ENSAE
University of Paris-Saclay
MASTER in ECONOMICS
Sujet d'examen — Written Examination
PUBLIC ECONOMICS — Course given by Robert Gary-Bobo

Academic year 2016-2017
January 2017; 2 hours. No lecture notes.

**ATTENTION PLEASE!** Choose the two questions **or** choose one of the two questions and the exercise. Answers must be in English or in French.

**Question A**. (10 points)
Discuss the notion of Local Justice. Present simple rules of fair distribution that can be used to share the benefit or the cost of a joint activity. Compare these rules.

**Question B**. (10 points)
What are the definitions of Utilitarianism, Rawlsian Egalitarianism and strict Egalitarianism? Explain the differences between these collective choice doctrines (or principles). Explain the consequences of choosing one of these principles or another in an illustrative example. Are there basic axioms that these principles do satisfy?

**Exercise**. *Crime control; local police* (10 points)
We consider a local public good model with two jurisdictions indexed by $j = 1, 2$. Each jurisdiction has a land endowment with area $A_j = 1$. There is a continuum of citizens with the same income, denoted $y$, but with a different willingness to pay for crime control (local police activity). The individual willingness to pay is denoted $\theta$ and assume that $\theta$ is uniformly distributed on the interval $[0, 2]$. The total size of the population, denoted $N$, is normalized in such a way that $N = 1$. The "number of citizens" in the interval $[0, t]$ is therefore $\int_0^t d\theta/2 = t/2$. In the economy, there are three goods: a single private consumption good (used as *numéraire*), land and a local public good (crime control). Consumers have a utility expressed as $u = \theta z + x + \ln(s)$ where $s$ is the quantity of land (or housing area) used; $x$ is private good consumption and $z$ is the crime control activity level. We assume that crime control in $j$, denoted $z_j$ is chosen in the interval $[0, 1]$. The per resident cost of crime control is equal to 1, so that the total cost of local police in $j$ is $N_j z_j$ if $N_j$ denotes the population of jurisdiction $j$. The land belongs to *absentee landlords* who, by definition, are living elsewhere, but pay a land tax to the jurisdiction in which they own land. The residents of jurisdiction $j$ must pay a rent $p_j$ for each square meter of land used. The total utility of the absentee landlords is just $p_1 A_1 + p_2 A_2 - T_L$ where $T_L$ is the total amount of land taxes. We will compare Pareto optima and free-mobility (*i.e.*, Tiebout) equilibria of the following particular form: there exists a cutoff willingness-to-pay $t$ such that all types $\theta \in J_1 = [0, t]$ reside in jurisdiction 1

and all types $\theta \in J_2 = [t, 2]$ reside in jurisdiction 2. We limit ourselves to this type of simple structure.

**Question 1.** *Pareto optima with two jurisdictions.* Since utilities $u$ are quasi-linear, we focus on Utilitarian Pareto optima in which the utilities of all citizens carry the same weight equal to 1 and the utility of absentee landlords has weight 0. The welfare function is therefore of the form $W = \int_0^2 u(\theta)d\theta/2$, where $u(\theta)$ is the utility of type $\theta$. There is an omnipotent social planner who maximizes $W$, observes $\theta$, can force each type to pay a tax $T(\theta)$ and impose a place of residence to everyone (*i.e.*, chooses $t$). In addition, each citizen $\theta$ will obtain a piece of land of size $s(\theta)$. The planner chooses $(t, s(\theta), T(\theta))$ subject to feasibility constraints $\int_0^t s(\theta)d\theta/2 = A_1$, $\int_t^2 s(\theta)d\theta/2 = A_2$, the budget constraint $N_1 z_1 + N_2 z_2 = \int_0^2 T(\theta)d\theta/2$, and $T_L = 0$. (Functions $T$ and $s$ are assumed piecewise continuous and integrable.)

**1a.** Give an expression for the social objective $W$ as a function of $(t, s(\theta), T(\theta))$.

**1b.** Show that since utilities are quasi-linear, we can eliminate $T$ (and the budget constraint) from the optimization problem.

**1c.** Explain why an optimal allocation divides land equally in each jurisdiction $j$, *i.e.*, $s(\theta) = s_j$ for all $\theta \in J_j$.

**1d.** Taking the above results into account, simplify the expression for $W$ and show that any optimal allocation with $0 < t < 1$ should be such that $z_1 = 0$ and $z_2 = 1$.

**1e.** Taking the above results (1a-1d) into account, show that $W$ can be expressed as a concave function of $t$ only and show that the optimal cutoff is $t = 1$. Give an economic interpretation for these results.

**Question 2**. *Free mobility equilibrium à la Tiebout.* We now show the existence of, and study, the free-mobility equilibrium of the economy of the form $J_1 = [0, \hat{\theta}]$, $J_2 = [\hat{\theta}, 2]$. Rental land markets are now open in each $j$. The absentee landlords pay a land tax $T_L$ to finance the local police activity. We assume that the citizens do not pay local taxes, that is, $T(\theta) = 0$ for all $\theta$: the budget constraint of a citizen is $y = ps + x$. For simplicity, we fix $\hat{z}_1 = 0$ and $\hat{z}_2 = 1$: we just try to decentralize the optimal choice of crime control.

**2a.** Compute the demand function for housing in each $j$. Prove that in land market equilibrium, we must have $p_j = N_j$ in each $j$.

**2b.** Give the expression of the (indirect) utility, denoted $U_j(\theta)$, of citizen $\theta$ in jurisdiction $j$ as a function of $\hat{z}_j$, $p_j$, $\theta$ and $y$.

**2c.** Explain why, in a free mobility equilibrium, we must have $U_1(\theta) \geq U_2(\theta)$ for all $\theta < \hat{\theta}$ and $U_2(\theta) \geq U_1(\theta)$ for all $\theta \geq \hat{\theta}$. Show that there exists an equilibrium with $\hat{z}_1 = 0$ and $\hat{z}_2 = 1$ and show that this implies $\hat{\theta} = \ln(N_2/N_1)$.

**2d.** Prove that $\hat{\theta} < 1$. Is the equilibrium socially efficient? Provide an economic interpretation for this result.

ENSAE-SCIENCES-PO-ECOLE POLYTECHNIQUE
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Academic year 2015-2016
22 January 2016; 2 hours.

**ATTENTION PLEASE!** Answer two questions **or** choose one of the two questions and the exercise. Answers must be in English or French.

**Question A**. (10 points)
The foundations of Utilitarianism.

**Question B**. (10 points)
The notion of free mobility equilibrium and Tiebout's conjecture about local public goods.

**Exercise**. (10 points)
*Problem of fair priorities*: There are two agents, Ann and Bob. Each of them has a problem that must be solved (or a file that must be treated, or a case that must be judged) by some public agent (resp. by a civil servant or a judge). We want to determine which of the two files must be treated first. Ann's file has a specific treatment time $t_A = 1$. Bob's file has a specific treatment time $t_B = 4$. There are two possible rankings (or priorities), $AB$ (Anna treated first) or $BA$ (Bob treated first). The utility of each agent is minus the waiting time, in other words, both agents want to minimize their waiting time. For instance, if $AB$ is chosen, Bob's waiting time is $t_A + t_B$.

*Question 1.* Assume that the civil servant can choose pure strategies only: $\sigma \in \{AB, BA\}$ (the choice is not random).

1a. Write the matrix of utilities $(u_A(\sigma), u_B(\sigma))$ for these choices (in the form of a simple table).

1b. Determine the utilitarian solution.

1c. Determine the Rawlsian egalitarian solution(s).

1d. The Leximin solution is defined, in this case, as the Rawlsian solution with the highest utility for the most advantaged agent (if there is more than one Rawlsian solution). Determine the Leximin solution.

*Question 2. Random choice (Mixed strategies).* We now consider the possibility of choosing $\sigma \in \{AB, BA\}$ at random. Let $p$ denote the probability of $AB$ and $q = 1 - p$ be the probability of choosing $BA$. We assume that the public authority can choose $p \in [0, 1]$. Ann and Bob now want to minimize their expected waiting time.

2a. Draw the feasible utilities in the plane $(u_A, u_B)$. Determine and draw the set of Pareto-efficient utility vectors $(u_A, u_B)$.

2b. Compute the utilitarian solution (*i.e.*, the utilitarian value of $p$).

2c. Show that $t_A u_A(\sigma) + t_B u_B(\sigma) =$ constant for all $\sigma \in \{AB, BA\}$. From this result, derive the fact that $Eu_A = Eu_B$ (*i.e.*, equal expected utilities) implies

$$p = \frac{t_A}{t_A + t_B}.$$

2d. Determine the Rawlsian and the Leximin solutions when random choice of priority is permitted.

*Question 3.* The Nash bargaining solution depends on a point in utility space called the status quo. Let $u_0 = (u_{0A}, u_{0B})$ denote the status quo and assume that $u_0 = (-6, -6)$. The Nash solution is the value of $p$, denoted $p^N$ that maximizes the product (the rectangle area) $(Eu_A - u_{0A})(Eu_B - u_{0B})$. Compute $p^N$ and compare it to the Rawlsian and utilitarian solutions.

ENSAE-SCIENCES-PO-ECOLE POLYTECHNIQUE
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Academic year 2014-2015
examination 22 January 2015; 3 hours.

**ATTENTION PLEASE! Answer two questions or choose one of the two questions and the exercise** Questions A and B and the exercise are not divisible: for instance, you cannot choose the first half of question B and the beginning of question A plus the exercise. Answers must be in English or French.

**Question A**. (10 points)
The Clarke-Groves Mechanism.

**Question B**. (10 points)
Can we use Mechanism Design to solve the problems posed by externalities? Distinguish the case in which the agents are perfectly informed (but the social planner doesn't observe key personal characteristics) from the case in which each agent has private information about his(her) personal characteristics.

**Exercise**. (10 points)
*Utilitarian Allocation of Education*: There are $n$ agents indexed by $i = 1, ..., n$, and two private goods: private consumption and education. The utility $u_i$ of agent $i$ only depends on private consumption, denoted $x_i$. We assume that,

$$u_i(x_i) = \frac{x_i^{1-\alpha}}{1-\alpha}$$

where $\alpha \geq 0$. Resources, expressed in terms of the private consumption good, can be used to provide a quantity of education, denoted $e_i$, to each agent $i$. There are constant returns in the production of education and one unit of private good produces one unit of education for any agent. Education cannot be negative, *i.e.*, $e_i \geq 0$. Each agent $i$ is endowed with a talent $\theta_i$, where $\theta_i$ is a positive real number. (Talent can also be interpreted as the opposite of handicap, $-\theta_i$.) Education and talent combined determine an individual productivity denoted $w_i$. We assume that $w_i = f(\theta_i, e_i)$, where $f$ is a "production function". Total resources (or total income) in the economy are $\Sigma_i w_i = w$. The total cost of education is $e = \Sigma_i e_i$. We consider two cases:

*Case A*: $f(\theta_i, e_i) = \theta_i e_i^{\beta}$ ; *Case B*: $f(\theta_i, e_i) = (\theta_i + e_i)^{\beta}$; where $0 < \beta < 1$.

We assume that the social planner is utilitarian and we study the optimal utilitarian allocation of income for private consumption and of education resources to individuals. We consider *Case A* first.

*Question 1.*

1a. Define the social objective of the utilitarian planner. Write the resource constraint in the economy.

1b. Write the first order conditions for an optimum in this economy. (Use Lagrange multipliers to take constraints into account). Are these conditions necessary and sufficient for an optimum?

*Question 2.* In *Case A*,

2a. Show that it is always optimal to allocate at least a small quantity of education, $e_i^* > 0$, to every individual $i$, since $\theta_i > 0$. (Show that $e_i^* = 0$ is never optimal).

2b. Compute the optimal allocation of education $(e_i^*)_{i=1,\ldots,n}$ as a function of talent: we have $e_i^* = e^*(\theta_i)$ for all $i$.

2c. Each agent $i$ produces a surplus $s_i = f(\theta_i, e_i) - e_i$. Show that $s_i^* = f(\theta_i, e_i^*) - e_i^* > 0$ for all $i$.

2d. Are educational resources allocated equally? Give an interpretation of the result.

*Question 3.* Total surplus is defined as $s^* = \Sigma_i s_i^*$. In *Case A*,

3a. Compute the total surplus. Compute the optimal allocation of the private good $(x_i^*)_{i=1,\ldots,n}$.

3b. Is the optimal allocation egalitarian? Provide an interpretation of the result. Show that each agent benefits from the talents of all individuals.

3c. Does the optimal solution depend on parameter $\alpha$? What happens if $\alpha \to +\infty$? Explain the result.

*Question 4.* We now consider *Case B*.

4a. Write the first order conditions for an utilitarian optimum in *Case B*. (Pay attention to sign constraints: education cannot be negative.)

4b. Compute the optimal allocation of education $(e_i^*)_{i=1,\ldots,n}$ as a function of talent.

4c. Some agents will not receive a positive amount of education. Give the condition on $\theta_i$ under which an agent $i$ receives $e_i^* = 0$. Which agents receive education?

*Question 5.* We still consider *Case B*. Assume that talents are such that $e_i^* > 0$ for all $i$.

5a. Compute the total surplus of education $s^*$ and the optimal utilitarian allocation of consumption $(x_i^*)_{i=1,\ldots,n}$.

5b. Provide an interpretation of the result. Compare Case A and Case B.

5c. Provide an interpretation of the production functions in cases A and B. What is the essential difference? (Are education and talent substitutes or complements?).

ENSAE-SCIENCES-PO-ECOLE POLYTECHNIQUE
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Academic year 2013-2014
examination 6 February 2014; 3 hours.

**ATTENTION PLEASE! Answer two questions or choose one of the two questions and the exercise** Questions A and B and the exercise are not divisible: for instance, you cannot choose the first half of question B and the beginning of question A plus the exercise. Answers must be in English or French.

**Question A**. (10 points)
Can the inefficient provision problem posed by public goods be solved in the case of local public goods and clubs?

**Question B**. (10 points)
Can we define an equilibrium level of crime, using the tools of standard economic theory? (a) Show that, on the basis of a rational choice model, it is possible to describe criminal activity as a rational choice. (b) If individual choices themselves depend on the aggregate level of crime, show that the equilibrium, aggregate level of crime can be described as a fixed point. (c) Discuss the implications of such a model for public policies.

**Exercise**. (10 points)
*Time-Sharing Problem*: There are $n$ agents indexed by $i = 1, ..., n$, and a pure public good with $K$ variants indexed by $k = 1, ..., K$. The set of agents is denoted $N = \{1, ..., n\}$. To fix ideas assume that agents work in a common space (*e.g.*, a fitness room) where radio must be turned on one of $K$ available stations. (One of the options may just be turning the radio off; call this option the "off" station). As the agents have different tastes, they ask the manager to share the time fairly between stations. The manager chooses a list of time shares, denoted $x_k$, for each of the radio stations, with $\Sigma_{k=1}^{K} x_k = 1$ and $0 \leq x_k \leq 1$ for all $k$. Each agent $i$ has a utility $u_i$ which depends on $x = (x_1, ..., x_K)$.

Assume first a simple preference pattern: utilities are equal to 1 or 0 for a station that agent $i$ likes or dislikes, respectively. In addition, each agent likes exactly one station and dislikes the other $K - 1$ stations. Let $N_k$ denote the subset of fans of radio $k$. By definition $i$ is a fan of $k$ if and only if $i \in N_k \subset N$ and we assume that $u_i(x) = x_k$ if $i$ belongs to $N_k$. There are $n_k \geq 0$ fans of radio station $k$, and $\Sigma_{k=1}^{K} n_k = n$.

*Question 1.* Show that a utilitarian manager chooses the "tyranny of the majority": the station with the largest support is on all the time, *i.e.*, the utilitarian solution is $x_k = 1$ if $n_k > n_{k'}$ for all $k' \neq k$, and if there are ties, that is, if several radio stations have the same highest number of fans, any mixing between them is optimal. Provide a rigorous statement and a rigorous proof of this result.

*Question 2.* Assume now that the manager is a Rawlsian-egalitarian. Show that the manager will play each station $1/K$th of the time, so that everybody is happy at least $1/K\%$ of the time. Again, give a rigorous statement and proof of this result.

*Question 3.* Suppose now that the manager maximizes the *Nash product*, that is, the social welfare function $W(x) = \Pi_{i=1}^{n} u_i(x)$, where $\Pi_{i=1}^{n} u_i$ is the product of utilities from $i = 1$ to $i = n$. Compute the optimal $x$ in that case. Provide an interpretation of the result (hint: show that the solution can be interpreted as a *random dictatorship*).

*Question 4.* Compare the three solutions described above in questions 1, 2 and 3. Which of these solutions makes sense? Explain your preferences over the three solutions.

*Question 5.* We now consider a variant of the problem in which some agents may like several radio stations. Assume that $K = 5$ and that $n = 5$. The set of stations is $A = \{a, b, c, d, e, \}$. Let the nonnegative real number $u_{ik}$ denote the intensity of preference of $i$ for $k \in A$. The utility of agent $i$ is defined as follows, $u_i(x) = \Sigma_{k \in A} u_{ik} x_k$. Consider the following matrix of intensities $U = (u_{ik})$, where agents are rows and stations are columns,

$$U = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \end{pmatrix}$$

Compute the Rawlsian-egalitarian, the utilitarian and the Nash-product solutions of the time-sharing problem in this example.

*Question 6.* Assume now that each agent likes only one radio, so that $N_k$ is the subset of the fans of $k$ and the $N_k$ form a partition of the set of agents. The intensities $u_{ik}$ are positive real numbers when $i \in N_k$ and $u_{ik} = 0$ if $i \notin N_k$. In this case, show that the Nash product solution, that maximises $\Pi_{i=1}^{n} u_i(x)$, is independent of intensities but that the utilitarian solution is not. Is this property desirable?

ENSAE-SCIENCES-PO-ECOLE POLYTECHNIQUE
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Academic year 2012-2013
examination 7 February 2013; 3 hours.

**ATTENTION PLEASE! Choose and answer two questions only in the following set of three questions: A, B and C**. Questions A , B and C are not divisible: for instance, you cannot choose the first half of question C and the beginning of question A plus question B. Answers must be in English or French.

**Question A**. (10 points)

A1. Is it true that pay regulation in the public sector can lead to a deterioration of the quality of service? Explain why. You'll particularly focus on the case of UK hospitals and discuss the evidence on this question presented in a paper by Carol Propper and John Van Reenen ("Can Pay Regulation Kill?..." *Journal of Political Economy*; 2010; You have the right to use a copy of this paper during the exam).

A2. Explain the economic theory underpinning the results found by the authors.

A3. Decribe the data used by these authors. In particular, explain how they measure hospital quality and "outside wages" (i.e. private sector wages).

A4. Explain the estimation method and the identification strategy used to estimate the impact of pay regulation on hospital performance in Propper and Van Reenen (2010).

A5. Discuss the authors' results and robustness tests. Are these results credible?

**Question B**. (10 points)

Show that O.D. Hart's *Theory of Incomplete Contracts* can be applied to the choice between the public sector (civil servants) and the private sector (outsourcing to private companies) to carry out a public task, and thus to determine the "proper scope of government".

**Question C**. (10 points)

*Problem*: Consider a simple club with a large number $N$ of identical potential members. All potential members have the same income $m$ and the same quasi-linear utility function $U$ representing preferences over combinations of private goods (money), the public good (produced by the club) and quality due to congestion. Let $n$ denote the number of members ($n < N$). Index the $n$ members by $i = 1, ..., n$; denote $h_i$ the individual rate of club-good consumption (rate of use) of member $i$; then, by definition,

$$H = \Sigma_{i=1}^{n} h_i.$$

The utility of a club member is defined as,

$$u(x_i, h_i, H) = x_i + \alpha \ln(h_i) - \beta \ln(H),$$

where $x_i$ is private goods consumption, $\alpha$ and $\beta$ are positive, and $H$ is the total consumption of the $n$ club members. The budget constraint of member $i$ is simply $m = P_i + x_i$, where $P_i$ denotes the total payment made to the club by $i$. The cost function of the club is denoted $C$, and defined as follows,

$$C(H) = f + cH,$$

where $f > 0$ is a fixed cost and $c > 0$ is a unit congestion-cost parameter. We assume that club member $i$ non-cooperatively chooses his(her) club use rate $h_i$ in the set $[0, \overline{h}]$, where $\overline{h}$ represents the maximum feasible rate of use. The club members cooperate to choose the optimal number of members, denoted $n^*$, they also collectively decide the membership fees and user rates but, once these decisions are made, they cannot control the individual rates of use $h_i$. We finally assume that $0 < \alpha - \beta < c\overline{h}$, and that $f/\beta$ is an integer, strictly greater than 1.

C1. (*Optimal club size and rate of use*). Compute the optimal club size $n^*$, and optimal individual rate of use $h^*$, that maximizes any member's utility $U$, while assuming that the total cost $C$ is divided equally among club members. You will neglect the "integer problem" here, and treat $n$ as a real variable while maximizing $U$. The *effective* optimal club size $int[n^*]$ is the largest integer smaller than or equal to $n^*$. Maximizing $U$ in this fashion is optimal, provided that the $N$ identical agents in the economy can be allocated to $K$ identical clubs of size $int[n^*]$ exactly, *i.e.*, $int[n^*]K = N$. Again, we will neglect this division problem and assume in addition that the optimal size $n^*$ happens to be an integer (*i.e.*, that we're lucky enough to find $int[n^*] = n^*$).

C2. The club's ruling authority has chosen $n = n^*$ and each member pays a fixed contribution $F$ to cover the cost $C(n^* h^*)$. Each member is also supposed to consume exactly $h^*$ and no more.

C2a. (*Noncooperative best response)*. Show that $h^*$ is not a *Nash equilibrium* of the game in which each of the $n^*$ members chooses $h$ noncooperatively, *i.e.*, show that if all members except one consume $h^*$, then, the last member would like to consume more than $h^*$.

C2b. (*Nash equilibrium*) Compute the Nash equilibrium of this game and show that it is unique.

C3. To reduce the individual rate of use, the club's authority now imposes a fee that depends on the rate of use: each member $i$ will pay $P_i = ph_i$, where $p$ is the price per unit of consumption of the club good.

C3a. (*Linear pricing*) Compute the Nash equilibrium of the game induced among the $n$ members by the fee schedule $ph_i$. Show that the equilibrium is interior ($i.e.$, $0 < h_i < \overline{h}$) ) if $p$ is large enough.

C3b. (*Decentralization*). Show that $p$ can be set at a level $p^*$ such that the Nash equilibrium rate of use is $h^*$. Compute the value of $p^*$.

C3c. (*Deficit*) Show that this policy will typically leave a deficit $D$. Show that $D > 0$ if $h^* > 1$.

C4. (*Two-part tariff*) Show that the deficit and excess use problems can be solved by means of a linear and affine tariff schedule such that each member $i$ pays $P_i = F + ph_i$.

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Academic year 2011-2012
examination 7 February 2012; 3 hours.

**ATTENTION PLEASE! Choose and answer two questions only in the following set of three questions: A, B and C**. Questions A , B and C are not divisible: for instance, you cannot choose the first half of question C and the beginning of question A plus question B. Answers must be in English or French.

**Question A**. (10 points)

A1. Is it true that the quality of schools has an impact on house prices? Explain why.

The evidence on this question, in the Paris case, is presented in a paper by G. Fack and J. Grenet (*Journal of Public Economics*; 2010; You have the right to use a copy of this paper during the exam).

A2. Decribe the data used by these authors.

A3. Why are hedonic regressions of house prices leading to biased estimates? Explain the estimation method and the identification strategy used to estimate the impact of school quality on house prices in the paper by Fack and Grenet.

A4. How are private-sector schools treated in this research?

A5. Describe the authors' results. Are these results credible?

**Question B**. (10 points)

Can we solve the problem of externalities by means of well-designed economic mechanisms? You'll start your discussion by recalling Coase's famous "theorem" and then discuss the contributions of *Mechanism Design* that you know, distinguishing the cases of complete and incomplete (or asymmetric) information.

**Question C**. (10 points)

*Problem*: Consider a simple economy with $n$ consumers indexed by $i = 1, ...n$, a private good ("money") and a pure public good. The private good is used as an input to produce the public good. Each agent $i$ is endowed with $w_i$ units of the private good. The public good is produced by means of voluntary contributions (i.e., monetary gifts) of the agents, denoted $g_i$. The gifts are constrained to be nonnegative and smaller than $w_i$, i.e., $w_i \geq g_i \geq 0$. The agents consume the rest of their endowment of private good (private consumption), denoted $x_i$. Each $i$ has a utility function $u_i$ with two arguments, the private consumption $x_i$, and the quantity of public good produced $G$, that is $u_i(x_i, G)$, for all $i$. The quantity of public good is just the sum of contributions, that is, $G = \Sigma_{i=1}^n g_i$. The price of the private good is just equal to 1. Finally, each agent is subject to a budget constaint: the sum of private consumption and the voluntary contribution is equal to the endowment. An equilibrium in this economy is a *Nash equilibrium* of the noncooperative *voluntary contribution game* in which each agent $i$ is a player and chooses a feasible contribution $g_i$.

C1. Write the utility maximization problem that agent $i$ must solve to determine her(his) best contribution $g_i$, given the total contribution of others, denoted $G_{-i} = \Sigma_{j \neq i} g_j$. Show that this problem can be formally rewritten as a utility maximization problem in which agent $i$ chooses the bundle $(x_i, G)$ subject to a budget constraint, and subject to the contraint that $G$ is greater than a certain lower bound.

Assume that all agents have the same preferences $u_i(x_i, G) = x_i^{1-\alpha} G^{\alpha}$ where $0 < \alpha < 1$.

C2. In this case, *(i)* compute the demand functions of agent $i$ in the $(x_i, G)$ plane, taking the constraints into account; *(ii)* derive the *best-response* of agent $i$ to the contribution of other agents $j \neq i$; *(iii)* determine the conditions under which agent $i$ is a nonzero contributor, i.e., $g_i > 0$.

C3. Suppose in addition that $w_1 \geq w_2 \geq ... \geq w_n$. Agent 1 is the richest. Find the conditions for an equilibrium $g^* = (g_1^*, g_2^*, ..., g_n^*)$ in which only agent 1 contributes, and therefore $g_i^* = 0$ for all $i > 1$.

C4. Denote the total endowment $W = \Sigma_{i=1}^n w_i$. Compute the symmetric Pareto optimum in this economy.

C5. An interesting case is when the total endowment $W$ belongs to agent 1, i.e., $W = \Sigma_{i=1}^n w_i = w_1$: *(i)* show that equilibrium public production is Pareto-optimal in this case. *(ii)* Suppose then, more generally, that total wealth $W$ is divided equally between $k$ agents, $k > 1$. Compute the total production of public good, denoted $G_k$; *(iii)* is $G_k$ efficient? What happens if $k$ increases without bound?

Assume now that all agents have the following quasi-linear preferences: $u_i(x_i, G) = x_i + a_i \ln(G)$, where $a_i > 0$.

C6. Show that if $a_i = 1$ for all $i$, then, any $g$ such that $0 \leq g_i \leq 1$ for all $i$ and $\Sigma_{i=1}^n g_i = 1$ is a Nash equilibrium.

C7. Show that if $a_1 = 1$ and $a_j < 1$ for all $j \neq 1$, then the *unique* equilibrium is such that $G = g_1 = 1$. Prove existence *and* uniqueness.