SCREENING UNDER ADVERSE SELECTION

Some Optimal Regulation Theory

- Principal-Agent Theory in the pure Adverse-Selection Case.

- *Principal :* *Agent :*

State	Public Firm
Banker	Entrepreneur
Firm	Worker
Shareholder	Manager

- Pure Adverse Selection : hidden characteristics of the agent only.

- No hidden actions (No moral hazard).

- Important applications :
Optimal Regulation and *Optimal Taxation*.

- We study a basic model of the Principal-Agent relationship

- We look for the optimal contract between Principal and Agent

- The Principal is uninformed.

Must Read :

• Jean-Jacques LAFFONT and David MARTIMORT (2002), The Theory of Incentives : The Principal-Agent Model, Princeton University Press, Chapter 2, pp. 28-81.

Other important references :

• Davis P. BARON and Roger B. MYERSON (1982), "Regulating a Monopolist with Unknow Costs", *Econometrica*, 50, pp. 911-930

• Jean-Jacques LAFFONT and Jean TIROLE (1986), "Using Cost Observation to Regulate Firms", *Journal of Political Economy*, 94, pp. 614-41.

Basic Principal-Agent Model

- Agent : produces $q \geq 0$
- Surplus due to production is $S(q)$
- We assume that S is differentiable and $S' > 0$, $S'' < 0$, $S(0) = 0$.
- Marginal Cost of production is denoted $\theta \in \{\underline{\theta}, \bar{\theta}\}$ (Two types)
- Cost function : $C(q, \theta) = \theta q$
- $\underline{\theta} < \bar{\theta}$

- So, $\bar{\theta} =$ "inefficient type"

  $\underline{\theta} =$ "efficient type"

- Prior probability of types :

$$\text{Prob}\,(\theta = \underline{\theta}) = \nu$$

$$\nu \in (0, 1)$$

- Denote $\triangle\theta = \bar{\theta} - \underline{\theta}$ (the "spread")

Definition of a Contract

- A pair of functions $\theta \longmapsto (q(\theta), t(\theta))$

- $q(\theta) =$ production of type θ

- $t(\theta) =$ transfer of money from Principal to Agent

- *Utility of the Agent :*

$$U = t - \theta q$$

- *Utility of the Principal :*

$$V = S(q) - t$$

- *Social surplus :*

$$\begin{aligned} W &= U + V \\ &= S(q) - \theta q \end{aligned}$$

- *First-Best Allocation :*

maximize $(S(q) - \theta q)$

with respect to q, for all θ, s.t. $q \geq 0$

or,

$$\text{maximize } (S(q) - t)$$

with respect to (q, t)

subject to, $t - \theta q \geq U_o(\theta)$ (participation constraint)

and $q \geq 0$, for all θ.

- First-Order Conditions for First-Best Optimality :

$$\begin{cases} S'(\underline{q}^*) = \underline{\theta} \\ \\ S'(\bar{q}^*) = \bar{\theta} \end{cases} \Rightarrow \underline{q}^* > \bar{q}^*$$

- *Transfers :* to implement the first-best,

$$\begin{cases} \underline{t}^* = \underline{\theta}\underline{q}^* + U_0(\underline{\theta}) \\ \\ \bar{t}^* = \bar{\theta}\bar{q}^* + U_0(\bar{\theta}) \end{cases} \Rightarrow U = U_0(\theta) \text{ (No rents)}$$

- For simplicity, we assume that $U_o(\theta) = U_o = 0$

(reservation utilities do not depend on type)

- *The timing of the model :*

$t = 0$ Agent discovers θ

$t = 1$ Principal offers contract (q, t)

$t = 2$ Agent accepts or refuses

$t = 3$ Contract is executed.

Incentive Compatibility and Feasibility :

- A contract is an array : $\{(\bar{t}, \bar{q}), (\underline{t}, \underline{q})\}$

- The contract is *Incentive Compatible* if

$$\underline{t} - \underline{\theta}\underline{q} \geq \bar{t} - \underline{\theta}\bar{q} \qquad (\underline{IC})$$

and

$$\bar{t} - \bar{\theta}\bar{q} \geq \underline{t} - \bar{\theta}\underline{q} \qquad (\overline{IC})$$

- The contract is *Individually Rational* if

$$\underline{t} - \underline{\theta}\underline{q} \geq 0 \qquad (\underline{IR})$$

$$\bar{t} - \bar{\theta}\bar{q} \geq 0 \qquad (\overline{IR})$$

- $IR + IC =$ *feasible* contract

Monotonicity Property :

- Adding $\underline{IC}$ and $\overline{IC}$ yields,

$$(\bar{\theta} - \underline{\theta})\underline{q} \geq (\bar{\theta} - \underline{\theta})\bar{q}$$

$$\Rightarrow \qquad \underline{q} \geq \bar{q}$$

- If monotonicity holds, then, there exists $(\bar{t}, \underline{t})$ such that IC holds :

$$\underline{\theta}(\underline{q} - \bar{q}) \ \leq \ \underline{t} - \bar{t} \ \leq \ \bar{\theta}(\underline{q} - \bar{q})$$

$$(\underline{IC}) \qquad (\overline{IC})$$

INFORMATION RENTS :

- Denote $\overline{U} = \bar{t} - \bar{\theta}\bar{q}$; $\underline{U} = \underline{t} - \underline{\theta}\underline{q}$.

- We have,

$$\bar{t} - \underline{\theta}\bar{q} = \bar{t} - \bar{\theta}\bar{q} + \bar{q}\triangle\theta = \overline{U} + \bar{q}\triangle\theta$$

- It follows that $\underline{IC}$ is equivalent to,

$$\underline{U} \geq \overline{U} + \triangle\theta\bar{q}$$

- We also have,

$$\underline{t} - \bar{\theta}\underline{q} = \underline{t} - \underline{\theta}\underline{q} - \triangle\theta\underline{q} = \underline{U} - \triangle\theta\underline{q}$$

- Hence, $\overline{IC}$ is equivalent to

$$\overline{U} \geq \underline{U} - \bar{q}\triangle\theta$$

- And we see that $\underline{U} > \overline{U}$. The *informational rent* of type $\bar{\theta}$ is $\bar{q}\triangle\theta$.

The Principal's Problem :

(Second-best problem)

$$\max_{(\bar{t},\bar{q},\underline{t},\underline{q})} \nu(S(\underline{q}) - \underline{t}) + (1-\nu)(S(\bar{q}) - \bar{t})$$

subject to $\underline{IC}$, $\overline{IC}$, $\underline{IR}$, $\overline{IR}$.

Use change of variables :

$$U = t - \theta q$$

The Principal's problem rewritten :

$$\max_{(\underline{U},\underline{q},\bar{U},\bar{q})} \{\nu[S(\underline{q}) - \underline{\theta}\underline{q}] + (1-\nu)[S(\bar{q}) - \bar{\theta}\bar{q}] - [\nu\underline{U} + (1-\nu)\overline{U}]\}$$

subject to,

$$\begin{cases} \underline{U} \geq \overline{U} + \bar{q}\triangle\theta & (\underline{IC}) \\ \overline{U} \geq \underline{U} - \underline{q}\triangle\theta & (\overline{IC}) \\ \underline{U} \geq 0 & (\underline{IR}) \\ \overline{U} \geq 0 & (\overline{IR}) \end{cases}$$

We consider contracts without "shutdown", *i.e.*, $\bar{q} > 0$.

• If $\underline{IC}$ and $\overline{IR}$ hold, then $\underline{IR}$ is always satisfied :

$$\underline{U} \geq \overline{U} + \bar{q}\triangle\theta \geq \bar{q}\triangle\theta > 0.$$

• Both $\underline{IC}$ and $\overline{IR}$ must be binding at the second-best optimum.

• If not, let $\overline{U} = \varepsilon > 0$. Choose $d\varepsilon < 0$ and decrease $\overline{U}$ and $\underline{U}$ by $d\varepsilon$. Contradiction since $(\nu\underline{U} + (1-\nu)\overline{U})$ decreases.

• If $\underline{U} = \bar{q}\triangle\theta + \eta$, $\eta > 0$, then, decrease η by $d\eta < 0$. Contradiction.

• We conclude that $\underline{U} = \bar{q}\triangle\theta$ at the second-best optimum.

- *The Principal's Problem becomes :*

$$\max_{(\underline{q},\bar{q})}\{\nu[S(\underline{q})-\underline{\theta}\underline{q}]+(1-\nu)[S(\bar{q})-\bar{\theta}\bar{q}]-\nu\triangle\theta\bar{q}\}$$

(if we substitute $\bar{U}=0$ and $\underline{U}=\bar{q}\triangle\theta$ and we ignore $\overline{IC}$ for a while...)

- *The First-Order Conditions are*

$$\begin{cases} S'(\underline{q}^{**})=\underline{\theta} \\ \\ \{S'(\bar{q}^{**})-\bar{\theta}\}(1-\nu)=\nu\triangle\theta \end{cases}$$

- Distortion $=\nu\triangle\theta/(1-\nu)$

- Trade-off : EFFICIENCY

  vs.

  RENT EXTRACTION

- We finally check that $\overline{IC}$ also holds.

- From monotonicity : $\bar{q}^{**} \leq \underline{q}^{**}$.

The omitted $\overline{IC}$ constraint can be written

$$
\begin{aligned}
\overline{U}^{**} = 0 \quad & \geq \underline{U}^{**} - \triangle\theta \underline{q}^{**} \\
& \geq \triangle\theta \bar{q}^{**} - \triangle\theta \underline{q}^{**} \\
\Leftrightarrow \underline{q}^{**} \quad & \geq \bar{q}^{**} \ \text{(true)}.
\end{aligned}
$$

- Note : $\underline{q}^{**} = \underline{q}^{*} > \bar{q}^{*} > \bar{q}^{**}$.

This is because,

$$S'(\bar{q}^{**}) = \bar{\theta} + \frac{\nu}{1-\nu} \triangle\theta.$$

Proposition : The second-best optimal contract is such that,

$S'(\underline{q}^{**}) = \underline{\theta}$ *(''no distortion at the top'')*

$S'(\bar{q}^{**}) = \bar{\theta} + \frac{\nu}{1-\nu} \triangle\theta$ *(distortion)*

$\underline{U}^{**} = \bar{q}^{**} \triangle\theta$ *(informational rent for efficient types)*

$\overline{U}^{**} = 0$ *(full rent extraction for inefficient types)*

- Discussion : rôle played by $\frac{\nu}{1-\nu}$ and $\triangle\theta$.

Some Mechanism Design.

Suppose we have one Principal and n agents indexed $i = 1, ..., n$.

Agent i's type θ_i is drawn from a set Θ. Principal chooses a decision $x \in X$. (For instance $x = (q, t)$, a contract). The utility of agent i is $U(x; \theta_i)$

Definition 1 (Mechanism)

A mechanism is a pair (M, g) where M is a *message space* and $g : M^n \to X$ is an outcome function.

$$x = g(m_1, m_2, ..., m_n)$$

• We suppose that $\theta = (\theta_1, ..., \theta_n)$ is not observed by the Principal. Each θ_i is private information of agent i.

Definition 2 (Direct Mechanism)

A mechanism is *direct* if $M = \Theta$.

Definition 3 (Equilibrium in Dominant Strategies)

Let $m_i^* : \Theta \to M$ be agent $i's$ communication strategy

Then, $(m_i^*(\theta_i))_{i=1...n}$ is an *equilibrium in dominant strategies* if for all i, for all $(m_j^*(\theta_j))_{j\neq i} = m_{-i}^*(\theta_{-i})$, we have

$$U[g(m^*(\theta)), \theta_i] \geq U[g(m_{-i}^*(\theta_{-i}), \widehat{m}_i), \theta_i]$$

for all $\widehat{m}_i \in M$

- Notation : $\theta_{-i} = (\theta_j)_{j\neq i}$

$$m(\theta) = (m_1(\theta), ..., m_n(\theta))$$

Definition 4 (Revealing Mechanism)

A mechanism (M, g) is *revealing (in dominant strategies)* if $m_i^*(\theta_i) \equiv \theta_i$ is a *dominant strategy* for all $i = 1, ..., n$.

Note : (M, g) is a *direct* and *revealing mechanism if* $m_i^*(\theta_i) = \theta_i$ is an equilibrium is dominant strategies.

REVELATION PRINCIPLE

Theorem : If mechanism (M, g) implements decision $f : \Theta \to X$ in dominant strategies, that is, $g(m^*(\theta)) = f(\theta)$ for all θ, m^* being a $n-$tuple of dominant strategies, then, (Θ, f) is *revealing* in *dominant strategies*.

Remark : If (M, g) chooses $f(\theta)$ for all θ, then, there exists a revealing mechanism which does the same job ; *i.e.*, (Θ, f).

Proof of the Revelation Principle :

- There exists a $n-$tuple of dominant strategies m^* such that $g[m^*(\theta)] = f(\theta)$ for all θ, by assumption.

- $U[g(m_i^*(\theta_i), m_{-i}), \theta_i] \geq U[g(m_i', m_{-i}), \theta_i]$,

for all $m_i' \in M$, all $m_{-i} \in M^{n-1}$.

- So, in particular, for all i and θ_i,

$$U[g(m_i^*(\theta_i), m_{-i}^*(\theta_{-i})), \theta_i] \geq U[g(m_i^*(\theta_i'), m_{-i}^*(\theta_{-i})), \theta_i]$$

for all θ_i', all θ_{-i}.

- Now, $g[m^*(\theta)] = f(\theta)$, then, for all i, all θ_i,

$$U[f(\theta), \theta_i] \geq U[f(\theta_i', \theta_{-i}), \theta_i]$$

for all θ_i', θ_{-i}

- We conclude that f is *truthfully implementable* by the direct revealing mechanism (Θ, f).

- Agents report their types $\theta_i \in \Theta$ directly.

- Agents have no incentive to make false reports (in a very strong sense).

- There is no loss of generality in constraining optimal contracts to be *incentive compatible (i.e., revealing).*

Other equivalent interpretation :

If g implements f and

$$g : M \to X$$

is not revealing (and not direct).

Then $\tilde{g} = g \circ m^*$ is a direct and revealing mechanism that also implements f.

$$\Theta \xrightarrow{m^*} M \xrightarrow{g} X$$

$$\tilde{g}(.) = g \circ m^*(.)$$