An IV Model of Quantile Treatment Effects

Readings Group - LMI

January 5, 2011
Introduction

Model
   Framework
   Theorem
   Assumptions

Identification
   Quick Overview of Identification Issues
Quantile Regression (QR) and IV

- QR useful to study heterogeneous impact of variables on outcome distribution.
- BUT QT inconsistent and incapable of measuring a causal effect if endogenous variables of interest.
- Ch. & Ha. (2005) develop a model of Quantile Treatment Effects (QTE) and prove identification.
Main Features of Ch. & Ha. (2005)

- Restricting conditions on the rank across treatment states.
- Identification of the true QTE through IV method without functional form assumptions.
- Address the endogeneity for discrete or continuous variables.
- Rank invariance and rank similarity (weaker) imply a testable moment condition
IVQR Model I

- $D$ is treatment (exposed as binary but authors claim that it is wlog) and $Y_d$ is potential outcome for realization $d$. Observed individual characteristics are $X = x$.

- Potential outcomes are measured by the Quantile Treatment Response function, noted, for some $\tau \in (0, 1)$:

$$q(d, x, \tau) \text{ (QTR)}$$

- Quantile Treatment Effects is, for some $\tau \in (0, 1)$:

$$q(1, x, \tau) - q(0, x, \tau) \text{ (QTE)}$$

- In general, for discrete case or continuous case, QTE are $q(d', x, \tau) - q(d, x, \tau)$ or $\frac{\partial q(d, x, \tau)}{\partial \tau}$. One integrates on $\tau$ for average effects.
Endogeneity and selection bias: realized treatment $D$ is correlatively selected with potential outcomes $Y_D$.

Ch. & Ha. (2005) shows how one can estimate the quantiles of latent outcomes through a non-linear quantile-type conditional moment restriction and with an instrument $Z$:

$$
P[Y \leq q(D, X, \tau)|X, Z] = \tau$$ (1)

$Z$ affects $D$ and is independent of $Y_D$. 

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**An IV Model of Quantile Treatment Effects**
Rank variable

- Skorohod representation of latent outcomes:
  \[ Y_d = q(d, X, U_d) \] where \( U_d \sim U(0, 1) \) where \( q(d, X, \tau) \) is the \( \tau \)-quantile of \( Y_d \mid X \).

- The idea is that \( q \) corresponds to \( F^{-1} \): \( U_d \) is the rank variable responsible for the heterogeneity (or unobserved characteristic e.g. “ability”).

\[
\begin{align*}
1 & \quad U_d \quad 0 \\
\{ & \quad Y \mid D, X \quad \}
\end{align*}
\]
Moment restriction

- Rank invariance (resp. similarity): $U_d|X, Z$ are equal (have same distribution conditional on $X, Z$ and some random vector $V$). $V$ plays a role in the selection of the treatment.

- Adding 4 main conditions (see assumptions below) jointly holding, the testable implication of the model is: $\forall \tau \in (0, 1)$,

$$
\mathbb{P}[Y \leq q(D, X, \tau)|X, Z] = \mathbb{P}[Y < q(D, X, \tau)|X, Z] = \tau \quad (2)
$$

- Intuition of the proof: Rank invariance means $U_d = U|X, Z$ and leads to

$$
\{ Y \leq q(D, X, \tau) \}\text{ equivalent to } \{ U \leq \tau \}
$$

- This theorem gives conditions to recover the quantiles of potential outcomes.
Assumptions

- A1: Conditional on $X = x$, $\forall d$, $Y_d = q(d, x, U_d)$ where $q(d, x, .)$ strictly increasing and $U_d \sim \mathcal{U}(0, 1)$.
- A2: $\forall d$, $U_d \perp Z|X = x$.
- A3: $D = \delta(Z, X, V)$ for $\delta$ unknown and $V$ random vector.
- A4: Rank invariance (a) or rank similarity (b).
- A5: $D, X, Z, Y = q(D, X, U_D)$ are observed variables.
Discussion

- A2: Conventional independance restriction.
- A3: Representation of the selection mechanism and $V$ describes the difference in treatment for observationnally identical individual (same $Z, Y, Z$ but different $D$).
- A4a (RI): Same $U$ (unobserved factor) leads to same rank across all treatment states. Choice of $X$ may be crucial!
- A4b (RS): RI leads to degenerate potential outcomes but one can imagine true multivariate unobserved factors. RS allows the rank $U_d$ to change across treatment spaces $d$. 
Moment equation

- To non-parametrically identify a function $\mu(.)$, the idea is to write moment equation as a linear IV condition:

$$E(Y - \mu(D)|Z) = 0$$

- Identification conditions are often full rank conditions on the Jacobian of the moment equation (differentiation wrt $Y$).

- In the case of $D, Z$ both binary, vector of moment equations writes with $y_D = (1 - D)y_0 + Dy_1$

$$\Pi(y = \begin{pmatrix} y_0 \\ y_1 \end{pmatrix}) = \begin{pmatrix} P[Y \leq y_D|Z = 0] - \tau \\ P[Y \leq y_D|Z = 1] - \tau \end{pmatrix}$$
Global Identification

- Identification means that $y$ is the unique solution of $\Pi(y) = 0$ and is achieved under a full rank condition (FRC) on $\Pi'(y)$.
- Intuition of the FRC: impact of $Z$ on $(Y, D)$ is sufficiently rich (diversity of different treatment spaces conditional on $Z$).
- Monotone likelihood ratio condition is equivalent to FRC:

$$\forall y = \begin{pmatrix} y_0 \\ y_1 \end{pmatrix}, \quad \frac{f_{Y,D}(y_1, 1|Z = 1)}{f_{Y,D}(y_0, 0|Z = 1)} \geq \frac{f_{Y,D}(y_1, 1|Z = 0)}{f_{Y,D}(y_0, 0|Z = 0)}$$

- Partial Identification (i.e. Identification Region without FRC): see set-inference approach literature.
Conclusion

- Endogeneity in Q-reg
- Comparison with Abadie, Angrist, Imbens (2002) : see slides
- Identification of a non parametric model of QTE, causal (structural) interpretation of QTE.
- Large framework : discrete or continuous treatment but continuous outcomes.