restricted set of 218 strata. We observe that although the data were generated using models with constant hazards, the overall estimated hazard function is decreasing. Furthermore, the hazard function of simulated durations is close to the nonparametric estimate of the hazard function of actual data, which indicates that the estimation fit is quite satisfactory.

5. INVESTIGATING STATE–DEPENDENCE IN A COMPETING–RISKS FRAMEWORK

Our previous analysis has shown that a significant fraction of product–outlet units is characterized by nonconstant hazards. Some theoretical models such as that proposed by Dotsey et al. (1999) show that state-dependence results in an increasing unconditional hazard whereas others put a special emphasis on the asymmetry between price increases and decreases. Here we test formally for state-dependence using an approach related to that of Caballero and Engel (1993b). This approach has two main features.

First, state-dependence is modeled by assuming that the adjustment hazard depends on the gap between the current price of the firm and a “frictionless price.” The frictionless price is approximated by the average price of the same item in the economy, as measured by the CPI. Second, the hazard functions are allowed to differ for price decreases and price increases. An obvious reason is that the impact of some covariates on the probability of a price change clearly differs in these two cases. Indeed, accumulated inflation since the last price change, when positive, is expected to lower the probability of a price decrease, whereas it is expected to have the opposite effect for a price increase. Another option would be to consider the absolute value of the “price gap.” An advantage of our approach is to allow for asymmetry in price-setting. Such an asymmetry has been documented with macro data by Caballero and Engel (1993b). In addition, recent specific surveys about firms’ pricing behaviors that have been conducted in the euro area (see Fabiani et al. 2006; Loupias and Ricart 2006, for France) suggest that, with regard to price adjustments, firms react differently when their production costs (or the demand for their product) rise or decrease. Firms report to react faster to a rising cost and a lowering demand than to changes going the other way round.

A competing-risks duration framework then appears to be particularly relevant as it encompasses those issues. Moreover, this framework may also help us in explaining the decreasing pattern of some estimated hazards, which could result from another type of heterogeneity, namely state-heterogeneity. One can indeed suspect that time-dependence has a different profile according to the nature of the event ending the spell (either a price increase or a decrease).

5.1 Multiple Outcomes as Competing Risks

In our dataset, the end of a price spell may correspond to four different events: an increase in the price of the item, a decrease in the price of the item, a product replacement (the item ceases to be sold and is replaced in the dataset by another equivalent item), or right-censoring (the spell is ongoing beyond the end of the observation period).

Formally, let us denote $T_1$ the latent duration associated with a price increase, $h_1(T_1)$ its hazard function, $f_1(T_1)$ its density function, and $S_1(T_1)$ its survivor function. Analogously, let us denote $T_2$ the latent duration associated with a price decrease, $h_2(T_2)$, $f_2(T_2)$, and $S_2(T_2)$ being its hazard, density, and survivor functions, respectively. Finally, let us denote $T_3$ the latent duration associated with a product replacement, $h_3(T_3)$, $f_3(T_3)$, and $S_3(T_3)$ being its hazard, density, and survivor functions, respectively. When the spell termination corresponds to a price increase, we know that the duration of the spell is shorter than the latent durations corresponding to either a price decrease or a product replacement: $T_1 \leq T_2$ and $T_1 \leq T_3$. When the price spell is right-censored, which corresponds either to the end of the observation period or to a decision of the statistical office to stop observing this particular item, then we have $\min(C, T_1, T_2, T_3) = C$, where $C$ denotes the latent duration associated with right-censoring.

Let us define the joint survivor function of the first three latent durations as

$$S(t_1, t_2, t_3) = \Pr(T_1 > t_1, T_2 > t_2, T_3 > t_3).$$

If $(T_1, T_2, T_3)$ are stochastically independent, then

$$S(t_1, t_2, t_3) = \prod_{k=1}^{3} S_k(t_k),$$

where $S_k(t_k)$ is the survivor function of the $k$th spell.
\[ S_k(t_k) \] being the marginal survivor function of the \( k \)th latent duration. In the sequel, our maintained assumption is that \((T_1, T_2, T_3)\) are conditionally independent given the covariates, namely

\[
T_k \mid T_k' \{z_{it}\}_{t=0}^{\tau} \quad \forall k' \neq k. \tag{3}
\]

The initial dataset contains right-censored spells, left-censored spells, as well as both right- and left-censored spells. The case of exogenous right-censoring is rather straightforward. Indeed, what is known about a spell that is right-censored in month \( t_i \) is that its complete (latent) duration is equal to or higher than \( t_i \) months. Its contribution to the likelihood function is then

\[
l_i(t_i) = S(t_i, t_i, t_i) = \prod_{k=1}^{3} S_k(t_i). \tag{4}
\]

Many spells in the sample, however, are either left-censored or both right- and left-censored. In general, the statistical treatment of left-censored spells induces more difficulties than that of right-censored spells. As was discussed in Section 3.2.3, left-censored spells have been discarded from the sample used for estimation. One first reason is that the sample is made of thousands of spells for each product type and outlet type, so that we are able to discard the left-censored spells without substantial information loss. Furthermore, left-censoring is independent of the duration of price spells. In the present context, and contrarily to what often occurs in unemployment-duration studies, left-censoring does not concern a particular subpopulation with specific characteristics. We have checked the absence of bias when disregarding left-censored spells by performing a simulation study, based on a data-generating process approximating the generation of a longitudinal price dataset.

5.2 Accounting for Time-Varying Covariates

We test for state-dependence in a reduced-form specification by testing for the influence of “price deviation” on the probability of a price change, in the vein of Caballero and Engel (1993a,b). Formally, the probability of a price increase is \( h(p_t^* - p_t) \), where \( p_t^* \) is the logarithm of the target price, \( p_t \) is the logarithm of the price set at date \( t \), and \( h(\cdot) \) is an increasing function (a similar framework holds for price decreases). At date \( t = 0 \), the price has been set by the firm at level \( p_0 \). By definition, \( p_t = p_0 \) at any date \( t \) between date 0 and the date of the next price change. The price deviation \( p_t^* - p_t \) at date \( t \) is not observable but it can be approximated by the accumulated sectorial inflation rate. This proxy can be rationalized in the following way. Assume that the target price is proportional to the price of competitors. Assume that the sectorial CPI index \( P_t \) is itself a proxy for the price of competitors in the sector so that \( p_t^* = m + \pi_t \), where \( \pi_t \) is the logarithm of \( P_t \) and \( m \) is an idiosyncratic constant. Assuming further that the price is set at the target price at \( t = 0 \), then \( p_t = p_0^* = m + \pi_0 \), where \( \pi_0 \) is the logarithm of \( P_0 \), so that the price gap is \( p_t^* - p_t = \pi_t - \pi_0 \), which is the accumulated rate of inflation since the beginning of the spell. Accumulated inflation is here defined as the growth rate in the sectorial price index from the month preceding the beginning of the spell to the month preceding the current month. This reflects the delay in the release of the CPI, and here precludes any simultaneity issue. We use sectorial price indices at the Coicop 5 level of aggregation since price indices are not available at the six-digit Coicop. We also assume that \( p_t^* \) may be affected by other time-varying covariates, which include:

1. a dummy variable for the Euro cash change-over which occurred in January 2002. The impact on the frequency of price changes is well documented (see, e.g., INSEE 2003; Baudry et al. 2007). This dummy is expected to raise both the probability of a price increase and that of a price decrease, for instance, if the retailer decides to set psychological prices in euros;
2. two dummy variables for the increase in the VAT rate which occurred in August 1995 (from 18.6% to 20.6%). Indeed, many outlets are closed in August and the VAT rate change may have been postponed to September in those outlets. For this reason, we incorporate two dummies, one for August 1995, the other for September 1995. Their coefficient is expected to be positive for price increases and negative for price decreases;
3. a dummy variable for the VAT rate decrease in April 2000 (from 20.6% to 19.6%), with coefficients expected to be of the opposite sign to those above.

The hazard function of the duration \( T_k \) \((k = 1, 2, 3)\) for the \( i \)th spell is assumed to be of the proportional hazard form, specified as:

\[
h_{ki}(\tau) = h_{*,k} \exp(z_{it} \alpha_k), \tag{5}
\]

where \( h_{*,k} \) is a baseline hazard function that is assumed to be constant over the interval \([t - 1, t]\), \( z_{it} \) is the value at time \( \tau \) of the vector of time-varying covariates, and \( \alpha_k \) is a vector of (unknown) parameters associated with the vector of covariates \( z_{it} \). We assume that the variables \( z_{it} \) do not vary over the time interval \([t - 1, t]\), namely \( z_{it} = z_{it-1}, \forall t - 1 < t. \) A noticeable feature is that we allow the hazard to be nonzero even when the price deviation is zero. This feature provides additional empirical flexibility, and can be rationalized in a menu cost model with multiproducts (see Midrigan 2006). Also the functional form is different from the quadratic one used by Caballero and Engel (1993b), but more in line with specifications used in duration analysis. The corresponding likelihood function is given in Appendix B.

Because our measure of the price deviation is an approximation, we cannot put too much emphasis on the interpretation of the estimated coefficients. Still, we interpret any significant response of the price change probability to covariates as an indication of state-dependence. Other potentially relevant time-varying covariates may be taken into account, such as, for instance, the aggregate rate of inflation, inflation variability, or cyclical indicators (such as sectorial or aggregate industrial production, and so on). Some of these covariates (e.g., cost or demand indicators at the product level) are simply not available in our dataset. Including other available covariates, such as the aggregate rate of inflation and the inflation variability, would create some difficulties, because these covariates are potentially correlated with the sectorial accumulated inflation rates. In addition, our systematic approach to stratifying the data would make specification search hardly feasible.
5.3 Empirical Set-up

As in Section 4, we impose constraints both on the minimum number of observations (spells) in each stratum and on the model itself. More precisely, we require that each stratum contains at least 120 observations. In addition, because we consider multiple outcomes, at least 30 exits toward the relevant destination are required. Consistent with our conditional independence assumption, we consider separately the different outcomes because there are strata for which there are few price decreases or product replacements. Under these criteria, the number of estimated models is \( N = 309 \) for spells ending with a price increase, \( N = 229 \) for spells ending with a price decrease, and \( N = 197 \) for those ending with a product replacement. The average number of observations per model is 362.2.

Estimation is performed using the GAUSS software constrained maximum likelihood procedure, by maximizing the likelihood function described in Appendix B. The hazard is constrained to be zero when there are no exits during a given month. Otherwise, we also restrict the hazard parameters to be positive by setting \( h_l = \exp(b_l) \), \( l = 1, \ldots, 14 \), and optimizing over the \( b_l \)'s. Here again, we restrict the baseline hazard to be constant from a duration of 14 months onward. For each estimate, we perform tests on the shape of the hazard function and on the effect of covariates.

5.4 An Example

To give an example, Table 4 presents estimates for the stratum “haircut for women.” First, the baseline hazard rate is seen to be different for price increases and price decreases. Concerning price increases, a Wald test of the assumption of a constant baseline hazard is rejected at the 5% level. However, when allowing for one peak in the baseline hazard function at month 12, the Wald test \( p \)-value is .147, so that the hazard constancy is not rejected. For price decreases the hazard constancy is not rejected either. Turning to parameter estimates, the estimated parameter of cumulative inflation is positive but not significant for price increases, whereas it is negative and significant at the 10% level for price decreases. In addition, the dummy for the euro cash change-over has a massive impact for both price increases and price decreases. The two dummies for VAT increases have also a marked impact on the instantaneous probability of a price

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Hazard for price increases</th>
<th>Hazard for price decreases</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline hazard (log)</td>
<td>Estimates</td>
<td>St. errors</td>
</tr>
<tr>
<td>( b_1 )</td>
<td>–5.140</td>
<td>.463</td>
</tr>
<tr>
<td>( b_2 )</td>
<td>–4.719</td>
<td>.386</td>
</tr>
<tr>
<td>( b_3 )</td>
<td>–4.187</td>
<td>.336</td>
</tr>
<tr>
<td>( b_4 )</td>
<td>–4.038</td>
<td>.322</td>
</tr>
<tr>
<td>( b_5 )</td>
<td>–3.624</td>
<td>.283</td>
</tr>
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<td>( b_6 )</td>
<td>–4.004</td>
<td>.340</td>
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<tr>
<td>( b_7 )</td>
<td>–3.682</td>
<td>.323</td>
</tr>
<tr>
<td>( b_8 )</td>
<td>–3.711</td>
<td>.340</td>
</tr>
<tr>
<td>( b_9 )</td>
<td>–3.998</td>
<td>.383</td>
</tr>
<tr>
<td>( b_{10} )</td>
<td>–3.638</td>
<td>.342</td>
</tr>
<tr>
<td>( b_{11} )</td>
<td>–3.875</td>
<td>.388</td>
</tr>
<tr>
<td>( b_{12} )</td>
<td>–2.988</td>
<td>.272</td>
</tr>
<tr>
<td>( b_{13} )</td>
<td>–3.170</td>
<td>.326</td>
</tr>
<tr>
<td>( b_{14} )</td>
<td>–3.388</td>
<td>.186</td>
</tr>
</tbody>
</table>

Time-varying covariates

- Cumulated inflation
- January 2002 (euro change-over)
- August 1995 (VAT increase)
- September 1995
- April 2000 (VAT decrease)

Wald test for constant hazard

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Wald stat.</th>
<th>( p )-value</th>
<th>Wald stat.</th>
<th>( p )-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b_1 = \cdots = b_{12} )</td>
<td>31.93</td>
<td>.002</td>
<td>10.31</td>
<td>.413</td>
</tr>
<tr>
<td>( b_2 = \cdots = b_{12} )</td>
<td>17.88</td>
<td>.057</td>
<td>3.29</td>
<td>.857</td>
</tr>
<tr>
<td>( b_1 = \cdots = b_5 = b_7 = \cdots = b_{12} )</td>
<td>26.35</td>
<td>.003</td>
<td>3.31</td>
<td>.913</td>
</tr>
<tr>
<td>( b_1 = \cdots = b_{11} )</td>
<td>14.62</td>
<td>.147</td>
<td>3.31</td>
<td>.855</td>
</tr>
<tr>
<td>( b_2 = \cdots = b_{11} )</td>
<td>7.81</td>
<td>.554</td>
<td>3.29</td>
<td>.772</td>
</tr>
</tbody>
</table>

Number of spells | 563 | 563 |
Log-likelihood at the maximum | –946.79 | –191.75 |

*Note: The (six-digit) Coicop code for this item is 121112. The estimated parameter is the logarithm of the baseline hazard \( h_i = \exp(b_i) \). St. error: standard error.
### Table 5. Tests on the estimated parameter associated with accumulated inflation

<table>
<thead>
<tr>
<th></th>
<th>Price increases</th>
<th>Price decreases</th>
<th>Product replacements</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>% positive and</td>
<td>% negative and</td>
<td>% positive and</td>
</tr>
<tr>
<td></td>
<td>significant</td>
<td>significant</td>
<td>significant</td>
</tr>
<tr>
<td></td>
<td>% positive and</td>
<td>% negative and</td>
<td>% positive and</td>
</tr>
<tr>
<td></td>
<td>significant</td>
<td>significant</td>
<td>significant</td>
</tr>
<tr>
<td></td>
<td>weighted</td>
<td>weighted</td>
<td>weighted</td>
</tr>
<tr>
<td>All sectors</td>
<td>.453</td>
<td>.122</td>
<td>.137</td>
</tr>
<tr>
<td></td>
<td>.435</td>
<td>.198</td>
<td>.214</td>
</tr>
<tr>
<td>By type of good</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Food</td>
<td>.601</td>
<td>.099</td>
<td>.111</td>
</tr>
<tr>
<td>Energy</td>
<td>.571</td>
<td>.167</td>
<td>.358</td>
</tr>
<tr>
<td>Durable goods</td>
<td>.200</td>
<td>.056</td>
<td>.102</td>
</tr>
<tr>
<td>Clothes</td>
<td>.000</td>
<td>.170</td>
<td>.176</td>
</tr>
<tr>
<td>Other manuf. goods</td>
<td>.297</td>
<td>.154</td>
<td>.093</td>
</tr>
<tr>
<td>Services</td>
<td>.300</td>
<td>.250</td>
<td>.167</td>
</tr>
<tr>
<td>By type of outlet</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hypermarkets</td>
<td>.456</td>
<td>.100</td>
<td>.043</td>
</tr>
<tr>
<td>Supermarkets</td>
<td>.566</td>
<td>.100</td>
<td>.135</td>
</tr>
<tr>
<td>Traditional corner shops</td>
<td>.426</td>
<td>.185</td>
<td>.230</td>
</tr>
<tr>
<td>Services</td>
<td>.293</td>
<td>.235</td>
<td>.067</td>
</tr>
<tr>
<td>Others</td>
<td>.442</td>
<td>.114</td>
<td>.132</td>
</tr>
</tbody>
</table>

NOTE: Models are estimated with piecewise-constant hazard functions. Each column reports the percentage of strata in which the null of nonstate dependence (i.e., \( a_1 = 0 \), where \( a_1 \) is the parameter of cumulative inflation) is not rejected at the 5% level using a \( t \)-test. "Weighted" indicates that results are aggregated across strata using CPI weights.


increase, whereas the dummy for the VAT decrease fails to be significant in the model for price decreases. Haircut prices thus exhibit a positive pass-through of VAT increases but no negative pass-through of VAT decreases. This asymmetry may, though, be consistent with a menu cost model in which the inflation trend is positive.

#### 5.5 Overall Results

Overall test results are reported in Tables 5–7. In reporting the results of the competing-risks model we focus on the parameters describing the impact of covariates. With regard to the baseline hazard function, we basically find the same kind of results as with the simple piecewise-constant model documented in Section 4 (though the level of the hazard for decreases and increases taken separately is obviously lower). In particular we observe, as in Section 4, a strong degree of heterogeneity in the baseline hazard function, and a rather large share of strata for which a constant hazard is not rejected. Detailed results are not reported but are available in a working paper version (Fougère, Le Bihan, and Sevestre 2005).

### Table 6. Tests on the parameter associated with the euro cash change-over dummy

<table>
<thead>
<tr>
<th></th>
<th>Price increases</th>
<th>Price decreases</th>
<th>Product replacements</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>% positive and</td>
<td>% negative and</td>
<td>% positive and</td>
</tr>
<tr>
<td></td>
<td>significant</td>
<td>significant</td>
<td>significant</td>
</tr>
<tr>
<td></td>
<td>% positive and</td>
<td>% negative and</td>
<td>% positive and</td>
</tr>
<tr>
<td></td>
<td>significant</td>
<td>significant</td>
<td>significant</td>
</tr>
<tr>
<td></td>
<td>weighted</td>
<td>weighted</td>
<td>weighted</td>
</tr>
<tr>
<td>All sectors</td>
<td>.447</td>
<td>.374</td>
<td>.275</td>
</tr>
<tr>
<td></td>
<td>.585</td>
<td>.454</td>
<td>.439</td>
</tr>
<tr>
<td>By type of good</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Food</td>
<td>.260</td>
<td>.318</td>
<td>.167</td>
</tr>
<tr>
<td>Energy</td>
<td>.222</td>
<td>.100</td>
<td>.286</td>
</tr>
<tr>
<td>Durable goods</td>
<td>.571</td>
<td>.500</td>
<td>.107</td>
</tr>
<tr>
<td>Clothes</td>
<td>.833</td>
<td>.500</td>
<td>.088</td>
</tr>
<tr>
<td>Other manuf. goods</td>
<td>.375</td>
<td>.378</td>
<td>.412</td>
</tr>
<tr>
<td>Services</td>
<td>.857</td>
<td>.378</td>
<td>.495</td>
</tr>
<tr>
<td>By type of outlet</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hypermarkets</td>
<td>.104</td>
<td>.163</td>
<td>.360</td>
</tr>
<tr>
<td>Supermarkets</td>
<td>.160</td>
<td>.319</td>
<td>.125</td>
</tr>
<tr>
<td>Traditional corner shops</td>
<td>.797</td>
<td>.667</td>
<td>.238</td>
</tr>
<tr>
<td>Services</td>
<td>.789</td>
<td>.667</td>
<td>.467</td>
</tr>
<tr>
<td>Others</td>
<td>.355</td>
<td>.400</td>
<td>.280</td>
</tr>
</tbody>
</table>

NOTE: Models are estimated with piecewise-constant hazard functions.

5.5.1 The Impact of the Sector-Specific Cumulative Inflation. Estimation results for the impact of the accumulated inflation on the probability of a price change are summarized in Table 5 and Figure 5. These provide two complementary ways of looking at the results. Table 5 documents the significance and sign of the estimated coefficients, which provide an indication of the importance of state-dependence in price-setting behaviors. Table 5 shows that with regard to price increases, the estimated inflation coefficient is frequently positive, as expected, and is statistically significant in about 45% of cases. State-dependence thus appears to be important to explain price rises. On the contrary, the coefficient of accumulated inflation is rarely significant for price decreases and product replacements. Thus price reductions and product replacements are not driven by this variable, reflecting an asymmetry in price adjustment.

Processed food products and energy appear to be largely sensitive to inflation in their sector, in contrast with other products. This result should not be taken to imply that inflation does not affect price changes in other sectors. Although their revision schedule does not heavily depend on the inflation rate, the magnitude of price revisions is likely to depend on the prevailing or expected inflation rate (as it is, for instance, predicted by Calvo’s model). Another interesting result is that the response of price changes to inflation is more systematic for hyper- and supermarkets than it is for traditional outlets. The proportion of significantly positive coefficients is larger in the former groups.

The other way to look at our estimation results is to analyze the magnitude of the impact of the accumulated inflation on the likelihood of a price change. This magnitude is clearly strongly heterogeneous, as indicated by Figure 5. The average impact of the accumulated inflation on the probability of a price increase is most often, as expected, positive. This impact varies a great deal and can even be negative (though often not statistically significant) in a nonnegligible fraction of cases.

5.5.2 The Impact of the Euro Cash Change-Over. The first striking result from Table 6 is that, on the whole, the euro cash change-over has had a quite symmetric impact on price increases and decreases. The proportion of significant coefficients is respectively 58.5% and 45.4% for increases and decreases, considering weighted figures. The magnitudes of average effects (not reported) are also very close. However, some differences emerge at a lower level of disaggregation. First, increases in prices have been more frequent for clothes and services than for other types of goods. Second, price increases seem to have occurred mainly in traditional outlets, for which we get most of the significant estimated coefficients and a larger magnitude of the impact. It must be noticed that the frequency of price decreases has also been increased in those outlets. This might reflect the search for psychological prices, leading to both increases and decreases in prices expressed in euros. At the opposite, the coefficients are almost never significant for hyper-
supermarkets. For instance, in hypermarkets the cash change-over indicator is significant in 7.8% of estimated models for price increases and 13.9% of models for price decreases. This can be seen as a confirmation that, at least at the very time of the euro cash change-over, hyper- and supermarkets fulfilled their commitment not to change their prices during the three months before and the three months after the change of numeraire.

5.5.3 The Impact of VAT Changes. As documented in Table 7, the main VAT rate changes also appear to have an impact on the occurrence of price changes. As expected, the coefficients of the dummies associated with the 1995 increase of the VAT rate are most often significant for explaining price increases (56.4% of cases). Results for the dummy characterizing the subsequent month, which captures muted effects of a VAT increase, are also often significant. Symmetrically, the dummy for the VAT rate decrease that occurred in April 2000 is significant in a majority of models for price decreases, namely 54.1% of cases. Note that the effect of a VAT increase on the probability of a price decrease, and of the VAT decrease on the probability of a price increase, not reported for brevity, are in general not statistically significant. It is also striking that the VAT decreases are generally more often transmitted to prices in large outlets than in smaller ones, thus confirming the lower importance of state-dependence in the price-setting behaviors of the latter group of outlets.

5.6 Unobserved Heterogeneity

Omitting unobserved heterogeneity is known to produce biased estimates in competing-risks duration models, as well as in single-spell duration models. A crucial issue is then to assess if the results reported in the previous subsection are robust to introducing unobserved heterogeneity. To test for the presence of unobserved heterogeneity in our competing-risks models, we tried to estimate two usual specifications, namely the one-factor loading model and the bivariate discrete distribution (see, for instance, the survey by Van den Berg (2001), for a detailed presentation). In the one-factor loading model, the hazard function of the duration $T_k (k = 1, 2, 3)$ for the $i$th spell is assumed to be of the following mixed proportional hazard (MPH) form:

$$h_{ki}(\tau) = h_{kr} \exp(z_{ir} \alpha_k + y_{ki} v_1)$$

for $i = 1, \ldots, N$ and $k = 1, \ldots, 3$, (6)

where $v_1$ is an iid realization of a standard normal $N(0, 1)$ random variable and $y_{ki}$ is an unknown parameter indexed by $k$ (i.e., the type of outcome). In the MPH model in which the unobserved heterogeneity follows a bivariate discrete distribution, the hazard function of the duration $T_k (k = 1, 2, 3)$ for the $i$th spell has the form

$$h_{ki}(\tau) = h_{kr} \exp(z_{ir} \alpha_k + \tilde{y}_{ki})$$

for $i = 1, \ldots, N, k = 1, \ldots, 3, j = 1, 2$, (7)

where $\tilde{y}_{kj}$ is a discrete random variable with two possible values, denoted $\gamma_{k1}$ and $\gamma_{k2}$. For the sake of simplicity, we assume that the hazard rate of the duration of a price spell that ends with a product replacement is not affected by unobserved heterogeneity, which means that $\gamma_3 = 0$ for the one-factor loading model and $\gamma_{31} = \gamma_{32} = 0$ in the model with a bivariate discrete unobserved heterogeneity. This implies that in the latter model, the probabilities of the couples of points of support are defined as $p_1 = \Pr(\tilde{y}_{i1} = \gamma_{i1}, \tilde{y}_{i2} = \gamma_{i2}), p_2 = \Pr(\tilde{y}_{i1} = \gamma_{i1}, \tilde{y}_{i2} = \gamma_{i2}), p_3 = \Pr(\tilde{y}_{i1} = \gamma_{i1}, \tilde{y}_{i2} = \gamma_{i2})$, and

$$p_4 = \Pr(\tilde{y}_{i1} = \gamma_{i1}, \tilde{y}_{i2} = \gamma_{i2}) = 1 - p_1 - p_2 - p_3.$$

For identifiability reasons, we impose the usual restriction setting $\gamma_{11} = 0$. For insuring statistical significance, and because of the large number of parameters to be estimated (around 40), we have restricted our analysis to the 66 strata including each more than 300 observations, with at least 100 price increases and 100 price decreases. For both models, convergence of the maximum likelihood procedure is very difficult to reach. In most of the cases, either (at least) one parameter $\gamma_k$ tends to infinity (or to zero), or one probability $p_j (j = 1, \ldots, 4)$ tends to zero. To overcome this problem, we tried to estimate more constrained models, for instance, the model with a bivariate discrete unobserved heterogeneity in which one probability (either $p_2$ or $p_3$) is assumed to be zero. The problem still remained. Finally, we tried to estimate models in which discrete unobserved heterogeneity is assumed to act on one hazard function only. Convergence was reached only for nine strata in which discrete unobserved heterogeneity was assumed to influence only the hazard rate for price increases. In these nine cases, we find that a very small fraction of observations (corresponding to a probability between 5% and 10%) has a low hazard rate, the remaining part (between 90% and 95% of observations in the stratum) having a hazard rate for price increases which is close to the hazard rate estimated for the model with no unobserved heterogeneity (see Fig. 6 for an example). This is in accordance with the result obtained by Willis (2006) who found that, in the analysis of magazine prices, a model in which the random unobserved heterogeneity has a discrete distribution with two mass points “does not identify significant heterogeneity.”

Thus it is impossible to detect unobserved heterogeneity (at least by estimating usual MPH competing-risks models) after having disaggregated the overall sample into more homogeneous strata. We have still to be prudent: as it is documented in the literature, the flexibility of the piecewise-constant hazard model makes it difficult to allow simultaneously for unobserved heterogeneity and state-dependence in the price-setting behaviors of the latter group of outlets.

Figure 6. Hazard functions with and without discrete heterogeneity. An example: cheese in supermarkets, price increases. (— hazard without heterogeneity; — hazard at the first mass point; —— hazard at the second mass point.)
heterogeneity (on this issue, see Baker and Melino 2000). Further research should focus on the specification and on the estimation of other types of models. However, our robustness exercises seem to suggest that, under the maintained assumption of a MPH competing-risks model with flexible piecewise-constant hazard functions, there is no stochastic dependence between price increases and price decreases that is caused by (omitted) unobserved heterogeneity.

6. CONCLUSION

This article has analyzed price stickiness by estimating duration models at a very disaggregated level. Four main results should be emphasized.

First, at the outlet-type product level, the assumption of a nondecreasing baseline hazard function for price changes cannot be rejected in about 75% of cases. Thus, the decreasing pattern that emerges with pooled data is a consequence of imperfectly accounting for heterogeneity in pricing behaviors.

Second, not only the level but also the shape of the hazard function for price changes is found to vary across sectors and types of outlets. Two typical patterns are the fully constant baseline hazard, consistent with Calvo’s model, and the partially constant hazard function with modes located at durations 1, 6, and 12 months. With regard to modes in the hazard function, sectorial specificities emerge: the hazard functions for energy products exhibit marked peaks at one month, while those for services exhibit peaks around 12 months.

Third, there is evidence of state-dependence in a large number of cases. The probability of a price change is found to be affected by cumulated inflation. In addition, the effects of covariates, when statistically significant, vary across sectors and outlet types.

Fourth, there is an asymmetry in the probability of a price change: the determinants for price increases differ from those that affect price decreases. State-dependence plays a more significant role in price increases than in price decreases.

The above results were obtained using a reduced-form approach. In the absence of the estimation of a structural model, we cannot address issues like the size of menu costs, or the welfare consequences of price stickiness. Our results, however, have some potential implications for macroeconomic modeling and monetary policy evaluation. First, the extent of heterogeneity in price stickiness suggests that pricing rules should be modeled as mixtures of flexible, Calvo-type, Taylor-type, and state-dependent behaviors. The models estimated in our article are obviously nonnested and cannot provide an accurate typology of existing pricing rules. We may, however, give an approximate evaluation based on the results of the tests carried out in our empirical analysis. Let us consider the strata that include enough price increases and decreases for estimating competing-risks duration models. There are 211 such strata. For 148 of these strata, we are not able to reject a constant hazard, when allowing for a peak at month 12. These 148 strata, which account for around 40% of the economy when we rescale results using CPI weights, are consistent with a time-dependent (Calvo-type) behavior. Among the other strata, 39, accounting for around 30% of CPI-weighted units, can be characterized as state-dependent (in general because price increases are state-dependent). The remaining strata, which account for 20% of CPI weights and in general correspond to decreasing hazard functions, are apparently in contradiction with the main prediction of standard models of price stickiness.

Such a mixture of behaviors may matter for macroeconomic dynamics. State-dependent agents are, for instance, more likely to react promptly to a (large) shock, while time-dependent agents react after some lag, inducing a delay in the impact of monetary policy. For instance, Dotsey and King (2005) found that output persistence is lower in a state-dependent model than in a time-dependent model, other things being equal. Heterogeneity in price stickiness also matters for the evaluation of monetary policy. A recent literature has characterized the optimal monetary policy in presence of heterogeneous price durations (see, e.g., Aoki 2001; Woodford 2003, chap. 4). The main message conveyed by this literature is that an optimal policy should put a larger weight on sectors in which prices are more rigid. Results that we have obtained suggest that defining an optimal policy when some agents are time-dependent and others state-dependent is a relevant issue.

ACKNOWLEDGMENTS

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APPENDIX A: DERIVATION OF LIKELIHOOD FUNCTIONS

Let us consider a price spell, beginning at time $t_0 = 0$ and lasting until time $T$. The date $T$ is not known exactly, although it is observed to be located between dates $t - 1$ and $t$. For convenience, we shall consider that dates $t - 1$ and $t$ correspond to the end of months $t - 1$ and $t$, respectively, then we know only that the price spell has lasted more than $t - 1$ months and at most $t$ months. We aim at characterizing the probability for a price change to occur after some time has elapsed since the previous price change. The survivor function $S(t)$ at $T = t$ is the probability for a spell to last at least $t$ months:

$$S(t) = Pr(T \geq t) = 1 - F(t),$$

where $F(\cdot)$ and $S(\cdot)$ are the cumulative density function and the survivor function of the duration $T$ of a price spell, respectively. Because our duration data are grouped on a monthly basis, the
typical likelihood contribution \(l(t)\) is the probability for a spell to terminate between the end of month \(t - 1\) and the end of month \(t\). This probability is given by

\[
l(t) = \Pr(t - 1 \leq T < t) = F(t) - F(t - 1) = S(t - 1) - S(t) = \exp[-H(t - 1)] - \exp[-H(t)],
\]

where \(H(t) = \int_0^t h(\tau) d\tau\), the last equality resulting from the relationship between the survivor function and the cumulated hazard, namely \(\ln S(t) = -H(t)\) (see, e.g., Kalbfleisch and Prentice 2002). Let us recall that the cumulated hazard function \(H(t)\) is defined by

\[
H(t) = \int_0^t h(\tau) d\tau,
\]

where \(h(\tau)\) is the hazard function of the duration \(T\) defined as

\[
h(\tau) = \lim_{\Delta \downarrow 0} \frac{1}{\Delta} \Pr(\tau \leq T < \tau + \Delta | T \geq \tau). \quad (11)
\]

Under this observation scheme, the baseline hazard \(h\) is not identified, unless we make some parametric assumption about the interval \([t - 1, t]\). The simplest assumption is to consider that this baseline hazard is constant over this interval, namely \(h(\tau) = h_t, \forall t - 1 \leq \tau < t\), although it may vary over different intervals indexed by \(t\). This assumption generates the well-known piecewise constant hazard model (see, e.g., Meyer 1990). Then, accounting for time discretization, the probability for a spell to end during month \(t\) is then given by

\[
l(t) = \Pr(t - 1 \leq T < t) = \exp[-H(t - 1)] - \exp[-H(t)] = \exp\left[-\sum_{s=1}^{t-1} h_s\right] - \exp\left[-\sum_{s=1}^{t} h_s\right]. \quad (13)
\]

The likelihood contribution of a price spell that is right-censored in month \(t\) is

\[
l(t) = S(t) = \exp\left[-\sum_{s=1}^{t-1} h_s\right]. \quad (14)
\]

Consequently, if we denote \(c_i\) the dummy variable taking value 1 if the \(i\)th price spell is not right-censored, and 0 otherwise, the log-likelihood function for a sample of \(N\) iid price spells is given by

\[
\ln L = \sum_{i=1}^{N} c_i \ln \left[1 - \exp\left(-h_{t-1}\right)\right] - \sum_{s=1}^{t-1} h_s. \quad (15)
\]

However, it may be unduly restrictive to assume that the hazard function is not affected by individual heterogeneity, particularly by individual unobserved heterogeneity. The most usual way to incorporate this type of heterogeneity is to assume that it is randomly distributed. Consider that the conditional hazard for observation \(i\) is given by

\[
h_i(t|\theta_i) = \theta_i h_t,
\]

where \(\theta_i\) is a random variable. The conditional survivor function given unobserved heterogeneity may be derived as

\[
S(t|\theta_i) = \exp\left[-\int_0^t h(t|\theta_i) \, ds\right].
\]

To estimate this model, we have to “integrate out” this conditional survivor function over \(\theta_i\) whose density function is \(\mu(\theta)\):

\[
S(t) = \int_0^\infty S(t|\theta) \mu(\theta) \, d\theta.
\]

Following Lancaster (1979) and Meyer (1990), we can assume that \(\theta\) is Gamma-distributed with mean 1 and variance \(\sigma^2\). Then we get

\[
S(t) = \left[1 + \sigma^2 H_0(t)\right]^{-1/\sigma^2},
\]

where

\[
H_0(t) = \sum_{s=1}^{t-1} h_s.
\]

Then the log-likelihood can be written as

\[
\ln L = \sum_{i=1}^{N} \ln \left[\left(1 + \sigma^2 \sum_{s=1}^{t-1} h_s\right)^{-1/\sigma^2}\right] - c_i \left[1 + \sigma^2 \sum_{s=1}^{t} h_s\right]^{-1/\sigma^2},
\]

namely

\[
\ln L = \sum_{i=1}^{N} \ln \left[1 + \sigma^2 \sum_{s=1}^{t-1} h_s\right]^{-1/\sigma^2} - c_i \left[1 + \sigma^2 \sum_{s=1}^{t} h_s\right]^{-1/\sigma^2}.
\]

**APPENDIX B: THE LIKELIHOOD FUNCTION OF THE COMPETING–RISKS DURATION MODEL**

If the hazard function is piecewise constant and if there is no unobserved heterogeneity, the likelihood contribution of a spell ending in month \(t_i\) with a type \(k\) event is

\[
l_k(t_i; \alpha) = \left[S_k(t_i - 1|z_{ik} h_0^{(k-1)}) - S_k(t_i|z_{ik} h_0^{(k)})\right]
\times \prod_{k' \neq k} S_{k'}(t_i|z_{ik} h_0^{(k)})
\times \left[1 - \exp\left[-\sum_{s=1}^{t-1} h_{k,s} \exp(z_{ik} \alpha_k)\right]\right]
\times \exp\left[-\sum_{s=1}^{t_i} h_{k,s} \exp(z_{ik} \alpha_k)\right]
\times \prod_{k' \neq k} \exp\left[-\sum_{s=1}^{t_i} h_{k',s} \exp(z_{ik} \alpha_{k'})\right]. \quad (16)
\]
For a spell ending in month $t_i$ with a type-$j$ event ($j \neq k$), the likelihood contribution is

$$l_j(t_i; \alpha) = \left[ 1 - \exp[-h_{k_i} \exp(z_{i} \alpha_j)] \right] \times \exp \left[ - \sum_{s=1}^{t_i} h_{k_s} \exp(z_{i} \alpha_j) \right] \times \prod_{k' \neq k \neq j}^{t_i} \exp \left[ - \sum_{s=1}^{t_i} h_{k'_s} \exp(z_{i} \alpha_{k'}) \right].$$

In this second type of contribution, parameters depending on the destination indicator $k$, namely $h_{k_s}$ and $\alpha_k$, appear only in the marginal survivor function

$$S_k(t_i|z_{i0}, \alpha_k) = \exp \left[ - \sum_{s=1}^{t_i} h_{k_s} \exp(z_{i} \alpha_k) \right].$$

This is also the case for right-censored spells because

$$l_c(t_i; \alpha) = \exp \left[ - \sum_{s=1}^{t_i} h_{k_s} \exp(z_{i} \alpha_k) \right] \times \prod_{k' \neq k} \exp \left[ - \sum_{s=1}^{t_i} h_{k'_s} \exp(z_{i} \alpha_{k'}) \right].$$

Consequently,

(1) as we have three possible terminating events (except right-censoring denoted by $c_i = 0$), we get $K = 3$ additively separable log-likelihood subfunctions with expressions:

$$\ln L_k = \sum_{i=1}^{N} c_i \ln (k_i = k) \times \ln \left( 1 - \exp[-h_{k_i} \exp(z_{i} \alpha_k)] \right) - \sum_{s=1}^{t_i} h_{k_s} \exp(z_{i} \alpha_k)$$

$$- \sum_{i=1}^{N} c_i \ln (k_i = k') \times \sum_{s=1}^{t_i} h_{k'_s} \exp(z_{i} \alpha_{k'})$$

$$- \sum_{i=1}^{N} (1 - c_i) \sum_{s=1}^{t_i} h_{k'_s} \exp(z_{i} \alpha_{k'})$$

where $1(\cdot)$ is an indicator function equal to 1 if the expression in parentheses is true, 0 otherwise;

(2) thus the total likelihood function may be written as

$$\max_{\theta} \ln L = \sum_{k=1}^{3} \max_{h_{k_s}, \alpha_k} \ln L_k;$$

(3) when maximizing the 4th subfunction $\ln L_k$ with respect to $[h_{k_s}, \alpha_k]$, price spells terminated by events $k' \neq k$ contribute only through their marginal survivor function $S_k(t_i|z_{i0}, \alpha_k)$. Thus they can be treated as right-censored spells.

REFERENCES


