Groupe de lecture

*Instrumental Variables Estimates of the Effect of Subsidized Training on the Quantiles of Trainee Earnings*

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*Econometrica (2002)*

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Objectives
Using IV to estimate the effect of treatment on the quantiles of an outcome distribution.

Illustration
Estimate the effect of training over the Job Training Partnership Act (JTPA).
Notation

- \( Y \) : continuously distributed scalar **outcome** variable \( \rightarrow \) earnings
- \( D \) : binary **treatment** indicator \( \rightarrow \) program **participation**
- \( Z \) : binary **instrument** \( \rightarrow \) randomized **offer of training**
- \( X \) : vector of **covariates**

**NB:** \( Z \neq D \)

- some people who were offered training does not receive it
- some people who were not offered training receive it anyway
Notation

- Potential outcomes: $Y_0, Y_1$ indexed against $D$.
- Potential treatment status: $D_0, D_1$ indexed against $Z$. 
Assumptions

**Assumption 2.1**

(i) **Independence**: \((Y_0, Y_1, D_0, D_1)\) jointly independent of \((Z|X)\)

Condition d’exclusion

(ii) **Nontrivial Assignment**: \(P(Z = 1|X) \in (0, 1)\)

(iii) **First-Stage**: \(E(D_1|X) \neq E(D_0|X)\)

(iv) **Monotonicity**: \(P(D_1 \geq D_0|X) = 1 \rightarrow (\text{no defiers})\).
**Lemma 2.1**

2.1(i) \( \Rightarrow (Y_1, Y_0) \perp \perp (D|X, D_1 > D_0) \)

**Proof**

\((Y_0, Y_1, D_0, D_1) \perp \perp Z|X \Rightarrow (Y_0, Y_1) \perp \perp Z|X, D_1 = 1, D_0 = 0\)

and when \( D_1 = 1 \) and \( D_0 = 0 \), \( D \) can be substituted for \( Z \)
The population of compliers

Lemma 2.1: In the population of compliers, comparisons by $D$ conditional on $X$ have a causal interpretation.

Since $Z = 0 \Rightarrow D = 0$ (almost) in the JTPA, results for the population of compliers are also valid for the treated.

⇒ estimation on the population of compliers.
The QTE Model

Linear model for conditional quantiles:

\[ Q_\theta(Y|X, D, D_1 > D_0) = \alpha_\theta D + X' \beta_\theta \]

- \( Q_\theta(Y|X, D, D_1 > D_0) \) is the \( \theta \)-quantile of \((Y|X, D)\) for compliers.
- \( \alpha_\theta \) is the difference in the conditional \( \theta \)-quantile of \( Y_1 \) and \( Y_0 \) for compliers.
- \( \alpha_\theta \neq \) the \( \theta \)-quantile of the difference \((Y_1 - Y_0)\).
\[(\alpha_{\theta}, \beta_{\theta}) = \arg\min_{(\alpha, \beta)} E[\rho_{\theta}(Y - \alpha D - X'\beta)|D_1 > D_0]\]

where the check function is:

\[\rho_{\theta}(\lambda) = \lambda(\theta - 1\{\lambda < 0\}) = \begin{cases} 
\lambda \theta & \text{if } \lambda \geq 0 \\
\lambda(\theta - 1) & \text{if } \lambda < 0
\end{cases}\]
Lemma 2.1 $\Rightarrow$ solution to this problem has a causal interpretation.

**BUT** : The set of compliers is not identified $\Rightarrow$ the problem cannot be solved directly.
To involve observed quantities only:

\[ \kappa(D, Z, X) = 1 - \frac{D(1 - Z)}{1 - \pi_0(X)} - \frac{(1 - D)Z}{\pi_0(X)} \]

where \( \pi_0 = P(Z = 1|X) \)

- \( \kappa = 1 \) when \( D = Z \)
- \( \kappa \) negative otherwise.
κ allows us to estimate the proportion of compliers:

**Lemma 3.1 (Abadie (2000))**

Let $h(Y, D, X)$ be any real function of $(Y, D, X)$ such that $E|h(Y, D, X)| < \infty$. Given assumption 2.1,

$$E[h(Y, D, X)|D_1 > D_0] = \frac{1}{P(D_1 > D_0)} E[\kappa.h(Y, D, X)]$$
\[
(\alpha_\theta, \beta_\theta) = \arg\min_{(\alpha, \beta)} E[\rho_\theta(Y - \alpha D - X' \beta)|D_1 > D_0]
= \arg\min_{(\alpha, \beta)} E[\kappa \rho_\theta(Y - \alpha D - X' \beta)]
\]
In practice, they use the conditional expectation given $U = (Y, D, X)$:

$$
\kappa_{\nu} = E[\kappa|U] = 1 - \frac{D(1 - \nu_{0}(U))}{1 - \pi_{0}(X)} - \frac{(1 - D)\nu_{0}(U)}{\pi_{0}(X)}
$$

avec $\nu_{0}(U) = E[Z|U] = P(Z = 1|Y, D, X)$

**Lemma 3.2**

Under assumption 2.1, $\kappa_{\nu}(U) = P(D_1 > D_0|U)$
**Proof**

The product $D.(1 - Z)$ differs from 0 only if $Z = 0$ and $D_0 = 1$. By monotonicity, $D_0 = 1$ implies $D_1 = 1$. Thus:

$$E[D.(1 - Z)|U] = P(D(1 - Z) = 1|U)$$
$$= P(D_1 = D_0 = 1|U).P(Z = 0|D_1 = D_0 = 1, U)$$
$$= P(D_1 = D_0 = 1|U).P(Z = 0|D_1 = D_0 = 1, Y_1, X)$$
$$= P(D_1 = D_0 = 1|U).P(Z = 0|X)$$

Similarly, $E[(1 - D).Z|U] = P(D_1 = D_0 = 0|U).P(Z = 1|X)$

$$\kappa_\nu(U) = E\left[1 - \frac{D(1 - Z)}{P(Z = 0|X)} - \frac{(1 - D)Z}{P(Z = 1|X)}\right]|U$$
$$= 1 - P(D_1 = D_0 = 1|U) - P(D_1 = D_0 = 0|U)$$
$$= P(D_1 > D_0|U)$$
Estimation strategy

• First step: non parametric estimation of $\nu_0(U)$ and $\pi_0(X) \Rightarrow$ estimate of $\kappa_{\nu}(U)$

• Second step: estimation of

$$
\hat{\delta}_\theta = \arg\min_{\delta} \frac{1}{n} \sum_{i=1}^{n} \{ \hat{\kappa}_{\nu}(U_i) \geq 0 \} . \hat{\kappa}_{\nu}(U_i) . \rho_\theta(Y_i - W'_i \delta)
$$

where $W = (D, X')'$ and $\delta_\theta = (\alpha_\theta, \beta_\theta)$
Asymptotic properties

**THEOREM 3.1**
Under assumptions 2.1 and 3.1 (and some conditions),

\[ n^{\frac{1}{2}} (\hat{\delta}_\theta - \delta_\theta) \xrightarrow{d} N(0, \Omega) \]

**THEOREM 3.2**
Under assumptions 2.1 and 3.1 (and some conditions),

\[ \hat{\Omega} \xrightarrow{p} \Omega \]
The JTPA

- From October 1983 to the late 1990’s.
- Largest component = **Title II**, which supports training for the economically disadvantaged.
- Applicants were **randomly selected** for JTPA treatments within sites.
- **Outcome** = **sum of earnings** in the 30th months period after assignment.
The JTPA

JTPA services offer:

• classroom training in occupational skills and/or basic education
• on-the-job training and/or job search assistance
• other services.
The JTPA

JTPA eligibility:

- long term use of welfare
- high school drop out
- 15 or more recent weeks of unemployment
- limited English proficiency
- physical or mental disability
- reading proficiency < 7th grade
- arrest record
Estimation

- $Y$: 30-month earnings

- $D$: enrollment for JTPA services

- $Z$: offer of services
  - 60% of those offered training actually received JTPA services.
  - less than 2% of the control group received JTPA services.

- $X$: dummies for Black and Hispanic applicants, dummy for high-school graduates, dummies for married applicants, age-group dummies, dummy for unemployment.
Estimation

- **OLS** as benchmark for **Quantile regression**
- **2SLS** as benchmark for **Quantile Treatment Effect**, with the randomized offer of treatment as an instrument.
Results

- **Table II** : QUANTILE REGRESSION and OLS : gap in quantiles by trainee status is much larger below the median than above. BUT not necessarily have a causal interpretation.

- **Table III** : QUANTILE TREATMENT EFFECT and 2SLS :
  - Men : no evidence of an impact of the treatment at low quantiles, whereas effects above the median are large and significant.
  - Women : significant effects of the treatment at every quantile, with the largest proportional effects at law quantiles.