

# **MOTIVATION AND MARKETS**

## **Self-Enforcing Contracts under Perfect Monitoring**

- This Chapter is based on the work of MacLeod and Malcomson (1989, 1998)
- See Bentley MacLeod and James Malcomson (1998), “Motivation and Markets”, *American Economic Review*, **88**, p 388-411.
- One of several origins of this theory : Carl Shapiro and Joseph Stiglitz (1984), “Equilibrium Unemployment as a Worker Discipline Device”, *American Economic Review*, **74**, p 433-444.
- Theory of Efficiency Wages
- Unemployment as as Discipline Device
- Repeated Interaction : Repeated Games and Folk Theorem used to explain Labor Contracts.

- The role of subjective performance assessments in the management of labor contracts
- Bonus, commission, piece rates, and promotions
- An “intriguing” story : the case of highly paid traders who quit First Boston Bank in 1993 because bonuses were smaller than they felt they had been promised (despite receiving compensation of over 500,000 dollars !)
- Find conditions for a self-enforcing agreement

- For some jobs, the cost of a vacant position is high...
- Firms must be able to replace workers quickly
- If the firm can replace a worker immediately, this implies that the firm doesn't earn a rent on such workers (*i.e.*, high salaries)...
- ... and the threat of job loss provides incentives.
- The same rent must go to new employees (otherwise firm would fire old workers to hire new ones : this would destroy motivation).
- This explains *efficiency wages*

- Other important case : jobs for which workers are in *short supply* (example : computer engineers, traders in financial sector, soccer stars,...).
- These workers can easily find another job : the threat of firing cannot be a motivator
- This implies performance pay ; payment of bonuses.

- Performance is subjective (observable but not verifiable) thus not enforceable in court...
- We must find conditions under which firms will keep their promise to pay bonuses : a practical possibility is the threat of workers quitting (or exerting low effort in the future).
- In this case the firm receives a rent from continued employment.

## *Self-Enforcing Agreements*

1. Dynamic incentives (Value functions and constraints)
2. Market equilibrium (Unemployment and Performance Pay).

- Employment is modeled as a long-term relationship governed by an informal agreement between a worker and a firm (also called *relational contract*).
- Time is discrete  $t = 1, 2, \dots$
- Beginning of period  $t$  : agreement on compensation package : fixed wage + possible bonus.
- Worker chooses effort  $e_t \in \{0, 1\}$ , firm decides if bonus  $b_t$  is paid. Bonus cannot be made a legally binding contract tying pay to performance.
- End of period  $t$  : either party may decide to leave. Firm can fire the worker or refuse to pay bonus ; worker can quit or shirk (low effort).



- The Wage is denoted  $W_t$ ;  $v > 0$  is the disutility of effort; the worker's payoff at time  $t$  is

$$W_t - ve_t$$

- Unemployed workers receive payoff  $u_t > 0$  per period.
- The profit from a job employing a worker in period  $t$  is

$$pe_t - W_t,$$

where  $p$  is the marginal product of a worker.

- Firms do not retain workers who always shirk because  $u_t > 0$  implies that  $W_t > 0$  and  $e_t = 0$  implies that profit is negative.
- There are  $L$  identical workers and free entry of firms with identical jobs.
- If there are  $J_t$  jobs at time  $t$ , the sunk cost of creating an additional job vacancy is  $c(J_t)$ , where  $c$  is continuous and non-decreasing.
- $c$  represents the capital equipment required plus the costs of reorganization.

- At the beginning of period  $t$ , firms with vacancies and unemployed workers meet on the job market. Firms make offers; workers may reject offers. If an offer is rejected, worker and firm reenter the market. Their expected utilities are then,  $\bar{U}_t$  and  $\bar{\Pi}_t$  for worker and firm, respectively.
- The pay in period  $t$  is the base wage  $w_t$  that the firm pays whatever the performance and a bonus  $b_t$  paid if the worker doesn't shirk. The bonus cannot be legally enforced.
- There is an exogenous separation rate : at the end of each period the relationship may end for exogenous reasons with probability  $(1 - \rho) > 0$ .

- The discounted expected utility of a worker employed in period  $t$  for a match starting at time  $\tau$  is defined as follows,

$$U_t = W_t - v + \delta[\rho U_{t+1} + (1 - \rho)\bar{U}_{T+1}]$$

for all  $t \geq \tau$ .

- $\delta \in (0, 1)$  is the discount factor
- The firm's discounted profit from employing the worker (if both abide by the agreement) is as follows :

$$\Pi_t = p - W_t + \delta\rho\Pi_{t+1}$$

- If a job becomes unprofitable, the subsequent profit from that job is zero.

### *Individual rationality constraints*

- *IRW* :  $U_t \geq \bar{U}_t$  for all  $t \geq \tau$ ,
- *IRF* :  $\Pi_t \geq \bar{\Pi}_t$  for all  $t \geq \tau$ ,
- The payoffs are discounted payoffs in a repeated game played by the worker and the firm.
- We will consider equilibria in the repeated game. The agreement can be supported as a subgame-perfect Nash equilibrium in the repeated game if deviations by either players are punished by some actions. It can be shown that, in this context, there is no loss of generality if we consider the most severe punishments.

## *Deviations and Punishments*

- The most severe penalties are given by the alternatives currently available on the market.
- Anonymous market assumption : We assume that prospective employers and employees do not know why the previous matches came to an end.
- Firms and workers cannot establish an external reputation. It is in practice difficult to obtain information from former employers.
- It follows that  $\bar{U}_t$  and  $\bar{\Pi}_t$  are the current market alternatives, no matter what the reason for separation.

## *Incentive Compatibility Constraints*

- To deter shirking, the expected utility of high effort must be at least as great as the utility from zero effort (zero effort implies  $b_t = 0$ ,  $W_t = w_t$  :

$$U_t \geq w_t + \delta \bar{U}_{t+1}$$

for all  $t \geq \tau$ .

- Substituting the recursive definition of  $U_t$  above yields, condition *ICW*,

$$\delta \rho(U_{t+1} - \bar{U}_{t+1}) \geq v - b_t$$

for all  $t \geq \tau$ .

- $\delta\rho(U_{t+1} - \bar{U}_{t+1})$  represent gains from continuation for the worker. Note : if bonus is large, future rents from continuation are not necessary. If  $b_t = 0$ , the *rent*  $U_{t+1} - \bar{U}_{t+1}$  must be large enough.
- Since the bonus is not legally enforceable, profits from paying the bonus and continuing the match must be at least as great as from not paying,

$$\Pi_t \geq p - w_t + \delta\rho\bar{\Pi}_{t+1}.$$



- Substituting the recursive definition of  $\Pi_t$  above yields, condition *ICF*,

$$\delta\rho(\Pi_{t+1} - \bar{\Pi}_{t+1}) \geq b_t.$$

- Intuition : the firm will not pay the bonus  $b_t$  unless the future expected gains from employment exceed that bonus.
- We assume here that if the firm doesn't keep their promise to pay the bonus, the employee believes that the firm will never do it in the future. In a similar way, if the worker is caught shirking, the firm believes that (s)he will always do so in the future.
- If something goes wrong : the relationship "sours" and both sides believe it cannot continue : the worker quits the firm or the firm fires the worker.

## *Necessary Condition for a Self-Enforcing Contract*

- Consider the sum of the left-hand sides of *ICW* and *ICF* : the bonus term cancels to yield a necessary condition for a self-enforcing contract, *i.e.*, *IC*,

$$\delta\rho(U_{t+1} + \Pi_{t+1} - \bar{U}_{t+1} - \bar{\Pi}_{t+1}) \geq v.$$

for all  $t$ .

- The wage and bonus terms cancel in this expression, and the individual rationality constraints *IRW* and *IRF* are also independent of the way in which surplus is divided.

- MacLeod and Malcomson (1989) have shown that the conditions  $IC$ ,  $IRF$  and  $IRW$  are necessary and *sufficient* for the existence of a self-enforcing contract. If these conditions hold, there exists a subgame-perfect Nash equilibrium of the repeated game that supports the agreement between the firm and the worker.
- We must have  $\rho > 0$ , that is, no predictable end of the relationship, and it is possible to find a wage  $w$  and a bonus  $b$  such that  $ICW$  and  $ICF$  are both satisfied.
- $v > 0$  implies that worker and firm cannot be both indifferent between keeping promises and deviating. Moreover  $IC$  says that in any period  $t$ , incentive compatibility depends only on future returns to the relationship.

## *Consequences of IC*

- In the model of efficiency wages, the firm receives no rents, so that  $\Pi_t = \bar{\Pi}_t$  and *IC* implies,

$$U_t \geq \bar{U}_t + \frac{v}{\delta\rho}.$$

- This implies that utility  $U_t$  is always greater than the best outside option  $\bar{U}_t$  (*i.e.*, going back to labor market).
- Sustaining cooperation also implies that the firm doesn't try to reduce the wage  $w$  at the beginning of every period. In subgame-perfect equilibrium, these deviations can be punished by low effort or a decision to quit.

- Conversely if the worker receives no rent, that is,  $U_t = \bar{U}_t$ , then, by *IC*, we must have,

$$\Pi_t \geq \bar{\Pi}_t + \frac{v}{\delta\rho}.$$

- This implies that the firm receives profits at  $t$  greater than the next best alternative (fire the employee and go back to the job market).
- Again, only future wages and bonus payments are important.

## *Stationary Market Equilibrium*

- We now consider conditions under which a stationary equilibrium is possible
- The cost of creating a job is a constant  $c$ . The revenue product of a job depends on total employment  $E$  : that is,  $p = p(E)$  and  $p(\cdot)$  is a decreasing function.
- There are  $L$  workers and  $J$  jobs ;  $(1 - \rho)J$  jobs are destroyed for exogenous reasons. To preserve stationarity, assume that exactly  $(1 - \rho)J$  new jobs are created at the beginning of each period. Finally, employment is  $E = \min\{L, J\}$ .

## *Computation of Value Functions*

- Total pay is  $W = w + b$ ;  $ICF$  and  $ICW$  are satisfied.
- Let  $\pi$  be the probability of finding a job during a given period (if unemployed), and if there is unemployment in equilibrium.
- Stationary equilibrium utilities are a solution of the system,

$$\begin{aligned}U &= W - v + \delta\rho U + \delta(1 - \rho)\bar{U}, \\ \bar{U} &= \pi U + (1 - \pi)(u + \delta\bar{U}).\end{aligned}$$

- The solution of this linear system is as follows,

$$U = \frac{(W-v)(1-\delta(1-\pi))+\delta(1-\rho)(1-\pi)u}{(1-\delta)(1-\rho\delta(1-\pi))},$$

$$\bar{U} = \frac{\pi(W-v)+(1-\delta\rho)(1-\pi)u}{(1-\delta)(1-\rho\delta(1-\pi))}.$$

- Let  $\gamma$  be the probability of finding a worker during a given period to fill a vacant job.
- Stationary equilibrium profits are a solution of the system,

$$\Pi = p - W + \delta\rho\Pi,$$

$$\bar{\Pi} = \gamma\Pi + (1 - \gamma)\delta\rho\bar{\Pi}.$$

- This yields the solution,

$$\Pi = \frac{p-W}{1-\rho\delta},$$

$$\bar{\Pi} = \frac{\gamma(p-W)}{(1-\delta\rho)(1-\rho\delta(1-\gamma))}.$$



- Now we have two cases : Efficiency wages à la Shapiro-Stiglitz, or Performance Pay equilibria à la MacLeod-Malcomson.
- If there is unemployment, *i.e.*,  $E = J < L$ , then, all vacant jobs can be filled straightaway, so  $\gamma = 1$  and  $\Pi = \bar{\Pi} = (p - W)/(1 - \delta\rho)$ .
- If all workers are employed, *i.e.*,  $E = L < J$ , then a worker can find a new job straightaway, so that  $\pi = 1$  and  $U = \bar{U} = (W - v)/(1 - \delta)$ .
- There can be no equilibrium with  $J = L$  since  $v > 0$ .

## *IR constraints in Stationary Equilibrium*

- *IRW* becomes,  $U \geq \bar{U}$ , it is easy to check that this is equivalent to,

$$w + b \geq v + u.$$

- *IRC* becomes,  $\Pi \geq \bar{\Pi}$ , this is equivalent to,

$$p \geq W.$$

*Efficiency Wages à la Shapiro-Stiglitz : Equilibrium with Unemployment*

- In this case we have  $\gamma = 1$  and  $\Pi = \bar{\Pi} = (p(J) - W)/(1 - \delta\rho)$ .
- The probability of finding a job is the ratio of newly created jobs divided by the number of unemployed workers, that is, with  $L \geq J$ ,

$$\pi = \pi(J) = \frac{(1 - \rho)J}{L - \rho J} = \frac{1 - \rho}{(L/J) - \rho}.$$

*Two conditions for a stationary unemployment equilibrium :*

(i), firms must create enough jobs to replace the destroyed ones, so  $\bar{\Pi}$  must be equal to the cost  $c$  of creating a new job (it's a zero-profit condition).

(ii), neither firms nor workers must be able to gain by cheating. Firms would always cheat on a bonus since  $\Pi = \bar{\Pi}$  (they have nothing to lose if they do not keep their promises). So  $b = 0$  and  $w$  must be above the market-clearing level, that is, with  $IC$ ,

$$U - \bar{U} \geq \frac{v}{\delta\rho}.$$

*A Market Equilibrium is a pair  $(w, J)$ , such that,*

*(i), using the expression  $\Pi$ , we get,*

$$\frac{p(J) - W}{1 - \delta\rho} = c.$$

*(ii), using the expressions for  $U$  and  $\bar{U}$ , and substituting in  $IC$ , we find,*

$$w \geq u + \frac{v}{\delta\rho(1 - \pi(J))}.$$

- An equilibrium must lie on the zero-profit line

$$w = p(J) - (1 - \delta\rho)c,$$

with  $w$  above the upward-sloping curve  $u + v/\delta\rho(1 - \pi(J))$ , and  $J < L$ .

- $1/(1 - \pi(J))$  is increasing, equal to 1 if  $J = 0$ , and equal to  $\infty$  if  $J = L$ .

- Draw picture in the  $(w, J)$  plane.
- There is an infinity of efficiency-wage equilibria corresponding to different values of  $w$  and  $J$  along the zero-profit line. Each  $w$  corresponds to a subgame-perfect equilibrium.
- The intersection of the two curves has the highest employment : it is the most efficient equilibrium.
- A downward shock on  $p(J)$  will force downward adjustments of wage  $w$  and employment  $J$  (...).

## *Performance Pay (i.e., Bonus) Equilibria*

- In this case we have  $L < J$  or  $\pi = 1$  and

$$U = \bar{U} = (W - v)/(1 - \delta).$$

- There are  $(1 - \rho)L$  workers looking for a job due to exogenous job destruction, and there are  $J - \rho L$  vacant jobs.

- The probability of a vacancy being filled during the period is therefore,

$$\gamma = \gamma(J) = \frac{(1 - \rho)L}{J - \rho L}.$$



*Conditions for Bonus Equilibrium are :*

(i), the expected post-entry profit per worker is equal to the cost of creating a new job  $c$ . So  $\bar{\pi} = c$  (again, the zero-profit condition).

(ii), neither firms nor workers must be able to gain by cheating. Since workers can always get another job, *i.e.*,  $U = \bar{U} = (w + b - v)/(1 - \delta)$ , they will shirk unless they get a bonus  $b \geq v$ . So any equilibrium of this type must have performance pay, with *IC*,

$$\pi - \bar{\pi} \geq \frac{v}{\delta\rho}.$$

*A Market Equilibrium with Performance Pay is a pair  $(w, J)$ , such that,*

*(i), using the expression for  $\bar{\Pi}$ , we get,*

$$\frac{\gamma(J)(p(J) - w - b)}{(1 - \delta\rho)(1 - (1 - \gamma(J))\delta\rho)} = c.$$

*(ii), using the expressions for  $\Pi$  and  $\bar{\Pi}$ , and substituting in  $IC$ , we find the *no-cheating constraint NCC*,*

$$w + b \leq p(J) - \frac{v}{\delta\rho} + v(1 - \gamma(J)).$$

- When  $J = L$ ,  $\gamma(L) = 1$  and we have  $\gamma(J) \rightarrow 0$  when  $J \rightarrow \infty$ .
- Without loss of generality we can choose  $b = v > 0$ . The *NCC* becomes,

$$w \leq p(J) - \frac{v}{\delta\rho} - v\gamma(J).$$

- The zero-profit condition becomes,

$$w = p(J) - v - c(1 - \delta\rho) \left[ \frac{1 - (1 - \gamma(J))\delta\rho}{\gamma(J)} \right]$$

- So  $NCC$  is an upward sloping curve in the  $(J, w)$  plane because when  $J$  increases without bound,  $\gamma$  decreases : it becomes harder to fill a vacancy and wages  $w$  may increase. Along  $NCC$ , we have,

$$\frac{dw}{dJ} = p'(J) - v\gamma'(J).$$

This derivative is positive if  $p' \simeq 0$ .

- The zero-profit line is downward sloping, along  $\bar{\Pi} = c$ , we have,

$$\frac{dw}{dJ} = p'(J) + \frac{c(1 - \rho\delta)^2\gamma'(J)}{\gamma(J)^2} < 0.$$

- Draw a picture of this case in the  $(J, w)$  plane.
- The unique efficient performance pay (or bonus) equilibrium is given by the intersection of  $NCC$  and the zero-profit,  $\bar{\Pi} = c$  condition. This point has less vacancies and higher wages  $w$  than the others (*i.e.*, the minimal value of  $J$  compatible with  $NCC$  and zero-profit).
- An unanticipated but permanent fall of the curve  $p(J)$  leaves the efficient number of jobs unchanged at  $L$  but requires a fall in  $W$ . Pay responds to such a shock but employment and vacancies remain unchanged (...)