MONOPOLY PRICING

Industrial Organization B

THIBAUD VERGÉ

Autorité de la Concurrence and CREST-LEI

Master of Science in Economics - HEC Lausanne (2009-2010)
Benchmark Case: Competitive Firm

- Denote by $p$ the price and by $q$ the quantity.
- The firm’s profit then writes as:
  $$\pi = pq - C(q)$$

Perfect Competition

- The firm is then price-taker, that is, $p$ is fixed.
- Chooses the quantity $q$ so as to maximize its profit.
- Supply is given by $C'(q) = p$.
- In equilibrium the price is determined by: Supply = Demand.
- That is, the equilibrium quantity $q^C$ is such that $P(q^C) = C'(q^C)$. 
Competitive Firm

Net Consumer Surplus

Profit

$p$

$q$

$P(q)$

$C'(q)$

$q^C$

$p^C$
Monopolist’s Profit Maximization

Link between price and quantity
- Demand function: \( q = D(p) \)
- Inverse demand function: \( p = P(q) \)

Profit maximizing quantity
- The monopolist’s profit writes as: \( \pi(q) = P(q)q - C(q) \)
  - First-order condition (assuming concavity of the profit function):
  \[
P'(q)q + P(q) = C'(q)
\]
  - Marginal Revenue
  - Marginal Cost
Monopolist’s Profit Maximization

The diagram illustrates the profit-maximizing behavior of a monopolist. The monopolist's demand function is represented by the blue line labeled $P(q)$, and the marginal revenue function is represented by the green line labeled $MR(q)$. The monopolist's profit is maximized at the quantity $q^M$ where $MR(q) = P(q)$.

The marginal cost function is represented by the red line labeled $C'(q)$. The monopolist sets the price $p^M$ at the quantity $q^M$, which is higher than the cost $p^C$ at the quantity $q^C$. This indicates that the monopolist is able to charge a price above marginal cost, which is a characteristic of monopoly pricing.
Profit Maximization
The linear demand case

Linear Demand, Constant Marginal Cost

- Assume \( D(p) = a - p \) and \( C(q) = cq \)
- with \( a > 0, c \geq 0 \).

- We thus have \( P(q) = a - q \)
- Profit writes as \( \pi(q) = q(a - q) - cq = (a - c - q)q \ldots \)

- \( \ldots \) and is maximum for \( q = q^M = \frac{a-c}{2} \)
- Monopoly profit is then equal to \( \pi^M = \frac{(a-c)^2}{4} \)
Welfare analysis - The deadweight loss

Two effects...

- The monopoly price is higher than the competitive price ($p^M > p^C$)
- Consumer surplus decreases

- Profit increases
- (i.e. shareholders benefit)

No ambiguity overall: Loss > Gain
Welfare Analysis

Net Consumer Surplus

Welfare Loss

Profit

$C'(q)$

$P(q)$

$q^M$

$q^C$

$p^M$

$p^C$
Profit Maximizing Price

Link between price and quantity

- Demand function: \( q = D(p) \)
- Inverse demand function: \( p = P(q) \)

Profit Maximizing Price

- The monopolist’s profit writes as:
  \[
  pD(p) - C(D(p))
  \]
  Revenue - Cost

- First-order condition:
  \[
  D(p) + pD'(p) = D'(p)C'(D(p))
  \]
  Marginal Revenue = Marginal Cost
Lerner Index

**Rewriting the first-order condition**

- \[ D'(p) [p - C'(D(p))] = -D(p) \]
- or

\[
\frac{p - C'(D(p))}{p} = \frac{-D(p)}{pD'(p)} = \frac{1}{\varepsilon}
\]

- Lerner Index
- Inverse of the Price Elasticity of Demand

**Interpretation**

- The higher the price elasticity of demand, the lower the monopolist’s mark-up
- Limit case: if \( \varepsilon \to +\infty \), then \( p^M = C' = p^C \)
A multi-product monopolist

**Some Notations**

- The monopolist sells \( n \) different goods index by \( i = 1, 2, \ldots, n \).
- It charges prices \( p = (p_1, p_2, \ldots, p_n) \)
- and sells quantities \( q = (q_1, q_2, \ldots, q_n) \) where the demand functions are \( q_i = D_i(p) \).
- Cost of production is \( C(q) \).

**Profit Maximization**

The monopolist’s profit is:

\[
\Pi(p) = \sum_{i=1}^{n} p_i D_i(p) - C(D_1(p), \ldots, D_n(p))
\]
First-order Conditions

In the general case, the first-order conditions write as:

For any \( i = 1, 2, \ldots, n \):

\[
\left( D_i + p_i \frac{\partial D_i}{\partial p_i} \right) + \sum_{j \neq i} p_j \frac{\partial D_j}{\partial p_i} = \sum_{j=1}^{n} \frac{\partial C}{\partial q_j} \frac{\partial D_j}{\partial p_i}
\]

Separable Costs

- We suppose here that \( C(q) = \sum_{i=1}^{n} C_i(q_i) \).
- We can thus simplify the f.o.c:

\[
\left( D_i + (p_i - C'_i) \frac{\partial D_i}{\partial p_i} \right) = - \sum_{j \neq i} (p_j - C'_j) \frac{\partial D_j}{\partial p_i}
\]
Dependent Demand, Separable Costs

First-order conditions

\[
\frac{p_i - C_i'}{p_i} = \left( 1 + \sum_{j \neq i} \left( \frac{p_j - C_j'}{p_i D_i} \right) D_j \varepsilon_{ji} \right) \frac{1}{\varepsilon_{ii}}
\]

where:

- \( \varepsilon_{ii} = -\frac{p_i}{D_i} \frac{\partial D_i}{\partial p_i} \) is the own-price elasticity of demand; and
- \( \varepsilon_{ji} = \frac{p_i}{D_j} \frac{\partial D_j}{\partial p_i} \) is the cross-price elasticity of demand for good \( j \) with respect to the price of good \( i \).

Comparison with the single-product firm

- If goods are substitutes i.e. \( \varepsilon_{ji} > 0 \), then \( \frac{p_i - C_i'}{p_i} > \frac{1}{\varepsilon_{ii}} \).
- If goods are complements i.e. \( \varepsilon_{ji} < 0 \), then \( \frac{p_i - C_i'}{p_i} < \frac{1}{\varepsilon_{ii}} \).
Independent Demands, Dependent Costs

Learning by Doing
- A monopolist produces at dates \( t = 1 \) and \( t = 2 \).
- At date \( t \), demand is \( q_t = D_t (p_t) \).
- At date \( t = 1 \), total cost is \( C_1 (q_1) \).
- At date \( t = 2 \), total cost is \( C_2 (q_2, q_1) \) with \( \frac{\partial C_2}{\partial q_1} < 0 \).

Intertemporal profit
The monopolist thus maximizes its total profit:

\[
p_1 D_1 (p_1) - C_1 (D_1 (p_1)) + \delta (p_2 D_2 (p_2) - C_2 (D_2 (p_2), D_1 (p_1)))
\]

where \( 0 \leq \delta \leq 1 \) is the discount factor.
At date $t = 2$, the monopolist equalizes marginal revenue and marginal cost, i.e.

$$\frac{p_2 - C'_2 (D_2 (p_2) , (p_1))}{p_2} = \frac{1}{\varepsilon_2}$$

At date $t = 1$:

$$\frac{p_1 - C'_1 - \delta \frac{\partial C_2}{\partial q_1}}{p_1} = \frac{1}{\varepsilon_1} \Rightarrow \frac{p_1 - C'_1}{p_1} < \frac{1}{\varepsilon_1}$$

Remark

The firm would underproduce (at both dates) if it were run by two consecutive managers each maximizing the short-term profit.
Durable goods

60% of all production

Questions

- Durability of the good (endogenous: chosen by the producer).
- Inter-temporal competition: new vs. used good.
- Asymmetric information about quality.
Selling over several periods

The monopoly competes against itself

Durable good

If bought today, the good can be consumed (i.e. creates utility) today but also tomorrow.

Main problem

- Denote by $p_t$ the price in period $t = 1, 2$
- Consumers who have already bought at date $t = 1$ do not buy again at date $t = 2$
- To attract new consumers at date $t = 2$ . . .
- . . . the monopolist must lower its price: $p_2 < p_1$

But if consumers anticipate this, are they still willing to buy at date $t = 1$?
A simple two-period model

Consumers

- Each Consumer buys one unit of the good or nothing
- The consumer’s (inter-temporal) net utility is:

\[ u = \begin{cases} 
(1 + \delta)v - p_1 & \text{if (s)he buys in the first period,} \\
\delta(v - p_2) & \text{if (s)he buys in the second period,} \\
0 & \text{if (s)he does not buy.} 
\end{cases} \]

- The “valuation” \( v \) is uniformly distributed over \([0, 1]\)
- \( \delta \) is the discount factor

The monopolist

- Constant marginal cost, normalized to 0.
Commitment not to change the price

**Assuming** $p_2 = p_1 = p$

- If the price is not changed (lowered) in period 2, nobody buys at that stage.
- If consumer $v$ buys the product (at $t = 1$), any consumer with a higher reservation price ($v' > v$) also buys.
- The indifferent consumer is $\hat{v}(p_1) = \frac{p_1}{1+\delta}$.
- The demand is thus: $1 - \frac{p_1}{1+\delta}$
- and the monopolist’s profit is: $p_1 \left( 1 - \frac{p_1}{1+\delta} \right)$

**Optimum**

- The optimal price is therefore: $\hat{p}_1 = \frac{1+\delta}{2}$
- And the profit is $\hat{\Pi} = \frac{1+\delta}{4}$.
Without commitment

For a given $p_1$, find the optimal $p_2^* (p_1)$

- If $v$ buys at $t = 1$, then $v' > v$ does too
- Denote by $\tilde{v}_1 (p_1)$ the indifferent consumer
- If $v > \tilde{v}_1 (p_1)$, then $v$ buys at $t = 1$
- If $v < \tilde{v}_1 (p_1)$, then $v$ doesn’t buy at $t = 1$
- Therefore $D_2 (p_2; p_1) = \max [\tilde{v}_1 (p_1) - p_2, 0]$
- The second period profit is then: $p_2 (\tilde{v}_1 (p_1) - p_2)$

The second period optimal price is thus

\[
p_2^* (p_1) = \frac{1}{2} \tilde{v}_1 (p_1)
\]
Determining $\tilde{v}_1$

\[
(1 + \delta) \tilde{v}_1 - p_1 = \delta (\tilde{v}_1 - p^*_2(p_1))
\]

utility if purchase at $t=1$

utility if purchase at $t=2$

therefore

\[
\tilde{v}_1(p_1) = \frac{1}{1 + \frac{\delta}{2}} p_1
\]
First period optimal price (II)

The total profit writes as

\[ \Pi_{1+2}(p_1) = p_1 \left( 1 - \tilde{v}_1(p_1) \right) + \frac{\delta}{4} \tilde{v}_1(p_1)^2 \]

that is

\[ \Pi_{1+2}(p_1) = p_1 \left( 1 + \left( -\frac{2}{2+\delta} + \frac{\delta}{(2+\delta)^2} \right) p_1 \right) = p_1 \left( 1 - \frac{4 + \delta}{(2 + \delta)^2} p_1 \right) \]

Optimal price

\[ p_1^* = \frac{(2 + \delta)^2}{2(4 + \delta)} \]
## Summary and comparison

### Without commitment

\[
p_1^* = \frac{(2 + \delta)^2}{2(4 + \delta)} , \quad p_2^* = \frac{1}{2} \frac{2 + \delta}{4 + \delta} < p_1^* \quad \text{and} \quad \Pi^* = \frac{1}{4} \frac{(2 + \delta)^2}{4 + \delta}
\]

### Comparison

\[
\Pi^* < \hat{\Pi} \quad \text{and} \quad p_2^* < p_1^* < \hat{p}
\]
Example: $\delta = 1$

**Without commitment**

- Do not buy.
- Buy at $T=2$ (at price 0.3)
- Buy at $T=1$ (at price 0.9)

Profit = $0.9 \times 0.4 + 0.3 \times 0.3 = 0.45$

**Commitment not to lower price**

- Do not buy.
- Buy at $T=1$ (at price 1)

Profit = $1 \times 0.5 = 0.5 > 0.45$
Coase Conjecture

The monopolist competes against itself

- Two variables: the number of periods and the discount factor
- If the number of periods is infinite and if $\delta \to 1$
- Then: $\Pi^* \to 0$
- That is: $p_1^* = 0$ (otherwise nobody buys - waiting is costless)

Evading the Coase Problem

- Committing itself to a sequence of prices (credibility? reputation?)
- Leasing
- Money-back guarantee ("most-favored customer" clause)
- New consumers
- Planned obsolescence (new versions, upgrades, ...)

Durable Good

Coase conjecture
Leasing rather than selling

Leasing

- One price per period
- The consumer doesn’t own the product
- The good is durable for the monopolist only
- Denote by $p_t$ the price for period $t$
- Consumer leases if $p_t \leq v$, demand is thus: $D_t(p_t) = 1 - p_t$
- Profit for period $t$ is thus $\Pi_t(p_t) = p_t (1 - p_t)$
- Optimal prices: $\overline{p_1} = \overline{p_2} = 1/2$
- Total profit: $\Pi_L = \frac{1}{4} + \delta \frac{1}{4} = \frac{1+\delta}{4} = \widehat{\Pi}$