ENSAE, University of Paris-Saclay, Master in Economics

Academic Year 2016-2017

Theory of contracts and incentives. Course given by Robert Gary-Bobo.

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duration: x hours
No lecture notes

Question (10 points).

Present the Principal-Agent relationship under pure adverse selection. To illustrate your presentation, you may focus on a simple model with two unobservable agent types à la Laffont-Martimort. Explain the difference between second-best and first-best optimal contracts. Discuss the role played by incentive and individual rationality constraints.

Exercise (10 points). *Tournaments with risk-neutral agents.*

Consider a firm with two identical workers, indexed by $i = 1, 2$. The firm lasts one period. Each agent i chooses a level of effort a_i. Effort is not observed by the employer. The employer and the workers observe a performance measure q_i, which is simply agent i's output here. We assume that $q_i = a_i + \epsilon_i$ where ϵ_i is a random variable with a normal distribution. We assume that variables ϵ_i are identically and independently distributed, with a zero mean and variance σ^2. Recall that the probability density of ϵ_i is the function $f(\epsilon_i) = (2\pi\sigma^2)^{-1/2} \exp(-(1/2)(\epsilon_i/\sigma)^2)$. Let H denote the cumulative distribution function of $\epsilon_1 - \epsilon_2$. Let $h = H'$ denote the density of this distribution (and recall that the difference of two normal random variables is itself normal). Let $C(a_i)$ denote the cost of effort for agent i. Function C is assumed nonnegative, strictly increasing, strictly concave and continuously differentiable. Let C' denote the marginal cost of effort.

The utility of agent i is $u_i = w_i - C(a_i)$ where w_i is agent i's compensation (workers are therefore risk-neutral). Each worker wants to maximize his(her) expected utility. A worker will not accept a contract if his expected utility falls below the individual rationality level u_0, representing the best outside option of both agents. The firm's profit is simply $\Pi = q_1 + q_2 - w_1 - w_2$. The firm seeks to maximize the expected value of profit.

Question 1. First Best. The first-best optimal allocation of effort, maximizes the expected social surplus. Show that the first-best effort of worker i, denoted a_i^* solves $C'(a_i^*) = 1$ for $i = 1, 2$.

Question 2. Piece-rate contracts. Assume that the employer proposes linear piece-rate contracts to both agents, that is, compensation is computed as a function of performance as follows: $w_i = \alpha + \beta q_i$, where α is a base wage and $0 \leq \beta \leq 1$. We now study the implementation of the first-best by means of these contracts.

2a. Show that the first-best allocation of effort can be implemented only if $\beta = 1$.

2b. Show that the individual rationality (*i.e., IR*) constraint of each agent i can be satisfied, and give the expression of the best value of α from the point of view of the employer as a function of u_0 and a_i^*.

Question 3. Tournament. To provide effort incentives, the employer promises to pay the most productive agent a bonus. In other words, at the end of the period, there is a winner and a loser. The winner is the agent i such that $q_i > q_{3-i}$ (or such that $q_i = \max_j\{q_j\}$). The winner receives compensation $\alpha + b$, where b is a bonus or "prize". The other agent obtains only the base wage α. Let p_i denote the probability of winning of agent i.

3a. Write the expression of p_i as a function of (a_1, a_2).

3b. Pose agent i's expected utility maximization problem and write the first-order necessary conditions for an optimal choice of a_i by agent i.

Question 4. Symmetric Nash equilibrium of the tournament. Agents being identical we look for a Nash equilibrium in terms of the effort variables such that $a_1 = a_2$.

4a. Show that the first-best allocation of effort can be implemented in Nash equilibrium and compute the appropriate value of the bonus b as a function of h and a_i^*.

4b. Show that the value of α can be chosen by the employer to satisfy the IR constraints. Give an expression of the best value of α (from the point of view of the employer) as a function of H, h, a_i^* and u_0.

4c. Show that in symmetric equilibrium the winner is chosen by chance (a flip of a coin).

Additional questions: We now consider a two-period model and a two-stage tournament. There are 4 agents in the first period $i = 1, 2, 3, 4$, and they form two teams of two agents: agent 1 competes with agent 2 and agent 3 competes with 4, forming two "semi-finals". The losers of the semi-finals are fired at the end of the first period and find a job elsewhere; the winners of the semi-finals obtain a bonus b_1 and compete during period 2. The winner at the end of period two obtains a bonus b_2, and the loser of the final receives bonus b_1. All agents in period $t =$ receive a base wage α_t, $t = 1, 2$. Agents exert effort in both periods with the same cost C, effort cost is additively separable over periods and there is no discounting.

Question 5. Two-stage tournament: last stage. This game must be solved for its subgame-perfect Nash equilibrium. To this end, it must be solved backwards.

We start by the study of the equilibrium in period 2. Let p_{it} denote the probability of winning the bonus b_t of agent i in period t. Let a_{it} denote the choice of effort of i in period t.

5a. Suppose that Agent 1 and Agent 3 have won the semi-finals. Give the expression of agent 1's expected utility in period 2 and write the first-order condition for agent 1's best choice of effort a_{12} in the second period.

5b. Determine the value of the difference $b_2 - b_1$ that implements the first-best level of effort in period 2.

Question 6. Two-stage tournament: first stage. Using the results of question 5, we can now study the first stage (and therefore, solve the complete game).

6a. Give the expression of Agent 1's expected utility in period 1, and write the first-order condition for the best choice of first-period effort a_{11}.

6b. Determine the value of the first-period bonus b_1 that implements the first-best level of effort in period 1, given that $b_2 - b_1$ is set optimally and that we have a symmetric equilibrium. Show that we must have $2b_1 < b_2$. Provide an economic interpretation of this result.

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Exercise Session 2

PC2

1. Demand for insurance

An individual is endowed with initial wealth w and he may lose L in case of an accident, which can happen with probability p. This agent can sign an insurance contract that reimburses q euros in case of an accident. The contract implies the payment of a premium that is proportional to the indemnity, *i.e.*, πq, where π is such that $p \leq \pi < 1$. The agent's attitude towards risk is captured by the von Neuman Morgenstern utility function, $U(x) = -\exp(-rx)$, with $r > 0$.

1. Determine the indemnity (*i.e.*, the value of q) chosen by the individual ?

2. How does q vary with r ?

3. Suppose that competition among insurance companies lead them to charge a premium equal to the expected value of the reimbursement (*i.e.,* $\pi = p$). What is then the insurance policy chosen by the individual ?

4. Derive the previous results using a picture in the plane (R_1, R_2), in which R_1 and R_2 denote respectively the agent's net wealth when an accident has or has not occurred.

2. Discriminating Monopolist under Incomplete Information

A monopoly produces one good and faces a population of consumers. There are two types of consumers in the population, but the monopolist doesn't observe these types. The utility of a consumer of type θ for a quantity q bought at a price p is defined as,

$$U(q, p; \theta) = \theta v(q) - p,$$

where v is strictly concave, strictly increasing and twice continuously differentiable function for all $q \geq 0$, and $v(0) = 0$. The type θ belongs to a set $\{\underline{\theta}, \overline{\theta}\}$, where $0 < \underline{\theta} < \overline{\theta}$ and the prior probability of type $\underline{\theta}$ is $\Pr(\underline{\theta}) = \alpha$. The monopoly proposes a *menu of contracts* (p, q) to the consumers. A menu is a pair $\{(\underline{p}, \underline{q}), (\overline{p}, \overline{q})\}$. Consumers choose in the menu a pair (p, q) to consume q units at price p. Another possible interpretation is that q is a quality index: in this case the consumers choose one unit of a good of quality q at price p in the menu. The cost of a unit of quantity (or quality) is c, where $c > 0$

is a parameter. The monopolist wants to choose a menu that maximizes the expected profit per consumer.

Question 1. Write the expected profit function of the monopolist, assuming that type $\overline{\theta}$ chooses $(\overline{p}, \overline{q})$ and type $\underline{\theta}$ chooses $(\underline{p}, \underline{q})$.

Question 2. Consumers can always decide not to consume the good supplied by the monopolist; in this case, the utility is zero, *i.e.,* $U(0, 0; \theta) = 0$. Consumers will not "participate" and buy the good if the price is too high. Write the participation constraints for the two types of consumer. These constraints are denoted $\underline{IR}$ and $\overline{IR}$ for types $\underline{\theta}$ and $\overline{\theta}$ respectively (IR stands for *individual rationality*).

Question 3. The monopolist wants each of the types to choose a different element in the menu, type $\overline{\theta}$ must be willing to choose $(\overline{p}, \overline{q})$, and type $\underline{\theta}$ must be willing to choose $(\underline{p}, \underline{q})$. Write the self-selection constraint for each type of consumer; constraints are denoted $\underline{IC}$ and $\overline{IC}$ for types $\underline{\theta}$ and $\overline{\theta}$ respectively (IC stands for *incentive compatibility*).

Question 4. Show the *production monotonicity* property: an optimal menu of contracts necessarily satisfies $\overline{q} \geq \underline{q}$: the high-type consumer should consume more (or a higher quality) than the low-type consumer. (Hint: add the IC constraints)

Question 5. Show that if $\underline{IR}$ and $\overline{IC}$ are satisfied, then $\overline{IR}$ is also satisfied.

Question 6. Show that $\underline{IR}$ and $\overline{IC}$ must be binding at the optimum. (Hint: if not, increase $\underline{p}$ or $\overline{p}$ by an $\varepsilon > 0$).

Question 7. Substitute $\underline{IR}$ and $\overline{IC}$, expressed as equalities, in the objective function and maximize profits, while ignoring $\underline{IC}$ (you'll check later that $\underline{IC}$ indeed holds). Write the first-order conditions for an optimal menu of contracts.

Question 8. Show that $\underline{IC}$ is satisfied by the solution of the relaxed profit-maxization problem studied in Question 7.

Question 9. What would be the choice (p, q) if there was only one type? How that this choice is efficient. Using the results of Question 7, show that there is no distortion at the top (*i.e.*, high types enjoy an efficient choice of (p, q)); show that the high types also enjoy an informational rent; show that the quantity (or quality) consumed by the low type is distorted downwards, as compared to the efficient allocation obtained when there is only one type. Discuss the rôle of α and $\delta = (\overline{\theta} - \underline{\theta})$ in the distortion and rents. Provide an economic interpretation of the results.

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Exercise Session 3

PC3

1. Adverse selection with a type-dependent participation constraint

A principal delegates the production of q units of a given good to an agent. The value for the principal of these q units is $S(q)$ where S is a continuously differentiable fucntion, $S' > 0$, $S'' < 0$ and $S(0) = 0$. The agent has a cost function $C(\theta, q) = \theta q$, where θ belongs to $\{\underline{\theta}, \bar{\theta}\}$. We denote $\Delta\theta = \bar{\theta} - \underline{\theta} > 0$. The agent can be either efficient ($\underline{\theta}$) or inefficient ($\bar{\theta}$) with respective probabilities ν and $1 - \nu$.

A contract between Principal and Agent is a pair of functions $\theta \to (q(\theta), t(\theta))$ where $q(\theta)$ is a level of production and $t(\theta)$ the transfert received by the agent of type θ. When choosing a contract the agent is informed about his type θ. The utility of the agent if he produces q and receives a transfer t is,

$$U = t - \theta q,$$

and the principal's benefits are

$$S(q) - t.$$

The efficient agent's outside utility level is $U_0 > 0$, and the inefficient agent's outside utility level is 0. *Nota Bene*: this is the only change in the model studied during the course: a type-dependent individual rationality level.

Question 1. Write the agent's participation constraints, denoted $\underline{IR}$ and $\overline{IR}$ for types $\underline{\theta}$ and $\bar{\theta}$ respectively.

Question 2. (*Complete information*) In this question only, the principal perfectly observes the agent's type. Characterize the first-best optimal contract $((\bar{t}^*, \bar{q}^*), (\underline{t}^*, \underline{q}^*))$. Provide an economic interpretation. *Note*: we denote $\bar{q}^* = q^*(\bar{\theta})$, $\underline{q}^* = q^*(\underline{\theta})$, etc.

Question 3. (*Asymmetric information*) Assume now that the agent's type θ is private information. Explain why the principal can restrict to direct revealing (or direct truthful) contracts without loss of generality. Write the agent's incentive compatibility constraints, denoted $\underline{IC}$ and $\overline{IC}$ for types $\underline{\theta}$ and $\bar{\theta}$ respectively. Show that they imply the monotonicity property $q(\underline{\theta}) \geq q(\bar{\theta})$.

Denote by $((\bar{t}^{**}, \bar{q}^{**}), (\underline{t}^{**}, \underline{q}^{**}))$ the second-best optimal menu of contracts when $U_0 = 0$. We characterized this contract when $U_0 = 0$, during the course.

Question 4. Suppose now that $U_0 > 0$.

4.a. Assume that $U_0 < \Delta\theta\overline{q}^{**}$. Without any calculations, prove that the optimal contract is the same as when $U_0 = 0$, and that $\underline{IC}$ and $\overline{IR}$ are binding.

4.b. Assume now that $\Delta\theta\overline{q}^{*} < U_0 < \Delta\theta\underline{q}^{*}$. Show that the optimal contract is such that $q(\theta) = q^*(\theta)$, (first-best efficient productions), and that $\underline{IR}$ and $\overline{IR}$ are binding. Give an economic interpretation.

4.c. Assume in this question that $\Delta\theta\overline{q}^{**} < U_0 \leq \Delta\theta\overline{q}^{*}$. Show then that the solution is such that $\underline{IR}$, $\underline{IC}$ and $\overline{IR}$ are binding. Show that, in this case, there are less distortions of the production, as compared to case 4a above.

4.d. (*Countervailing incentives*) Assume finally that $U_0 \geq \Delta\theta\underline{q}^{*}$. Show that in this case, the solution may be such that $\overline{IC}$ and $\underline{IR}$ are binding. (Hint: to be more precise, this case appears if $U_0 \geq \Delta\theta q^c$ where $q^c > \underline{q}^*$. Show that we may now obtain upward distortions of the production.

2. Optimal Regulation of a Firm with a Continuum of Types.

A public firm produces a quantity q; the social value of this quantity is $U(q) = q-(1/2)q^2$, and the firm's total cost is $C(q;t) = tq$, where t is a parameter, unobserved by the public authority (*i.e.*, the principal), but known to the firm. Social surplus is defined as $W(q, s) = U(q)-s$, where s is a transfer of money paid to the firm. The prior distribution of *type* t is uniform on interval $[0, 1]$. The firm receives from the principal a transfer $s(\widehat{t})$, function of a report $\widehat{t}$, and a production requirement $q(\widehat{t})$. Production q is observed by the principal. The firm's profit is defined as $\pi(t) = s(t) - tq(t)$. A firm cannot be forced to make losses and therefore we require $\pi(t) \geq 0$ for all t (*participation* constraint). The firm can decide to make a false report, that is , report $\widehat{t} \neq t$. We now study pairs of functions $(q(t), s(t))$, called contracts, that satisfy the incentive compatibility or *revelation constraint*,

$$s(t) - tq(t) \geq s(\widehat{t}) - tq(\widehat{t})$$

for all $(t, \widehat{t})$ in $[0, 1]^2$.

Question 1. The first-best optimum can be realized if the principal observes t. The first-best contract, denoted $(q^*(t), s^*(t))$ maximizes the net surplus $W(q, s)$, under the participation constraint. Compute the first-best $(q^*(t), s^*(t))$ and provide an economic explanation for the result.

Question 2. Suppose now that the type t is not observable. A second-best optimum is a contract $(q(t), s(t))$ that maximizes expected surplus $E(W(q(t), s(t)))$ subject to revelation and participation constraints. We look for an optimal contract in the class of continuously differentiable functions $(q(t), s(t))$ (almost everywhere). In fact, here, we will obtain continuously differentiable contracts.

a) Compute the optimal report $\theta(t)$, that is, the type $\theta(t)$ that the firm will report to maximize profits when the real type is t, under the first-best contract $(q^*(t), s^*(t))$. Show that the firm would make a false report for almost all values of t. Provide an economic interpretation.

b) Consider now an incentive compatible contract $(q(t), s(t))$. Write the first-order and second-order necessary conditions for truthful revelation, that is, to ensure that $\widehat{t} = t$ for all t.

c) Show that if the necessary conditions for truthful revelation derived above are satisfied for all t, the revelation constraint implies $s'(t) = tq'(t)$ for all t (condition $IC1$) and $q'(t) \leq 0$ for all t (condition $IC2$).

It is easy to show that $IC1$ and $IC2$ are sufficient conditions for revelation ($IC1$ and $IC2$ imply that the revelation constraint holds for all t). To show this, express s and t as the integral of their derivatives (you can skip this question).

Question 3. Using the results of question 2 above, assuming that we have a revealing contract,

a) Show that $s(t)$ can be expressed as a function of $q'(t)$; (Hint: Integrate s' from t to 1.);

b) Derive the expression $\pi(t) = \pi(1) + \int_t^1 q(\theta)d\theta$ (Hint : Integrate by parts).

c) Find a simple condition for the participation constraint to be satisfied for all t.

Question 4. We now compute the second-best optimal solution.

a) Pose the constrained optimization problem of the principal, which is to maximize $EW(q, s)$ with respect to contract $(q(.), s(.))$, subject to participation and revelation constraints, using the transformed expression of these constraints (*i.e.*, the results of questions 2 et 3 above).

b) Show without any computation that it is optimal to choose $\pi(1) = 0$, and that the participation constraint is then satisfied.

c) Eliminate $s(t)$ from the problem, using its expression as a function of $q(t)$ (we then get rid of $IC1$). Simplify the expression of EW so obtained, using integration by parts (Hint: apply this to a double integral appearing in the expression of EW). We find an expression of EW as an integral, with respect to t, ranging from 0 to 1, of a quadratic function of $q(t)$.

d) To compute the second-best solution $q^{**}(.)$, maximize EW while ignoring $IC2$. It happens that $IC2$ will be satisfied by the solution, as can be checked (and don't forget that production is nonnegative).

e) Compare $q^{**}(.)$ et $q^*(.)$. Draw the production schedules in the (t, q) plane. What is the effect of asymmetric information? Provide an economic interpretation of the result.

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Exercise Session 4

PC4

1. Efficiency wages

1.1. First part : 1 project

Part 1. Consider a Principal-Agent situation in which the principal P has a project requiring an investment I and that delivers a return $x = X$ with probability p and $x = 0$ with probability $1 - p$. The probability of success p depends on Agent A's effort, denoted e ; this effort cost the agent $C(e) = e$ and can take two values, 0 and $E > 0$. The probability of success of the project is $p = p_0$ if $e = 0$ and $p = p_E$ if $e = E$.

We assume that,

$$p_E > p_0 > 0,$$
$$p_E X - E > p_E X - \frac{2p_E E}{p_E - p_0} > I > p_0 X.$$

Question 1. Is the project profitable ? Give a qualified answer.

Question 2. The agent chooses the effort freely and the principal does not observe this choice. The principal only observes the realized return. She proposes a contract consisting in a fixed wage w and a bonus b ; total compensation is therefore given by

$$\begin{array}{lll} w & \text{if} & x = 0, \\ w + b & \text{if} & x = X. \end{array}$$

The principal and the agent are risk neutral. The principal maximizes the expected return net of the investment cost and of the agent's compensation. The agent maximizes his expected gain, net of the cost of effort. Under which condition is the agent induced to exert effort $e = E$?

Question 3. The agent has no initial wealth, so that the contract must satisfy $w \geq 0$ and $w + b \geq 0$ (the *Limited Liability* constraint). Determine the optimal contract and the associated gains for the principal and the agent.

1.2. Second part : 2 projects

Question 4. Suppose now that the principal has two identical projects, project 1 and project 2, whose probabilities of success are independent. For each project $i = 1, 2$, the agent can exert an effort $(e_i = E)$ or not $(e_i = 0)$; the total cost of effort is equal to $C(e_1, e_2) = e_1 + e_2$.

Suppose moreover that the principal offers a contracts of the form (w, b_1, b_2), where w represents the fixed wage and b_i the bonus paid in case of success of project i. Determine the optimal contract for the principal and the associated gains.

Question 5. Suppose now that the principal offers a contract (w, B), where w reprensents the fixed wage and B the bonus that is paid only if both projects are successful. The agent's compensation is therefore

$$\begin{array}{ll} w & \text{if} \quad x_1 = 0 \text{ or } x_2 = 0, \\ w + B & \text{if} \quad x_1 = x_2 = X. \end{array}$$

Determine the optimal (w, B) for the principal and the associated gains.

Question 6. Compare the results of questions 4 and 5. Which type of contract is the most profitable for the principal?

2. Linear incentives with several signals

A risk-neutral employer wants to sign an incentive contract with a salesman (hereafter called the agent) who is risk-averse. The agent must exert some effort $a \geq 0$, which yields a random gross profit for the firm equal to $x = a + \epsilon$, where ϵ follows a normal distribution with zero mean and variance σ^2. The monetary cost of effort for the agent is quadratic, equal to $a^2/2$. The employer wants to maximize his net expected profit $\mathbb{E}[x - w]$, which is the expected difference between the gross profit x and the salesman's wage compensation w. The agent wants to maximize the expected utility of his compensation net of the cost of effort, *i.e.*, $z = w - a^2/2$. His Von Neumann Morgenstern utility function is $U(z) = -e^{-rz}$, with $r > 0$.

The employer does not observe the agent's effort a, but he observes the gross profit x. We will restrict attention to contracts within the class of linear contracts $w(x) = \alpha + \beta x$, with $\beta \geq 0$. Suppose the agent's reservation utility is U_0 and he will refuse any contract that delivers less than this level of utility in expectation. Let us define $U_0 = -e^{-rw_0}$.

1. Preliminaries.

Write down the expected utility of the agent when he exerts effort a under a contract characterized by (α, β). Show that the maximization of expected utility is equivalent to maximization of the certainty equivalent $\mathbb{E}[w] - (r/2)\mathbb{V}(w) - a^2/2$, where $\mathbb{E}$ is the expectation operator and $\mathbb{V}$ the variance. Hint: Recall that if z is a normal random variable with mean μ and variance σ^2, then $E(e^z) = e^{\mu + (\sigma^2/2)}$.

2. First-best optimum.

We focus first on the optimal contract under *perfect information* (first best). Determine the effort and the contract parameters (a^*, α^*, β^*) that the employer would impose if she were able to observe the level of effort? Show that the participation constraint is binding at the optimum.

3. Second-best optimum.

We now focus on the optimal linear contract for the employer, denoted $(\alpha^{**}, \beta^{**})$, and the associated effort level, denoted a^{**}, when *effort is not observable by the employer*. Write down the program that determines this optimal contract and solve it. What is the profit loss for the employer, as compared to the first-best situation? Give an interpretation of the results.

4. Second-best optimum with two imperfect signals.

Suppose now that the employer observes two different measures of the agent's activity, the random variables $x = a + \epsilon$ and $y = a + \eta$, in which (ϵ, η) follows a bivariate normal distribution with mean $(0, 0)$, variances $Var(\epsilon) = \sigma_\epsilon^2$, $Var(\eta) = \sigma_\eta^2$, and covariance $Cov(\epsilon, \eta) = \sigma_{\epsilon\eta}$. We look for the optimal contract in the class of linear contracts $w(x, y) = \alpha + \beta x + \gamma y$. Suppose that the gross profit equals $(x + y)/2$ in this case.

4.a. Compute the variance of the compensation under the contract (α, β, γ).

4.b. Write down the maximization problem that determines the optimal contract, with the appropriate constraints.

4.c. We first focus on the symmetric case: $Var(\epsilon) = Var(\eta) = \sigma^2$ and we let ρ denote the correlation coefficient between both measures. Determine the optimal contract and associated effort in this case, that is, $(\alpha^{**}, \beta^{**}, \gamma^{**})$. Discuss the result as a function of ρ and σ. What happens when the two measures are perfectly correlated? What happens if the two measures are perfectly anti-correlated ?

4.d. We finally focus on the case in which $\sigma_{\epsilon\eta} = 0$ and in which the variances of x and y may be different. Give a condition that the ratio β/γ must satisfy when the contract is optimal and give an interpretation of the result.

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Exercise Session 9

PC9

1. The Groves and Loeb Mechanism

(Adapted from Jean-Jacques Laffont, *Fundamentals of Public Economics*, MIT Press, 1988). Consider an economy with N consumers, two private goods and a pure public good. The utility function of consumer i is given by

$$U_i(x_{i1}, x_{i2}, y) = x_{i1} + \theta_i(y + \sqrt{x_{i2}}),$$

where x_{i1} (respectively x_{i2}) indicates the quantity consumed of good 1 (resp. good 2) by consumer i and where y is the quantity consumed of the public good. $\theta_i \in R_+$ is a taste parameter known only to consumer i. The values of θ_i can be assumed to be the result of independent draws from an unknown distribution with finite mean and variance. The endowment of good 1 to consumer i is w_{i1}, $i = 1, ..., N$. Good 2 is produced using good 1 according to a constant returns technology: one unit of good 1 used as input produces one unit of good 2 as output. The public good is also produced using good 1; the cost function expressed in units of good 1 associated with this production process is $N(y^2/2)$. A public authority chooses y. Bankruptcy issues are ignored.

1. Compute the Pareto-optimal level of the public good when all consumers consume strictly positive amounts of good 1. This production level is denoted $y(\theta)$.

2. Assume that the cost of producing the public good is shared equally among all consumers and that good 2 is produced by competitive firms. We shall retain this framework until the end. The price of good 1 is normalized to 1 without loss of generality.

2a. What is the equilibrium price of good 2? [*Hint*: exploit the constant returns to scale assumption].

2b. Determine the *indirect utility function* of consumer i, denoted $V_i(\theta_i, y)$, as a function of θ_i and y [*Reminder:* this is the utility derived from the consumption of private goods when the utility $U_i(x_{i1}, x_{i2}, y)$ is maximized with respect to x_i under the budget constraint].

2c. Let the profile of preference parameters be denoted $\theta = (\theta_1, ..., \theta_N)$. Then, consider the game in which each agent announces a taste parameter $\widetilde{\theta}_i$, knowing that a public authority, hereafter called the *public decision maker,* will choose a level of output for the public good according to the decision rule,

$$y = y(\widetilde{\theta}) = \frac{\sum_{i=1}^{N} \widetilde{\theta}_i}{N}. \tag{1.1}$$

By assuming, for simplicity, that each agent can announce one response $\widetilde{\theta}_i$ in the set of real numbers R and that each agent knows the θ_j of all other agents, show that the game does not have a *Nash equilibrium* in general. We shall ignore the fact that y could be negative, and neglect the sign constraint on y.

3. We now want to characterize the revelation mechanisms with transfers $t_i(\theta)$ measured in terms of good 1, that allow the public decision maker to elicit the truthful reports, $\widetilde{\theta}_i = \theta_i$, as *dominant strategies*. Formally we consider direct reveleation mechanisms $\theta \to (y(\theta), t_1(\theta), ..., t_N(\theta))$.

3a. Assuming that the transfer functions t_i of the mechanism are differentiable, show that mechanisms leading to dominant strategy equilibria with truthful revelation exist.

3b. Show that, in this example, budget-balanced mechanisms exist, that is, whow that we can find transfer functions such that $\Sigma_i t_i(\theta) = 0$ for all θ. [Hint: Find a balanced set of transfer functions in the class of quadratic functions of $(\theta_1, ..., \theta_N)$, with square and rectangular terms.]

4. Instead of using the mechanisms of question 3, suppose now that the public decision maker estimates the preference parameter θ_i from an observation of the quantity x_{i2} of good 2 consumed by agent i, and chooses the corresponding level of output of the public good $y(\theta)$, according to formula (1.1). Knowing this decision rule, the agents behave strategically and change their consumption of good 2. Compute the social loss that results from this behavior and show that it tends to zero when N tends to infinity.

SCIENCES PO
Department of economics
PhD Program
Year 2012
Course of Robert Gary-Bobo
Written examination. 14 May 2012.

Choose QUESTION 1 or QUESTION 2.

QUESTION 1. *Exercise*

We consider a Principal-Agent relationship under moral hazard, risk-neutrality and limited liability. The Agent's output is random and can take any of n values q_i, with $i = 1, ...n$, and $q_1 < q_2 < ... < q_n$. The Agent chooses an unobservable level of effort e in $\{0, 1\}$. The probabilities of the outcomes are denoted $\pi_{ie} = \Pr[q_i \mid e]$. We assume $\pi_{i1} > \pi_{i0} > 0$ for all i. A contract is a vector $(w_1, w_2, ..., w_n)$, where w_i is the Agent's compensation in state q_i. In state q_i, the utility of the risk-neutral Agent is $u(w_i, e) = w_i - \psi e$, where $\psi > 0$ is a cost parameter. The Agent's best outside option has a utility $u_0 = 0$. The value of q_i for the risk-neutral Principal is $S_i = S(q_i)$. The Principal's payoff in state q_i is $(S_i - w_i)$. There are limited liability constraints (hereafter LL); in other words, the Agent cannot be subjected to negative payments; we therefore impose

$$w_i \geq 0 \qquad \text{for all} \qquad i = 1, ..., n. \tag{LL}$$

Question 1.1. We assume that effort $e = 1$ is optimal for the Principal, even under incomplete information.

1.1a. Write the Agent's incentive constraint, called IC, and the Agent's participation constraint, called IR. Write the Principal's optimization problem, which determines the optimal contract $(w_1^*, w_2^*, ..., w_n^*)$ under IR, IC and LL.

1.1b. Show that LL and IC imply IR, so that the IR constraint can be ignored.

1.1c. Write the first-order conditions for an optimal contract. Denote λ the Lagrange multiplier of IC and μ_i the Lagrange multiplier of the constraint $w_i \geq 0$, $i = 1, ..., n$.

Question 1.2. Assume that the ratios π_{i0}/π_{i1} are all different.

1.2a. Show that $w_j^* > 0$ if and only if

$$\frac{\pi_{j1} - \pi_{j0}}{\pi_{j1}} = \max_i \left\{ \frac{\pi_{i1} - \pi_{i0}}{\pi_{i1}} \right\}. \tag{ML}$$

Then, show that there is a unique index j such that $w_j^* > 0$ and that for all $i \neq j$, we have $w_i^* = 0$.

1.2b. Show that IC must be binding at the optimum and compute the value of w_j^*.

1.2c. Compute the expected rent of the Agent under the optimal contract.

Question 1.3. *Interpretation.*

1.3a. What is the meaning of the ratios π_{i0}/π_{i1}?

1.3b. Why is the Agent rewarded only in state q_j where j is such that ML holds?

1.3c. Show that the property: for all (k, i),

$$\frac{\pi_{k1} - \pi_{k0}}{\pi_{k1}} \geq \frac{\pi_{i1} - \pi_{i0}}{\pi_{i1}}$$

if and only if $k \geq i$, called *Monotone Likelihood Ratio Property*, implies that w_i^* is a non-decreasing function of q_i.

QUESTION 2. *Principal-Agent Model under Pure Adverse Selection.*

2a. Describe the basic assumptions of a Principal-Agent model when the Agent has a type that is not observable by the Principal, but no hidden action.

2b. Define the contract between Principal and Agent in this framework. Is it a direct revealing mechanism? (Apply the Revelation Principle to justify the approach).

2c. Describe the revelation and participation constraints.

2d. What is the main tradeoff faced by the Principal in this type of problem?

2e. What are the main results of the theory in this case? How does the optimal contract look like?

ENSAE-SCIENCES-PO-ECOLE POLYTECHNIQUE
MASTER « ECONOMICS AND PUBLIC POLICY »
Sujet d'examen - Written Examination-ECO 568
MICROECONOMICS OF INFORMATION- Course given by R.Gary-Bobo
Academic year 2013-2014 - examination 24 March 2014 - 3 hours.

ATTENTION PLEASE! No course notes, no laptops, no cell phones. **Answer the questions and solve the exercise.** Answers must be in English or French.

Questions. (10 points) *Signaling Games.*

Q1. Define a signaling game with two players, a finite number of actions and messages. Give a description of a signaling game in extensive form.

Q2. Define the equilibrium concept used to solve a signaling game.

Q3. Explain, if needed by means of an example, that there are separating and pooling equilibria in signaling games.

Q4. Which principles can be used to select some equilibria in these games, when there are several equilibria?

Exercise. (10 points) *A simple model of credit and banking with moral hazard.*

There is a population of identical risk-neutral entrepreneurs needing money to fund a project lasting one period. The investment required for each project is $K = 1$. There are also many risk-neutral lenders. The interest rate on funds used by lenders is $i = 0$, for simplicity (so that a lender lending $K = 1$ must at least get back $(1 + i)K = K = 1$ at the end of the period). The entrepreneurs have the choice between a "good technology", and a "bad" technology. At the end of the period, the good technology produces a net return $X = G$ with probability p_g (this event is called success) or $X = 0$ with probability $1 - p_g$ (this event is called failure); the bad technology produces a net return $X = B$ with probability p_b (success) or $X = 0$ with probability $1 - p_b$ (failure). Assume that the net present value (*i.e.*, NPV, $p_xX - 1$) of a project is positive only for good-technology projects: we have, $p_gG > 1 > p_bB$, but $B > G > 0$ and hence $p_g > p_b$. The event of success (*i.e.*, returns are not zero) is verifiable by outsiders but the choice of technology is not observable, and outsiders cannot distinguish if the return was B or G. In a loan contract, the entrepreneur can promise to repay R only in case of success, and repays 0 in case of failure (since we assume that entrepreneurs have no other source of cash).

The advantage of bankers over ordinary lenders is that they incur costs to observe the choices made by entrepreneurs (this is called *monitoring*) while the ordinary lenders do not. We will study two kinds of credit markets: (a), the case of direct competitive lending, and (b), the case of banking with monitoring.

Question 1. In the competitive, direct lending case, there is no monitoring, and the entrepreneurs will choose the good technology only if repayment R is such that choosing the good technology yields a higher expected profit.

1a. Write the expression for the entrepreneur's expected profit and the condition under which he or she chooses the good technology.

1b. From the latter condition, show that there exists a critical threshold value R_c such that the good technology is chosen if and only if $R \leq R_c$. Check that $R_c < G$.

1c. The probability of success of a project, denoted $p(R)$, is a function of R. Find the value of $p(R)$.

Question 2.

2a. We have a competitive equilibrium in the direct lending market only if $p(R)R = 1$. Explain why.

2b. Show that a competitive equilibrium exists only if $p_g R_c > 1$. Explain that this happens when the moral hazard problem is not too important (and that R_c is a measure of the extent of the latter problem).

2c. Show that the credit market collapses (*i.e.*, we have a no-trade equilibrium) if $p_g R_c < 1$, because all funded projects would be bad and would make losses on average.

Question 3. Now, we introduce a monitoring technology. By definition, a banker pays an additional cost C to prevent entrepreneurs from choosing the bad technology. We assume perfect competition between banks. Let R_m denote the repayment on a bank loan in case of success (m stands for "monitoring").

3a. Show that in a competitive equilibrium with active bankers (using the monitoring technology), we must have $p_g R_m = 1 + C$.

For bank lending to appear, two conditions are needed: *(i)*, R_m has to be smaller than G; and *(ii)*, direct lending, which is less costly, has to be impossible.

3b. Express condition *(i)* above and show that the monitoring cost has to be less than the NPV of the good project. Give the expression of condition *(ii)* above.

3c. Show that bank lending appears at equilibrium for intermediate values of p_g, namely, when

$$\frac{1+C}{G} < p_g < \frac{1}{R_c},$$

provided that the interval is not empty. Under which conditions is this interval non-empty? Show that the rate at which bank loans are proposed is $(1+C)/p_g$.

3d. If $p_g > 1/R_c$, in equilibrium, all entrepreneurs issue direct debt at rate $1/p_g$. Explain why.

3e. If $p_g < (1 + C)/G$ the credit market collapses. Explain why.

ENSAE
Cours de Théorie des Contrats et des Incitations
de Robert Gary-Bobo
Examen écrit de juin 2011
Durée : 3heures. Aucun document n'est autorisé.

QUESTION DE COURS (5 points)
Risque moral et aversion pour le risque dans la relation Principal-Agent.

PROBLEME (15 points)
On considère un principal (une firme) et un agent (son unique employé). L'agent choisit une action $a \in [0,1]$ à chaque période t. Le temps est discret, l'horizon est infini et les deux protagonistes ont le même taux d'escompte r (et donc le même facteur d'escompte $\delta = 1/(1+r)$). L'action a est inobservable pour le principal. La valeur de l'action pour la firme, notée $y \in \{0,1\}$, ne peut prendre que deux valeurs (succès ou échec). On suppose que la probabilité $\Pr(y = 1 \mid a) = a$. Le coût de l'action a pour l'agent est $c(a) = \gamma a^2$; avec le paramètre réel $\gamma > 0$. Mais la contribution de l'agent est trop complexe pour être mesurée objectivement. La variable y est une observation du principal et de l'agent, mais qui ne peut pas être vérifiée par une partie extérieure au contrat, donc par un juge, en cas de contentieux. Il existe par ailleurs une mesure de performance objective, observable et vérifiable, notée $p \in \{0,1\}$, qui ne prend elle aussi que deux valeurs 0 et 1, et qui peut faire l'objet d'un contrat explicite. Cette mesure de performance est malheureusement imparfaite pour les raisons suivantes. Avant de choisir a, l'agent reçoit un signal μ qui n'est pas observé par le principal, avec la propriété que la probabilité $\Pr(p = 1 \mid a) = \mu a$. Le signal μ est une variable aléatoire non-négative de moyenne $E(\mu) = 1$ et de variance $var(\mu)$ finie, et on suppose pour simplifier que la distribution de μ et le paramètre γ sont tels que l'on a toujours $\mu a < 1$. Les événements $\{y = 1\}$ et $\{p = 1\}$ sont indépendants sachant (μ, a). Nous allons étudier des contrats fondés sur le paiement par le principal d'une prime dite "explicite", notée β et payée ssi $p = 1$, cet événement étant, si nécessaire, vérifiable par le juge, et d'un bonus dit "implicite", payé au bon vouloir du principal si $y = 1$, ce dernier événement n'étant pas vérifiable, comme on l'a dit plus haut. En plus, l'agent reçoit un salaire de base contractuel s vérifiable. La rémunération R de l'agent est $R = s$ si $(p, y) = (0, 0)$; $R = s + b$ si $(p, y) = (0, 1)$; $R = s + \beta$ si $(p, y) = (1, 0)$; et $R = s + b + \beta$ si $(p, y) = (1, 1)$. L'agent n'accepte pas de contrat dont la valeur espérée par période serait inférieure à w_0, paramètre qui représente une opportunité extérieure permanente, disponible à chaque période. Le paiement de la firme durant toute période t est de forme $\pi = y - R$. Tandis que le paiement de l'agent est à chaque période de forme $u = R - c(a)$.

Question 1. Considérons le problème statique, qui dure une période. Déterminer l'action a^0 optimale de premier rang, c'est à dire, celle qui maximise le surplus espéré de la firme et de l'agent, et qui serait mise en oeuvre si y était observable et vérifiable. Pourquoi cette action de premier rang ne dépend-elle pas de μ?

Question 2. On étudie maintenant le problème de l'agent. Calculer l'action $a^*(\mu, b, \beta)$ choisie par l'agent, celui-ci sachant le signal μ, et les termes du contrat (b, β). On suppose que l'agent est neutre vis-à-vis du risque. A quelles conditions l'action choisie est elle plus petite que a^0?

Question 3: Etablir la contrainte de participation de l'agent dans le problème qui dure une période, sachant qu'on doit avoir $E(u) \geq w_0$.

Question 4. Ecrire et calculer le profit espéré *ex ante* $E(\pi)$ du principal au cours d'une période, c'est à dire avant que l'agent n'observe le signal. Exprimer ensuite ce profit espéré en supposant que la contrainte de participation de l'agent est saturée. On notera $V(b, \beta)$ le profit espéré dans ce dernier cas, et on le désignera comme "valeur du contrat (b, β)".

Question 5. On étudie maintenant un contrat statique explicite reposant uniquement sur la mesure de performance observable p. Dans cette question on pose donc a priori $b = 0$. Calculer β^*, la valeur de la prime qui maximise $V(0, \beta)$. Calculer aussi $V(0, \beta^*)$. Quel est l'impact de $var(\mu)$ sur ces résultats? Donner une interprétation économique.

Question 6. On s'intéresse maintenant à un contrat implicite reposant uniquement sur la promesse de payer un bonus b si et seulement si $y = 1$. On pose donc $\beta = 0$ dans cette question. Le principal et l'agent jouent un jeu répété et on considère des stratégies dites *simples*: les deux parties coopèrent tant qu'aucune d'entre elles n'a renié sa promesse (c'est à dire que l'agent travaille pour la firme, tandis que la firme paie le bonus b si $y = 1$), et cessent de coopérer pour toujours sinon (l'agent quitte le principal, les profits issus de cette relation sont nuls pour toujours si le principal ne tient pas ses promesses de payer le bonus b).

6a. Déterminer la valeur espérée du contrat implicite $V(b, 0)$.

6b. On cherche l'équilibre du jeu répété qui donne le plus grand profit espéré par période au principal, avec ces stratégies dites simples. Ecrire la contrainte sur b qui assure que le principal a intérêt à payer b lorsque $y = 1$, dite contrainte d'incitation, sachant la stratégie simple de l'agent. Montrer que cette contrainte revient à vérifier qu'une parabole est plus grande qu'une droite passant par zéro. On s'aidera au besoin d'une représentation graphique.

6c. Déterminer b^*, le meilleur bonus pour le principal, sous contrainte d'incitation. Montrer qu'on doit distinguer trois cas suivant la valeur de r et de w_0: un cas où la solution n'existe pas, un cas où la solution est un optimum de second rang, et un cas où la solution est l'optimum de premier rang.

6d. Donner une interprétation économique des résultats obtenus.

ENSAE, University of Paris-Saclay, Master in Economics

Academic Year 2015-2016.

Theory of contracts and incentives. Course given by Robert Gary-Bobo.

examen écrit — Written examination
January 2016

duration: 2 hours.
No lecture notes

Question (10 points).

The theory of implicit, self-enforcing labor contracts.

Describe the framework in which the theory can be presented and the basic assumptions. Discuss the notion of self-enforcing contract and the role of incentives constraints. Explain the main results of this theory.

Exercice (10 points). *Yardstick Competition under Complete Information.*

There are n identical firms indexed by i where $i \in N = \{1, \dots, n\}$. Firms are risk-neutral and regulated by the government. Each firm i faces the same demand function $q(p_i)$ in a separate market, where p_i denotes the regulated price of firm i. We assume that $q(p) = max\{0, b - p\}$, where b is a parameter. Firm i's marginal cost is a constant denoted c_i and $c_i \in [0, c_0]$. There are no fixed costs. Each firm can reduce c_i below its maximal value c_0 by means of a costly cost-reduction effort. More precisely, a firm's cost is c if the firm incurs expenditures denoted $R(c)$. The cost-reduction expenditure function is defined as follows.

$$R(c) = \alpha + \frac{\gamma}{2}(c_0 - c)^2,$$

where α and γ are positive parameters. The government gives a subsidy T_i to firm i. The profit of firm i, denoted V_i, is defined as sales revenue minus production cost, minus the cost-reduction expenditure, plus the subsidy. The consumers' surplus in firm i's market is denoted S_i and defined as follows:

$$S_i = \int_p^{+\infty} q(z)dz - T_i.$$

Total consumer surplus is the sum of the S_is. We assume that $\gamma c_0 > b > c_0 > 0$. The functions q and R are common knowledge of the firms and the government.

Question 1. We study the first-best optimum. It is defined, in each market i, as the vector (p_i, c_i, T_i) that maximizes S_i subject to the constraint $V_i \geq 0$.

1a. Show that one can equivalently maximize total welfare $W_i = S_i + V_i$ with respect to (p_i, c_i) and eliminate T_i.

1b. Write the first-order conditions for a first-best optimum in market i. Provide an economic interpretation of these conditions.

1c. Compute the first best values (p^*, c^*) as a function of problem parameters.

Question 2. We now study the implementation of the first-best in Nash equilibrium strategies by means of a simple mechanism (*i.e.*, Shleifer's Mechanism). We suppose that the government can only set regulated prices p_i and subsidies T_i for each i. The regulator announces a pricing rule and a rule to compute subsidies, as a function of observed choices $(c_1, \ldots, c_n)$, and observed expenditures $(R(c_1), \ldots, R(c_n))$. Each firm $i \in N$ then chooses a strategy c_i in $[0, c_0]$ noncooperatively.

2a. Show that cost-of-service regulation doesn't work. In other words, show that if the regulator sets $p_i = c_i$ and, say, $T_i = R(c^*)$, the equilibrium is $c_i = c_0$ for all i.

2b. Show that if $n \geq 2$ and the regulator sets

$$p_i = \frac{1}{n-1} \sum_{j \neq i} c_j \qquad \text{and} \qquad T_i = \frac{1}{n-1} \sum_{j \neq i} R(c_j)$$

for all i, then, the first-best is implemented in Nash equilibrium, that is, the unique Nash equilibrium strategy of firm i is $c_i = c^*$ and it follows that $p_i = c^*$.

Question 3. We finally study a possible generalization. Suppose that firms are no longer identical and that there is a finite set of firm types Θ. Each firm i has a type θ_i drawn in this set Θ, which can be interpreted as a vector of *observable* characteristics. The constant marginal cost of firm i is now defined as $c_i + \beta(\theta_i)$, where β is a function with nonnegative values. Cost-reduction expenditures are the same as above. Define the subsets $A(\theta) = \{i \in N | \theta_i = \theta\}$. Assume that there are at least 2 firms in $A(\theta)$ for all θ.

3a. Compute the first-best optimum, denoted $(c^*(\theta), p^*(\theta))$, in this case.

3b. Find a rule for the computation of transfers, denoted $T_i(\theta)$, and a rule for regulated prices, denoted $p_i(\theta)$, that allows implementation of the first-best as a Nash equilibrium among firms, and generalizing the mechanism studied above. (Hint: consider averages within subsets $A(\theta)$.)

ENSAE

Cours de Théorie des Contrats et Incitations

Examen du 29 avril 2015. Ecrit 3h sans documents.

Question de cours (10 points). Expliquez le modèle de réglementation optimale de la firme en information asymétrique et ses principaux résultats. (On pourra s'appuyer sur une version simple du modèle de Laffont et Tirole.)

Exercice (10 points). Modèle de rationnement de crédit de Tirole (2006).

Un entrepreneur souhaite investir dans un projet risqué et s'adresse à un marché des capitaux où il rencontre de nombreux prêteurs en situation de concurrence. Le montant de l'investissement est noté I. L'entrepreneur ne possède qu'une somme A et $A < I$. Il faut donc qu'il emprunte $I - A$.

La réussite du projet est affectée par l'effort inobservable de l'emprunteur. Le projet réussit ou échoue ; l'effort est élevé ou faible. Le projet a une probabilité de succès **P** si l'effort est élevé et une probabilité de succès $p < \mathbf{P}$ si l'effort est faible. On pose $d = \mathbf{P} - p$ et on suppose $p > 0$, $\mathbf{P} < 1$. Le projet rapporte un revenu R en cas de succès, 0 en cas d'échec. Lorsque l'effort de l'emprunteur est faible, il gagne des bénéfices privés de montant $B > 0$. Son utilité est la somme espérée de ses gains et des bénéfices privés. Les bénéfices privés sont nuls en cas d'effort élevé.

L'entrepreneur et les prêteurs potentiels sont neutres vis à vis du risque. Ils ont un facteur d'escompte égal à 1 c'est à dire pas de préférence pour le présent — les revenus futurs ont le même poids que les coûts aujourd'hui. Les prêteurs peuvent se procurer des fonds à taux d'intérêt nul, ils peuvent prêter si le taux d'intérêt moyen sur leur prêt est au moins égal à zéro. La concurrence entre prêteurs va pousser leur intérêt (et leur profit) espéré vers zéro. Enfin, la responsabilité de l'entrepreneur-emprunteur est limitée : il ne peut pas perdre plus que A (sa part du revenu R ne peut pas être négative).

Un contrat de prêt entre l'emprunteur entrepreneur et le prêteur est caractérisé par une partage de R, soit $R = R_1 + R_2$ où R_1 est la part de l'emprunteur et R_2 celle du prêteur. Ils gagnent 0 en cas d'échec du projet.

Question 1. Supposons l'effort élevé.

1a. Quelle est la contrainte de participation (contrainte de profit nul ou contrainte IR) des prêteurs ?

On suppose $\mathbf{P}R - I > 0 > pR - I + B$.

1b. Quelle est la signification économique de cette hypothèse ? Quel est ici le surplus social associé au projet ? Quelle est la valeur actuelle nette (ou valeur espérée nette) du projet ?

Question 2.

2a. Ecrire la contrainte d'incitation à l'effort de l'emprunteur (désignée ci-dessous comme contrainte IC).

2b. Quelle est la part du revenu R la plus élevée qui puisse être promise au prêteur ?

Question 3. On ne prête qu'aux riches.

3a. Montrer que le projet ne peut être financé par le prêteur que si A est plus grand qu'un seuil A_0 que l'on déterminera.

On suppose que $I > \mathbf{P}(R - B/d)$.

3b. Quelle est la signification économique de cette hypothèse ?

3c. Montrer que si $A < A_0$, un projet de valeur espérée nette positive ne peut pas être financé.

3d. Montrer que si $A > A_0$, en offrant au prêteur le plus petit rendement possible, noté R^*_2, l'entrepreneur est incité à choisir un effort élevé.

3e. Calculer l'utilité espérée de l'emprunteur quand $R_2 = R^*_2$, et montrer qu'il reçoit tout le surplus social du projet d'investissement.