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Chapter IV

Local Public Goods and Clubs

The goal of this chapter is to provide an introduction to the theory of local public goods and clubs. The advanced literature on these topics is rather technical, and the basic points of these theories are familiar from elementary textbook presentations. We'll try to do more than the most basic presentation, without developing the full technical apparatus (existence theorems, etc.)

Local public goods are public goods with a limited scope in geographical space. In simple terms, local public goods are goods that you can consume or enjoy only if your residence is located in a certain place where the goods are produced: a village, a city, a jurisdiction, with a given territory. For instance, street lights at night, a park in your neighborhood, etc. In the standard theory of local public goods, the citizen-consumers vote to choose the bundle (the vector) of public goods produced in their jurisdiction. But the preferences of the population, and hence the majority, are endogenous because citizens are free to move to another place: citizens can "vote with their feet". Policy changes can also be induced by citizen mobility.

The theory of local public goods has a pioneer: Charles Tiebout. In a 1956 note, published in the *Journal of Political Economy*, entitled "A Pure Theory of Local Expenditures", he formulated the famous *Tiebout conjecture*. His main idea is that the inefficiency problem in the provision of public goods should be solved in the case of local public goods since the free mobility of citizens forces towns and communities to compete. Competition between jurisdictions should be the driving force yielding an optimal allocation of local public goods. If a community imposes high taxes, has high real estate prices and mediocre public goods, the citizens can vote for change or move to a town with a better supply of local public goods and (or) lower taxes. In fact, in a free mobility regime, the citizens self-select according to their preferences: old people move to sunny and quiet places where schools are bad; young married couples choose communities that support good quality schools, etc... This is because the seniors don't want to pay taxes for the betterment of schools (this is not an invention, see Epple, Romano and Sieg, *J. Pub. Econ.* (2012)). When they move to another community, the citizens must not only pay the local taxes, but also pay for their residence (either renting a home or buying a house or a piece of land). It follows that the free mobility model yields a theory of land (and real estate) prices. House prices reflect (and capitalize) the quality of the local public goods and taxes.

In fact, it is easy to show that the Tiebout conjecture is false in general; it may be true under very strong assumptions (see Truman Bewley, *Econometrica* (1981), Suzanne Scotchmer, *Handbook of Pub. Econ.* (2002)). The externalities imposed by free mobility are the main source of inefficiency. The Tiebout conjecture would be true, for instance, if there were n communities and n types of consumer preferences (or less). Then, each type of consumer would settle in a given community (perfect sorting). Each community would choose the bundle of local public goods that maximizes the residents' utility: the residents being identical in each jurisdiction, they would support the local policies unanimously.

The theory of clubs proposes a different approach to public goods. This approach can be generalized to take the form of an extension of general equilibrium theory (see Ellickson, Grodal, Scotchmer and Zame, in *Econometrica* (1999)). The seminal paper on clubs is due to James Buchanan, "An Economic Theory of Clubs", *Economica* (1965). Again, the idea is that citizens can self-organize and form groups that will provide the local and(or) excludable public goods that they need. The groups will find a way of sharing the production costs among themselves. Therefore, there would be a limited need for public intervention, because there would be no essential market failure. Clubs are an important form of private provision of public goods. The right-wing vision that the government should not interfere has a tangency point here with the left-wing view according to which people should be free to form associations everywhere, to self-organize, and cooperate to produce the public goods that they need. Many clubs can form and compete, hence, a competitive theory of club formation and pricing can be developed. A general equilibrium with clubs can be shown to be efficient (in fact, a club equilibrium is in the *core* of the club economy). We discuss below the difference between club equilibria and the free-mobility equilibria à la Tiebout.

1. Local public goods and free mobility equilibrium

We will study this question with the help of an example due to Suzanne Scotchmer (2002).

An example with 2 jurisdictions.

Assume that there exist 2 jurisdictions (or 2 communities) indexed by $j = 1, 2$; each community has a land area $A_j = 1$. The citizens have the same utility functions but different incomes. The income, denoted y, is the type of an individual here. We assume that $y \in [0, 1]$ and y is distributed uniformly over $[0, 1]$. Formally, a jurisdiction j is a subset J_j of agents included in the set of possible income-types $[0, 1]$. And jurisdictions form a partition of this set, that is, $J_1 \cap J_2 = \emptyset$ and $J_1 \cup J_2 = [0, 1]$. The J_j are assumed *measurable* but we will consider only sets with a simple structure like sub-intervals. Let N_j be the number of inhabitants in j. Formally, N_j is the measure of the set J_j (*i.e.*, simply the length of J_j if this set is an interval, since the distribution of y is assumed uniform).

Utility functions.

There is a single aggregate private good in the economy, that is used as a *numéraire*, with quantities denoted x. There is a single public good in each j, say, a garden with flowers of size z_j. The land consumed by an individual (or the area of the person's apartment, measured in square meters) is denoted s. The utility of an individual consuming (x, s, z) is denoted u, and defined as follows,

$$u(x, s, z) = x + b(z) + f(s),$$

where b and f are strictly increasing, strictly concave and continuously differentiable functions representing the utility for the public good and housing, respectively.

In each jurisdiction, there is an income tax with a flat rate t_j that is used to pay for the cost of the public good. There is a price p_j for a unit of land (or equivalently, p_j can be interpreted as a rent per square meter). Citizen-consumers are subjected to the budget constraint,

$$y = t_j y + x + p_j s.$$

We assume that one unit of private good is needed to produce one unit of the public good in each j (*i.e.*, the marginal cost of production of z_j is equal to 1).

Pareto-Optimal Allocation with two jurisdictions

We first study the structure of a Pareto-optimal allocation in this economy with 2 jurisdictions (a first-best optimum). To this end, we assume that there exists a benevolent planner that has the power of choosing z_j in each j, allocate the land surface $s_j(y)$ to each inhabitant of type y in j, decide who (which types y) will reside in $j = 1$ and $j = 2$ and finally, tax each type of individual freely by choosing the tax $\tau(y)$ of type y. Remark that the planner can freely redistribute the income by means of an income tax $\tau(y)$ and therefore, it does not matter how individuals are distributed among the two jurisdictions, provided that the house size s_j and public good production z_j is the same in each j. The only important thing for the allocation of housing is the total number $N_j = \int_{J_j} dy$ of residents in each j, since if N_1 is larger than N_2, jurisdiction 1 will be more congested than jurisdiction 2 and the land allocation of each inhabitant of 1 will be smaller. It follows that we can consider optimal allocations with 2 jurisdictions of the form, $J_1 = [0, \bar{y}]$, $J_2 = (\bar{y}, 1]$ where $\bar{y}$ is a cutoff type, to be determined. We also assume that the benevolent planner is utilitarian, willing to maximize $\int_0^{\bar{y}} U(y,1)dy + \int_{\bar{y}}^1 U(y,2)dy$ where $U(y,j)$ is the final utility of type y in jurisdiction j. The private good consumption of each type is $x(y) = y - \tau(y)$ and $U(y,j) = y - \tau(y) + b(z_j) + f(s_j(y))$. As a consequence of these assumptions, the planner wants to maximize

$$\int_0^{\bar{y}} [y - \tau(y) + b(z_1) + f(s_1(y))]\, dy + \int_{\bar{y}}^1 [y - \tau(y) + b(z_2) + f(s_2(y))]\, dy$$

subject to feasibility (resource) constraints,

$$\int_0^{\bar{y}} s_1(y)dy = 1 \quad \text{and} \quad \int_{\bar{y}}^1 s_2(y)dy = 1,$$

(since the total quantity of land is 1 in each j) and,

$$\int_0^1 \tau(y)dy = z_1 + z_2,$$

since the planner can use national resources to fund local public goods (and therefore redistribute between jurisdictions). It is easy to see that the planner should maximize $\int_0^{\bar{y}} f(s_1(y))dy$ subject to $\int_0^{\bar{y}} s_1(y)dy = 1$ and maximize $\int_{\bar{y}}^1 f(s_2(y))dy$ subject to $\int_{\bar{y}}^1 s_2(y)dy = 1$ for fixed $\bar{y}$. Since f is strictly concave, the optimal allocation of land is egalitarian in each

j, so that $s_j(y) = s_j^*$ for all j. It follows that we have,

$$s_1^* = \frac{1}{N_1} = \frac{1}{\bar{y}} \quad \text{and} \quad s_1^* = \frac{1}{N_2} = \frac{1}{1-\bar{y}}.$$

Note that we can eliminate $\int_0^1 y dy = 1/2$, a constant, from the maximization problem. Substituting the values of s_j and the resource constraints in the social objective function above yields the following problem.

$$\max_{(\bar{y}, z_1, z_2)} \int_0^{\bar{y}} \left(b(z_1) + f\left(\frac{1}{N_1}\right)\right) dy + \int_{\bar{y}}^1 \left(b(z_2) + f\left(\frac{1}{N_2}\right)\right) dy - z_1 - z_2.$$

We can also easily compute the integrals and the problem becomes

$$\max_{(\bar{y}, z_1, z_2)} \bar{y} \left(b(z_1) + f\left(\frac{1}{\bar{y}}\right)\right) + (1-\bar{y}) \left(b(z_2) + f\left(\frac{1}{1-\bar{y}}\right)\right) - z_1 - z_2.$$

Maximization with respect to z_j yields Samuelson's equation in each jurisdiction j,

$$\bar{y} b'(z_1^*) = 1 \quad \text{and} \quad (1-\bar{y}) b'(z_2^*) = 1, \tag{1}$$

or,

$$b'(z_1^*) = \frac{1}{N_1} \quad \text{and} \quad b'(z_2^*) = \frac{1}{N_2}.$$

To obtain the optimal value of the cutoff $\overline{y}$, we maximize the above objective with respect to $\bar{y}$. Recall that $N_1 = \bar{y}$ and $N_2 = 1 - \bar{y}$. It follows that we can maximize the objective with respect to N_1 with $N_2 = 1 - N_1$. Taking the derivative with respect to N_1, yields the first-order necessary condition for optimality,

$$b(z_1^*) + f\left(\frac{1}{N_1^*}\right) - \left[b(z_2^*) + f\left(\frac{1}{N_2^*}\right)\right] = f'\left(\frac{1}{N_1^*}\right)\frac{1}{N_1^*} - f'\left(\frac{1}{N_2^*}\right)\frac{1}{N_2^*}. \tag{2}$$

Interpretation. The direct benefits of the person of type $\overline{y}$ who moves from jurisdiction 1 to jurisdiction 2 must be just equal to the spatial congestion effects (reduction in $s_2 = 1/N_2$) generated by the move. Type $\bar{y}$ moving from 1 to 2 frees some space in $j = 1$ and squeezes other residents in $j = 2$.

More precisely assume that a small group of consumers moves to $j = 2$; So jurisdiction 1 looses a small mass $dN_1 < 0$. We have $dN_1 = -dN_2$. The change in the size of housing consumption in $j = 2$ can be written, $ds_2 = (-1/N_2^2) dN_2 < 0$. This implies

$dU_2 = -f'\left(1/N_2\right)\left(1/N_2^2\right)dN_2$, but there are N_2 individuals in $j = 2$, so that the total utility loss in jurisdiction 2 is

$$N_2 dU_2 = -f'\left(\frac{1}{N_2}\right)\frac{1}{N_2}dN_2.$$

The same type of reasoning shows that the utility gain in $j = 1$, due to the availability of more space for the remaining residents, is given by

$$N_1 dU_1 = -f'\left(\frac{1}{N_1}\right)\frac{1}{N_1}dN_1.$$

This shows that the terms $f'(1/N)(1/N)$ measure an externality due to congestion.

Define function L as follows.

$$L(N_j) = f\left(\frac{1}{N_j}\right) - f'\left(\frac{1}{N_j}\right)\frac{1}{N_j}$$

Remark that this function is decreasing since $L'(N) = (1/N)^3 f''(1/N) < 0$. Equation (2) can be rewritten

$$b(z_1) - b(z_2) = L(N_2) - L(N_1). \tag{3}$$

We can find the optimum by solving (1) and (2) simultaneously: take $z = z_1^* = z_2^*$ and $1/2 = N_1^* = N_2^*$. We have a symmetric optimum with $\bar{y} = 1/2$ and $b'(z^*) = 2$. Note that $z_1 = z_2$ implies $N_1 = N_2$ and conversely. Given our assumptions on b and f (and $L' < 0$), if a solution of (1) and (2) is such that $z_1 > z_2$ then $N_1 > N_2$ (by Eq. (3)). This would typically not be a solution. We can now study the free-mobility equilibrium in this economy and compare the equilibrium allocation with the optimal allocation.

Free-Mobility Equilibrium with 2 Jurisdictions and Majority Voting

To define a free mobility equilibrium, we need two elements: a land market (or housing market) and a rule to choose the local public good production z_j in each j.

Demand for Housing: We assume that each resident y of j must rent a piece of housing area $s_j(y)$ at rental price p_j on a competitive market for housing. This could also be a competitive market for land. It is now easy to determine the demand for housing of a resident of j. Each individual in j maximizes u subject to the budget constraint. This is equivalent to maximizing $f(s) - p_j s$ for all y. The demand for housing of a given citizen will be independent of income because of quasi-linearity (this is a simplification of course). The first-order condition determining s_j is simply $f'(s_j) = p_j$. There is one unit of land in j, so that, in equilibrium, we must have $N_j s_j = 1$. It follows that equilibrium prices are directly related to the size of the population,

$$f'(1/N_j) = p_j \tag{4}$$

for all j.

Majority Voting (Application of the Median Voter Theorem): The public decision in each j is a rate of income tax t_j. This tax rate determines the amount of public good z_j produced. Since a unit of money produces a unit of public good, we have $z_j = Y_j t_j$, where $Y_j = \int_{J_j} y dy$ is the total income in j. The indirect utility of a citizen with income y in j is $V_j = (1 - t_j)y + f(s_j) - p_j s_j + b(z_j)$, using Eq. (4) and substituting s and z with equilibrium values, we have

$$V_j(y, t_j) = y(1 - t_j) + b(t_j Y_j) + L(N_j).$$

To apply the Median Voter Theorem, we can now check that the citizen's preferences over t_j are single-peaked. Dropping the index j to lighten notation, we can define the *ideal tax* of citizen y, as the solution $t(y)$ of the following program,

$$\max_t \{(1 - t)y + L(N) + b(tY)\}$$

The first-order derivative of the indirect utility with respect to t is

$$\frac{\partial V}{\partial t} = -y + b'(tY)Y,$$

the second derivative is

$$\frac{\partial^2 V}{\partial t^2} = b''(tY)Y^2 < 0.$$

We conclude that V_j is strictly concave, and therefore single-peaked. The peak is reached for y if $t(y)$ solves the first-order condition $y = b'(tY)Y$, the solution of which is typically unique since b is concave. We can rewrite this as $y/Y_j = b'(t_j(y)Y_j)$ in each j or

$$\frac{y}{\hat{y}_j} = N_j b'(t_j(y)Y_j) \tag{5}$$

where $\hat{y}_j = Y_j/N_j$ is the mean income in j. It is easy to check that since $b'' < 0$, we have $t_j'(y) < 0$. The ideal tax is a decreasing function of income y in each j.

Equilibrium with Income Stratification: In this type of model, the free-mobility equilibrium may not exist or there may be several equilibria. It happens that in this example we have an equilibrium with two jurisdictions that is socially stratified: jurisdiction 1 is inhabited by citizens with incomes smaller than a certain cutoff $\tilde{y}$ (to be determined) and the rest of the citizens is in jurisdiction 2. We now define and compute a free-mobility equilibrium with $J_1 = [0, \tilde{y}]$ and $J_2 = [\tilde{y}, 1]$. Given that V_j is single-peaked, and given that the set of taxes is one-dimensional (*i.e.*, $0 \leq t_j \leq 1$), we can apply the Median Voter Theorem to the choice of t_j in each j. The ideal tax of the median voter is $t_j^m = t_j(y_j^m)$, where y_j^m is the median income in j. This tax rate t_j^m is a Condorcet winner (*i.e.*, it cannot be defeated by a majority supporting an alternative choice t_j'). The median income in $j = 1$ is equal to the mean income in J_j since the distribution is symmetric (in fact, it is assumed uniform in this simple case). We have $y_j^m = \hat{y}_j$, and we have,

$$y_1^m = \frac{\tilde{y}}{2} \qquad \text{and} \qquad y_2^m = \frac{1 + \tilde{y}}{2}.$$

Note that

$$Y_1 = \int_0^{\tilde{y}} y dy = (1/2)\tilde{y}^2 \qquad \text{and} \qquad Y_2 = \int_{\tilde{y}}^1 y dy = (1/2)(1 - \tilde{y}^2).$$

Using Eq. (5) with $y = y_j^m$, given that $y_j^m = \hat{y}_j$, we obtain that t_1^m and t_2^m must solve,

$$1 = N_1 b' \left(\frac{t_1^m \tilde{y}^2}{2} \right) \qquad \text{and} \qquad 1 = N_2 b' \left(\frac{t_2^m (1 - \tilde{y}^2)}{2} \right).$$

Remark that Samuelson's equation precisely requires $N_2 b'(z_2) = 1$ and $N_1 b'(z_1) = 1$. Therefore, majority voting yields optimal productions *conditional on* $\tilde{y}$. Clearly, $t_1^m = 2z_1/\tilde{y}^2$, and $t_2^m = 2z_2/(1 - \tilde{y}^2)$ solve the Samuelson equations, if (z_1, z_2) solves them. This result is due to the fact that the distribution of income is uniform: it is not a general property.

Definition of a free mobility equilibrium (or Tiebout equilibrium):
A free-mobility equilibrium is an array $(s_1, s_2, p_1, p_2, z_1, z_2, t_1, t_2, \tilde{y})$ and two jurisdictions $J_1 = [0, \tilde{y}]$ and $J_2 = (\tilde{y}, 1]$ with total populations $N_1 = \tilde{y}$ and $N_2 = 1 - N_1$, such that,

(i) $f'(s_j) = p_j$ and $N_j s_j = 1$ for all j (demand and supply of housing are equal in each jurisdiction),

(ii) $t_j = t_j^m$ or t_j solves $y_j^m/\hat{y}_j = N_j b'(t_j^m Y_j)$, with $Y_j = \int_{J_j} y dy$, for all j (the tax rate chosen in each jurisdiction cannot be defeated by majority voting in this jurisdiction),

(iii) $z_j = t_j Y_j$ for all j (no waste in the production of public goods in each jurisdiction),

(iv) $V_j(y, t_j) \geq V_{3-j}(y, t_{3-j})$ for all y in J_j, $j = 1, 2$ (the citizens of a given jurisdiction cannot increase their utility by moving individually to the other jurisdiction).

In other words, in a free-mobility equilibrium: markets clear, the tax chosen by majority voting is the tax rate preferred by the median voter in each jurisdiction and nobody wants to move.

Condition *(iv)* in the above definition implies that at point $\tilde{y}$, the frontier citizen must be indifferent between $j = 1$ and $j = 2$. If not, the frontier citizen, and some of his neighbors in the income distribution, would like to move to another jurisdiction. It follows that in a free-mobility equilibrium, we must have, $V_1(\tilde{y}, t_1^m) = V_2(\tilde{y}, t_2^m)$, or equivalently,

$$(t_2^m - t_1^m)\tilde{y} + b(z_1) - b(z_2) = L(N_2) - L(N_1). \tag{6}$$

If we compare Eq. (6) with the optimality condition Eq. (3) above, it is easy to see that they are both incompatible. Remark that V_j is linear with respect to y for each j. Condition *(iv)* in the definition of an equilibrium implies $V_1(y, t_1^m) - V_2(y, t_2^m) > 0$ for all $y < \tilde{y}$ and $V_1(y, t_1^m) - V_2(y, t_2^m) < 0$ for all $y > \tilde{y}$. This is possible only if

$$t_1 > t_2,$$

if the tax rate is higher in the jurisdiction with the smallest median income (*i.e.*, $j = 1$). It is now easy to check that the optimal allocation cannot be a free mobility equilibrium. If we choose $\tilde{y} = 1/2$ and $z_1^* = z_2^*$, then, Eq.(6) implies $t_1 = t_2$ and we have just shown that this cannot be an equilibrium.

If $N_1 = N_2 = 1/2$, then, in the poor jurisdiction, citizens will choose a tax rate t_1 such that $2 = b'(t_1/8)$ while in the rich jurisdiction, the tax rate will solve $2 = b'(3t_2/8)$ (using Eq. (5), and $Y_1 = 1/8$, $Y_2 = 3/8$). It follows that we would have $3t_2^m = t_1^m$. In the rich jurisdiction, tax rates would be a third of their value in the poor jurisdiction. The rich can collectively pay for their school (or their park) with a smaller proportion of their income. As a result, many residents of $j = 1$ will want to move to $j = 2$ to enjoy lower taxes. This will cause an increase in real estate prices p_2. Higher housing prices in $j = 2$ will in turn act as a brake on changes of location. The optimum $\overline{y} = 1/2$ and $b'(z_1) = b'(z_2) = 2$ is unstable since the marginal resident $\bar{y}$ of jurisdiction 1 wants to move to jurisdiction 2 to save on his tax, or, in other terms, to *avoid the subsidy that he implicitly makes to lower income residents.* In equilibrium, land prices capitalize the differences in public services and in tax rates. In equilibrium, the rich jurisdiction will have more residents, smaller taxes and higher housing prices than the poor jurisdiction.

This example shows why free-mobility equilibria are not Pareto-optimal, in general.

Solution of an example: We can try to solve for the free-mobility equilibrium with specific functional forms for b and f.

Example :

$$b(z) = 2\sqrt{\frac{z}{\beta}} \quad \text{with} \quad \beta > 2; \qquad \text{and} \qquad f(s) = -\frac{1}{2s}.$$

This is a reasonable model. We have

$$b'(z) = \frac{1}{\sqrt{\beta z}}.$$

The Samuelson equations (*i.e.*, $z_j = (b')^{-1}(1/N_j)$) yield, $z_1 = \tilde{y}^2/\beta$ and $z_2 = (1 - \tilde{y})^2/\beta$. Then, $t_j = z_j/Y_j$ yields

$$t_1^m = \frac{2}{\beta} \qquad \text{and} \qquad t_2^m = \frac{2(1 - \tilde{y})}{\beta(1 + \tilde{y})}.$$

So, we can easily check that $t_2^m < t_1^m < 1$.

Next, we have $L(1/s) = f(s) - sf'(s) = -(1/2s) - s(1/(2s^2)) = -1/s$. It follows that $L(N) = -N$. Given these results, we can write the Tiebout equilibrium condition, Eq. (6),

$$(t_2^m - t_1^m)\bar{y} + 2\sqrt{\frac{z_1}{\beta}} - 2\sqrt{\frac{z_2}{\beta}} = N_1 - N_2,$$

$$\Leftrightarrow \qquad \frac{2}{\beta}\left[\frac{1 - \tilde{y}}{1 + \tilde{y}} - 1\right]\tilde{y} + \frac{2}{\beta}(\tilde{y} - (1 - \tilde{y})) = 2\tilde{y} - 1$$

$$\Leftrightarrow \qquad 0 = 4\tilde{y}^2 + (\beta - 2)(2\tilde{y} - 1)(1 + \tilde{y})$$

$$0 = 2\beta\tilde{y}^2 + (\beta - 2)\tilde{y} - (\beta - 2).$$

We find a quadratic equation. This equation's discriminant is $\Delta = (\beta-2)^2 + 8\beta(\beta-2) > 0$, positive since $\beta > 2$, and the only positive solution is finally,

$$\tilde{y} = \frac{-(\beta-2) + \sqrt{(\beta-2)^2 + 8\beta(\beta-2)}}{4\beta}.$$

It is easy to check that when $\beta \to 2^+$, then $\tilde{y} \to 0$ (jurisdiction 1 becomes very small and very poor when β is close to 2); at the same time, $t_2^m \to 1$ (the taxation rate becomes close to 100% in both jurisdictions, but $t_1^m > t_2^m$ for $\beta > 2$: this means that a reasonable value for β is larger than 8, say).

At the same time, the ratio of public good productions can be written

$$z_1/z_2 = [\tilde{y}/(1-\tilde{y})]^2$$

and it can also become very small, but if we pick $\beta = 3$, for instance, we find $\tilde{y} = 1/3$, $t_2/t_1 = 1/2$ and $z_1/z_2 = 1/4$. It follows that $j = 1$ has tax rates that are twice as large but only one fourth of the public production of $j = 2$. Everybody would prefer to move to $j = 2$ but the house prices are much larger. Recall that $p_j = f'(1/N_j)$, that is, in our example, $p_1 = 1/18$ and $p_2 = 1/6$: house prices are three times larger in $j = 2$. This is how equilibrium is achieved. The smallest incomes are "priced out" of the rich jurisdiction: they can enjoy large houses (twice as large), but in a place with bad schools. The distortion with respect to the symmetric optimum is due to residency choices, since the median voter's favorite tax rate leads to optimal choices conditional on $\tilde{y}$ in each jurisdiction.

There is scope for public (central) intervention. If we could force some citizens with incomes near $\tilde{y}$ to go to $j = 1$, this would be a Pareto-improvement. These people arriving in $j = 1$ would increase the average income there, causing an increase of z_1, and a decrease of t_2, so that the rich *and* the poor would be happier. That's why the Paris "bobos" should receive subsidies to leave Paris and to settle in the eastern or northern suburbs!

2. Theory of Clubs

A club is any group of individuals (members) who form an organization (or association) to produce and share the consumption of a good or service that is within their reach (called a club good). The club members share the production costs of the club good in one way or another. The club good must be a public good of certain kind: a *public good with exclusion* (non-members cannot consume the good). The members can enjoy the good in the same way to a certain extent (the good is public), but there are *congestion* effects: when the size of the club is too large, the quality of the club good is lower. This typically leads to a small efficient size: enough members to share the cost, but no too many to limit congestion. A school, a gym, a swimming pool are typical examples. Another specific property of clubs is the fact that the characteristics of any member can influence the satisfaction (affect the utility) of other members: clubs may generate consumption externalities for their members. Consider for example a dancing club, in which the boys hope to meet girls. The satisfaction of male (and female) members may depend on the proportion of female members.

Club pricing is a key question. Member fees can be used to cover the costs and to influence the characteristics of members to manage the consumption externalities. The club pricing possibilities are also an important difference with local public goods. In the case of local goods, if a community is viewed as a club, the role of membership prices is played by land prices (or housing rents) and taxes, and a jurisdiction has more limited possibilities of discriminating among citizens by means of prices, since tax discrimination is limited by law in general. Jurisdictions are tied to a location, to some specific piece of land, while clubs can be duplicated, like firms, provided that members can afford to pay for the price of inputs.

Clubs happen to be an efficient way of producing many public goods (with exclusion) and may therefore solve many problems without the need for government intervention. If we let people form clubs of all sorts freely, the general equilibrium theory of clubs predicts that a competitive equilibrium with clubs is Pareto efficient (extension of the first welfare theorem) and even a stronger result: the set of competitive equilibria coincides with the *core* of the club economy, under fairly standard assumptions.

The simplest model of a club is easy to describe. Assume that all individuals are identical: they have the same utility U and there is a single private good (with quantities denoted x) and a single club good, the quantities of which are denoted g. Let n be the number of club members. Then we assume that $U = U(x, g, n)$. Each individual has an endowment of private good ω and let $Z(n, g)$ denote the total quantity of resources needed to produce g. If club members share the cost equally, the optimal club size n^* and the optimal production g^* maximize

$$U\left(\omega - \frac{Z(g,n)}{n},\, g,\, n\right)$$

with respect to (g, n). Suppose now that the Euclidean division of the total population N by n^* is k^*, that is, $n^*k^* = N$ and k^* is an integer. Then, everybody becomes a member of a club; all clubs have the same structure (n^*, g^*), and this allocation is in the core. Indeed, if a group of n' individuals decides to form a different club (n', g'), we must have

$$U\left(\omega - \frac{Z(g',n')}{n'},\, g',\, n'\right) \leq U\left(\omega - \frac{Z(g^*,n^*)}{n^*},\, g^*,\, n^*\right),$$

showing that no coalition can increase the utility of its members. If the Euclidean division of N by n^* has a remainder r, *i.e.*, $N = k^*n^* + r$, then, the r unmatched individuals may offer higher membership fees to join a club. The core may be empty. But if N is large and n^* is small (the typical situation), then r is negligible and the club allocation is approximately efficient.

Clubs and the market

To avoid these integer problems, we model club economies as economies with a continuum of consumers (following Robert Aumann (1964)). The set of consumers is described by an atomless distribution λ over some set of types A, endowed (as usual) with a σ-algebra of measurable subsets. Each consumer type $a \in A$ is a point in an infinite set, "like a drop in the ocean". We consider individuals and clubs as price-takers on the market. We will study the notion of a competitive price for club membership, and the general competitive equilibrium with clubs. In the economy, there are private goods and clubs. We assume that there exists L private goods. A bundle of goods is a vector $x \in \mathbb{R}^L_+$. Individuals a and clubs c can trade in private goods on competitive markets. Let $p \in \mathbb{R}^L_+$ denote the vector of prices for private goods, $p = (p_1, \ldots, p_L)$. The value of a bundle is the product $p \cdot x = \Sigma_{h=1}^{L} p_h x_h$. Each agent a has an initial endowment of private goods $\omega_a \in \mathbb{R}_+^L$. The total aggregate endowment is $\bar{\omega} = \int_A \omega_a d\lambda(a) < \infty$ (and of course we assume that $a \mapsto \omega_a$ is integrable). We assume that $\bar{\omega}_h > 0$ for all $h = 1, \ldots, L$.

Club type. A club is described by the number and characteristics of its members and by a particular activity. There is a *finite* set of club activities Γ and an activity is denoted γ. Individuals are endowed with *external characteristics* that belong to a *finite* set Θ. An element $\theta \in \Theta$ is a complete description of characteristics that are by definition observable by other agents (gender, age, height, weight, education, etc.). A club profile is a vector $\pi = (\pi(\theta))_{\theta \in \Theta}$ where $\pi(\theta)$ is the number of members of type θ. Let $|\pi|$ denote the total number of members of a club with profile π. A club type is simply a pair $c = (\pi, \gamma)$. We assume that a club type c belongs to a set $\mathcal{C}$ that is finite and that club size $|\pi|$ is always bounded for $c \in \mathcal{C}$. It follows that clubs are like individuals, there are always small (infinitesimal) relative to society and the market as a whole. A club can have dozens or hundreds (or maybe even thousands) of members but cannot comprise a third or a tenth of the population.

Clubs have a technology and consume inputs of private goods. Let $Z(\pi, \gamma)$ denote the input vector ($Z \in \mathbb{R}_+^L$) representing the cost of club $c = (\pi, \gamma)$. Define the *per capita* input cost $z(\pi, \gamma) = (1/|\pi|)Z(\pi, \gamma)$.

Club pricing. Since the set of clubs is finite, we can index all possible club types by $k = 1, ..., K$, where $K = |\mathcal{C}|$. There is a labeling $k \mapsto c_k = (\pi_k, \gamma_k)$. We can also define a *membership* as a vector (θ, c) where c is a club type and θ is the external characteristic of a particular individual. We will define a price $q(\theta, c)$ for each external individual characteristic in each possible club $c = (\pi, \gamma)$. Let q denote the vector of prices $(q_k(\theta))$ where $q_k(\theta) = q(\theta, \pi_k, \gamma_k)$ for each (θ, k). An agent a has the right to become a member of several clubs. The membership fees facing agent a depend on θ_a, his(her) external characteristics, and we define $q(\theta_a)$ as the vector of all prices $q_k(\theta_a)$ where $\pi_k(\theta_a) > 0$.

Choice of club memberships. Let μ_a describe agent a's choice of club memberships. A membership is not divisible so that μ_a maps the set of club types $\mathcal{C}$ into $\{0, 1\}$, where

0 means "not member", and 1 means "member". It follows that μ_a can be viewed as a function, or as a K-dimensional vector of zeros and ones, $\mu_a \in \{0,1\}^K$. It also follows that the cost of individual a's memberships can be written

$$q(\theta_a) \cdot \mu_a = \Sigma_k q_k(\theta_a)\mu_a(c_k),$$

where $\mu_a(c_k) = 0$ if θ_a is not admitted in club type c_k (*i.e.*, if $\pi_k(\theta_a) = 0$). There is an upper bound on the number of possible clubs chosen by an individual.

Description of the club economy $\mathcal{E}$. To complete the description of the economy we define the utility u_a of individual a. The utility is defined over the set of bundles (x_a, μ_a) of private goods and club memberships choices, $u_a(x_a, \mu_a)$. A state of the economy is a function $a \mapsto (x_a, \mu_a)$ giving the choices of every a. Finally, a club economy $\mathcal{E}$ is a function $a \mapsto (\omega_a, u_a, \theta_a)$.

Assumption 1. The utility functions are continuous and strictly monotone increasing with respect to private goods x_a, and measurable with respect to (a, x_a, μ_a). The function $a \mapsto (\omega_a, u_a, \theta_a)$ is assumed measurable.

Measurability is a purely technical assumption. Strict monotonicity and continuity are restrictions with an economic meaning.

Feasible and consistent states. A state (x, μ) is *feasible for a* if $x_a \geq 0$ and $\pi_k(\theta_a) = 0$ implies $\mu_a(c_k) = 0$ and it is *feasible* if it is feasible for (almost) all a and satisfies in addition the resource equation,

$$\bar{x} + \int_A \sum_k z(c_k)\mu_a(c_k)d\lambda(a) = \bar{\omega}.$$

This means that the total consumption of private goods by individuals, $\bar{x} = \int_A x_a d\lambda(a)$, and the total use of private goods as inputs by clubs (the integral in the above equality) is equal to the total endowment $\bar{\omega}$.

The state (x, μ) is called *consistent* if there are enough clubs of each type to accommodate members as a consequence of individual choices, given that each club must have the right combination of member characteristics. Define the total number of members in club type c as $\bar{\mu} = \int_A \mu_a(c)d\lambda(a)$. In fact we need to define the total number of members with the external characteristic θ in a club of type c. In other words, we need to sum on the sets of individuals a, such that their external characteristic is $\theta_a = \theta$, and define the aggregate membership for each θ as follows,

$$\bar{\mu}(\theta, c) = \int_A \mathbf{1}(\{a|\theta_a = \theta\})\mu_a(c)d\lambda(a),$$

where the indicator function $\mathbf{1}(\{a|\theta_a = \theta\}) = 1$ if $\theta_a = \theta$ and 0 otherwise. Then, the state (x, μ) is by definition *consistent* if and only if there exists a real number $\alpha(c)$ for each club type $c = (\pi, \gamma)$ in $\mathcal{C}$ such that

$$\bar{\mu}(\theta, c) = \alpha(c)\pi(\theta).$$

In this expression, $\alpha(c)$ can be interpreted as the "number" of clubs of type c. It follows that there are $\alpha(c)\pi(\theta)$ individuals with characteristic θ in clubs of type $c = (\pi, \gamma)$. This number must be just equal to the total "number" of agents with this characteristic θ that chose a club of type c, that is, $\bar{\mu}(\theta, c)$. Intuitively, $\alpha(c)$ is in fact the number of clubs of type c divided by the total population, and $\bar{\mu}(\theta, c)$ is the frequency of individuals with characteristic θ and choosing c in the population).

Definition of a Competitive Equilibrium with Clubs.

We can now define a general equilibrium with clubs. A competitive equilibrium is a *feasible* and *consistent* state (x, μ) and a price system (p, q) with $p > 0$, such that, for (almost) all a,

(i), (x_a, μ_a) is feasible for a and satisfies a's budget constraint, that is

$$p \cdot x_a + q(\theta_a) \cdot \mu_a = p \cdot \omega_a;$$

(ii), each individual a maximizes utility subject to the budget constraint, that is, if (x'_a, μ'_a) is feasible for a and $u_a(x'_a, \mu'_a) > u_a(x_a, \mu_a)$, then $p \cdot x'_a + q(\theta_a) \cdot \mu'_a > p \cdot \omega_a$;

(iii), the profit of each club is zero: for every $c \in \mathcal{C}$, we have $p \cdot Z(c) = \Sigma_\theta \pi(\theta) q(\theta, c)$.

The zero-profit condition is a competitive equilibrium condition in an economy with free entry in the club activities. Clubs are created according to needs in equilibrium and these clubs compete with each other for members and for resources (inputs). Feasibility and consistency ensure that we are in a general equilibrium state: demand equals supply for every private good (feasibility) and for every club (consistency). An important aspect of club pricing is that membership fees q depend on external characteristics θ (price discrimination) and that prices q_k are not required to be positive. In fact we can have zero or negative prices. A good example of this is when the entry into night clubs is free for (single) women, or when good students are admitted in a PhD program with a grant (scholarship).

First Theorem of Welfare Economics: Efficiency of Competitive Equilibria with Clubs.

We are now ready to study equilibria. Before the statement of the important results, we need a few assumptions. In particular, we will make sure that all individuals can survive in equilibrium with the help of their endowment.

Assumption 2. (*Desirability of endowments*). For every agent a, we have $u_a(\omega_a, 0) > u_a(0, \mu_a)$ for any feasible μ_a.

The assumption means that each agent would prefer to remain isolated (no club memberships) and consume his(her) endowment rather than to belong to any set of club without any private goods.

In addition to this we need an *irreducibility* assumption. We say that a state (x, μ) is *club-linked* if, whenever for almost all a, $x_{ah} = 0$ for some private goods $h \in H \subset \{1, \ldots, N\}$, then, for almost all a, there exists a number r such that $u_a(\omega_a + r\delta_j, 0) > u_a(x_a, \mu_a)$ where δ_j is a bundle with zeros everywhere except a one at entry j, with j not in H. This assumption says that if the entire endowment of some private goods is used as an input for clubs, then there exists another good j, and a sufficiently large consumption of that good, such that $(\omega_a + r\delta_j, 0)$, a bundle with no clubs, is preferred to (x_a, μ_a). The economy $\mathcal{E}$ is called *irreducible* if every feasible state is *club-linked*. We assume,

Assumption 3. (*Irreducible economy*). The economy $\mathcal{E}$ is irreducible.

We can now state an extension of the First Theorem of Welfare Economics.

Theorem 1. (*Ellickson, Grodal, Scotchmer, Zame (1999)*). Assume that endowments are desirable. Then, every competitive equilibrium with clubs is Pareto-optimal.

The proof of this result is close, in essence, to the standard proof of the First Theorem (for such a standard proof, see the microeconomics textbooks of A. Mas-Colell, M. Whinston, and J. Green, or D. Kreps), except that we are working with a continuum economy here (this technology is presented in the book of Werner Hildenbrand, *Core and Equilibria of a Large Economy* (1974)). We conclude that a competitive equilibrium with clubs is efficient: clubs produce an efficient quantity of public goods and membership fees (club prices) internalize all externalities. This is a nice result, but recall that there are no *pure* public goods in this economy: clubs can solve all public production problems. In addition, we can prove that a club equilibrium exists in $\mathcal{E}$ under Assumptions 1-3 (see Ellickson *et al.* (1999)).

The Core of a Club Economy. In fact we can prove a stronger result: the set of equilibria coincide with the core. Recall the definition. An allocation (a feasible and consistent state (x, μ) here) is in the *core* if a coalition (measurable subset) of agents $B \subseteq A$ cannot improve the utility of its members, using the resources of the coalition, *i.e.*, $\int_B \omega_a d\lambda(a)$, only. In other words, no coalition B can profitably block the allocation by deciding to form an isolated sub-economy with clubs. We know that the competitive equilibria of an ordinary private property economy are in the core. The result can be extended to club economies.

Theorem 2. (*Ellickson, Grodal, Scotchmer, Zame (1999)*). Assume that endowments are uniformly bounded above and desirable and that $\mathcal{E}$ is irreducible. Then, the core of the club economy coincides with the set of competitive equilibrium states.

The proof of this result is technical, see Ellickson *et al.* (1999). It is an extension of the results proved in Hildenbrand (1974).

Some illustrative examples

The study of examples will help understanding the general theory.

Example 1: Swimming Pools and Crowding

Consider a simple economy with a single private good and a single type of club. There is a continuum of consumers indexed by y, where $y \in [0, 10]$. Index y is in fact the endowment of consumer y, that is, $\omega_y = y$. It follows that there are rich ($y = 10$) and poor ($y = 0$) consumers. Consumers have the option of constructing a swimming pool alone, or in a club (or no swimming pool). The cost of a pool is 6 units of private good. The utility of an individual consuming x units of private good with no pool is $u(x, 0) = x$. The utility of a consumer consuming x and a pool with n users is $u(x, n) = 4x/n$. No one belongs to more than one club and the price of the private good is equal to 1.

The pool cost will be shared equally because the size n is the only thing that matters. The price of club of size n is therefore $q_n = 6/n$. The no pool option has utility $u(y, 0) = y$ for type y and sharing a pool with n members has utility $u(y - q_n, n) = (4/n)(y - 6/n)$.

We will show that the equilibrium of this economy is characterized by social stratification (or stratification by wealth). Let $u_{(n)}$ denote the utility of a pool with n members. For instance, agent y prefers a pool with 1 member to pool with 2 members if and only if $u_{(1)} \geq u_{(2)}$, or equivalently, if $4(y - 6) \geq (4/2)(y - (6/2))$ or $y \geq 9$.

In general, we have $u_{(n)} \geq u_{(n+m)}$, with $n \geq 1$ and $n + m \geq 1$ iff

$$\frac{4}{n}\left(y - \frac{6}{n}\right) \geq \frac{4}{n+m}\left(y - \frac{6}{n+m}\right) \Leftrightarrow y\left(\frac{1}{n} - \frac{1}{n+m}\right) \geq 6\left(\frac{1}{n^2} - \frac{1}{(n+m)^2},\right)$$

$$\Leftrightarrow y \geq 6\left(\frac{1}{n} + \frac{1}{(n+m)}\right).$$

The right-hand side of the above inequality is a decreasing function of m and n. Club size n is optimal for consumer y if and only if,

$$\begin{aligned} &u_{(n)} \geq u_{(n+m)} \qquad n, m \geq 1 \\ &u_{(n)} \geq u_{(n-p)} \qquad n \geq 1, \quad p \leq n-1 \\ &u_{(n)} \geq u_{(0)} = y, \end{aligned}$$

that is, iff

$$6\left(\frac{1}{n-1} + \frac{1}{n}\right) \geq y \geq 6\left(\frac{1}{n} + \frac{1}{n+1}\right),$$

and

$$\frac{4}{n}\left(y - \frac{6}{n}\right) \geq y$$

From the above inequalities, with $n = 1$ we obtain $y \geq 9$ and $y \geq 8$, so that $y \geq 9 \Rightarrow u_{(1)} \geq u_{(2)}$ and $u_{(1)} \geq u_{(0)}$. For $n = 2$ we obtain $9 \geq y \geq 5$ and $y \geq 6$, so that

$y \in [6,9)$ implies $u_{(2)} \geq u_{(1)}$ and $u_{(2)} \geq u_{(0)}$. Finally, to check that $u_{(0)} > u_{(n)}$ for $y < 6$, we need to check that $y > (4/n)(y - (6/n))$, or equivalently, that $(4-n)y < 24/n$ when $y < 6$. It immediately follows that $n \geq 4$ implies $u_{(0)} > u_{(n)}$ for all y, and that $u_{(0)} \geq u_{(n)}$ for $n = 1, 2, 3$ if $y < 6$. We conclude that in equilibrium, the rich (that is, agents with $y \in [9, 10)$) will have their own swimming pool ($n = 1$). The middle range agents (with $y \in [6, 9)$)) will share a swimming pool ($n = 2$) and the poor will have no pool ($n = 0$ when $y \in [0, 6)$). Note that the result is efficient, but this does not mean that the result is fair: the club economy does not solve the inequality or redistribution problems miraculously. So, if you dislike the result, you should redistribute the income: you may then reach a situation in which everybody shares a pool with somebody else. This is possible if there are enough resources. If you equalize income by redistribution and if the mean income happens to be $\bar{y} = 5$, nobody will build a pool; the pools will disappear. If $9 > \bar{y} > 6$ we will have $n = 2$ for everybody. The example is drawn from the work of Ellickson *et al.* (1999).

Example 2 : Housing and Segregation

Example 2 is also drawn from Ellickson *et al.* (1999). Assume that there is a continuum of consumers, uniformly distributed on $[0, 1]$. Consumers a in $[0, .3]$ are *blue* (denoted B) and consumers a in $(.3, 1]$ are *green* (denoted G). There is a single private good. Each consumer has a private good endowment $\omega_a = 2$. The clubs are houses here. All agents have the option of constructing and using a duplex (a house with two apartments). The cost of a duplex is 2 units of private good. The preferences are defined as follows. No housing and x units of the private good yields utility $u_B(x; 0) = x = u_G(x; 0)$ where u_B (resp. u_G) is the utility of blue (resp. green) agents.

There are consumption externalities in the clubs. The utilities derived from a duplex depend on the characteristics of the occupant of the other half of the duplex. Using obvious notations, we assume,

$$u_B(x; BB) = 4x, \qquad u_B(x; BG) = 6x$$

$$u_G(x; GG) = 6x, \qquad u_G(x; BG) = 4x$$

it follows that the Bs like to share a house with the Gs, but the Gs prefer segregated housing.

Price System. There will be prices for the various kinds of club: $q(BB)$ for a duplex with two Bs; $q(GG)$ for a duplex with two Gs. In a duplex of the BG category, with two different types of consumers, the price may depend on the characteristic of the club member: $q_B(BG)$ is the price paid by a B consumer for membership in a BG club, and similarly, $q_G(BG)$ is the price paid by a G consumer for a BG duplex.

Computation of Equilibrium. The housing prices (club membership fees) for each type of duplex must sum to 2 units to cover the building cost. It follows that we must have in equilibrium $q(BB) = 1 = q(GG)$ and $q_B(BG) + q_G(BG) = 2$.

B and G consumers can obtain utility $u_G = u_B = 2$ by choosing not to build a duplex (they stay in a hotel, say). A consumer of type B obtains utility $u_B(2-1, BB) = 4$ in a segregated BB duplex and utility $u_B(2-q_B(BG); BG) = 12-6q_B(BG)$ in an integrated BG duplex.

The G consumer obtains $u_G(2-1, GG) = 6$ with segregation and $u_G(2-q_G(BG); BG) = 8-4q_G(BG))$ with integration.

Now, if integrated housing exists in equilibrium, then, the utility derived by Bs and Gs in integrated housing cannot be less than in integrated housing, implying,

$$12 - 6q_B(BG) \geq 4 \qquad \text{and} \qquad 8 - 4q_G(BG) \geq 6.$$

This in turn implies $q_B(BG) \leq 8/6 = 4/3$ and $q_G(BG) \leq 2/4 = 1/2$. It follows that

$$q_B(BG) + q_G(BG) \leq \frac{4}{3} + \frac{1}{2} = \frac{11}{6} < 2.$$

This contradicts the requirement $q_B(BG) + q_G(BG) = 2$, to cover the cost of the duplex.

We conclude that the only equilibrium has all consumers living in segregated housing, with prices $q(BB) = q(GG) = 1$. The prices for integrated housing, with zero demand, are constrained by $q_B(BG) \geq 4/3$, $q_G(BG) \geq 1/2$, $q_B(BG) + q_G(BG) = 2$. Segregation is an equilibrium phenomenon; it is driven by the agents' preferences here.

Example 2, continued. Segregation is not necessarily an equilibrium. Suppose for instance that $u_B(x; BG) = 10x$ and everything else is the same as before.

If no integrated housing exists in equilibrium then, we must have, $10(2 - q_B(BG)) \leq 4$

for the B consumers and $4(2 - q_B(BG)) \leq 6$ for the G consumers. This is equivalent to $q_B(BG) \geq 16/10 = 8/5$ and $q_G(BG) \geq 1/2$. Therefore,

$$q_B(BG) + q_G(BG) \geq \frac{8}{5} + \frac{1}{2} = \frac{21}{10} > 2.$$

This result is inconsistent with the equilibrium property that total membership fees are equal to cost. This implies that in equilibrium, some blue and some green consumers live in integrated housing. For consistency, since the proportion of green consumers is higher, some green consumers must live in segregated housing. This in turn implies that in equilibrium, green consumers must be indifferent between segregation and integration. Hence, we must have, $8 - 4q_G(BG) = 6$, implying $q_G(BG) = 1/2$, and finally, $q_B(BG) = 2 - (1/2) = 3/2$. The surprising result is that B consumers must pay more than (twice as much as) G consumers to live in integrated housing. The G consumers are "subsidized" by Bs to accept integration.

At these equilibrium prices, $q_B(BG) = 3/2$, $q_G(BG) = 1/2$ and $q_B(BB) = q_G(GG) = 1$, the B consumer still prefers integrated housing to segregated housing since $10(2-3/2) \geq 4$, that is, $10 \geq 8$ (which is true).

In this example, for consistency, a fraction 3/7th of the green consumers live in integrated housing (recall that the proportion of blue consumers in the population is 30%), and the remainder lives in segregated housing. All the blue consumers live in integrated housing.

Some Exercises

Exercise 1 *Another example: free mobility and externalities (Scotchmer (2002))*

We have an economy with a continuum of consumers characterized by their incomes $y \in [0, 1]$. There is one private good, the quantity of which is denoted x. The land area is denoted s. There are two jurisdictions, indexed $j = 1, 2$, with land area normalized to 1. Each j is characterized by a subset J_j that forms a partition of the set of incomes $[0, 1]$. We assume that the utility of a citizen in j is specified as follows: $u(x, s, \hat{y}_j) = x + ln(s) + \hat{y}_j$, where $\hat{y}_j$ is the mean income in j, that is, $\hat{y}_j = (1/N_j) \int_{J_j} y dy$ and N_j is the mass of citizens in J_j. Agents like to be grouped with high-income people. Assume that the distribution of y is uniform on $[0, 1]$.

Question 1. Compute the symmetric Pareto-optimum with 2 jurisdictions of the form $J_1 = [0, \bar{y}]$ and $J_2 = (\bar{y}, 1]$. Prove that $\bar{y} = 1/2$.

Question 2. Show that $\bar{y} = 1/2$ is not a free-mobility equilibrium. Show that mobility generates a negative externality. Assume that there is a competitive land market with price p_j per unit of area in each j. Compute the free mobility equilibrium with two jurisdictions of the form $J_1 = [0, \tilde{y}]$ and $J_2 = (\tilde{y}, 1]$ and show that $\tilde{y} < 1/2$.

Question 3. Show that another efficient partition is $J_1 = [0, 1/4) \cup [3/4, 1]$, and $J_2 = [1/4, 3/4)$. Is this partition a free mobility equilibrium?

Exercise 2. *A marriage is a club* (Ellickson *et al.* (1999))

Consider a club economy with a continuum of consumers, uniformly distributed on $[0, 1]$. Consumers in $[0, \beta)$ are males; consumers in $[\beta, 1]$ are females, and $0 < \beta < 1$. There are 2 private goods. The endowment of each consumer is $\omega = (10, 10)$. Let the subscript s denote a single and the subscript m denote a married person. The only club type is a marriage with a male and a female, denoted m. The preferences of males (M) and females (F) are specified as follows:

$$u_M(x_1, x_2; s) = x_1; u_F(x_1, x_2; s) = x_2$$

$$u_M(x_1, x_2; m) = u_F(x_1, x_2; m) = \frac{5}{2}\sqrt{x_1 x_2}.$$

Denote q_M and q_F the gender-specific prices for club membership (marriage prices). A marriage is assumed costless: $Z(m) = 0$. Denote $\bar{m}$ the mass of married males.

Question 1. Solve for the general competitive equilibrium of this club economy as a function of β, assuming $\beta < \frac{1}{2}$. We recommend that you choose the following price normalization for the prices of private goods: $p_1 + p_2 = 1$. Show that in equilibrium,

$$q_F^* = 10 - 8\sqrt{\frac{\beta}{1 - \beta}}.$$

Question 2. Show that if $\beta = 1/5$, men are indifferent between marriage and being a single ($\bar{m}$ is undetermined).

Question 3. Show that if $\beta \in (1/5, 1/2)$, then $\bar{m}$ is uniquely determined and all men are married.

Question 4. Show that if $\beta < 1/5$, there are no marriages in equilibrium (find equilibrium marriage prices q_F, q_M such that nobody wants to be married). Provide an interpretation of these results.