Families, Bankers and the Intergenerational Transmission of Wealth*

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Abstract

We propose and study a new form of financial product, called *intermediated family contract*. These contracts are used to share the family's wealth and income among its members and to transmit wealth from one generation to the next, with the help of a banker-insurer, according to a multi-period plan. Family arrangements can be defined as a bundle of bilateral contracts signed simultaneously with a bank by several members of a family. These contracts may be contingent on a number of publicly observable events, like a person's death; they can also be analyzed as a bundle of financial products, combining equity release and home loans (reverse and standard mortgages), annuitization (longevity insurance) and consumption smoothing. A secondary benefit of such a product may be tax optimization. The optimal intermediated family arrangements are studied under three main assumptions: parents and children are risk-averse; parents are altruist to a certain degree; the elderly are owner-occupiers and strongly attached to their homes. We used calibrated simulations to study the welfare and comparative statics properties of the optimal contract in a population of families. Simulations provide bounds on the surplus that can be extracted by the intermediary, by means of a family contract, *i.e.*, on the profitability of a given family for the banker-insurer. An intermediated family contract can typically increase family welfare, and markedly increase the size of the children's home, as compared to ordinary solutions.

Keywords : Aging; Annuities; Bequests; Household Finance; Housing Equity; Inter-vivos transfers; Longevity Risk; Real Estate; Reverse Mortgage; Taxation.

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1 Introduction

In the present article, we propose and study a new kind of financial product, called the *intermediated family contract*. This contract can be used to share the family's wealth and income, and to transmit wealth from one generation to the next, with the help of a banker and (or) an insurer. Intermediated family contracts can be analyzed as a bundle of financial products, potentially combining equity release and home loans (reverse and standard mortgages); annuitization (longevity insurance) and consumption smoothing. The motivation for such arrangements comes from the fact that a majority of seniors is endowed with illiquid wealth in the form of one or several houses. The elderly typically want to stay in their homes, insure themselves against risks and transmit (part of) their wealth to their children. We show that intermediated family contracts can improve family welfare for several reasons. In essence, they improve risk-sharing, and allow the younger generations to buy substantially larger homes. Among the problems solved by intermediated family contracts, the most important is the specification of an intertemporal plan to share the parents' estate with the children. This is why these contracts may also embody, or implement, elements of a person's will and testament. A secondary benefit of such a product may be to save on some taxes.

The family agreements that we study can be defined as a bundle of contracts, simultaneously signed by a bank and several members of a family. In legal terms, it may not be necessary to sign a multilateral agreement, but the banker may require that each of the family members involved signs a two-sided contract with the bank at the same moment, and the contracts may be contingent on a number of publicly observable events, like a person's death.

We study these intermediated family contracts under several key assumptions. First, the older member of the family (hereafter called the *mother*), is supposed to be an altruist. The mother wants to maximize a weighted sum of her utility and the utilities of her children. Second, a number of risks, and in particular, the mother's longevity risk, as well as market risks like house-price variability, can be diversified by a banker-insurer, who would hold large bundles of family contracts. Third, we assume that the mother owns her residence and that she is strongly attached to her house: the disutility of moving, or the disutility of house downsizing, are supposed to be very high. The banker can therefore also offer a liquidity service, unlocking the mother's housing equity by means of a reverse mortgage, a home reversion plan (or a French *viager* contract — see below), while permitting the mother to stay in her home. The money released can be used to schedule gifts to the next generation, and the children can at the same time borrow to buy their own homes.

As a starting point, we characterize the set of optimal intermediated family contracts in a simple model with a single risk, *i.e.*, longevity risk, a finite horizon, discrete time, risk-averse and altruist agents. The optimal solution may in general depend on the tax system, but we show that the mother's home equity should be entirely liquidated by means of a home reversion plan (or a *viager* contract)), if the tax rate on bequests is positive. In addition, direct *inter-vivos* gifts to the children should be zero, but the children's mortgage payments can be subsidized. Finally, the mother's assets must be annuitized and simultaneously, the children will be hedged against the mother's longevity risk. The model can be generalized to study various related questions. In particular, the analysis can be extended to study the case of multiple children or heirs and multiple risks, including the long-term care and health risks.

We then calibrate the model and explore the properties of family contracts by means of numerical simulations. We discuss the welfare gains of these contracts, and show how the surplus can be extracted and shared by financial institutions. The simulations are calibrated using French data on income and wealth, and we focus on well-to-do seniors living in the Paris region as an illustrative example. Our simulations provide an evaluation of the profitability of improvements in the efficiency of risk-sharing due to the intervention of financial intermediaries in family arrangements.

The contribution of the paper is not to emphasize the usefulness of equity release instruments, that has been studied by various authors (see below), but to explore the consequences of optimal risk-sharing between family members in terms of welfare and product design, for the needs of applications in household finance. Family contract models of the kind proposed here can also be applied to the study of public policies and taxation.

A well-known observation is the small magnitude of asset decumulation, or dissaving, among retired households. Except for Social Security and some pension-related assets, housing equity is the most important asset of a large fraction of older citizens in Europe and in the United States. It seems that the elderly do not wish to downsize their housing consumption and are reluctant to borrow against home equity to finance consumption. These facts have been discussed in the literature. The behavior of the elderly can be explained, to a large extent, by the combination of a bequest motive and precaution against the risk of large health expenditures (see, Venti and Wise (1990), Dynan *et al.* (2004), De Nardi *et al.* (2010)). Further study of this question shows that, in the absence of changes in household structure (for instance, the death of a spouse), or other shocks, like the need to enter a nursing home, most elderly families are unlikely to move. Therefore, it is reasonable to assume that retired individuals are strongly attached to their homes (on these questions, see Venti and Wise (1990), (2004)). Psychological factors certainly play a rôle, as introspection may reveal, but home-ownership is also objectively providing a form of insurance against various risks. Firstly, ownership is a hedge against upward variations in future house prices; secondly, liquidating the house and entering the rental market exposes the household to rent variability, and this latter risk cannot be insured. Sinai and Souleles (2005) have shown that home-ownership rates are higher in areas with greater variability of rental prices.

It seems relatively well-established that the commitment of the seniors to their homes is due to the fact that these homes are a store of value. More precisely, home ownership is a substitute for *long-term care insurance*[1]. Medicaid and, more generally, public insurance systems may be responsible for some crowding out of the demand, but the main reason seems to be the fact that home ownership plays the role of long-term care insurance for many people (see Thomas Davidoff

[1]There is a literature on the reasons for which long-term care insurance is underdeveloped (see, *e.g.*, Brown and Finkelstein (2007)).

(2010)).

Venti and Wise (2004) put the emphasis on another important fact: the apparently limited recourse to housing equity-release instruments, and mainly to reverse-mortgage contracts that are available in the United States since the mid 90s. Many older households have substantial wealth locked in illiquid housing. They should normally like to release this wealth to finance consumption[2]. In many countries, there is a significant segment of "income-poor" and "house-rich" individuals[3]. Reverse mortgages in the US, the UK and other countries (to a certain extent, *viager* contracts in France) attract a limited number of clients only, in spite of important potential benefits.

The stylized fact that equity release loans are characterized by low take-up rates may have to be revised because the reverse-mortgage market witnessed substantial growth in the mid 2000s, particularly in the United States (see Hui Shan (2011), Sinai and Souleles (2008)). Higher house prices, generating windfall gains for many homeowners, may finally persuade large fractions of the potential borrowers to engage into equity release. Yet, in some countries, such as France, the reverse mortgage market is totally underdeveloped[4]

Economic agents typically make mistakes and do not always behave as predicted by normative theory, but when explanations are offered to them, in general, they understand the benefits of new products. Even if they don't understand the contracts well, at some point, they simply start to mimic successful neighbors who did choose these products before them. But either kind of learning takes time. For that reason, the development of reverse mortgages may become impressive in a few years from now, even in financially under-developed countries, simply because they are useful and enhance welfare, as predicted by normative theory, and since people will eventually understand their usefulness. However limits to the development of equity-release contracts can also be due to the presence of bequest motives, that we discuss below.

The *annuity puzzle* is an important behavioral anomaly (see, *e.g.*, the survey of Benartzi *et al.* (2011)). Older households do not *annuitize* their wealth, while theory predicts that they should[5]. More precisely, Davidoff *et al.* (2005) show that, in the case of incomplete markets, rational consumers will generally want to annuitize a significant fraction of their wealth. There is also a debate on the reasons for which these markets are underdeveloped, and this is still an open research question. Scott, Watson and Hu (2011) study annuity design problems and suggest that some innovations may increase participation in the annuity market and allow for significant welfare improvements. Again, bequest motives, the desire to keep real estate assets as a hedge

[2]See, *e.g.*, Fratantoni (1999).

[3]For international comparisons of housing adjustment decisions, see Chiuri and Jappelli (2010).

[4]The French *crédit viager hypothécaire* has been legalized by an act passed in 2006. This kind of loan is strictly regulated and French banks cannot advertise it. In contrast, the *viager* contract is not limited (see below, in Section 2).

[5]The classic result is due to Yaari (1965): a risk-averse agent without a bequest motive should completely annuitize her wealth, that is, sell her assets in exchange for a sequence of certain payments that will be terminated after the end of the agent's life. The use of annuities is rational because they protect old agents against the risk of outliving their income. On the theory of annuities in general, see Sheshinski (2008)

against large shocks, as well as the rigid or irreversible consequences imposed by a limited set of available options, may explain incomplete annuitization and limited participation.

On these themes, see Campbell (2006), who argues that if some observed facts are unquestionably behavioral anomalies in a certain sense, their origin is often unclear. It may be that theory, and some well-known economic models, are too simple to capture the complexity of individual decision problems; the data sources are too limited to convincingly identify some effects; and of course, some —but not necessarily a majority of— individuals lack knowledge and simply make mistakes. It follows that the data may lead to rejection of a "textbook model" but that a more sophisticated theory relying on rationality assumptions would fit the data reasonably well.

In the following, another important point is the importance of parental altruism and bequest motives. Economists hesitate about the observed bequests: are they random, purely unintentional transfers or do they reflect the existence of a bequest motive? Economic theory also hesitates about the appropriate model for the bequest motive. In this context, *pure altruism* means that utility functions depend on the utility of children, leading to dynastic and recursive formulations of utility[6]. The distinction between the various sources of inheritance flows is of course crucial for the theory of taxation. For instance, if bequests are mainly due to accidents, they can be taxed with no incidence on savings[7]. There exists an econometric literature on family transfers and the pure altruism hypothesis. The latter assumption is rejected by some tests (see *e.g.*, Altonji *et al.* (1997)), but, currently, there is a lack of consensus on this question. The evidence concerning the existence of a bequest motive seems to support the hypothesis (see, *e.g.*, Kopczuk and Lupton (2007), Ameriks *et al.* (2011)).

In the following, as a motivation, Section 2 provides some descriptive statistics, based on European data, namely, the SHARE survey, and briefly recalls elements on the institutional setup: the existing housing-equity release instruments. The data strongly suggest that there are inefficiencies in the intergenerational transmission of wealth. Nowadays, many households inherit too late, between 50 and 60. Parental transfers (*i.e.*, gifts and bequests) trigger house acquisitions in a significant way. Section 3 is devoted to our family-contract model: we characterize optimal contracts. Section 4 presents various numerical simulations of the model. We discuss the possibility of welfare improvements and the potential profitability of intermediated family contracts for the banks. Concluding remarks are gathered in Section 5.

2 Inheritance and Home Acquisition: Some Related Facts

We begin with a presentation and discussion of some descriptive statistics and econometrics, based on SHARE data. SHARE is a large scale survey of the health and wealth of persons aged 50 and more in Europe. In the following, we particularly use the first wave of SHARE,

[6] The importance of the "joy of giving", altruism and bequest motives has been recognized a long time ago by economists (see, *e.g.*, Becker (1974), Andreoni (1989), Abel and Warshawsky (1988), Laferrère and Wolff (2006)).

[7] On estate and gift taxation, see, *e.g.*, Kaplow (2001), Cremer and Pestieau (2006), Diamond (2006), Kopczuk (2013).

with more than 30000 individuals surveyed between 2004 and 2005, in 12 countries. We will focus on features of the survey that are directly relevant to our later analysis: real property and intergenerational transfers (bequests and inter-vivos gifts)[8].

Each surveyed individual has a principal residence. Table 1 gives the distribution of the housing status. In the sample, 70% of the households are owners of their principal residence. Table 2 explains how the residence was acquired. We see that the house was received as a bequest or a gift in 10% of the cases.

A glance at the distribution of the age at which the residence was acquired is useful, to shed light on access to the status of owner-occupier. Fig. 1 gives the histogram of this distribution, based on SHARE data[9]. On Figure 1 99% of the mass is above 19 years old, the mean is 41 and the median is 39. We see that roughly 60% of the studied sample bought their house between 30 and 50, while 20% became owners before 30.

Table 1: Ownership and Tenancy Status

	Count	%
owner	14,307	69.8
member of a cooperative	463	2.3
tenant	4,563	22.3
subtenant	74	0.4
rent free	1,071	5.2
other	14	0.1

Source: SHARE data set, first wave.

We now study transfers and try to see if the data suggests that transfers played a role in helping the households becoming owner-occupiers. The European SHARE data shows that 70% of the households are owner-occupiers and that 30% received a transfer. As we will see, the typical individual buys a home 10 years before receiving a bequest from his(her) parents. The data doesn't indicate if the money received, or the assets received by households take the form of

[8]We work with a sample of 20505 individuals, called *principal respondents*. Each principal respondent is attached to a household. This is due to the double structure of the SHARE survey: the data provide information on individuals, but also on the household to which they belong (generally on the spouse). Among these individuals, we more particularly studied those who received money from their parents. On the SHARE survey and methodology, see Börsch-Supan et al.(2005), Börsch-Supan and Jürges (2005).

[9]The age is that of the "principal respondent" in each household. Strictly speaking, Fig. 1 gives the probability of acquisition at an age denoted t_h, given that the age a at the moment of the survey is greater than 50 and given $t_h \leq a$. It follows that the reported frequencies are biased estimates of the probabilities of acquiring a home at ages $t_h > 50$, because data are censored for individuals of age $a > 50$ that would buy their homes only in the future, at an age $t_h > a$. The associated survival distribution of the acquisition age could be estimated by the Kaplan-Meier method, but the estimates of probabilities would be imprecise for ages greater than 50. The reader must remember that probabilities of acquisition after 50 are somewhat under-estimated by the histogram frequencies.

Figure 1: Histogram of age at which house was acquired

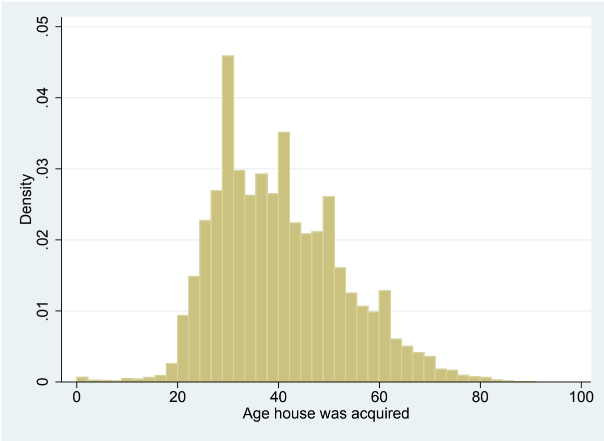


Figure 2: Age at which first transfer was received

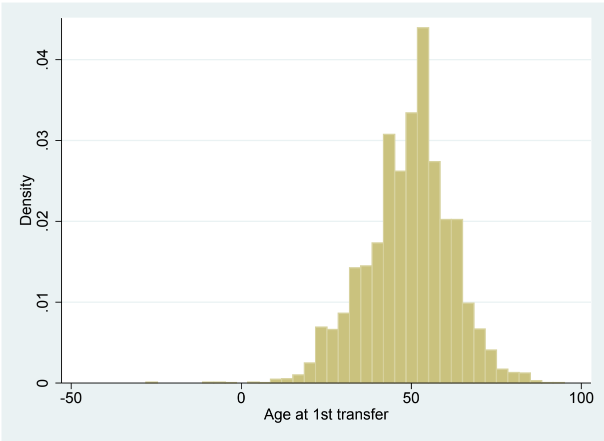
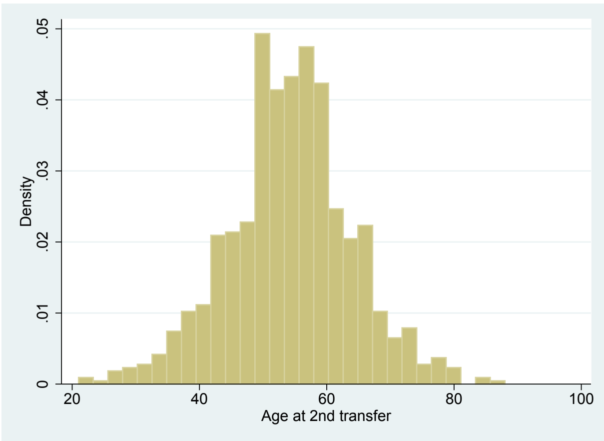


Figure 3: Age at which second transfer was received



Source: SHARE data set, first wave.

Table 2: How the residence was acquired

	Count	%
purchased or built solely with own means	11,612	78.8
purchased or built with help from family	1,009	6.8
received as a bequest	1,446	9.8
received as a gift	201	1.4
acquired through other means	463	3.1

Source: SHARE data set, first wave.

a bequest or of an *inter-vivos* gift. This is why we describe gifts and bequests simply as *transfers* below. These transfers are concentrated. Most of the surveyed individuals never received a transfer greater than 5000 euros.

We found 6274 individuals who received a transfer greater than 5000 euros at least once in their life[10].

The survey records a list of at most five transfers per individual, with the time at which they were received. In the sample, we observe that 81% of the individuals who received at least a transfer received exactly one such transfer and that 97.5% received less than two transfers. Thus, we are not missing anything important by neglecting the third transfer, etc. We now look at the distributions of the ages at which the principal respondent received the first and the second transfers (conditional on the fact that these transfers do exist). Presumably, the vast majority of all recorded transfers are bequests.

Figure 2 depicts the distribution of the age at which the first transfer (not necessarily the greatest) was received[11]. On Fig. 2 we see that people receive help from their parents at 50 on average. The median age is also around 50. In addition, 75% of the individuals receive their first transfer after 42. We can safely conclude that help coming from the parents, or bequests, arrive late in life. Presumably, young adults would have needed help during their thirties. The data show that they have to wait until they themselves have raised their children and bought a house, before receiving a transfer that is very likely to be a bequest[12]. The median age of the second transfer is slightly larger, at 54. Its standard deviation is smaller than that of the distribution reported on Figure 2 (10 years instead of 12.5), and 75% of the individuals receive their second transfer after 48.

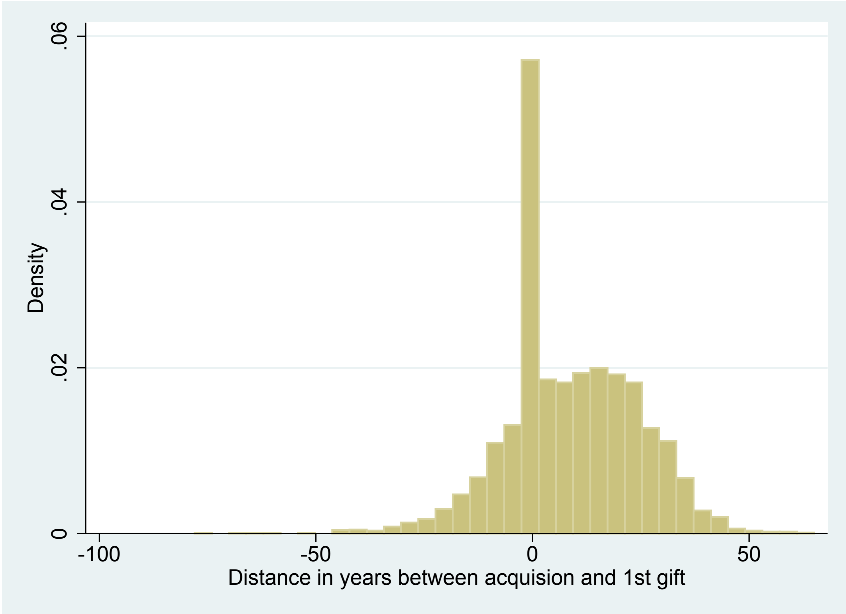
We now study the age at which the main property was acquired as possibly determined or

[10]There are various missing data problems: we selected the largest sub-sample without missing observations of the relevant variables.

[11]In this figure and the next, the same *caveat* is needed: the displayed frequencies are biased estimates of the corresponding probabilities, because data is censored, for persons with ages strictly greater than 50 at the time of the survey may receive a transfer in the years following the survey (but Kaplan-Meier survival estimates would not be very precise for old ages).

[12]It is not easy to know which of the transfers reported in the data are in fact bequests, when we do not know if parents are deceased.

Figure 4: Distribution of distance between house acquisition and first transfer



Source: SHARE data set, first wave. Distribution of distance, in years, between the year of house acquisition and the year the first transfer was received

caused by the transfers. There is a possible relationship between inter-generational transfers and property acquisition. Figure 4 depicts a remarkable distribution, namely that of the difference (in years) between the main residence's acquisition date and the moment of the first transfer. The distribution displayed on Fig. 4 is obviously the mixture of a mass at point 0 (hereafter called the "peak") and a regular bell-shaped density. The distribution's mean is 9 years, with a standard deviation of 15 years, and 50% of the individuals wait at least 7 years, after the transfer, before the acquisition of their property.

It is interesting to study what characterizes individuals in the peak of Fig. 4. Out-of-peak and within-peak individuals are strikingly different. In essence, 83% of the off-peak individuals purchased or built their houses "solely with their own means", while 50% of the within-peak individuals in fact obtained their property directly as a bequest. In the whole population, 7% of the individuals declare that they purchased their home with the help of their family, as shown by Table 2.

Finally, we estimated the role of transfers in access to the owner-occupier status. This is done by means of a linear probability model, regressing the ownership indicator on log-transfers, age, age squared and controls. Table 3 displays the result of three variants of this regression. Take for instance column (2) in this table. In this variant we control for age, age squared and age at first transfer. The profile of the probability of access to ownership is clearly a concave (quadratic) function of age, with a maximal probability at age 40. Given this, the probability of becoming an owner-occupier decreases further with the age of the individual (*i.e.*, principal respondent) at the moment of the first transfer. Since the log-transfer itself has a significant and

Table 3: Ownership explained by gifts

	(1) Owner	(2) Owner	(3) Owner
Log total transfer	0.0403***	0.0395***	
	(0.0052)	(0.0057)	
Age	0.0193**	0.0162*	0.0189**
	(0.0085)	(0.0093)	(0.0088)
Age^2	-0.0002***	-0.0002**	-0.0002***
	(0.0001)	(0.0001)	(0.0001)
Log total income	-0.0035	-0.0016	-0.0018
	(0.0041)	(0.0045)	(0.0042)
Gender	-0.0769***	-0.0899***	-0.0973***
	(0.0130)	(0.0143)	(0.0134)
Age at first transfer		-0.0017***	-0.0023***
		(0.0006)	(0.0006)
Constant	0.0499	0.2011	0.5462*
	(0.2990)	(0.3253)	(0.3037)
R^2	0.04	0.05	0.03
Number of observations	4,022	3,472	3,914

Source: SHARE data set, first wave. Linear probability model.
Significance: *** p-value<0.01, ** p-value<0.05, * p-value<0.1.
Method: OLS.

positive coefficient, we conclude that large and early transfers play a significant role in access to ownership, more or less as expected.

We now explore the possibility of improving efficiency in the transmission of income and wealth between parents and children, taking into account the risks associated with the longevity of parents. This is made possible, in particular by housing equity release instruments.

2.1 Housing equity release contracts: home reversion, lifetime mortgages and *viager*

Various forms of equity release contracts can be used, in many countries, to increase the income and pensions of the seniors. It is also well-known that the recourse to these contracts is still limited. Insufficiently developed financial markets, the tax system and behavioral problems are among the main reasons for the slow growth of these financial markets. Another important reason for this is just the fact that these markets have been opened recently. For instance, in the United States, the reverse mortgage programs started in 1989: this is relatively recent if it takes a generation to get accustomed to these instruments. In continental Europe, the underdevelopment of home equity release markets may also be due to regulation, which is sometimes very strict.

The UK, Canada and Australia have a relatively more developed equity release activity. In the UK for instance, there exist several important types of equity release contracts: the *lifetime mortgage* and the *home reversion*. The lifetime mortgage is a loan secured on the borrower's home. The loan is repaid by selling the property after the borrower's death. The borrower retains ownership of the house. Some of the value of the property can be ring-fenced as an inheritance for children. Borrowers can choose to make repayments, to pay only interest, or to let the accrued interest 'roll-up'. The lifetime mortgage may embody several elements of insurance: caps on the interest rate (if the rate is not fixed) and most importantly, there is a *no negative equity guarantee*. According to this guarantee, when the property has been sold, if the amount is not enough to repay the loan to the lender, the estate (*i.e.*, the heirs or the borrower) will not be liable for the difference between sales revenue and total debt. The latter provision entails an element of life insurance (because of the longevity risk of the borrower) combined with insurance of the real-estate market risks (*i.e.*, house-price risk) — this is a key reason for the existence of a minimum borrower age. The borrower has the right to remain in the property for life, or until he or she needs to move to long-term care. The released money can be drawn as a lump sum or in smaller amounts through time, or both.

Under the *home reversion* plan, part or all of the person's home is sold to a specialized intermediary in return for a lump sum or regular payments. The seller has the right to continue living in the property until death, rent free, but agrees to maintain and insure the property. Again, a percentage of property can be ring-fenced to form a bequest. At the end of the home reversion plan, the property is sold by the intermediary and the proceeds are shared according to the remaining proportions of ownership. The no negative equity guarantee applies. Again, a home reversion plan embodies an element of life insurance: the provider bears the seller's longevity risk and the future house price risks.

In the United States and Canada, *reverse mortgages* are closely related contracts. Again, such a contract allows elders to access equity built up in their homes and defer payment until after death. Interest is added to the loan balance each month and can grow to exceed the value of the home, but the estate is generally not required to repay any loan balance.

France has a traditional and original form of contract closely related to home reversion called the *viager* contract. This form of contract is based on the breakdown of property rights in French civil law. The property of an asset, and in particular, that of a house, can be decomposed in three parts: *usus* (*i.e.*, the right to use the good), *fructus* (*i.e.*, fruit: the right to earn the asset's income or rents) and *abusus* (the right to sell or dispose of the good). The *bare property* is by definition the ownership of a house, without usus and fructus, the latter being the rights to inhabit and (or) earn the associated rents. Under the viager contract, the buyer pays a lump sum, called *bouquet*, and commits to pay a sum of money every year or month, called *rent*, to the seller until the seller dies. The buyer is the *bare owner* of the house until the seller's death; retrieves full ownership when the seller passes away. The seller becomes the usufructuary (that is, is entitled to usus and fructus) when the viager is signed, for life. The buyer is clearly bearing the seller's longevity risk. The value of the house at the moment of the seller's death is also uncertain. Bare ownership means that the property is not yielding any rents. It follows that the discounted, expected (actuarial) value of the periodical payments received by the seller cannot be greater than the discounted expected value of the future rents that can be earned after the seller's death. In France, most of the viager contracts are signed between individuals. This market is underdeveloped because individual buyers have to bear the longevity risk of the seller, which is a typically sub-optimal way of sharing risks. In addition, there is a risk of default on the part of the buyer. These risks should obviously be borne and diversified by insurance companies and banks. The volume of viager sales could grow if powerful, deep-pocket intermediaries did invest a sufficient amount of capital in this market, because intermediated viager contracts could render the same services as home reversion plans[13].

3 A Model of the Intergenerational Transmission of Wealth

We now describe and analyze a simplified model of family contracts, allowing the study of intergenerational transmission. The basic model has three agents and a finite number of periods: the mother, the daughter and the banker-insurer. The mother owns a house and is entitled to a (nonrandom) stream of income. The daughter wishes to buy a home but she might not have enough assets, or a sufficiently high income, to buy a house that is spacious enough. The mother would like to give something to her daughter: to a certain degree at least, the mother is an altruist. At the same time, the mother wants to stay in her house; she doesn't want to move or downsize her consumption of housing. The banker offers to sign a contract with the mother and the daughter simultaneously. Each of the two contracts can be seen as two-party agreements, involving the signature of the banker and one client only, but the banker agrees to sign each of the contracts only if the other contract is signed in the appropriate terms. Alternatively, one can view the two contracts as a single agreement between three parties. There is no difference between the two views at least as long as contract renegotiation is not considered[14].

In the model, there is a single source of risk, namely, the risk affecting the duration of the mother's life. The mother may pass away at the end of any period or survive, and will pass away before the end of the next to last period for sure. For the sake of simplicity, we assume that the daughter lives until the last period for sure and passes away. The mother's after-tax financial and housing wealth is entirely transmitted as a bequest to the daughter at the beginning of the period immediately following her death.

The banker-insurer offers a reverse mortgage to the mother and simultaneously, offers a mortgage contract to the daughter to buy her home[15]. Under the terms of this arrangement, the

[13] A few economists advocated the development of intermediated *viager* markets in France (see Masson (2014)). The economic literature on the French traditional *viager* market is recent and, to the best of our knowledge, papers on this topic are rarities (see Février *et al* (2012)).

[14] If renegotiation may happen, then of course, it is important to specify if unilateral renegotiations are permitted (with the bank) or if the agreement of more than two parties is needed to renegotiate. We are not considering renegotiation here.

[15] The contract offered to the mother by the banker can also be interpreted as a French *viager* contract.

mother receives a downpayment and a sequence of (yearly or monthly) payments of the bank in exchange for her house. The payments stop when the mother dies (annuitization) and the mother keeps the right to stay in her house until the end. The full ownership of the house that was initially sold is transferred to the bank at the moment of the mother's death. The house is then liquidated, at the spot price, on the real estate market. This bundle of contracts simultaneously allows for *(i)*, equity release (transforming an illiquid asset into cash, or a sequence of cash payments); *(ii)*, an inter vivos gift to the daughter; *(iii)*, an ordinary bank loan to the daughter, *(iv)* income annuitization for the mother; and *(v)*, independently of the existence of a bank loan, a certain amount of consumption smoothing through income insurance for the daughter.

3.1 Basic assumptions

There exists a competitive spot market and a rental market for housing in each period t. The spot price of a square meter in period t is denoted q_t. The rent of a square meter available in period t is denoted R_t. We assume that these prices are non-random and that the agents have perfect foresight, for simplicity. We will see later that these assumptions can be relaxed and how restrictive they are.

Let y_{0t}, resp. y_{1t}, denote the net disposable income of the mother, resp., of the daughter in period t. These variables are non-random and perfectly foreseen. For instance, to fix ideas, one can assume that the mother is retired and earns a constant pension payment y_0, while the daughter is a civil servant with lifetime employment and a non-risky income profile (y_1). This is of course for the sake of simplicity. We focus on longevity risk, on the life insurance problem, on the intergenerational transfer and housing-equity release problems. Generalizations to cases in which incomes, real estate prices and rents are random are of course possible, and are not particularly difficult. For instance, in the case of income risk, the optimal intermediated family contract would depend on the expected discounted value of the family's income (in other words, the family's permanent income). It follows that the actuarial budget constraint defined below can be reinterpreted as depending on the expected values of the mother's and the daughter's incomes, properly discounted.

The mother owns a house of size $H_0 > 0$. House size is expressed in units of physical area, *i.e.*, in square meters. The daughter buys a house of size H_1; she may also rent square meters in period t.

We now define contingent consumption. There are two goods in the economy at each period: a consumption good and housing services. The mother's consumption in period t is denoted c_{0t}. The consumption profile of the daughter, if the mother survives in period t, is denoted c_{1t}. The mother dies in period T, drawn at random. The continuation consumption profile of the daughter, from $t = T + 1$ on, is denoted $(c_{1t}(T))$. In other words, for $t > T$, $c_{1t}(T)$ denotes the contingent consumption of the daughter.

The daughter dies at the beginning of period $\mathcal{T} + 2$ (or at the end of $\mathcal{T} + 1$) ; she sells her house on a futures market in period $\mathcal{T} + 1$. The daughter has no children and no bequest motive, and as a consequence, she liquidates her housing capital entirely before death (and will be allowed to borrow against the value of her home). For simplicity, the daughter doesn't adjust ownership after period 1: H_1 is fixed in periods $t = 1, \ldots, \mathcal{T} + 1$.

The daughter can also rent additional square meters at each date t. Again, we consider contingent housing consumption. Let h_t denote rented housing in square meters if the mother survives in period t, and let $h_t(T)$ denote rented square meters in periods $t > T$ if the mother passes away at the end of period T.

The mother and the daughter are endowed with additively separable, instantaneous Von Neumann-Morgenstern utility functions for consumption and housing. Let $u(c)$ denote the instantaneous utility of consumption (the same for the mother and the daughter, to keep the model simple). Let v_0, resp v_1 denote the separable utility of housing services for the mother and the daughter, respectively. Let β_0, resp. β_1 denote the discount factors of the mother and the daughter, respectively. Let r be the bank's discount rate (*i.e.*, the rate at which the bank itself can borrow). The bank's discount factor is denoted

$$\beta = \frac{1}{1+r}.$$

The mother's instantaneous utility for housing consumption is denoted $v_0(h; H_0)$, where h represents current consumption and H_0 is the reference point, *i.e.*, the house owned by the mother at the beginning of period 1.

We assume the following.

Assumption 1.
(a) Utility v_0 is continuous, strictly increasing, strictly concave and differentiable almost everywhere with respect to current housing consumption h. We assume the existence of a kink at point (H_0, H_0). The left-hand side partial derivative of v_0 with respect to the first variable h is higher than its right-hand derivative.
(b) The utilities u and v_1 are twice continuously differentiable, strictly increasing and strictly concave functions.
(c) $\lim_{c \to 0} u'(c) = +\infty$; $\lim_{h \to 0} (\partial v_0 / \partial h)(h; H) = +\infty$, $\lim_{h \to 0} v_1'(h) = +\infty$.

The assumption relative to v_0 allows us to formally represent the psychological fact that the mother clings to her house, as a kind of "habit formation" model for housing. The concavity assumptions formally capture risk aversion as usual.

We now consider the financial contracts and the taxes.

1. The mother can sell a fraction of her property to the bank under the provision that the transfer of ownership will be effective only after the mother's death, and that she has the right to stay in her house until she dies. Let z_0 denote the fraction of the house sold at $t = 1$ as a home reversion contract. The mother can also downsize her housing consumption and sell a fraction z_t, on the spot market in period t, if she survives in period t. We assume that $z_t \geq 0$

and obviously, the mother cannot sell more than her house,

$$0 \leq \sum_{t=0}^{t=T} z_t \leq H_0.$$

Given Assumption 1, the instantaneous utility of the mother in period t can be written as follows,

$$v_0\left(H_0 - \Sigma_{\tau=1}^{t} z_\tau; H_0\right) + u(c_{0t}).$$

The instantaneous utility of the daughter is given by the following expression,

$$v_1(H_1 + h_t) + u(c_{1t}).$$

2. The mother receives a sequence of payments from the bank in period t, also as a consequence of the sale. The net transfer of the bank to the mother in period t is denoted M_{0t}. This amount of money is of course paid only if the mother survives (*i.e.*, this is an annuity).

3. The net transfer from the bank to the daughter is denoted M_{1t}, if the mother survives in period t, and denoted $M_t(T)$, if the mother dies in period T and $t > T$. The bank may lend money to the daughter to buy her house in period $t = 1$, and the transfers may embody loan repayment charges.

4. Let x_t denote the direct transfer from the mother to the daughter (a gift of the mother in period t is a nonnegative x_t). Let τ_g denote the tax rate on inter vivos transfers.

5. When the mother dies, any remaining property (not already sold to the bank or on the real estate market) is given to the daughter in the form of a bequest and the spot market value is taxed. Let τ_b denote the tax rate on bequests.

The family is endowed with a welfare function. This is tantamount to assuming that the mother is a pure altruist, and a benevolent dictator, in her family. This is formalized as follows. Let U denote the expected utility of the mother; let V denote the expected utility of the daughter.

Assumption 2. The preferences of the family over contingent resource allocation are represented by the utility $U + \gamma V$, where $\gamma > 0$ is a parameter.

Parameter γ can be interpreted as measuring the mother's degree of concern for the welfare of her daughter. Indeed, maximizing $U + \gamma V$ subject to the zero-profit constraint of the banker, the budget constraints of the mother and the daughter, and the sign and feasibility constraints, will yield a Pareto-optimal contract in the economy comprising the mother, the daughter and the banker. The set of Pareto-optimal arrangements (with zero profit for the bank) is obtained when γ varies in the set of positive real numbers. Instead of a zero-profit constraint, we can impose that the bank's expected profit must be larger than, or equal to, a given constant π_0, and that the utility of the daughter be larger than or equal to a given value V_0. If we maximize the utility of the mother subject to these constraints, we obtain a Pareto-optimal allocation (and a Pareto-optimal contract). Then, varying π_0 and V_0, we can describe all the Pareto-optima in the economy with three agents: mother, daughter, banker.

Let us now define the left derivative of v_0

$$\frac{\partial v_0(h, H_0)}{\partial h}^{-} = \lim_{h \to H_0, h < H_0} \frac{v_0(h, H_0) - v_0(H_0, H_0)}{h - H_0}.$$

The value of this left derivative at point (H_0, H_0) represents the strength of the mother's psychological ties to her house. It may also represent other unmodeled reasons for willing to remain an owner occupier.

Recall that the daughter dies at the end of $\mathcal{T} + 1$, and that the mother dies at the end of period T, which is random. We assume that $1 \leq T \leq \mathcal{T}$. The daughter will outlive her mother by at least one period. Let p_T be the unconditional probability of the mother dying at T. Thus, we have,

$$\sum_{T=1}^{\mathcal{T}} p_T = 1.$$

The cumulative distribution is denoted P_t, that is, by definition,

$$\Pr(T < t) = \sum_{T=1}^{t-1} p_T = P_t.$$

The survival function is denoted S_t. By definition again, we have,

$$S_t = \Pr(T \geq t) = 1 - P_t.$$

To pose the optimal-contract problem, we consider the contingent consumptions of private good and housing services defined above. The family contract becomes contingent with respect to the mother's death event, *i.e.*, to the drawing of $T \in \{1, \ldots, \mathcal{T}\}$. The mother's utility, conditional on death at date T, is denoted U_T, and can be written,

$$U_T = \sum_{t=1}^{T} \beta_0^{t-1} \left[v_0 \left(H_0 - \sum_{\theta=1}^{t} z_\theta \right) + u(c_{0t}) \right]. \tag{1}$$

The mother's expected utility is simply

$$U = \sum_{T=1}^{\mathcal{T}} p_T U_T, \tag{2}$$

As is well-known, letting the survival probability appear, we can easily rewrite this expression as follows,

$$U = \sum_{t=1}^{\mathcal{T}} \beta_0^{t-1} S_t \left(v_0 \left(H_0 - \sum_{\theta=1}^{t} z_\theta \right) + u(c_{0t}) \right). \tag{3}$$

The daughter's utility conditional on T is denoted V_T, and by definition, we have,

$$V_T = \sum_{t=1}^{T} \beta_1^{t-1} \left[v_1(H_1 + h_t) + u(c_{1t}) \right] + \sum_{t=T+1}^{\mathcal{T}+1} \beta_1^{t-1} \left[v_1(H_1 + h_t(T)) + u(c_{1t}(T)) \right], \tag{4}$$

and

$$V = \sum_{T=1}^{\mathcal{T}} p_T V_T. \tag{5}$$

Using the fact that $S_{\mathcal{T}+1} = 0$, again, we can rewrite V as follows,

$$\begin{aligned} V = &\sum_{t=1}^{\mathcal{T}+1} \beta_1^{t-1} S_t \left[v_1(H_1 + h_t) + u(c_{1t}) \right] \\ &+ \sum_{T=1}^{\mathcal{T}} p_T \sum_{t=T+1}^{\mathcal{T}+1} \beta_1^{t-1} \left[v_1(H_1 + h_t(T)) + u(c_{1t}(T)) \right]. \end{aligned} \tag{6}$$

3.2 The actuarial budget constraint

We now consider budget constraints and derive the actuarial resource constraint. Recall that (M_{jt}) denote the transfers from the banker to the mother ($j = 0$) and to the daughter ($j = 1$). The transfer to the mother stops after death, that is, $M_{0t} = 0$ if $t > T$. Given these definitions, the mother's budget constraints can be written as follows,

$$c_{0t} + x_t \leq M_{0t} + y_{0t} + q_t z_t, \tag{7}$$

for all $t \leq T$. The daughter's budget constraints, for $t \leq T$, are written as follows,

$$c_{1t} + R_t h_t + q_1 H_1 \delta_{1t} \leq M_{1t} + y_{1t} + (1 - \tau_g) x_t, \tag{8}$$

where $\delta_{1t} = 1$ if $t = 1$ and 0 otherwise. The family's aggregate budget constraint now becomes,

$$c_{0t} + c_{1t} + R_t h_t + q_1 H_1 \delta_{1t} + \tau_g x_t \leq M_t + y_t + q_t z_t, \tag{9}$$

again for $t \leq T$, where, to simplify notation, we denote

$$M_t = M_{0t} + M_{1t} \qquad \text{and} \qquad y_t = y_{0t} + y_{1t}.$$

After T, the bank's transfers to the daughter at t depend on T, and are denoted $M_t(T)$. The daughter's budget constraints have a different structure for $t > T$. If $t > T + 1$ and $t < \mathcal{T} + 1$, we have,

$$c_{1t} + R_t h_t(T) \leq M_t(T) + y_{1t}. \tag{10}$$

At time $t = T + 1$ the daughter receives the mother's bequest and pays the inheritance tax. We have,

$$c_{1,T+1}(T) + R_{T+1} h_{T+1}(T) \leq M_{T+1}(T) + y_{1,T+1} + (1 - \tau_b) q_{T+1} \left(H_0 - \Sigma_{t=0}^{T} z_t \right). \tag{11}$$

At time $t = \mathcal{T} + 1$ the daughter's house is sold at price $q_{\mathcal{T}+2}/(1 + r)$ and the daughter dies at the end of period $\mathcal{T} + 1$. If $T < \mathcal{T}$ we have the final budget constraint,

$$c_{1,\mathcal{T}+1}(T) + R_{\mathcal{T}+1} h_{\mathcal{T}+1}(T) \leq M_{\mathcal{T}+1}(T) + y_{1,\mathcal{T}+1} + q_{\mathcal{T}+2} \frac{H_1}{(1 + r)}. \tag{12}$$

The banker's expected profit, conditional on T, is denoted π_T. For $1 \leq T \leq \mathcal{T}$, we have,

$$\pi_T = \frac{q_{T+1} z_0}{(1+r)^T} - \sum_{t=1}^{T} \frac{M_t}{(1+r)^{t-1}} - \sum_{t=T+1}^{\mathcal{T}+1} \frac{M_t(T)}{(1+r)^{t-1}}. \tag{13}$$

To simplify notation, we now use the discount factor $\beta = (1+r)^{-1}$. The bank's expected profit is by definition,

$$\pi = \sum_T p_T \pi_T.$$

The bank's profitability constraint is simply $\pi \geq 0$. This can be rewritten,

$$\pi = z_0 X - \sum_{t=1}^{\mathcal{T}} \beta^{t-1} S_t M_t - \sum_{t=1}^{\mathcal{T}} p_T \sum_{t=T+1}^{\mathcal{T}+1} \beta^{t-1} M_t(T) \geq 0, \tag{14}$$

where the unit expected value of the home sold at random date $T+1$ is given by the following expression,

$$X = \sum_{T=1}^{\mathcal{T}} \beta^T p_T q_{T+1}. \tag{15}$$

Now, if we substitute the agent's budget constraints, expressed as equalities, in the profitability constraint $\pi \geq 0$, taking into account the fact that $S_1 = 1$ and $S_{\mathcal{T}+1} = 0$, some easy algebra yields the following actuarial resource constraint,

$$\begin{aligned} q_1 H_1 + \sum_{t=1}^{\mathcal{T}} \beta^{t-1} S_t (c_t + R_t h_t + \tau_g x_t) + \sum_{T=1}^{\mathcal{T}} p_T \sum_{t=T+1}^{\mathcal{T}+1} \beta^{t-1} (c_{1t}(T) + R_t h_t(T)) \\ \leq z_0 X + Y + \sum_{t=1}^{\mathcal{T}} \beta^{t-1} S_t q_t z_t + (1-\tau_b) \sum_{T=1}^{\mathcal{T}} \beta^T p_T q_{T+1} \left(H_0 - \Sigma_{t=0}^{T} z_t\right) + q_{\mathcal{T}+2} \beta^{\mathcal{T}+1} H_1. \end{aligned} \tag{16}$$

The left-hand side of (16) is the sum of the values of all contingent goods, including gifts, the price of which depends on the tax rate τ_g. The right-hand side of (16) is the sum of all resources in expected present value terms. More precisely, the first term is the value of the fraction of the mother's house sold to the bank, $z_0 X$. The second term is the family's expected wealth, derived from income flows, that is, by definition,

$$Y = \sum_{t=1}^{\mathcal{T}+1} \beta^{t-1} (S_t y_t + P_t y_{1t}).$$

Note, at this point, that if the income profiles (y_{jt}) were random, the values of y_{jt} could be replaced with their expected values in the expression of Y, and the derivation of the optimal contract would be the same (but the values of M_{jt} would become income-contingent).

The third term on the RHS, that is, $\Sigma_{t=1}^{\mathcal{T}} \beta^{t-1} S_t q_t z_t$, is the expected value of revenues related to planned property-downsizing sales. The next to last term is the net expected after-tax value of inheritance,

$$(1-\tau_b) \sum_{T=1}^{\mathcal{T}} \beta^T p_T q_{T+1} \left(H_0 - \Sigma_{t=0}^{T} z_t\right).$$

The last term is the value of the daughter's house once resold after the daughter's death. It follows that the effective present price of property is

$$Q_1 = q_1 - q_{\mathcal{T}+2}\beta^{\mathcal{T}+1}. \tag{17}$$

For future reference, we define a variable X_t as follows, for all $t \geq 1$,

$$X_t = S_t^{-1} \sum_{T=t}^{\mathcal{T}} \beta^{T+1-t} p_T q_{T+1}. \tag{18}$$

Note that $X_1 = X$, as defined by (15) above. This new variable is the conditional expectation, discounted at date t, of the real estate price, that is, $X_t = \mathbb{E}_T[q_{T+1}\beta^{T+1-t} \,|\, T \geq t]$.

3.3 Optimal Intermediated Family Contracts

It is now possible to study the optimal family contracts. The optimal intermediated family contracts are obtained as solutions of the following program: maximize $U + \gamma V$ subject to the actuarial resource constraint (16), (hereafter ARC), the feasibility constraint $H_0 - \Sigma_{t=0}^{\mathcal{T}} z_t \geq 0$, and sign constraints on consumption c, housing ownership H_1, renting h, and partial liquidation z_t.

Let λ denote the Lagrange multiplier of the ARC. To simplify the analysis, we assume the following about real estate prices.

Assumption 3. The sequences of real estate prices (q_t) and rents (R_t) are deterministic and $Q_1 > 0$.

We can now state a number of results characterizing the optimal contract.

Proposition 1. Assume that the tax rate on inheritance is positive, *i.e.*, $0 < \tau_b \geq 1$. Condition (a), or condition (b) below are sufficient for property (c).
If, either

(a), $q_t \leq X_t$ for all $t \geq 1$,

or

(b), $(\partial v_0(H_0, H_0)/\partial h)^-$ is large enough,

then,

(c), $H_0 = z_0^*$ and $z_t^* = 0$ for all $t \geq 1$, where starred variables denote optimal values. In other words, the mother's house is sold entirely to the bank under a home reversion contract.

For proof, see the Appendix.

To interpret condition (a) in Proposition 1, recalling definition (18), assume that the mother is still alive and that $q_t > X_t$ at some time t. Then, if the mother's house had no consumption value, it should be sold or let at time t, because waiting implies expected losses. Assume that housing prices are increasing at a rate κ, that is, $q_t = (1 + \kappa)^{t-1}q_1$. Then, it is easy to check that $q_t \geq X_t$ will hold if $\kappa > r$, that is, if the rate of growth of prices is larger than the bank's base rate. Condition (b) says that if the marginal value of the mother's house is large enough at point H_0, she will prefer to stay in her house and refuse to downsize her housing consumption, even if house prices are falling sharply in the future. Both conditions can be combined and we would find that statement (c) of Proposition 1 holds if house prices are not falling too quickly and the mother is sufficiently attached to her home.

We now consider the daughter's choice of house size H_1^* and the daughter's tenure choice. The daughter is allowed to rent and (or) to buy a certain quantity of housing. We want to characterize the market conditions under which the daughter indeed at least weakly prefers to buy her home over renting. We obtain the following proposition.

Proposition 2

(i) The family prefers ownership of the daughter's house starting from $t = 1$ to renting during all subsequent periods only if the following tenure arbitrage inequality holds,

$$Q_1 \leq \sum_{t=1}^{\mathcal{T}+1} \beta^{t-1} R_t. \tag{19}$$

(ii) If rents are sufficiently high, that is, more precisely, if

$$R_t > Q_1 \frac{(1-\beta_1)}{(1-\beta_1^{\mathcal{T}+1})} \left(\frac{\beta_1}{\beta}\right)^{t-1}. \tag{20}$$

then, the daughter is not renting any housing area, *i.e.*, $h_t^* = 0$, $h_t^*(T) = 0$, and (19) holds as a strict inequality.

(iii) Assuming that (20) holds, the daughter's house size is uniquely determined by the following equation,

$$v_1'(H_1^*) = \frac{\lambda^*}{\gamma} \frac{(1-\beta_1)}{(1-\beta_1^{\mathcal{T}+1})} Q_1. \tag{21}$$

For proof, see the Appendix.

Remark that if $Q_1 < 0$, then, any household can make sure profits by just buying a house, staying in it until $\mathcal{T}+1$ and reselling it, because the net present value of the real estate investment would be positive, even if rents were zero.

If the housing market is competitive and if the standard arbitrage equation linking rents and prices holds (*i.e.*, if the arbitrage inequality (19) holds as an equality), renting and buying are equivalent decisions. In this case, we could easily assume that the daughter has (at least) a slight preference for ownership and derive the result that renting is zero. This preference for ownership may be well-justified, for various reasons that are not captured by our simple model. A simple way of introducing a bias towards ownership is to assume that the daughter's utility for housing services v_1 has two arguments, *i.e.*, we have $v_1 = v_1(h, H)$ instead of $v_1 = v_1(h + H)$, and the marginal utility of a square meter is higher when owned. We focus on daughters with at least a slight preference for ownership.

The next step is to study the optimal consumption profile of the mother and the daughter: this yields indications on the consumption smoothing or income insurance properties of the optimal family contract. We obtain the following proposition.

Proposition 3

The optimal consumption path is determined as follows.

(i) The Lagrange multiplier λ^* is positive and equal to the marginal utility of consumption of the mother in the first period,

$$u'(c_{01}^*) = \lambda^*.$$

(ii) The sharing rule is determined by γ as follows,

$$\gamma \beta_1^{t-1} u'(c_{1t}^*) = \beta_0^{t-1} u'(c_{0t}^*).$$

(iii) The inter-temporal allocation of consumption is determined by the Euler equations,

$$u'(c_{0t}^*) = u'(c_{01}^*) \left(\frac{\beta}{\beta_0} \right)^{t-1},$$

for all $t \leq \mathcal{T}$.

(iv) The contingent consumption $c_{1t}(T)$ doesn't depend on p_T: we find

$$u'(c_{1t}^*(T)) = (\lambda/\gamma)(\beta/\beta_1)^{t-1}$$

and therefore, given the Euler equation above, the *full insurance property* holds, that is, for all $t > T$,

$$c_{1t}^*(T) = c_{1t}^*,$$

The event of the mother's death at T has no impact on her daughter's consumption.

For proof, see the Appendix.

The sharing rules show how parameter γ, measuring the mother's altruism, determines the division of income between the two women. If $\beta_0 = \beta_1$, the rule boils down to equalization of weighted marginal utilities $\gamma u'(c_{1t}^*) = u'(c_{0t}^*)$. This is natural in this context of Utilitarian welfare maximization. The intertemporal allocation rules are standard "Euler equations", expressing consumption smoothing. The dynamics of consumption are determined by the discount factors β_1 and β_0. The optimal consumption is constant through time if $\beta_0 = \beta_1 = 1/(1+r)$. The last two properties express the fact that the daughter is completely insured against the consequences of the mother's risk of early death. The banker is clearly providing longevity insurance here.

Finally, we determine the optimal gifts. We have the following clearcut result: optimal gifts are zero because there is a tax on gifts.

Proposition 4. If the tax rate on inter vivos gifts τ_g is positive, the optimal gifts are zero, that is, $x_t^* = 0$ for all t.

For proof, see the Appendix.

The interpretation of the result is that, if the intermediated transfers between the mother and the daughter are not considered as straightforward gifts, then, these transfers coming from the bank are cheaper and should be preferred to any direct transfer of income (or property) between the mother and the daughter. The optimal family contract may be interpreted as a hidden transfer from the mother to the daughter by the tax authority. It is then possible to give a definition of the implicit transfers made through the bank intermediary, but to a large extent, this definition is a question of convention. The optimal intermediated contract may clearly play the role of a tax optimization instrument.

It is now possible, and easy, to compute the optimal family contract. The first thing that we need is the optimal value of λ^*. To solve the model completely, we need to compute the value of λ at the optimal solution. This can be done by first solving for the consumption profile as a function of λ. Then, the ARC provides an equation that determines λ^* — the entire solution depends on this key parameter. This equation is nonlinear and must be solved numerically, except in special cases. Going back to the period-by-period budget constraints for the mother and the daughter taken separately, we derive the optimal values of transfers with the bank, *i.e.*, the M^*_{jt} values.

By assumption, the marginal utility functions u' and v'_1 are invertible. Let ϕ denote the inverse of u' and ψ denote the inverse of v'_1. With these definitions, it is easy to solve for optimal consumption and housing as a function of λ^*. We have,

$$c^*_{01} = \phi(\lambda^*); \qquad c^*_{0t} = \phi\left[\lambda^*\left(\frac{\beta}{\beta_0}\right)^{t-1}\right];$$

$$c^*_{1t} = \phi\left[\frac{\lambda^*}{\gamma}\left(\frac{\beta}{\beta_1}\right)^{t-1}\right];$$

and so on, and

$$H^*_1 = \psi\left[\frac{\lambda^*}{\gamma}\frac{(1-\beta_1)}{(1-\beta_1^{\mathcal{T}+1})}Q_1\right].$$

We then substitute these expressions in the actuarial resource constraint (16) above, expressed as an equality since $\lambda^* > 0$ implies that this constraint is always binding. This yields,

$$Q_1H^*_1 + \sum_{t=1}^{\mathcal{T}+1}\beta^{t-1}S_tc^*_{0t} + \sum_{t=1}^{\mathcal{T}+1}\beta^{t-1}c^*_{1t} = H_0X + Y, \tag{22}$$

using again $S_{\mathcal{T}+1} = 0$, $P_1 = 0$, and $S_t + P_t = 1$. This is an equation in λ^* that can be solved numerically.

When λ^* is known, using the equations stated in Proposition 3, we derive the optimal consumption plan for the mother and the daughter. Finally, the optimal consumption plan in turn determines the optimal *annuity payment* M^*_{0t}, the optimal *net transfers* between the bank and the daughter, M^*_{1t}, the net contingent transfer $M^*_t(T)$ for $t > T$, the optimal size of the daughter's property H^*_1. Note that the optimal downpayment of the bank to the mother is now just a part of the first transfer to the mother, that is, a part of M^*_{01}. Given that the mother is risk averse, it will be optimal to smooth her consumption, and this can be implemented with a zero downpayment of the bank to the mother.

Given the optimal values of c^*_{jt} and H^*_1, we finally derive the optimal contract payments as follows.

Proposition 5. The optimal intermediated family contract is defined by the following list of variables.

$$M^*_{0t} = c^*_{0t} - y_{0t}; \tag{23}$$

$$M^*_{11} = c^*_{11} + q_1 H^*_1 - y_{1t}; \tag{24}$$

$$M^*_{1t} = c^*_{1t} - y_{1t}; \tag{25}$$

$$T < t < \mathcal{T} + 1 \Rightarrow M^*_t(T) = c^*_{1t} - y_{1t}; \tag{26}$$

$$M^*_{\mathcal{T}+1}(T) = c^*_{1t} - y_{1t} - \beta q_{\mathcal{T}+2} H^*_1; \tag{27}$$

The optimal contract is characterized by the absence of inheritance, full housing equity release on the part of the mother, full annuitization of family wealth, consumption smoothing for both family members. The optimal contract typically entails a loan to the daughter, as indicated by (24), so that we expect M^*_{11} to be very large, because the period 1 transfer to the daughter covers the price of the house $q_1 H^*_1$ and later transfers may be negative to repay the loan, principal and interest. Finally, the bank will "tax" (extract) the value of the daughter's house in the last period, as indicated by (27): this allows to smooth the daughter's consumption until the end. Intuitively, a downpayment (from the bank to the mother) could have been used by the mother in period $t = 1$ as an *inter-vivos* gift, to help the daughter buying her house, but the bank is implicitly doing this job on behalf of the mother, so that no explicit gift appears. As a consequence the taxation of this implicit gift would be problematic. In particular, the gift can now appear as a subsidization of the daughter's mortgage interest (or principal) repayments. In practice, it would be difficult to decide if the daughter is borrowing on good terms, or if the bank is in fact a middleman, executing a sequence of gifts to the daughter, planned by the mother. This shows that taxing gifts is difficult, except in the case in which the mother gives her house to the daughter. In this case, the observable gift can be taxed by the government.

3.4 Banking Monopoly Model

The basic family contract model can be turned "upside down" to study surplus extraction by the banker. Instead of maximizing $U + \gamma V$ subject to, say $\pi \geq 0$, or $\pi \geq \pi_{\min}$, where $\pi_{\min}$ is some reservation profit level for the bank, we study the profit maximization problem, that is, we maximize the expected profit π, subject to $U + \gamma V \geq W_0$, where W_0 is the best outside option of the family. We study this problem by means of micro-simulations in the next section. A difficulty stems from the fact that there are many ways of modelling the family's outside option W_0.

One possible definition of the outside option of the family is the best arrangement that the family can carry out without any help from the bank. But this point of view is too extreme. It is more reasonable to define the outside option as the best arrangement that the mother and the daughter can achieve with the help of a limited set of services from the bank (for instance, just a banking account and standard mortgage loans, etc.). In addition, it seems natural to assume that family members are subjected to the inheritance tax, and that the mother can transfer money in cash to her daughter without paying taxes on the inter vivos transfer (simply because the cash transfers are not observable). This type of arrangement will typically be suboptimal, insofar as risk sharing is imperfect. As a consequence, there will be a surplus generated by the intermediated family contract, and the bank will be able to extract this surplus from the family, under the banking monopoly assumption. We have chosen to explore the consequences of a particular definition of the outside option. The merit of the proposed approach is that it represents what many people do in the real world[16].

Thus, to model the outside option of the family, our assumptions are the following:

1. The daughter buys a first house at time $t = 1$ with the help of a standard mortgage loan, and resells this house at the beginning of period $T + 1$, after her mother's death.

2. The daughter buys a second house with the help of the mother's bequest, with the proceeds of the sale of the first house, net of anticipated reimbursement of the first mortgage, and if needed, with a second mortgage loan.

3. Mother and daughter cannot borrow, except in the form of a standard mortgage loan. Both family members have no intention to save, except under the form of mortgage repayments.

4. Mother and daughter can share the total income of the family $y_{0t} + y_{1t}$ within each period, by means of direct transfers in cash.

5. The daughter is myopic: the first house is bought under the assumption that the mother will never die.

The familiar consequence of these assumptions is that there will be a shock on the daughter's consumption and housing services consumption after the mother's death. The daughter is therefore not well insured (more precisely, her rate of consumption is not smoothed), and given her nonzero degree of risk aversion, the allocation of resources used as an outside option will be inefficient. Another important effect of the contract is of course to release the value of the mother's house, allowing the daughter to buy a larger house sooner. These properties in turn imply that the banker can extract some surplus by offering a family contract allowing a less inefficient allocation.

[16] Any good alternative choice for the outside option would typically be characterized by second-best constraints, bearing on the intertemporal allocation of income and wealth within the family. For instance, the best outside option problem could be the result of weighted utility maximization subject to a sequence of budget constraints, instead of a single intertemporal, actuarial resource constraint, as defined above. These problems are technically much more difficult to solve by numerical methods, in a micro-simulation context.

4 Housing, Welfare and Profitability: Calibration and Numerical Simulations

We now endeavour to study the model by means of simulations. We first specify the model completely by choosing functional forms for utility functions. We then calibrate the model, using appropriate actuarial life tables and values of parameters that are generally viewed as acceptable in the literature. Next, we generate a random sample of families reflecting these parameters. As an illustration, we choose to simulate well-to-do families of the Paris area, and therefore use French data to calibrate the model. We compute the optimal contract for each individual in the sample and compute the value of an outside option for each of these families as well. We compute the utilities and the profit of a bank underwriting the family contract. Finally, we study the properties of housing demand, consumption, expected utility and expected profit in the artificially generated sample of families. The optimal contract under study is the full-insurance family contract, exactly as defined in sub-section 3.1 above, with families comprising a mother and a daughter.

Simulations are computed under some simplifying assumptions. We assume that the income levels of the mother and of the daughter are constants, y_0 and y_1, respectively. As noted above, this assumption is harmless when we study the optimal family contract. We choose standard CRRA utilities, namely,

$$u(c) = \frac{c^{1-\sigma} - 1}{1-\sigma},$$

if $\sigma \neq 1$, where the index of relative risk aversion is $\sigma \geq 0$, and

$$v_1(H) = v_0(H) = \frac{\alpha(H^{1-\sigma} - 1)}{1-\sigma},$$

where $\alpha > 0$ is a parameter measuring the importance of housing for the daughter.

In addition, we assume that house prices are predictable, with a rate of growth κ, that is, $q_t = q_1(1+\kappa)^{t-1}$. Finally, we assume that the discount factors of the mother and the daughter are equal, *i.e.*, $\beta_1 = \beta_0$.

4.1 Computation of the Outside Option: the Myopic Contract

The myopic family contract (hereafter, the MFC) is an arrangement with the bank defined as follows.

The daughter buys a house of size H_1 at date $t = 1$ with a mortgage from the bank at a rate $\rho > r$. Define,

$$\delta = \frac{1}{1+\rho}.$$

The mortgage is a constant amortization loan that lasts during the daughter's entire life, *i.e.*, until date $\mathcal{T}+1$, for simplicity. The daughter borrows $q_1 H_1$ and plans to pay a constant amortization annuity A until $\mathcal{T}+1$. The constant repayment is $A q_1 H_1$ where

$$A = \frac{1-\delta}{1-\delta^{\mathcal{T}+1}}.$$

The family budget constraint is therefore,

$$Aq_1 H_1 + c_0 + c_1 = y_0 + y_1, \tag{28}$$

in any period $t \leq T+1$. The family's optimal consumption vector $(\hat{c}_0, \hat{c}_1, \hat{H}_1)$ is obtained as the bundle maximizing the per-period utility

$$u(c_0) + v_0(H_0) + \gamma[u(c_1) + v_1(H_1)],$$

subject to (28), where H_0, the mother's house size, is given. The solution is easy to compute, we get the explicit, closed-form solutions,

$$\hat{c}_0 = (y_0 + y_1)\Gamma; \tag{29}$$

$$\hat{c}_1 = (y_0 + y_1)(1/\gamma)^{-1/\sigma}\Gamma; \tag{30}$$

$$\hat{H}_1 = (y_0 + y_1)(Aq_1/\alpha\gamma)^{-1/\sigma}\Gamma; \tag{31}$$

where

$$\Gamma = \frac{1}{1 + (1/\gamma)^{-1/\sigma} + (Aq_1/\alpha\gamma)^{-1/\sigma}}.$$

The mother dies at random time T. The daughter inherits the mother's house. The mother's house is sold at time $T+1$ and the net proceeds are $q_{T+1}H_0(1-\tau_b)$. In addition, the daughter sells her first house and obtains $q_{T+1}\hat{H}_1$. The daughter then reimburses the remainder of her loan's principal to the bank at date $T+1$. Let K_{T+1} denote the remaining capital at time $T+1$, we must have,

$$K_{T+1}\delta^T + Aq_1\hat{H}_1 \sum_{t=1}^{T} \delta^{t-1} = q_1\hat{H}_1.$$

Some elementary algebra yields,

$$K_{T+1} = q_1\hat{H}_1 \frac{1 - \delta^{\mathcal{T}+1-T}}{1 - \delta^{\mathcal{T}+1}}. \tag{32}$$

We finally obtain the daughter's wealth at date $T+1$, denoted D_{T+1},

$$D_{T+1} = q_{T+1}(\hat{H}_1 + (1-\tau_b)H_0) - K_{T+1}. \tag{33}$$

The daughter then buys a second house of size $\hat{H}_2$ at time $T+1$. She borrows

$$L_{T+1} = q_{T+1}\hat{H}_2 - D_{T+1} \tag{34}$$

from the bank at rate ρ. This loan is reimbursed by means of constant repayment annuities until $\mathcal{T}+1$. If $L_{T+1} < 0$, the daughter in fact receives a constant annuity payment from the bank at rate ρ, as a reward for her capital. We assume that the rate does not depend on the sign of L_{T+1} (this is for the sake of simplicity, avoiding a kink at point $q_{T+1}\hat{H}_2 = D_{T+1}$). The constant repayment annuity for the second loan is $A_2(T)L_{T+1}$, where

$$A_2(T) = \frac{1-\delta}{1 - \delta^{\mathcal{T}+1-T}}. \tag{35}$$

The daughter sells her house during the last period of her life and obtains a constant annuity, at rate r, during periods $T+1 \leq t \leq \mathcal{T}+1$ in exchange for the value of this sale. This is to make sure that the total expected and discounted value available to the family as a whole is the same in the MFC and in the optimal contracts. More precisely, the value of the second house is $\beta q_{\mathcal{T}+2}\hat{H}_2$ at time $\mathcal{T}+1$. Recall that $\beta = (1+r)^{-1}$, where r is the bank's discount rate. Let $A_1(T)$ denote the annuity received from $T+1$ until $\mathcal{T}+1$ in exchange for one unit at time $\mathcal{T}+1$. We must have,

$$A_1(T)\sum_{t=T+1}^{\mathcal{T}+1}\beta^t = \beta^{\mathcal{T}+1}.$$

From this we derive

$$A_1(T) = \frac{(1-\beta)\beta^{\mathcal{T}-T}}{1-\beta^{\mathcal{T}+1-T}}. \tag{36}$$

The daughter therefore earns an additional income $A_1(T)\beta q_{\mathcal{T}+2}\hat{H}_2$ in every period from $T+1$ until the end of her life.

Putting the above elements together, we can write the daughter's budget constraint, for each period $t \geq T+1$, as follows,

$$\hat{c}_{12} + Q_2\hat{H}_2 = y_1 + A_2(T)D_{T+1}, \tag{37}$$

where $\hat{c}_{12}$ denotes the daughter's consumption level after $T+1$, and where Q_2 is the effective price of housing services after $T+1$, that is, precisely,

$$Q_2 = A_2(T)q_{T+1} - A_1(T)\beta q_{\mathcal{T}+2}. \tag{38}$$

It is possible to check that $Q_2 > 0$ if $(1+\kappa)/(1+r)$ is sufficiently small and ρ is sufficiently close to r. We assume that $Q_2 > 0$ holds. To find $(\hat{c}_{12}, \hat{H}_2)$, we maximize the daughter's utility $u(\hat{c}_{12}) + v_1(\hat{H}_2)$ subject to (37), knowing T. This yields the following solution,

$$\hat{c}_{12}(T) = \frac{y_1 + A_2(T)D_{T+1}}{1+\alpha^{1/\sigma}Q_2^{1-1/\sigma}}, \tag{39}$$

$$\hat{H}_2(T) = \frac{y_1 + A_2(T)D_{T+1}}{1+\alpha^{1/\sigma}Q_2^{1-1/\sigma}}. \tag{40}$$

We can now use the solutions $(\hat{c}_0, \hat{c}_1, \hat{H}_1, \hat{c}_{12}, \hat{H}_2)$ obtained above to compute $\hat{U}+\gamma\hat{V}$, the expected utility of the family under the MFC.

We can also compute the profit of the bank under the MFC. After some computations, for given T and given ρ, the present value of the profit (*i.e.*, the net present value of the MFC) can be written,

$$\hat{\pi}_T(\rho) = \hat{H}_1 q_1\left[A\frac{(1-\beta^T)}{(1-\beta)} - 1\right] + \beta^T K_{T+1} + \beta^T L_{T+1}\left[A_2(T)\frac{(1-\beta^{\mathcal{T}+1-T})}{(1-\beta)} - 1\right]. \tag{41}$$

We finally compute the expected NPV of the myopic contract for the bank as follows,

$$\hat{\pi}(\rho) = \sum_{T=1}^{\mathcal{T}} p_T\hat{\pi}_T(\rho). \tag{42}$$

4.2 Calibration

The key parameters of the model are $(\beta_0, \sigma, \gamma, \kappa, r)$, the psychological discount factor (recall that we assume $\beta_0 = \beta_1$), the degree of risk aversion, the degree of altruism of the mother, the (real) rate of growth of house prices and the base rate of interest.

To calibrate the model completely, we need to specify the joint probability distribution of income and housing assets, that is, the joint distribution of (H_0, y_0, y_1). We assume that these variables are jointly log-normal, *i.e.*, that $(\ln(H_0), \ln(y_0), \ln(y_1))$ is normally distributed. We will simulate a sample of homeowners in the Paris area. To calibrate the key parameters, we consider a survey of the French Statistical Institute (*i.e.*, INSEE) on income and wealth (*i.e.*, the *Enquête Patrimoine* 2010). We consider the subsample of households with a reference person aged between 50 and 70, living in the Paris region, being the owners of their main residence (at least). We study the area of their homes in square meters and the total yearly income of the household.

To determine the covariance of $\ln(y_0)$ and $\ln(y_1)$, we use the literature on intergenerational transmission of earnings and wealth. Lefranc and Trannoy (2005) have estimated the correlation between the daughter's and the mother's income with the help of French data. The standard approach to these questions is to specify a simple econometric model of the form,

$$\ln(y_1) = a_0 + a \ln(y_0) + \epsilon,$$

where ϵ is an independent normal error term. If $\ln(y_1)$ and $\ln(y_0)$ have the same variance, the regression coefficient a is equal to the correlation coefficient of the two logarithms. According to Lefranc and Trannoy (2005), a is somewhere between 0.25 and 0.45, closer to 0.4 in the recent years. We choose $a = 0.4$, and equal variances for the log-incomes. The variance of $\ln(y_1)$ is taken from INSEE's *Enquête Patrimoine* 2010 and we find that the standard deviation of log-income is 0.83. We assume $Var(\ln(y_1)) = Var(\ln(y_0)) = 0.83^2 = 0.69$. From this we derive,

$$Var(\epsilon) = Var(\ln(y_1))(1 - a^2) = 0.69 \times (1 - 0.16) \simeq 0.58.$$

The standard deviation of ϵ is therefore $\sqrt{0.58} = 0.76$. From *Enquête Patrimoine* we also get $\mathbb{E}(\ln(y_0)) = 11$. We assume $\mathbb{E}(\ln(y_0)) = \mathbb{E}(\ln(y_1))$ for simplicity (this corresponds to a stationary environment). This assumption pins down a_0, we have

$$a_0 = (1 - a)\mathbb{E}(\ln(y_1)) \simeq 6.6.$$

We specify the distribution of H_0 in the same way, using the data from *Enquête Patrimoine* 2010. We run a regression of the form

$$\ln(H_0) = b_0 + b \ln(y_0) + \xi,$$

where ξ is an independent normal random error term, to model the correlation of H_0 and y_0. The coefficients are precisely estimated. We obtain $b = 0.25$, $b_0 = 1.88$ and the standard deviation of ξ is equal to 0.39.

Given these specifications, we draw a random sample of 500 families in the joint log-normal distribution so obtained, where the variances of $\ln(H_0)$ and $\ln(y_0)$ are chosen as indicated above.

There are $N = 500$ families. The mother's age is 70 in all families. We assume that all mothers pass away before the age of 100. We also use the appropriate INSEE (or INED) life tables to compute the survival function S_T and the probability p_T.

4.3 Simulations and Results

We can now treat the simulated data by means of a statistical software as if it was a sample of real observations, and we can generate as many samples as we wish by varying the values of the model's structural parameters, to perform sensitivity analysis.

Given this sample of families, we solve the model, for each of the 500 families in the sample, in 11 different scenarios. These scenarios are different combinations of key parameters. We study the effect of changing three parameters only: risk aversion, altruism and time preference. There is a 'central scenario' based on the choice $(\sigma, \gamma, \beta_0) = (.9, 1, .96)$ and we take $r = .03$, $\rho = .04$, $q_1 = 5250$ (in euros per square meter) with a real rate of growth of 1 percent per year, $\kappa = .01$. We then construct the other scenarios by changing the values of each parameter separately while the others are fixed at their central values. The values of σ are chosen in the set $\{.8, .9, 1.1\}$; the values of γ vary in the set $\{.5, 1, 1.5, 2\}$, and the values of β_0 are chosen in the set $\{.95, .96, .97, .98\}$. Table 4 gives the summary statistics of the empirical distribution of key exogenous variables in the simulated sample. The figures show that simulated observations

Table 4: Simulated Data

	Mean	Median	Std. Dev.
Size of the mother's house	111.70	101.54	57.78
Income of the mother	89,349	61,386	125,595
Income of the daughter	86,595	60,799	86,424
log Income of the mother	10.97	11.02	0.89
log Income of the daughter	11.02	11.02	0.83

Simulated data set, N=500. Yearly income is in euros. Areas are in square meters

represent relatively well-to-do individuals from the Paris region. Real estate prices q_1 are chosen as an average of the Paris region (average prices in the center of Paris would be closer to 9000 euros per square meter). Given the basic data describing each randomly drawn family, we compute the optimal, full-insurance contract as described in sub-section 5.1 above. We compute the numerical values of utilities for each family member and each family $i = 1, \ldots, 500$, that is, $U_i(\pi)$, $V_i(\pi)$, where $\pi \geq 0$ is the bank's expected profit from the contract. More precisely, the utility levels depend on the profit earned by the bank on each family i, denoted π_i. The latter variable is a summary, in present actuarial value terms, of the surplus extracted by the bank from family i: this profit can be subtracted from Y in the actuarial resource constraint (16) or

(22). Let then π_i^* denote the maximal profit of the bank with the optimal contract. It is easy to see that π_i^* is such that the family is indifferent between the optimal contract and the MFC (*i.e.*, myopic contract). Let $\hat{U}_i$, $\hat{V}_i$ and $\hat{W}_i = \hat{U}_i + \gamma \hat{V}_i$ denote the utility levels of the mother, the daughter and the family, respectively, under the MFC. The function $W_i = U_i + \gamma V_i$ is typically an indirect utility function that is increasing with respect to Y_i and hence decreasing with respect to π_i. It follows that π_i^* solves the equation,

$$U_i(\pi_i^*) + \gamma V_i(\pi_i^*) = \hat{W}_i. \tag{43}$$

Our simulation program computes π_i^* for each family i in the sample. We end up with a data set including exogenous data (H_{i0}, y_{i0}, y_{i1}), and endogenous data

$$(H_{i1}, U_i(\pi_i^*), V_i(\pi_i^*), \hat{U}_i, \hat{V}_i, \hat{H}_{i1}, \pi_i^*, \hat{\pi}_i(\rho))$$

where $\hat{H}_{i1}$ is the daughter's house size and $\hat{\pi}_i(\rho)$ is the profit of the bank under the MFC, defined above by (42). The distribution of expected profits π_i^* generated by the profit-maximizing family contracts is one of the outputs. Table 5 gives the main descriptive statistics of the simulated profits. Profit is nonnegative and substantial. The reader must keep in mind that the profit,

Table 5: Simulated profits

	Mean	Median	Std. Dev
Myopic contract	43,084	32,141	41,423
Optimal contract, with profit	164,376	150,000	110,905

Simulated data set, N=500. Central scenario $(\gamma, \sigma, \beta_0, \rho) = (1, .9, .96, .04)$).

expressed in euros, is the total discounted sum of inter-temporal expected benefits, earned by the bank over the entire life of a contract with one family, *i.e.*, 31 years, and that the chosen families are (relatively) rich Parisians. To better appreciate if the profits are really large, we can transform the median profits in Table 5 into an equivalent annuity at rate $r = .02$. That is, we compute $A\pi$ with $A = (1 - \beta)/(1 - \beta^{31})$. A simple computation shows that $A = .043$. In the myopic contract case, the bank extracts $0.043 \times 32,141 = 1382$ euros per year during 31 years, or 115 euros per month, from the family with median profitability. In the case of the optimal contract with profit maximization, we obtain $0.043 \times 150,000 = 6450$ euros per year or 537 euros per month. The reader must keep in mind that this last figure is an upper bound, that is sensitive to the choice of r. Under the optimal contract, profits are the maximal amount of surplus that can be extracted by the bank from a family, leaving the family indifferent between the old myopic contract and the new arrangement. It follows that the simulated profits are not a prediction of what would be observed, but show the existence of a potential and sizeable source of surplus.

We now look at the utilities, distinguishing three cases: the myopic contract, the optimal contract with zero profit and the optimal contract with maximal profit, as defined above. The summary statistics are given by Table 6. Figure 6 shows the distribution of $\ln(\pi_i^*)$ (optimal

Table 6: Simulated utilities

	Mean	Median	Std. Dev
Mother's utility			
Myopic contract	280.64	279.07	28.18
Optimal contract, zero profit	287.33	285.61	25.61
Optimal contract, maximal profit	285.01	283.23	26.71
Daughter's utility			
Myopic contract	412.37	408.12	43.43
Optimal contract, zero profit	413.11	408.54	42.51
Optimal contract, maximal profit	408.93	404.08	44.79
Family Welfare $U + V$			
Myopic contract	693.01	687.70	71.39
Optimal contract, zero profit	700.44	694.79	67.93
Optimal contract, maximal profit	693.94	688.69	71.31

Simulated data set, N=500. Central scenario $(\gamma, \sigma, \beta_0, \rho) = (1, .9, .96, .04)$.

contract, maximal profit) and $\ln(\hat{\pi}_i(\rho))$ (MFC). It is clear from Table 5 and Fig. 6 that the bank's profit increases very significantly as compared to the MFC profit. The MFC and the optimal contract with maximal profit cases can be interpreted as providing a lower and an upper bound on the profit of the bank per family.

To check how the model works, we ran the OLS regression of the bank's profit (per family) on the mother's and the daughter's income and on the mother's house size in square meters. Results are given in Table 7. Under the myopic contract, the adjustment is perfect because profit is a linear function of (y_0, y_1, H_0), with no constant, as shown by eq. 41. This is why we find $R^2 = 1$ here. The adjustment is not perfect in the case of the optimal contract, because the reduced form is nonlinear (and a log-linear specification would not yield a better fit). Under the optimal contract, all covariates are of course highly significant. There are striking differences between the two models (*i.e.*, the two columns on the right of Table 7): under the optimal contract, the mother's income becomes a negative factor, but the mother's house is transformed into a source of profit, while it is a negative profitability factor under the myopic contract. The latter property is due to the fact that the daughter will inherit her mother's house at some point in her life, and expects to use the money to invest in her own property, which reduces the amount of lending and profits. In contrast, under the optimal contract, the mother's house is annuitized, and the benefits of home equity-release are extracted by the bank.

We can compare the size of the daughter's house under the MFC and the optimal contract. Table 8 gives the summary statistics for the simulated values of H_1 and $\hat{H}_1$. Figure 6 depicts the densities of the logarithm of the daughter's house size under the myopic contract, under the optimal contract with zero profit and under the optimal contract with maximal profit. It is striking that the move from the myopic to the optimal contract typically shifts the distribution

Figure 5: Densities of simulated log-profits with myopic and optimal contracts

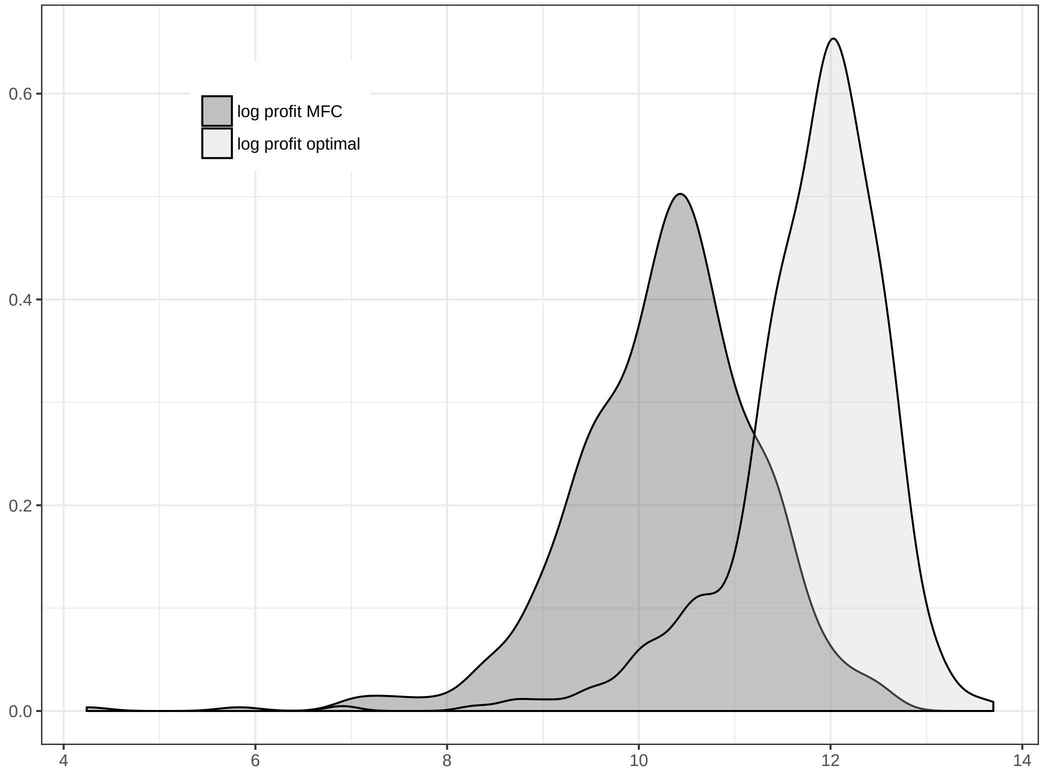


Table 7: Regression of profits

	Dependent variable: per family profit of the bank	
	Myopic contract	Optimal contract
Mother's income$\times 10^{-3}$	0.147***	−0.474***
	(0.000)	(0.033)
Daughter's income$\times 10^{-3}$	0.431***	0.501***
	(0.000)	(0.046)
Mother's house size	−54.996***	1,365.936***
	(0.000)	(69.587)
Constant	0.000***	11,698.050
	(0.000)	(8,459.730)
Observations	500	500
R^2	1.000	0.502

Note: Stars denote statistical significance; * for 10%; ** for 5%; *** for 1%.

of $\ln(H_1)$ to the right. But the difference between the zero profit and the maximal profit cases (under the optimal contract) is small in comparison. On average, the optimal contract will substantially increase the area of the house used by the daughter.

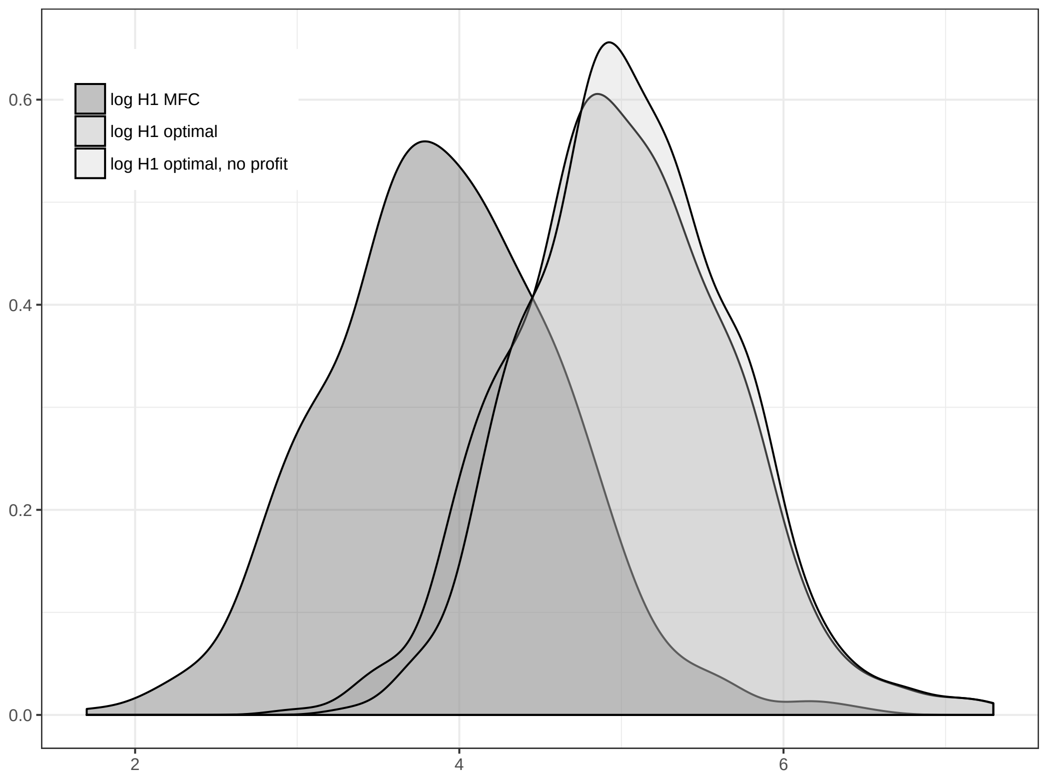
We also ran the regression of the daughter's house area $\hat{H}_1$ on (y_0, y_1, H_0), under the MFC, and of H_1^* on (y_0, y_1, H_0, π), under the optimal contract with maximal profit, using the simulated data as a sample. The results of an OLS regression are reported in Table 9. Again, the fit is perfect because the demand for housing is a linear function of (y_0, y_1, H_0, π), with a zero constant. In fact, with the Cobb-Douglas utility specification used here, the demand for housing, H_1^*, happens to be linear with respect to $XH_0 + Y - \pi$; see (21), (22) above.

Table 8: Simulated size of the daughter's house (in square meters)

	Mean	Median	Std. Dev
Myopic contract	65.14	47.85	61.20
Optimal contract, zero profit	196.48	151.71	157.99
Optimal contract, maximal profit	188.64	142.12	157.79

Simulated data set, N=500. Central scenario $(\gamma, \sigma, \beta_0, \rho) = (1, .9, .96, .04)$.

Figure 6: Densities of simulated log-size of the daughter's house



We now study the impact of some key parameters and compare profits in different scenarios. First, if we decrease the discount factor β_0, do we observe that the new distribution of the profit

Table 9: Regression of the Daughter's House Size

	Dependent variable: daughters' house size		
	Myopic contract (1)	Optimal contract zero profit (2)	Optimal contract maximal profit (3)
Mother's income$\times 10^{-3}$	0.376***	0.795***	0.795***
	(0.000)	(0.000)	(0.000)
Daughter's income$\times 10^{-3}$	0.376***	1.200***	1.200***
	(0.000)	(0.000)	(0.000)
Mother's house size	−0.000*	0.224***	0.224***
	(0.000)	(0.000)	(0.000)
Profit$\times 10^{-3}$			−0.059***
			(0.000)
Constant	0.000***	0.000***	−0.000***
	(0.000)	(0.000)	(0.000)
Observations	500	500	500
R^2	1.000	1.000	1.000

Note: Stars denote statistical significance; * for 10%; ** for 5%; *** for 1%.

π^* dominates the former in the sense of first-order stochastic dominance[17] (hereafter FOSD)? The answer is yes: a less patient family (a lower β_0) is more profitable for the bank, as shown by Figure 7. Fig. 8 shows the impact of β_0 on the daughter's house H_1. A more impatient family leads to a smaller house for the daughter, in the particular sense of FOSD. Next we look

Figure 7: Impact of β_0 on log-profits (Optimal contract)

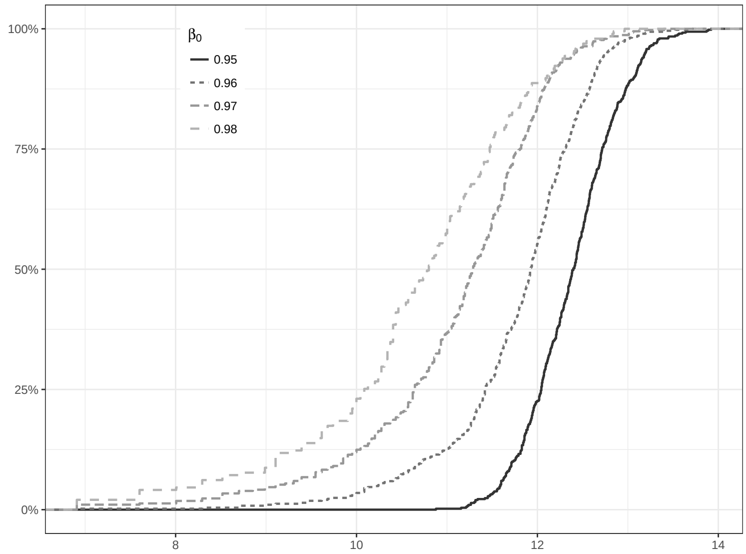
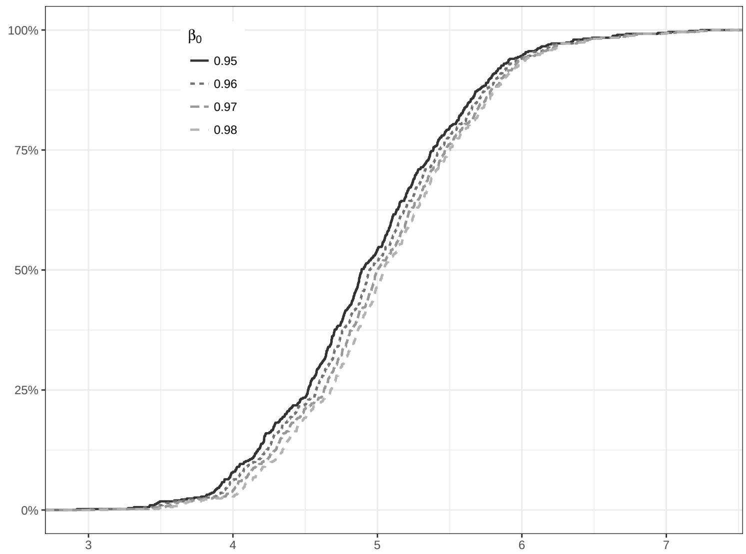


Figure 8: Impact of β_0 on log-size of the daughter's house (Optimal contract)



at the effect of increasing the weight γ of the daughter in the family welfare function. Fig. 9 gives the cumulative distribution of $\ln \pi^*$ in the sample for four values of γ: .5, 1, 1.5 and 2. We see that the distributions obtained with a smaller value of γ dominate the others in the sense of first-order stochastic dominance. This shows that a less altruist mother is more profitable for the bank. This is because an increase of γ is analogous to an increase in patience β_0, insofar as

[17] Recall that distribution F dominates distribution G in the sense of first-order stochastic dominance if the cdf of F is everywhere smaller than, or equal to, the cdf of distribution G.

the daughter lives longer than her mother, a higher γ increases the weight on the future, on the periods starting after the mother's death. If we now look at the distribution of H_1 (under the optimal contract, depicted on Fig. 10 we see that a more altruist mother leads to a distribution of H_1 that dominates the distribution obtained with smaller values of γ in the FOSD sense. Daughters with more altruistic mothers tend to buy a larger house.

Figure 9: Impact of γ on log-profits (Optimal contract)

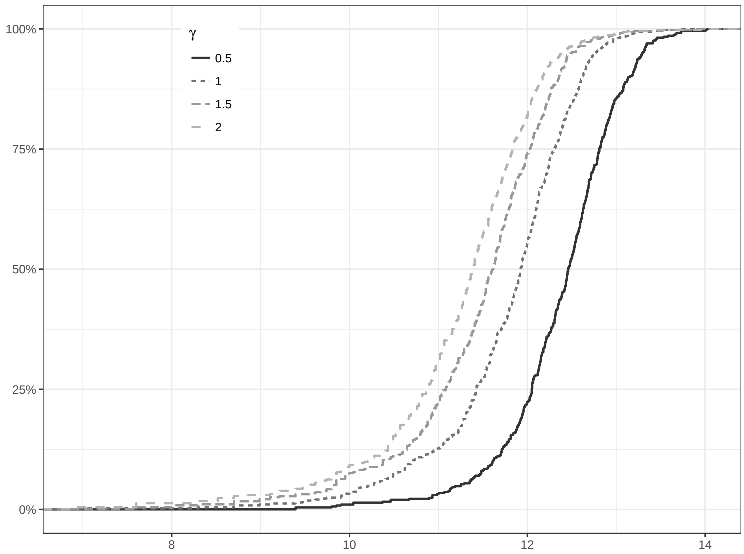
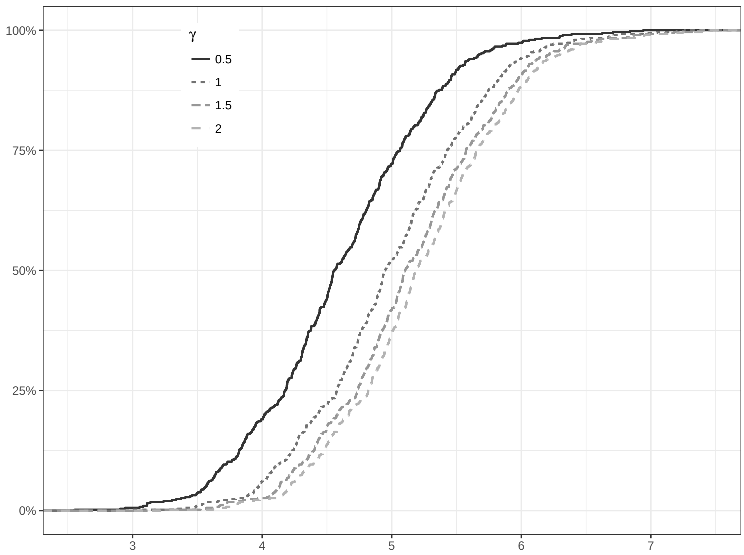


Figure 10: Impact of γ on log-size of daughter's houses (Optimal contract)



Next, we computed the model's solutions for three values of the risk-aversion parameter σ. The values are .8,.9, and 1.1. There is again a clear FOSD ranking of the distributions. Fig. 11 shows that more risk-averse families are more profitable for the bank, since the distribution for $\sigma = 1.1$ dominates the distribution obtained with $\sigma = .9$, etc. Fig. 12 shows the same type of FOSD ranking on the cdf of $\ln(H_1)$ and the results show that more risk-averse daughters end up with a larger house under the optimal contract. To interpret this result, note that, as in many models using the same common functional forms, σ plays a double role as coefficient of risk aversion and as a key parameter determining the demand for housing (the other important parameter being α). It is not difficult to check that the price-elasticity of the demand for housing, *i.e.*, the elasticity of H_1 with respect to Q_1 is equal to $-1/\sigma$. It follows that when σ increases, the size of the house increases under the optimal contract, for given Q_1. Finally, we performed

Figure 11: Impact of σ on log-profits (Optimal contract)

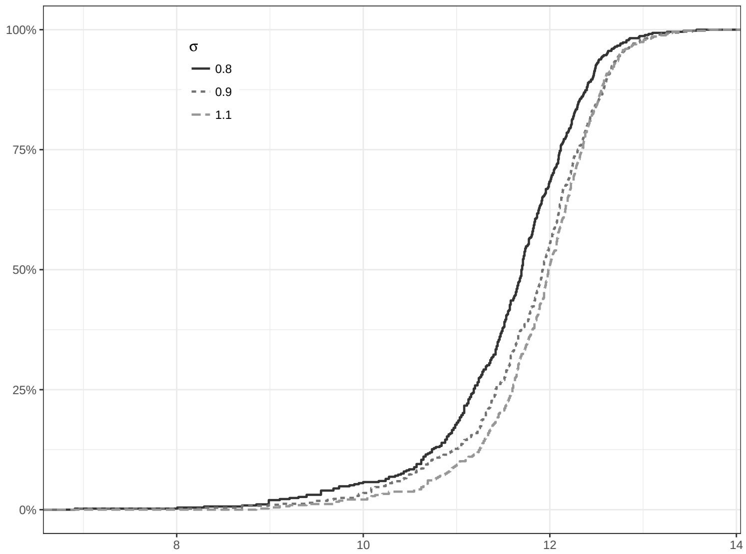
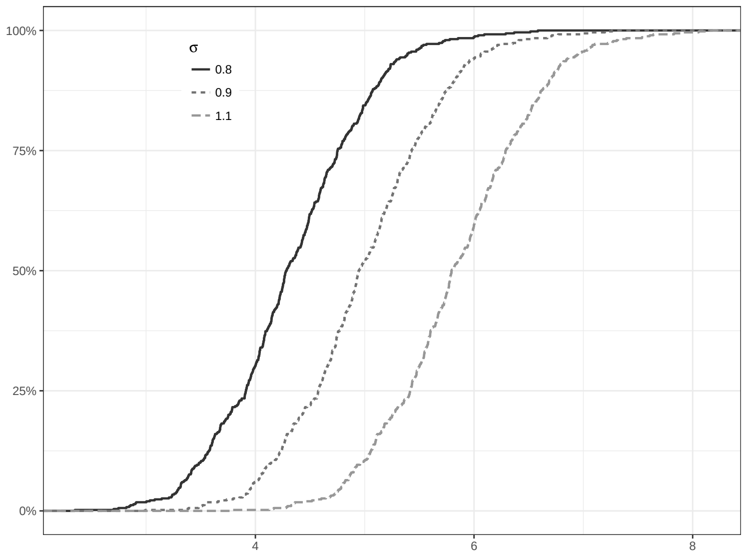


Figure 12: Impact of σ on log-size of the daughter's house (Optimal contract)



the same exercise with a change of ρ, the interest rate on home loans of the myopic contract. The values of ρ are .035, 0.04, 0.05. When ρ goes up, the profit also goes up (as shown by Fig. 13). The impact of ρ on the distribution of $\ln(H_1)$ is represented on Fig. 14. Remark that ρ affects H_1 only through the outside option of the family $\hat{W}$. The daughter's house size H_1 depends on $(Y - \pi^*)$, but when ρ increases, $\hat{W}$ obviously decreases, because properties become more expensive. This in turn allows for an increase of π^*, and H_1 should be reduced, but the latter impact is small.

Figure 13: Impact of ρ on log-profits (Optimal contract)

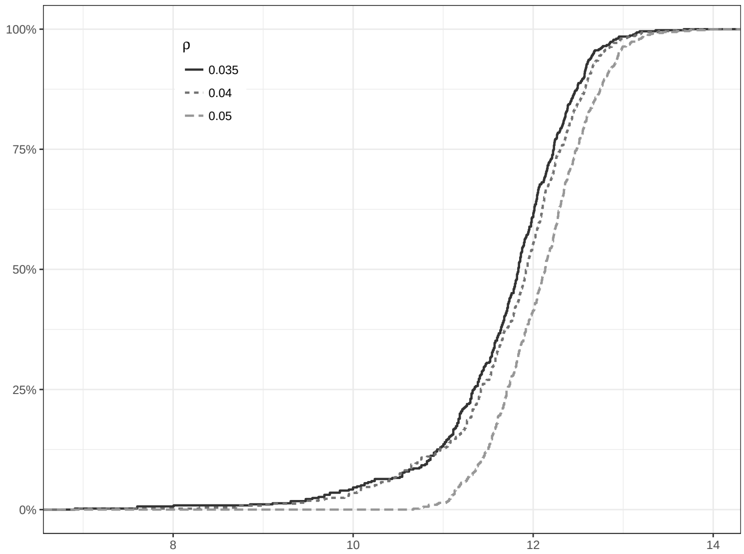
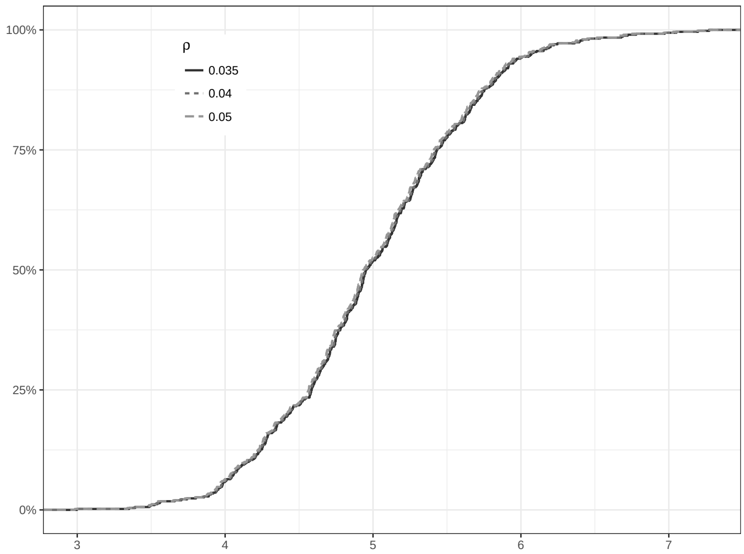


Figure 14: Impact of ρ on log-size of the daughter's house (Optimal contract)



Finally, Table 10 displays the results of numerical evaluations of some elasticities implied by the model, using the central scenario as a benchmark. Table 10 gives the impact of a 1% change in γ, β_0, σ and ρ on a number of variables. These results confirm the fact that the effects of β_0 and σ are much larger (in absolute value) than the effects of γ and ρ.

From this numerical exploration, we conclude that an optimal intermediated family contract can markedly improve the welfare of the family and, as shown by Table 8 above, may lead to a sizeable increase in the quality of housing of the younger generations.

5 Concluding Remarks. The Design of Family Contracts.

The above theory and the associated numerical analysis, based on a fully specified and calibrated version of our model, both show that there is room for improvements in the efficiency of family arrangements, relative to standard practices of inter-vivos gifts and inheritance. Our contribution is to study the combination of equity release contracts with more general insurance and household investment strategies, involving several members of a family and different generations. A banker and an insurer can jointly improve the efficiency of family arrangements, in particular, because they can radically improve risk-sharing. The banker-insurer can immunize the family against real-estate price risk and at the same time immunize the heirs (children or spouse, or both) against longevity risk, by means of structured inter-temporal transfers between parents and children. The banker-insurer acts as a middleman for family members. The gifts to children being in fact carried out by the financial intermediary, they can be made independent of the event of a parent's death. Under the terms of a family contract, the banker-insurer supplies the family with a bundle of services: liquidity, various kinds of loans and insurance; the financial intermediary satisfies the demand for bequest and gifts, due to the altruist motives of the parents, on behalf of the parents. The intermediated family arrangement can be interpreted as a bundle of bilateral contracts with the bank.

Another important question is the possibility of removing the behavioral obstacles to the use of equity-release instruments and annuitization of wealth. If the elderly have a bequest motive (based on pure altruism or some taste for giving), they could annuitize their wealth partially, and ring-fence part of their assets for inheritance. But if the children are just slightly impatient, then the money should be transmitted as soon as possible in the form of an inter-vivos gift. This could happen, say, at the latest of the two moments when the parents have completely repayed the mortgage on their residence and when their children reach adulthood and want to settle. In practice, given the facts studied in Section 2 above, the elderly members of the family may in fact wish to help their grandchildren, because they are typically still young when their own children want to buy a house. Home reversion plans can do the job of transmitting the present value of the parent's real estate assets sooner than the death of the last surviving member of the parent's household.

It seems rational to annuitize the real-estate assets, by means of a home reversion contract, only if the family seniors are at the same time well insured against the need for long-term care, *i.e.*, the risk of moving to a nursing home or the risk of a sharp increase in health expenditures. Our model should and could be extended to take into account the long-term care and health risks, at the cost of additional complexity. But it's clear that the banker-insurer can price the relevant risks with a reasonable degree of accuracy and solve the equity-release and annuitization problems completely. What remains is to structure the timing and sharing of the estate among family members, and in so doing, to insure the heirs against the father's (or the mother's) longevity risk. The banker-insurer can provide a service that the mother, or the father, cannot replicate without his help by just selling his (her) house on a home reversion market. On her own, the mother could share the annuity value of her house among her children under a home reversion contract. This move would insure the whole family against future house-price risk. But the money distribution, *i.e.*, transfers to children, would stop when the mother passes away. This is a risk if the children still have to repay mortgages at this moment. The banker-insurer can commit to continue subsidizing the children's asset building efforts during a time span that is completely independent of the mother's longevity. This is the key element in our theory of intermediated family contracts. There are of course some tax optimization aspects in the design of these contracts. Since the intermediated family arrangement can be interpreted as a bundle of bilateral contracts with the bank, it is unclear whether the transfers of purchasing power between family members, implemented by the bundle of contractual arrangements, can be interpreted as gifts (in legal terms), or if they must be viewed as cross-subsidies between several categories of the bank's clients, or even simply as promotional pricing aimed at new customers. It may be that inter-vivos gifts are taxed, in a given country, while cross-subsidies between clients or products are in fact a legally admissible form of (third-degree) price discrimination.

Assuming that the long-term care insurance problem has been solved, it is reasonable to expect that an intermediated family contract would help removing the behavioral obstacles to annuitization of the elderly's wealth, because annuitization becomes part of an encompassing solution to a family problem. Of course, further empirical research would be needed to really estimate the extent to which the latter claim is true.

Table 10: Elasticities

	Central scenario	$\Delta\gamma = 1\%$		$\Delta\beta_0 = \Delta\beta_1 = 1\%$		$\Delta\sigma = 1\%$		$\Delta\rho = 1\%$	
Profits									
Profit MFC	43084.3	43209.9	0.29%	43084.3	0.00%	47080.1	9.27%	44612.1	3.55%
Profit Optimal	164376.0	163078.0	-0.79%	79476.0	-51.65%	167876.0	2.13%	166312.0	1.18%
Simulated size of daughter's house									
H1 MFC	65.1	65.5	0.49%	65.1	0.00%	69.3	6.41%	64.8	-0.49%
H1 Optimal	187.0	187.8	0.43%	194.9	4.20%	196.7	5.17%	186.9	-0.06%
H1 Optimal zero profit	196.8	197.6	0.39%	199.7	1.47%	207.2	5.29%	196.8	0.00%
Simulated utilities									
Mother MFC	280.6	280.4	-0.08%	304.4	8.48%	265.7	-5.33%	280.6	0.00%
Mother Optimal	284.7	284.4	-0.09%	308.9	8.52%	269.3	-5.41%	284.6	-0.01%
Mother Optimal zero profit	287.3	287.1	-0.09%	310.5	8.07%	271.7	-5.43%	287.3	0.00%
Daughter MFC	412.3	412.6	0.06%	465.7	12.95%	390.6	-5.26%	412.3	-0.02%
Daughter Optimal	408.3	408.6	0.07%	460.9	12.89%	387.0	-5.20%	408.2	-0.01%
Daughter Optimal zero profit	413.1	413.4	0.06%	463.9	12.30%	391.6	-5.22%	413.1	0.00%
Family Welfare MFC	693.0	697.1	0.60%	770.2	11.14%	656.3	-5.29%	692.9	-0.01%
Family Welfare Optimal	692.9	697.1	0.60%	769.8	11.09%	656.3	-5.29%	692.9	-0.01%
Family Welfare Optimal zero profit	700.4	704.6	0.59%	774.5	10.57%	663.3	-5.30%	700.4	0.00%

6 References

ABEL, Andrew D., and Mark WARSHAWSKY (1988). "Specification of the Joy of Giving: Insights from Altruism", *Review of Economics and Statistics*, 70(1), 145-149.

ALTONJI, Joseph G., HAYASHI, Fumio and Laurence J. KOTLIKOFF (1997). "Parental Altruism and Inter Vivos Transfers: Theory and Evidence", *Journal of Political Economy*, 105(6), 1121-1166.

AMERIKS, John, CAPLIN, Andrew, LAUFER, Steven and Stijn VAN NIEUWERBURGH (2011), "The Joy of Giving or Assisted Living? Using Strategic Surveys to Separate Public-care Aversion from Bequest Motives", *Journal of Finance*, 66(2), 519-561.

ANDREONI, James (1989). "Giving wih Impure Altruism: Applications to Charity and Ricardian Equivalence", *Journal of Political Economy*, 97(6), 1447-1458.

BECKER, Gary S. (1974). "A Theory of Social Interactions", *Journal of Political Economy*, 82(6), 1063-1093.

BÖRSCH-SUPAN, Axel, A. BRUGIAVINI, H. JÜRGES, J. MACKENBACH, J. SIEGRIST and G. WEBER (2005), *Health, Ageing and Retirement in Europe. First Results from the Survey of Health, Ageing and Retirement in Europe.* Mannheim Research Institute for the Economics of Aging (MEA), Mannheim.

BÖRSCH-SUPAN, Axel and H. JÜRGES (Editors) (2005), *The Survey of Health, Ageing and Retirement in Europe - Methodology*, Mannheim Research Institute for the Economics of Aging (MEA), Mannheim.

BENARTZI, Shlomo, PREVITERO, Alessandro and Richard H. THALER (2011), "Annuitization Puzzles", *Journal of Economic Perspectives*, 25(4), 143-164.

BROWN, Jeffrey R. and Amy FINKELSTEIN (2007), "Why is the Market for Long-Term Care Insurance so Small?", *Journal of Public Economics*, 91(10), 1967-1991.

CAMPBELL, John Y. (2006), "Household Finance", *Journal of Finance*, 61(4), 1553-1604.

CHIURI, Maria-Concetta, and Tullio JAPPELLI (2010), "Do the Elderly Reduce Housing Equity? An International Comparison", *Journal of Population Economics*, 23, 643-663.

CREMER, Helmuth, and Pierre PESTIEAU (2006). "Wealth Transfer Taxation: a Survey of the Theoretical Literature", Chap. 16 in: S-C. Kolm and J. Mercier-Ythier, eds., *Handbook of the Economics of Giving, Altruism and Reciprocity*, North Holland, Amsterdam.

DAVIDOFF, Thomas (2010), "Home Equity Commitment and Long-Term Care Insurance Demand", *Journal of Public Economics*, 94, 44-49.

DAVIDOFF, Thomas, BROWN, Jeffrey R. and Peter A. DIAMOND (2005), “Annuities and Individual Welfare”, *American Economic Review*, 95(5), 1573-1590.

DE NARDI, Mariacristina, FRENCH, Eric and John B. JONES (2010). “Why Do the Elderly Save? The Role of Medical Expenses”, *Journal of Political Economy*, 118, 39-75.

DIAMOND, Peter A. (2006). “Optimal Tax treatment of Private Contributions for Public Goods with and without Warm-glow Preferences”, *Journal of Public Economics*, 90(4), 897-919.

DYNAN, Karen, SKINNER, Jonathan and Stephen ZELDES (2004). “Do the Rich Save More?”, *Journal of Political Economy*, 112, 397-444.

FRATANTONI, Michael C. (1999), “Reverse Mortgage Choices: A Theoretical and Empirical Analysis of the Borrowing Decisions of Elderly Borrowers”, *Journal of Housing Research*, 10(2), 189-208.

FÉVRIER, Philippe, LINNEMER, Laurent and Michael VISSER (2012), “Testing for Asymmetric Information in the *Viager* Market”, *Journal of Public Economics*, 96, 104-123.

KAPLOW, Louis (2001). “A Framework for Assessing Estate and Gift Taxation”, in W. G. Gale, J. R. Hines and J. Slemrod, eds., *Rethinking Estate and Gift Taxation*, Brookings Institution Press, Washington, D.C.

KOPCZUK, Wojciech (2013).“Taxation of Intergenerational Transfers and Wealth”, Chap. 6 in: A. Auerbach, R. Chetty, M. Feldstein and E. Saez eds., *Handbook of Public Economics*, vol.5, North Holland, Amsterdam.

KOPCZUK, Wojciech and Joseph P. LUPTON (2007). “To Leave or not to Leave: an Empirical Investigation of the Distribution of Bequest Motives”, *Review of Economic Studies*, 74(1), 207-235.

LAFERRÈRE, Anne and François-Charles WOLFF (2006), “Microeconomic Models of Family Transfers”, *in*: S-C. Kolm and J. Mercier-Ythier, eds., *Handbook of the Economics of Giving, Altruism and Reciprocity*, North Holland, Amsterdam.

LEFRANC, Arnaud and Alain TRANNOY (2005), “Intergenerational Earnings Mobility in France: is France more Mobile than the US?”, *Annals of Economics and Statistics (Annales d'Economie et de Statistique)*, no 78, 57-77.

MASSON, André (2014), “Un viager obsolète: des “viages” adaptés pour les classes aisées et les plus modestes”, Note de la chaire *Transitions démographiques, transitions économiques*, n°43bis, saison 3, February 2014, Caisse de Dépôts, Paris.

SCOTT Jason S., WATSON, John G. and Wei-Yin HU (2011), “What Makes a Better Annuity?” *Journal of Risk and Insurance*, 78(1), 213-244.

SHAN, Hui (2011), "Reversing the Trend: The Recent Expansion of the Reverse Mortgage Market", *Real Estate Economics*, 39(4), 743-768.

SHESHINSKI, Eytan (2008), *The Economic Theory of Annuities*, Princeton University Press, Princeton, New Jersey.

SINAI, Todd and Nicholas S. SOULELES (2005), "Owner-Occupied Housing as a Hedge Against Rent Risk", *Quarterly Journal of Economics*, 120(2), 763-789.

SINAI, Todd and Nicholas S. SOULELES (2008), "Net Worth and Housing Equity in Retirement", *in*: J. Ameriks and O. S. Mitchell, editors, *Recalibrating Retirement Spending and Saving*, Oxford University Press, Oxford, UK.

VENTI, Stephen F. and David A. WISE (1990), "But They Don't Want to Reduce Housing Equity", *in*: David Wise, editor, *Issues in the Economics of Aging*, NBER, University of Chicago Press, Chicago, Illinois.

VENTI, Stephen F. and David A. WISE (2004), "Aging and Housing Equity?", Chapter 3, *in*: David Wise, editor, *Perspectives on the Economics of Aging*, NBER, University of Chicago Press, Chicago, Illinois.

YAARI, Menahem E. (1965), "Uncertain Lifetime, Life Insurance, and the Theory of the Consumer", *Review of Economic Studies*, 32(2), 137-150.

7 Appendix: Proofs

Proof of Proposition 1

The optimal contract optimization problem has a concave objective and linear constraints. Thus, the first-order conditions are necessary and sufficient. To prove Proposition 1 (and the following ones) we write the Kuhn and Tucker conditions for optimality (maximizing the family's expected utility $U + \gamma V$ subject the ARC (*i.e.*, Eq. 16), feasibility and sign constraints. We introduce Lagrange multipliers: λ for the ARC constraint (16); μ for the feasibility constraint $H_0 \geq \Sigma_{t=0}^{\mathcal{T}} z_t$; ν_t, $t = 0, \ldots, \mathcal{T}$ for the z_t variables. We have $\nu_t \geq 0$, $z_t \geq 0$, and the complementary slackness conditions $\nu_t z_t = 0$. We also have $\mu(H_0 - \Sigma_0^{\mathcal{T}} z_t) = 0$. Taking derivatives with respect to z_0, we get the following first-order condition,

$$\nu_0 + \lambda \tau_b X = \mu. \tag{44}$$

From this equation, we derive that if $\lambda > 0$, and $0 < \tau_b \leq 1$, then $\mu > 0$, and therefore, $H_0 = \Sigma_{t=0}^{\mathcal{T}} z_t^*$ (the mother's house is entirely sold before death).

We now write the first-order conditions for the optimality of partial sales $(z_1, \ldots, z_{\mathcal{T}})$. For $t > 0$ we find,

$$\nu_t = \sum_{\tau=t}^{\mathcal{T}} \beta_0^{\tau-1} S_\tau \frac{\partial v_0}{\partial h}^{-} \left(H_0 - \sum_{\theta=1}^{\tau} z_\theta, H_0 \right) + \mu + \lambda(1-\tau_b) \sum_{T=t}^{\mathcal{T}} p_T \beta^T q_{T+1} - \lambda \beta^{t-1} S_t q_t. \tag{45}$$

Substituting (44) in (45) and rearranging terms, we obtain,

$$\nu_t = \sum_{\tau=t}^{\mathcal{T}} \beta_0^{\tau-1} S_\tau \frac{\partial v_0}{\partial h}^{-} + \nu_0 + \lambda \tau_b \sum_{T=1}^{t-1} p_T \beta^T q_{T+1} + \lambda \beta^{t-1} S_t (X_t - q_t). \tag{46}$$

We know that $\nu_0 \geq 0$, and $\lambda \geq 0$ by Kuhn and Tucker's Theorem, and $\tau_b > 0$ by assumption. Condition (a) then easily implies $\nu_t > 0$ and thus, by complementary slackness conditions, $z_t = 0$. If the left-derivative of v_0 at point H_0 is large enough, we obviously have $\nu_t > 0$ for all $t > 0$ at the optimum, even if $q_t > X_t$. We again conclude that $z_t = 0$ at the optimum for all $t > 0$. This in turn implies (with $\mu > 0$) that $z_0 = H_0$ at the optimum: the mother sells her entire house in the form of a home-reversion contract at time $t = 1$.

It can easily be shown that the multiplier of the ARC constraint is always positive; this is due to the fact that marginal utility u' is strictly positive and $\lambda^* = u'(c_{01}^*)$ (as shown in the proof of Proposition 3 below). *Q.E.D.*

Proof of Proposition 2

Assuming interiority, that is $H_1 > 0$ at the optimum, we take the derivative of the Lagrangian with respect to H_1 and we find the condition,

$$\gamma \sum_{t=1}^{\mathcal{T}+1} \beta_1^{t-1} S_t v_1' \left(H_1 + h_t \right) + \gamma \sum_{T=1}^{\mathcal{T}} p_T \sum_{t=T+1}^{\mathcal{T}+1} \beta_1^{t-1} v_1' \left(H_1 + h_t(T) \right) = \lambda Q_1. \tag{47}$$

Let now η_t (resp. $\eta_t(T)$) denote the Lagrange multiplier of the sign constraint $h_t \geq 0$ (resp. $h_t(T) \geq 0$). Kuhn and Tucker's Theorem tells us that these multipliers are nonnegative. Taking partial derivatives of the Lagrangian with respect to h_t and $h_t(T)$ and equating them to zero yields the following necessary conditions,

$$\gamma\beta_1^{t-1}S_t v_1'(H_1 + h_t) + \eta_t = \lambda\beta^{t-1}S_t R_t, \tag{48}$$

for all $t = 1, \dots, \mathcal{T}$, and with the condition $\eta_t h_t = 0$, and,

$$\gamma\beta_1^{t-1}p_T v_1'\left(H_1 + h_t(T)\right) + \eta_t(T) = \lambda\beta^{t-1}p_T R_t, \tag{49}$$

for $t > T$ and all $T = 1, \dots, \mathcal{T}$ with the condition $\eta_t(T)h_t(T) = 0$. Now, we substitute (48) and (49) in (47) and we eliminate the terms involving v_1'. Given that $\eta_t \geq 0$, $\eta_t(T) \geq 0$, assuming $\lambda^* > 0$, this yields,

$$Q_1 \leq \sum_{t=1}^{\mathcal{T}+1} \beta_1^{t-1}S_t R_t + \sum_{t=2}^{\mathcal{T}+1} \beta_1^{t-1}R_t \sum_{T=1}^{t-1} p_T = \sum_{t=1}^{\mathcal{T}+1} \beta^{t-1}R_t, \tag{50}$$

where we use the fact that $S_1 = 1 - P_1 = 1$ and $S_{\mathcal{T}+1} = 0$. Inequality (50) becomes an equality if the daughter rents at least part of her housing area, since $h_t > 0$, $h_t(T) > 0$ implies $\eta_t = 0$, $\eta_t(T) = 0$.

The tenure arbitrage condition (19) is only a necessary condition for $H_1 > 0$. To make sure that $h_t = 0$ for all t, and $h_t(T) = 0$ for all $t > T$, we need to assume that rents are sufficiently large, that is, using the conditions for optimality, we require $\eta_t > 0$ and $\eta_t(T) > 0$, for this implies $h_t = 0$ and $h_t(T) = 0$ by the complementary slackness conditions. From (48) and (49), we find that these requirements imply, for all t,

$$R_t > \frac{\gamma}{\lambda^*}\left(\frac{\beta_1}{\beta}\right)^{t-1} v_1'(H_1^*). \tag{51}$$

If R_t is sufficiently large, then $h_t = 0$, $h_t(T) = 0$ is the solution, and the daughter's house size $H_1^* > 0$ is determined by the following equation, derived from (47),

$$\gamma v_1'(H_1^*)\frac{(1 - \beta_1^{\mathcal{T}+1})}{(1 - \beta_1)} = \lambda^* Q_1, \tag{52}$$

and (52) is equivalent to (21).

Using (52) or (21), it is easy see that (51) is equivalent to,

$$R_t > Q_1\frac{(1 - \beta_1)}{(1 - \beta_1^{\mathcal{T}+1})}\left(\frac{\beta_1}{\beta}\right)^{t-1}. \tag{53}$$

Now, it is easy to use the inequalities (53) to find a lower bound for the discounted sum of rents, and we find that (19) holds as a strict inequality. *Q.E.D.*

Proof of Proposition 3

Taking the partial derivatives of the Lagrangian with respect to consumption variables and equating them to zero, we find

$$u'(c_{01}) = \lambda > 0.$$

This guarantees that $\lambda^* > 0$ since by Assumption 1, $u' > 0$. In addition, we find,

$$\begin{aligned} \beta_0^{t-1} u'(c_{0t}) &= \lambda \beta^{t-1} && (54) \\ \gamma \beta_1^{t-1} u'(c_{1t}) &= \lambda \beta^{t-1} && (55) \\ \gamma \beta_1^{t-1} u'(c_{1t}(T)) &= \lambda \beta^{t-1}. && (56) \end{aligned}$$

Combining the latter equations, we easily derive points 1-4 in the statement of Proposition 3 and the full insurance property. *Q.E.D.*

Proof of Proposition 4

Let α_t be the Lagrange multiplier of the sign constraint on gifts x_t, that is, $x_t \geq 0$, $\alpha_t \geq 0$ and $\alpha_t x_t = 0$. Kuhn-Tucker conditions yield $\alpha_t = \lambda \beta^{t-1} S_t \tau_g$. But $\lambda^* \tau_g > 0$ implies $\alpha_t^* > 0$ and therefore $x_t^* = 0$. *Q.E.D.*

Proof of Proposition 5

The proof of Proposition 5 is trivial: the expressions are mere restatements of the mother's and the daughter's period budget constraints (7)-(12). *Q.E.D.*