INTRODUCTION TO PRINCIPAL-AGENT THEORY UNDER MORAL HAZARD

Moral Hazard :

- Moral Hazard occurs in the Principal-Agent relationship when some *actions* of the agent are not perfectly observable.

- Contractual payment cannot depend on variables that are not observable by the Principal, the Agent, and by an outside (or third) party : a Judge.

- This poses the problem of performance measures. They may be more or less imprecise ”signals” of the Agent’s true activity.

Moral Hazard and Risk

• *Important property.* In situations of *Moral Hazard*, the probabilities of outcomes, success or failure of projects, etc., depend on hidden actions of the agents.

• Uninformed parties (*i.e.*, Principals) must take into account this fact.

• To create effort incentives, contracts will typically involve an element of profit-sharing, and therefore impose risk upon the agent.

• Risky compensation hurts the risk-averse agent. This generates transactions costs.

Application of the Theory

- Hierarchical Organization in Firms. Piece Rates. Compensation Policy.

- Delegation and Outsourcing ; Public Procurement.

- Theory of Labor Contracts.

- Theory of Financial Intermediation (Banking), Insurance Markets,...

- Financial Theory of the Firm. Corporate Finance.

A Model of the Principal-Agent Relationship :

- Grossman and Hart's model (1983).
- We follow the presentation by David Kreps (1990).
- Agent chooses an action (or effort) *a* in a set A.
- Principal observes an outcome s in a set S.
- $A = \{a_1, a_2, ..., a_N\}$ (A is a discrete finite set).
- $S = \{s_1, s_2, ..., s_M\}$ (S is also finite).

Formal Representation of Moral Hazard and Risk :

- Define $\pi_{nm} = Prob[s_m \mid a_n]$

- $\sum_m \pi_{nm} = 1$ for all n.

- ASSUMPTION 1 : We assume that $\pi_{nm} > 0$ for all n and m. Any outcome is possible, under every action.

- The Agent is potentially risk averse. We assume,

$$U(w, a) = u(w) - a$$

where w = wage.

- ASSUMPTION 2 : *Risk aversion :* $u : \mathbb{R} \to \mathbb{R}$ is strictly increasing, concave and continuously differentiable.

Definition of a Contract :

• A contract is a function $w : S \to \mathbb{R}$ or equivalently : $w = (w_1, w_2, ..., w_m)$, where w_m is compensation if outcome s_m is observed.

• *Note :* It is crucial here that the outcome s_m is *observable* and *verifiable*. The Principal, the Agent, and possibly a Judge, can observe s_m and check if $w_m = w(s_m)$ is paid to the Agent.

The Principal's Objective and Information

• The Principal is risk neutral and observes the outcome (signal) s_m only.

• His objective is expected profit, net of expected wages $E(w \mid a)$, that is,

$$B(a) - E(w \mid a)$$

• $B(a_n) = \sum_m \pi_{nm} s_m = E(s \mid a_n)$

• Note $E(w \mid a_n) = \sum_m \pi_{nm} w_m$

The Principal-Agent Problem

- Choose $(a, w) \in A \times \mathbb{R}^M$ so as to maximize

$$B(a) - E(w \mid a)$$

subject to the Agent's participation constraint,

$$E(u(w) \mid a) - a \geq U_o,$$

and subject to the incentive constraint,

$$E(u(w) \mid a) - a = \max_{\alpha \in A} \{E(u(w) \mid \alpha) - \alpha\}.$$

- Note : U_o is the Agent's best outside option.

First-Best Effort :

- Compute the minimal certain reward for a_n, denoted $C^o(a_n)$

$$C^o(a_n) = u^{-1}[a_n + U_o]$$

- $C^o(a_n)$ is the cost of action a_n if action a is observable.

- The First-Best effort is a solution of the problem

$$\underset{a \in A}{Max}[B(a) - C^o(a)]$$

- Let a^* be the first-best effort.

Transformation of the Principal-Agent Problem :

- Define, $x_m = u(w_m)$.
- Denote, $v = u^{-1}$.
- We have $w_m = v(x_m)$, and v is convex.
- Then,

$$\begin{aligned} E(w_m \mid a_n) &= \sum_m \pi_{nm} v(x_m) \\ E(u(w_m) \mid a_n) &= \sum_m \pi_{nm} x_m. \end{aligned}$$

Cost of Incentives :

- The cost minimization problem : for a given action a_n,

$$\text{Minimize } \sum_m \pi_{nm} v(x_m)$$

subject to the *participation constraint*,

$$\sum_m \pi_{nm} x_m \geq U_o + a_n \qquad (IR)$$

and s.t. the *incentive constraints*,

$$\sum_m \pi_{nm} x_m - a_n \geq \sum_m \pi_{\nu m} x_m - a_\nu \qquad (IC_\nu)$$

for all $\nu = 1 ... N$.

- Minimization is with respect to

$$x = (x_1, x_2, ..., x_M).$$

- The solution of the above problem is a cost function $C : A \to \mathbb{R}_+$

- $C(a_n) = \underset{w}{Min}\{E(w \mid a_n) \mid (IR)$ and $(IC_\nu)\}$,

the smallest expected wage required to obtain an effort level a_n from the Agent.

- *Second-Best Effort :* a solution of

$$\underset{a \in A}{Max}[B(a) - C(a)]$$

- We will show that the second-best solution may be different from a^* since $C(a) > C^o(a)$.

Efficient Risk-Sharing

• Assume that a_n is observable, then, the optimal contract problem is just

$$\underset{x}{Min} \sum_m \pi_{nm} v(x_m)$$

s.t.,

$$\sum_m \pi_{nm} x_m \geq U_o + a_n$$

- Write the first-order conditions (λ is a Lagrange multiplier) :

$$v'(x_m) = \lambda \text{ for all } m$$

(Borch's equation).

- This implies $x_1 = x_2 = ... = x_M = x^*$
(Full Insurance) or equivalently, $w_1 = w_2 = ... = w_M = w^*$.

- $$x^* = U_o + a_n$$

$$w^* = v(U_o + a_n).$$

Proposition 1 : If the agent is strictly risk averse (*i.e.,* $u'' < 0$), then, $C^o(a_n) < C(a_n)$ for any action a_n that is more costly than some other action, *i.e.,* such that $a_n > \underset{A}{Min}(a_\nu)$.

Proof : Simple. If agent is risk averse, the unique efficient risk-sharing implies *Full Insurance* (as shown above). A constant wage implies that chosen effort is $\underset{A}{Min}(a)$. Thus, IC_ν is binding for some ν...

Proposition 2 : The IR constraint is always binding at the optimum.

Proof : If not $\sum_m \pi_{nm} x_m > U_o + a_n$ choose $\varepsilon > 0$ and $x'_m = x_m - \varepsilon$ for all m. If ε small enough, IR and IC_ν are still satisfied. But expected wage $\sum_m \pi_{nm}\, v(x'_m)$ is smaller : contradiction.

- *Remark :* $u(w)$ is not bounded below here.

Taking IC_ν constraints into account

- *Lagrangian :*

$$\mathcal{L}(x,\lambda,\mu) = -\sum_m \pi_{nm} v(x_m) + \lambda \left(\sum_m \pi_{nm} x_m - a_n - U_o \right)$$

$$+ \sum_{\nu \neq n} \mu_\nu \left[\sum_m (\pi_{nm} - \pi_{\nu m}) x_m - a_n + a_\nu \right]$$

- *First-Order Conditions : for all m,*

$$v'(x_m) = \lambda + \sum_{\nu \neq n} \mu_\nu \left(1 - \frac{\pi_{\nu m}}{\pi_{nm}} \right)$$

(Mirrlees' equation)

- Interpretation : variable wage ; w_m = base wage + bonuses or penalties...

Special Case : Risk-Neutral Agent.

- Assume $u(w) \equiv w$ (risk-neutrality)

Proposition 3 : If the Agent is risk neutral, then, the first-best effort can be implemented,

$$\underset{a \in A}{Max}(B(a) - C(a)) = \underset{a \in A}{Max}(B(a) - C^o(a))$$

and

$$w_m = s_m - B(a^*) + C^o(a^*)$$

• *Proof :* Under the proposed contract, the Agent chooses a_n so as to maximize,

$$\begin{aligned} E[U(w,a_n)] &= \sum_m \pi_{nm} s_m - (B(a^*) - C^o(a^*)) - a_n \\ &= B(a_n) - a_n - \text{ constant}, \\ \text{and, } C^o(a_n) &= a_n + U_o, \text{ thus,} \end{aligned}$$

$$E[U(w,a_n)] = B(a_n) - C^o(a_n) + U_o - \text{ constant.}$$

This proves that the agent will choose $a_n = a^*$.

Q.E.D.

We have proved that

$$\begin{aligned} C(a^*) &= B(a^*) - B(a^*) + C^o(a^*) \\ &= C^o(a^*). \end{aligned}$$

Since : $B(a) - C(a) \leq B(a) - C^o(a)$

and : $B(a^*) - C^o(a^*) = \underset{A}{max}(B(a) - C^o(a))$.

We proved that :

$$\underset{A}{max}(B(a) - C(a)) = \underset{A}{max}(B(a) - C^o(a)).$$

Q.E.D.

Further Properties of the Optimal Contract

• The optimal contract trades off *insurance* and *incentives :* If the Agent is better insured, the *power of incentives* is reduced.

• But the contract also exploits the *information conveyed by the signal* s_m *on effort* a_n.

• Is it true that the optimal contract is a *profit-sharing contract* in the sense that :

$$w_1 < w_2 < ... < w_M$$

if

$$s_1 < s_2 < ... < s_M?$$

• Assume now that signals are ranked : $s_k < s_{k+1}$.

- Assume that effort levels are ranked : $a_1 < a_2 < ... < a_N$.

- Define $\Pi_{nm} = Prob(s \geq s_m \mid a_n)$

- *ASSUMPTION 3 :* If $\nu > n$ then $\Pi_{nm} \leq \Pi_{\nu m}$ for all $m = 1...M$, with a strict inequality at least for some m.

- *First-Order Stochastic Dominance :*

Increasing effort increases the probability of getting a higher outcome. This implies that $B(a_n)$ is increasing in n.

Example (Kreps)

- Three levels of profit :

$$s_1 = 1, s_2 = 2, s_3 = 10,000.$$

- Two levels of effort :

$$a_1 = 1 \text{ and } a_2 = 2.$$

- The probabilities π_{nm} are :

s	$Pr(s \mid a = 1)$	s	$Pr(s \mid a = 2)$
10,000	0.2	10,000	0.5
2	0.3	2	0.1
1	0.5	1	0.4

- Check that π_{nm} satisfy the FOSD assumption.

- Suppose that a_2 is the optimal effort level.

- Write Mirrlees' equations :

$$\begin{cases} v'(x_1) = \lambda + \mu\left(1 - \dfrac{0.5}{0.4}\right) = \lambda - \dfrac{\mu}{4} \\[2ex] v'(x_2) = \lambda + \mu\left(1 - \dfrac{0.3}{0.1}\right) = \lambda - 2\mu \\[2ex] v'(x_3) = \lambda + \mu\left(1 - \dfrac{0.2}{0.5}\right) = \lambda + \dfrac{3}{5}\mu \end{cases}$$

- $x_1 = u(w_1), x_2 = u(w_2)$, etc...

- Remark that w_m is non monotonic : $w_2 < w_1$ and $w_3 > w_2$!

- We need two additional assumptions.

- ASSUMPTION 4 : (MLRP)

If $a_n < a_\nu$ and $s_m < s_\mu$, we have

$$\frac{\pi_{\nu\mu}}{\pi_{n\mu}} \geq \frac{\pi_{\nu m}}{\pi_{nm}}$$

- The Monotone-Likelihood Property. Can we now prove

$w_{k+1} > w_k$? Not yet !

- Using Mirrlees' equations, we get,

$$v'(x_m) - v'(x_\mu) = \sum_\nu \mu_\nu \left[\frac{\pi_{\nu\mu}}{\pi_{n\mu}} - \frac{\pi_{\nu m}}{\pi_{nm}} \right]$$

and $\mu_\nu \geq 0$ (multipliers are ≥ 0).

- We want $w_\mu > w_m$ or $x_\mu > x_m \Rightarrow v'(x_\mu) > v'(x_m)$.

The difference above should be negative.

- We obtain the desired results if $\mu_\nu = 0$ for all $\nu > n$.

Convexity of Distribution Function :

- ASSUMPTION 5 :(CDF)

The cumulative distribution function of outcomes s is convex with respect to effort :

$$\Pi_{nm} = Prob(s \geq s_m \mid a_n)$$

is a concave function of a.

- Interpretation : increases in effort have decreasing marginal impact on the probabilities of better outcomes.

Proposition 4 : If u is strictly concave, (π_{nm}) satisfy FOSD, MLRP and CDF, then the optimal wage-incentive scheme has wages that are nondecreasing functions of the level of firm profits, *i.e.*,

$$w_1 \leq w_2 \leq ... \leq w_M.$$

Proof : (see Kreps (1990))

Readings :

- David KREPS (1990), *A Course in Microeconomic Theory,* Chapter 16, Harvester Wheatsheaf, pp. 577-624.

- Jean-Jacques LAFFONT and David MARTIMORT, (2002), *The Theory of Incentives : The Principal-Agent Model*, Princeton University Press, Chapter 4, pp. 145-186.

- *The source for the theory presented here :*

Sanford GROSSMAN and Oliver HART (1983), "An Analysis of the Principal-Agent Problem", *Econometrica*, 51, pp. 7-45.