Economic Scenario Generators and Incomplete Markets

C., GOURIEROUX [(1)] and A., MONFORT [(2)]

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[1]CREST and University of Toronto.
[1]CREST and Banque de France.

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Abstract

In Solvency 2 the valuations in the prudential balance sheets have to be market consistent. Since a large number of financial assets and insurance products are not traded on liquid secondary markets, the mark-to-market valuations have to be completed by technical standards defined by the supervision. We discuss this joint use of mark-to-market valuation and technical standards, explain how to perform coherent stress tests by simulation, and finally emphasize the potential role of the technical standard for economic policy.

Keywords : Solvency 2, Incomplete Market, Scenario, Technical Standard, Stress Test.

1 Introduction

An aim of Solvency 2 is to standardize and reinforce the prudential regulation for the insurance and occupational pensions sectors within the European Union[2]. This new regulation has to fix the frame to compute the technical provisions and the solvency capital requirements (SCR) of the insurance companies. Currently the official implementation of Solvency 2 for Europe is scheduled on January 1st, 2016. Solvency 2 is accompanied by Quantitative Impact Studies (QIS) which are used to analyze how the new tools are implemented by the firms and to modify and improve these tools, if they are revealed misunderstood, or inadequate [see EIOPA (2013) in this respect]. The fifth quantitative impact studies (QIS5) require the computation of the prudential balance sheets for the year 2009. The valuation of the lines of these balance sheets is based on notions of economic values, which have to be consistent with market values[3] (see Appendix 1).

What can be learned from QIS 5 [see e.g. Autorité de Contrôle Prudentiel (2011)]?

First the sector of insurance is much less concentrated than the banking sector. For France this sector included 440 small and medium size companies for a total of 484 companies. Moreover a large proportion of these companies did not have the technical knowledge and/or power to directly implement some methodologies appearing in Solvency 2, especially the methodologies based on financial pricing theory[4]. This explains why a large majority of these companies appealed to specialized consulting firms for the technical components of Solvency 2 (see Table 1 below).

Second the deadline given for the quantitative studies was rather short, with a clear impact on the development of these consulting firms. As seen in Table 1, one firm Barrie & Hibbert (now a component of Moody's) has 2/3 of the market for the so-called Economic Scenario Generators; then 3/4 of the market is covered once we account for the second consulting firm : FRACTALES. Even if the basic modelling and techniques proposed by Barrie & Hibbert, say, can be used with some flexibility by the insurance companies, there is a risk for the expected standardization of the new supervision to be

[2]We will not discuss the current implementation of this regulation in other countries as Canada, US, Japan....

[3]We will not discuss in this note recent adjustments appeared in the QIS 6 of 2013 and of the QIS 7 of 2014. They mainly concern the long term guarantee assessment.

[4]The same fact has been observed for other European countries [see EIOPA (2013)]

implicitly fixed by a consulting firm , instead of being chosen after discussions between the insurance companies, the consulting firms and the supervisors with the objective of a sustainable insurance sector.

Table 1 : The Market Shares for ESG (year 2014)

B&M, Moody's	66%
SOLVEO, FRACTALES	8%
Other external tool	6%
internal tool	20%

Finally, it has been noted that the current valuation methods were not very robust. They were very sensitive to the methods used to calibrate the models; in particular to short term market conditions. They also left too much room for arbitrary pricing of some specific risks by the insurance companies.

The aim of this note is to emphasize the difficulty in applying a market consistent valuation approach, when a lot of risk factors, such as inflation, real estate, longevity, lapses, have no derivatives highly traded on organized financial markets. In other words, applying a market consistent valuation approach does not mean that any line of the balance sheet can be valued at market price. This is this difficulty that we discuss in this paper.

In Section 2 we briefly review the modern pricing theory when only a part of all assets is tradable. In such an incomplete market framework, there exists a multiplicity of market consistent valuation formulas, and thus a multiplicity of market consistent prudential balance sheets. How to choose among this large number of possibilities ? This question is discussed in Section 3 on estimation, calibration and technical standards. Loosely speaking the available data on observable risk factors and on derivative prices can be used to fix some components of the model. However, other components (parameters) of the model have to be fixed arbitrarily. Therefore norms (technical standards) have to be chosen and applied uniformly by all insurance companies to avoid regulatory arbitrages, while keeping in mind country specificities [5]. We explain in Section 4 how to analyze the robustness of the valuation methodology by considering the effects of shocks on the prudential balance

[5]This explains the terminology "holistic balance sheet" used in EIOPA (2014).

sheets, on the technical provisions, and on the solvency capital requirement (SCR)[6]. These shocks can be introduced on the historical dynamics of the risk factors, on the way the market is pricing the derivatives, but also on the selected technical standards. These robustness analyses are a useful step for fixing the standards.

These analyses generally require simulations of the risk factors. These are often nested simulations to be done in a consistent way under the real world (historical distribution) and the pricing world (risk-neutral distribution). In Solvency II terminology the simulators have been called Economic Scenario Generators (ESG). Finally we discuss the potential use of the fixed technical standards for economic policy. Section 5 concludes. Two appendices are provided. Appendix 1 describes and discusses some articles and definitions in the prospect of the directive for Solvency 2. Appendix 2 gives the meaning of different acronyms.

2 Market Consistent Modelling

A market consistent modelling is proposed in QIS 5 at least for business lines including options or guarantees (see Appendix 1 ii)). This modelling assumes no arbitrage opportunity (see Appendix 1, TP.2.96). We introduce below this modelling with the standard terminology used in Finance, even if words such as incomplete market, risk-neutral (R.N.) distribution, or stochastic discount factor (s.d.f) have been avoided in the European directive.

2.1 The pricing formula

In modern Finance the valuation of (derivative) assets is based on the assumption of no arbitrage opportunity, that is on the impossibility to make surely positive profits on financial markets from an initial zero investment. When the information set of the investor is based on variables Y_t, this methodology leads to a valuation formula for a European derivative[7] paying $C(Y_{t+H})$ at time-to-maturity H, in which the price at date t is given by :

[6] or on the solvency ratio defined as the ratio of own funds/SCR.

[7] We consider European derivative for expository purpose, but a similar pricing formula is valid for derivatives with path dependent cash-flows $C(Y_{t+H}, Y_{t+H-1}, \ldots)$, say.

$$\pi_t(C;H) = E_t[M_{t,t+H}C(Y_{t+H})], \tag{2.1}$$

where $M_{t,t+H} = M_{t,t+1}M_{t+1,t+2}\dots M_{t+H-1,t+H}$ [see Harrison, Kreps (1979)].

$M_{t,t+1}$ is a short term stochastic discount factor for period $(t, t+1)$, that is a positive function of the information up to time $t+1$, and E_t denotes the conditional expectation given the information available at time t.

This conditional expectation is taken with respect to the historical distribution of variables (Y_t) (also called physical probability, or real world, or objective probability).

The pricing formula (2.1) can be applied to highly traded derivatives, but also to illiquid or even no traded assets. In the first case $\pi_t(C;H)$ is the observed market price, and equation (2.1) will imply restrictions on the pattern of the stochastic discount factor. For illiquid or non traded asset, formula (2.1) is used in the reversed way : from the knowledge of the s.d.f., we deduce for this derivative asset a quote, which is consistent with the market prices of other highly traded assets and with the quotes of the other non traded assets.

2.2 Risk-Neutral probability

The pricing formula (2.1) can in particular be used to price the unitary payoffs : $C(Y_{t+H}) = 1$. We deduce the value (or quote) at date t of the zero-coupon bond of term H as :

$$B(t, t+H) \equiv E_t(M_{t,t+H}). \tag{2.2}$$

In particular the short term riskfree interest rate r_t is given by :

$$\exp(-r_t) = B(t, t+1) = E_t(M_{t,t+1}). \tag{2.3}$$

By substitution in pricing formula (2.1), we get :

$$\pi_t(C;H) = E_t\{\exp(-r_t \dots - r_{t+H-1})\Pi_{h=1}^H \left[\frac{M_{t+h-1,t+h}}{E_{t+h-1}(M_{t+h-1,t+h})}\right] C(Y_{t+H})\}. \tag{2.4}$$

Let us denote by $p_{t+1|t}$ the historical transition of Y, that is the conditional density of Y_{t+1} given the current and lagged values of Y. It is easily checked that :

$$q_{t+1|t} = \frac{M_{t+1,t}}{E_t(M_{t+1,t})} p_{t+1|t}, \tag{2.5}$$

defines another transition density. The associated probability is called the risk-neutral probability and is usually denoted by Q. Thus the pricing formula (2.4) becomes :

$$\pi_t(C; H) = E_t^Q[\exp(-r_t \ldots - r_{t+H-1}) C(Y_{t+H})]. \tag{2.6}$$

The historical probability summarizes the uncertainty of the environment, whereas the risk-neutral probability has no such interpretation. This is just a convenient mathematical way for representing standardized prices of contingent assets.

2.3 A special case

In basic textbooks and pricing formulas, the methodology is often presented with a constant interest rate : $r_t = r$, independent of t. When $r = 0$, the pricing formula (2.6) becomes :

$$\pi_t(C; H) = E_t^Q[C(Y_{t+H})]. \tag{2.7}$$

This means that the payoff is valued (under Q) as if the investor were risk-neutral, which explains the terminology risk-neutral probability. We deduce :

$$\pi_t(C; H) = E_t^Q[\pi_{t+1}(C; H-1)], \tag{2.8}$$

by the Iterated Expectation Theorem. This is the martingale property of the price of this payoff under the risk-neutral probability (but not in general under the historical probability). When r is constant different from zero, the martingale property applies to the discounted payoff : $\exp(-rt)\pi_t(C, H)$.

3 Estimation, Calibration and Accounting Standards

3.1 The principle

The market consistent modelling introduced in Section 2 can be used for the valuation of the lines of the balance sheet and for the predictions of future balance sheets, when the historical dynamics and the stochastic discount factor (or the historical and risk-neutral dynamics) are given. How to fix these two components of the model in practice ?

In this respect the following remarks are important

i) It is not sufficient to only specify the risk-neutral (R.N.) probability, say. Indeed this R.N. probability is needed to value the derivative assets, but we can't infer from it the historical transition of Y, hence the predictions on Y without additional information on the stochastic discount factor. Similarly it is not sufficient to only specify the historical probability, as the stochastic discount factor would be needed in addition to it in order to price the derivatives.

ii) The historical and risk-neutral probabilities cannot be specified independently. Indeed their ratio satisfies condition (2.5) and the change of measure $q_{t+1|t}/p_{t+1|t} = M_{t+1,t}/E_t(M_{t+1,t})$ is constrained by the pricing formula (2.1) written for liquid assets.

iii) The choice has to be compatible with the available data, that are essentially the evolutions of the underlying variables Y_t, and the prices of liquid derivatives. Intuitively the first type of data is used to estimate (consistently) the historical probability or a part of it. The second type of data is used to derive constraints on the stochastic discount factor from pricing formula (2.1), once the historical probability is given. This is usually called the calibration step. The estimation of the R.N. probability, which mixes the historical probability and the s.d.f. by (2.5) may need the two types of data used in a coherent way.

iv) As usual in modelling, there is a tradeoff between flexibility and misspecification. A too constrained model, such as a model easy to use and leading to closed form valuation formulas, will likely be misspecified with significant biases, and possibly severe underestimation of the future risks for instance. A too flexible model often leads to imprecise results, that is to a large uncertainty on the reasonable levels of technical provisions and

Solvency Capital Requirement (SCR).

3.2 The identification issue

However the main difficulty is the impossibility to market price all derivatives of interest included in the balance sheets of the financial institutions and insurance companies. Indeed risk variables such as inflation, Gross Domestic Product (GDP) growth, unemployment, longevity, real estate... are not traded on deep, liquid and transparent (DLT) financial markets. This difficulty appears clearly in the market consistent modelling by means of pricing equation (2.1).

(*) The stochastic discount factor cannot be nonparametrically identified, since it is a (complicated) function of infinite dimension, which cannot be recovered from the finite number of derivative prices with liquid trades.

(**) If the s.d.f. is parametrically specified with a number K, say, of specific parameters [8] often interpreted as risk premia, we can distinguish the three following situations, according to the number p of derivatives used :
just identification, $p = K$: There is a unique s.d.f. solution.
overidentification, $p > K$: The model is too restricted and immediately rejected from the available data.
underidentification, $p < K$: It is not possible to deduce the s.d.f. in an unambiguous way. A number $K - p$ of risk-premia cannot be revealed from the data.

There is a tendency among practitioners and supervisors to privilegiate the case of just identification, which seems mathematically easier to implement. [9] But, by choosing a too small number K of parameters, there is a risk of misspecification (see the discussion in subsection 3.4). This risk appeared immediately in the first implementation of Solvency 2, where the insurance companies had the choice of the modelling including the data on which the calibration step is based. Even if they employed similar basic models (see e.g. the basic model described in Section 3.4), their results differed by the choice of the derivatives used at the calibration step. By simply modifying

[8] that are parameters independent of the parameters characterizing the historical distribution.

[9] A typical example is the calibration of the yield curve with the Smith-Wilson approach [see e.g. European Commission (2010) b].

this set of derivatives, these institutions might fix almost as they want the levels of their reserves.

In such a case, the best solution is certainly the choice of an underidentified model with an increased role of the supervisors. First note that the standard terminology "best estimates" introduced in the Solvency 2 terminology is very misleading. We can only look for a best estimate among a set of consistent estimates. However we are in a case where some risk premia are not identifiable, that is where the set of consistent estimates is empty. The only reasonable solution is to fix a standard (a norm) for the parameters, which are not identifiable, standard which will be later on applied uniformly by all institutions. This solution is typically followed in accounting, when proposing different standardized amortizing schemes for the equipments. These schemes are not assumed to fit the actual values of these equipments, for which a liquid second-hand market does not exist [see also the discussion in Moody's Analytics (2013) on the difference between producing a "realistic transaction price or some stabilized price anchored around economic fundamentals", or the discussion in Section 4.3.].

3.3 An example

Let us now illustrate the identification issue from a simple bivariate example where the underlying variables correspond to a stock index $S_t = Y_{1t}$ and a real estate index $RE_t = Y_{2t}$, say. We assume a zero riskfree rate $r = 0$. Thus we have only to specify the historical and risk-neutral probabilities, the s.d.f. being deduced by the change of measure.

Historical distribution : We consider a Gaussian VAR(1) model for the log-indexes :

$$\log Y_t \equiv \begin{pmatrix} \log Y_{1t} \\ \\ \log Y_{2t} \end{pmatrix} |Y_{t-1} \sim N\left[a + B \log Y_{t-1}, \Omega\right].$$

Risk-Neutral distribution : We consider another Gaussian VAR(1) model for the log-indexes, taking into account the martingale property under risk-neutral probability Q :

$$\log Y_t | Y_{t-1} \sim N[\log Y_{t-1} - \frac{1}{2}\begin{pmatrix} \sigma_{11} \\ \sigma_{22} \end{pmatrix}, \Sigma],$$

where $\Sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{pmatrix}$ and $\log Y_t \equiv \begin{pmatrix} \log Y_{1t} \\ \log Y_{2t} \end{pmatrix}$.

Note that the historical and risk-neutral volatility matrices Ω and Σ are not necessarily equal in a discrete time framework, in which the self-financed portfolio updating are performed at discrete dates.[10]

From historical (discrete time) data on S_t and RE_t, we can estimate the historical parameters a, B, Ω, by Gaussian maximum likelihood. Let us now consider the calibration step. Let us assume that there exist liquid derivatives written on the stock index. They can be used to derive the risk-neutral variance σ_{11}. Similarly we can derive the risk-neutral variance σ_{22} from the prices of derivatives written on the real estate index.

The difficulty is the absence of liquid derivatives written on both the stock and real estate indexes (i.e. quanto-derivatives). Thus σ_{12}, or equivalently the risk-neutral correlation : $r_{12} = \sigma_{12}/(\sigma_{11}\sigma_{22})^{1/2}$ is not identifiable.

Clearly the risk-neutral parameter r_{12}, which captures the value of risk-dependence between the stock and real estate risks is of primary importance. Several standards are possible, but they have to be clearly announced and their relevance has to be discussed. They are often hidden in the models currently implemented. Two norms appear in the proposed basic models :

(*) $r_{12} = 0$

(**) $r_{12} = \rho_{12}$, where $\rho_{12} = \omega_{12}/(\omega_{11}\omega_{22})^{1/2}$., where ω_{ij} is the generic element of matrix Ω.

The first norm is typically hidden, in the models assuming the independence between risk factors under the risk-neutral probability.

The second norm is followed in the pricing of credit derivatives developped by KMV-Moody's, or in the basic model for Solvency 2 proposed by KPMG (see Section 3.4).

3.4 A basic model

Let us now discuss the structures and the limitations of the basic models considered by the industry. For illustration purpose, we discuss below a

[10] Contrary to continuous time models with continuous portfolio updating [see e.g. Black, Scholes (1973)].

simplified model described in Plomb et al. (2013) and developed by KPMG. This model is rather representative of the basic models used in practice [see e.g. Baldvinodottir, Palenborg (2011)]. Of course, the consulting firms may also develop more sophisticated models, and their models may be modified and improved by the insurance companies, and they can be calibrated and estimated internally.

The basic model defines the risk-neutral dynamics of four variables that are the (instantaneous) short rate r_t, an equity index S_t, a real estate index RE_t and an inflation rate I_t. The RN dynamics is written in continuous time as :

$$\begin{cases} dS_t = S_t r_t dt + \sigma_S S_t dW_S(t), \\ dRE_t = RE_t r_t dt + \sigma_{re} RE_t dW_{re}(t), \\ dI_t = \lambda(\mu - I_t)dt + \sigma_i dW_i(t), \end{cases} \tag{3.1}$$

where :

$$dr_t = (\theta(t) - ar_t)dt + \sigma dW_r(t), \tag{3.2}$$

and the different Brownian motions may be correlated :

$$Corr(dW_t, dW_t') \equiv \begin{pmatrix} 1 & \rho_{S,re} & \rho_{S,i} & \rho_{S,r} \\ \cdot & 1 & \rho_{re,i} & \rho_{re,r} \\ \cdot & \cdot & 1 & \rho_{i,r} \\ \cdot & \cdot & \cdot & 1 \end{pmatrix}. \tag{3.3}$$

The dynamics for the equity and real estate indexes are simply Black-Scholes models [Black, Scholes (1973)], allowing for a stochastic interest rate and possibly dependence between the Brownian motions. The short term rate is assumed to satisfy an Hull-White model [Hull, White (1993)]. Finally the inflation rate follows and Ornstein-Uhlenbeck process with mean reversion.

As usual for the calibration/estimation step, this model is time discretized. The basic model involves different "parameters", that are λ, μ, a, the function $\theta(t)$, the volatilities and correlations. Their calibration/estimation is performed as follows :

- $\theta(t)$ is calibrated from a LIBOR swap rate curve.[11]
- Then parameters a, σ are calibrated from at-the-money swaption prices.
- Parameters $\sigma_S, \rho_{S,r}$ are calibrated from the prices of equity index options. But since these options are liquidly traded up to a maturity of two years, artificial option prices are created by extrapolation for larger time-to-maturity. The extrapolation technique is based on a stochastic volatility model, proposed by the CRO Forum [see CRO Forum (2010)].
- The real estate and inflation dynamic parameters, that are : $\sigma_{re}, \sigma_i, \rho_{re,i}, \rho_{re,r}, \rho_{i,r}$ are estimated by their historical counterparts.

Let us comment this basic model and its implementation. We see that :

- the historical model is not presented for the equity index, the real estate index and the instantaneous interest rate although it is needed for future projections.
- the dynamics of the inflation rate is not compatible with the "martingale property" expected in the R.N. world.
- the Hull, White's dynamics selected for the short term rate is rather constrained. This is a one-factor model used in order to be able to reconstruct easily the risk-neutral interest rate dynamics from swaption data. This is the well-known Jamshidian trick [Jamshidian (1989)], which is known to be very sensitive to market conditions.[12]
- The extrapolation method based on the stochastic volatility model is not consistent with the initial model[13] and also not market consistent.

[11]These swap curves have to be appropriately adjusted to remove credit risk. For instance it has been proposed to "reduce the inter-bank swap rates by 10 bps for all currencies to reflect the impact of credit risk". [see e.g. CFO Forum (2010), Section 2 and p10].

[12]The Jamshidian trick is no longer valid when the short term rate depends on two factors or more.

[13]Contrary to a requirement of QIS 5 [see CEIOPS (2010), 2.3] : "The extrapolated part of the riskfree interest rate curve... should be calculated according to the same procedures as the non extrapolated part".

- The historical R.N. correlation parameters are not distinguished. Surprisingly $\rho_{S,r}$ is estimated as a R.N. correlation, whereas the other correlations are estimated as historical correlations.

- The model (3.1)-(3.3) is oversimplified with likely a too small number of parameters. For instance, as already noted, the dynamic model for instantaneous interest rate is a restrictive one-factor model. It would be possible to introduce more factors, for instance a stochastic volatility in the interest rate equation [see e.g. Baldvinsdottir et al. (2011)].

4 Stress tests and scenarios

4.1 Principle of stress tests

We have seen that the main ingredients of a dynamic analysis of the risks of financial institutions and insurance companies are :

i) the modelling by means of the historical dynamic model and the s.d.f..

ii) the way the model is "estimated" including the calibration step and the technical standards.

The next step is the analysis of the robustness of the valuation methodology. This is done by considering how the results are impacted, when we change some features of the model. These changes can concern :

- the historical dynamics of one of the variable, such as the real estate for instance.

- the risk-neutral dynamics of one of the variable.

- a technical standard.

In a simple modelling such as the Gaussian VAR(1) modelling of Section 3.3, the effect of such changes can be derived in closed form. But in more complicated modelling it can be necessary to compute the effects by means of simulations. Some simulations have to be done under the risk-neutral dynamics to evaluate at each date the prices of derivatives. Other simulations have to be performed under the historical probability to compute the best estimate, the Value-at-Risk of the future value of the firm, and the SCR. Thus we need nested simulations.

Moreover, for stress tests, we also need nested simulations under the stressed historical distribution for analyzing the effect of changes of the historical dynamics of one variable. We can also need nested simulations under the stressed risk-neutral distribution if the change concerns the risk-neutral dynamics of one variable.

4.2 An example

To illustrate the nested simulations, let us consider two risk factors $Y_t = (Y_{1t}, Y_{2t})$ and let us assume a zero riskfree rate and a business line of the balance sheet at $t+1$ including a European derivative written on Y_t with payoff $C(Y_{t+2})$ at time $t+2$. The valuation at $t+1$ of this business line is $\pi(C,1) = E^Q_{t+1}[C(Y_{t+2})]$. If the process of risk factors is Markov under Q, this value is a function $h(Y_{t+1}, C)$, say, that depends on the past by means of Y_{t+1} only. How to evaluate at time t the risk on the future business line ?

If the process of risk factors is also a Markov process under the historical probability, the information at time t is summarized by Y_t.

From this given value $Y_t = y_t$, we can draw from the historical transition a future value Y^s_{t+1}, say. It is a real world scenario. Then from this simulated value, we can now draw S' simulations of Y_{t+2} : $\tilde{Y}^{s,s'}_{t+2}, s' = 1, \ldots, S'$, say, from the risk-neutral conditional density of Y_{t+2} given $Y_{t+1} = Y^s_{t+1}$. These R.N. simulations are used to compute approximately the European derivative price as :

$$h(Y^s_{t+1}, c) = \frac{1}{S'} \sum_{s'=1}^{S'} C(\tilde{Y}^{s,s'}_{t+2}).$$

This computation is replicated for $s = 1, \ldots, S$, providing for instance an approximation of the best estimate as :

$$BE_t \simeq BE^*_t = \frac{1}{S} \sum_{s=1}^{S} h(Y^s_{t+1}, C) = \frac{1}{SS'} \sum_{s=1}^{S} \sum_{s'=1}^{S'} C(\tilde{Y}^{s,s'}_{t+2}).$$

It would also be possible to compute approximately the market consistent technical provision :

$$TP_t = E^Q_t[C(Y_{t+2})],$$

by simulation. This could be done by R.N. simulation only :

conditional on $Y_t = y_t$, draw from the R.N. transition a value $\tilde{\tilde{Y}}^{s"}_{t+1}$, say, and then, conditional to $\tilde{\tilde{Y}}^{s"}_{t+1}$ and from the R.N. transition a value $\tilde{\tilde{Y}}^{s"}_{t+2}$, $s" = 1, \ldots, S"$.

The approximated level of technical provision is :

$$TP_t \simeq TP_t^* = \frac{1}{S"} \sum_{s"=1}^{S"} C(\tilde{\tilde{Y}}^{s"}_{t+2}).$$

Finally the risk margin is estimated by the difference : $TP_t^* - BE_t^*$.

4.3 Technical standard or new instrument of economic policy

Finally let us discuss the possible use of a technical standard and the interpretation of the associated stress tests. Let us illustrate this question with the example of the term structure of riskfree yields, which has to be defined up to a time-to-maturity of 100 years. The part of the yield curve up to maturity of about 10 years, say is calibrated from market data. After about 10 years, say, [14] an extrapolation method with an ultimate forward rate (UFR) is provided by the central institutions[15] (supervision authority or central bank). Before Solvency 2 the central institutions were setting a target rate, usually the overnight rate, that is a kind of short term rate. However on short term maturities the Central institution was competing with the market for short term maturities, which are rather liquid except during a liquidity crisis. Therefore in a standard situation this short term instrument is not very efficient. The Central institutions are now setting the UFR for deriving the prudential balance sheets, that is they decide of a standard for the large maturity section of the yield curve, for which the markets are not liquid, or even do not exist. This is a much more efficient instrument of economic policy. An increase of the UFR will improve the results of a firm with long maturity liabilities, whose valuation is based on the UFR, and with assets invested on liquid markets for instance in bonds with short or medium

[14]Entry points into the yield curve extrapolation depending on the currency and the swap markets have been suggested, such as 30 years for EUR, 50 years for GBP, 30 years for USD, 20 years for JPY, 15 years for CHF... But these limits do not account carefully of liquidity.

[15]called Solvency 2 policymakers in Hibbert (2012)

maturities and valued mark-to-market. This type of balance sheet can be encountered for insurance companies. Therefore by increasing the UFR, the Central institution has implicitly an instrument to impact the results of the insurance companies, their amount of required capital and their probabilities of failure. The impact is reversed for a credit institution offering long term loans such as loans to municipalities for instance. Clearly the objective function for deciding of the level of the UFR is not to be close to the true level of the UFR, which may even not exist. This is to manage the magnitude of the mismatch of maturities between the asset and liability components of the balance sheets, to ensure a sufficient role to insurance companies and credit institutions in the transformation of short term debt (resp. long term debt) into long term debt (resp. short term debt), or to get a reasonable balance between the sectors of insurance and credit.

5 Concluding remarks

In this paper we reviewed the main theoretical results on market consistent model in an incomplete market framework and their implications for the current risk supervision such as Solvency 2. We emphasize the needs for selecting appropriate technical standards for fixing the unobservable risk premia and for robustness checking of the selected standards.

Following the fifth quantitative impact studies, it has been decided to focus on market consistent modelling for the long term riskfree rate, especially important for measuring the effect of longevity risk, and for the supervision of pension funds. Currently the standards discussed in QIS6 follow the so-called "macroeconomic approach". They fix a given value for the long term (forward) rate, such as 4.2%, and an extrapolation model for completing the term structure to large maturities [see e.g. CRO Forum (2010)]. The current tested ones, such as the Swensson method [Swensson (1994) the Nelson-Siegel model (Nelson, Siegel (1987), currently followed by Barrie & Hibbert), or the Smith-Wilson approach [Smith, Wilson (2000), Thomas, Mare (2007), Sorensen (2008) FINANSTILSYNET (2010), QIS 5 (2010)] are surprisingly not compatible with some of the basic principles of Solvency 2 [see CEIOPS (2010), Basic principles 2.3]. Indeed,

- they are not arbitrage free, in particular not time consistent [see Filipovic (1999), Gourieroux, Monfort (2013), (2015)].

- they are not theoretically and economically sound.

In fact the tested methodologies have not yet taken into account the academic literature on interest rate models published in the twenty last years.

Finally other dimensions of incompleteness should also be carefully analyzed and tested, especially the risk dependence, requiring technical standard concerning common factors (often called systematic factors), the historical and risk-neutral correlations between these risk factors, or the recovery rates when a guarantee defaults [see Appendix 1, v) for rather ad-hoc standards proposed for the implementation of QIS 5].

R E F E R E N C E S

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Appendix 1

QIS 5 Technical Specifications

We provide in this appendix different definitions or principles proposed by the European supervisor (CEIOPS). They are given with their references in the document European Commission (2010). Since these definitions are written in a literary way, we also explain how they can be interpreted in mathematical terms and possibly misunderstood. The supervisor may also have introduced a specific terminology for notions, which are known under other names in actuarial studies, finance or statistic. We give these equivalent terminologies when necessary.

i) Mark-to-Market vs mark-to-model

V.17 i. "Identify assets and liabilities marked to market and assets and liabilities marked to model".

V.10 ii. "Where marking to market is not possible, mark to model techniques shoud be used... Undertakings will maximise the use of relevant observable inputs and minimise the use of unobservable inputs".

ii) Valuation

V3 "Assets should be valued at the amount for which they could be exchanged... Liabilities should be valued at the amount for which they could be transferred, or selected...".

This is the definition of the "fair value", which assumes implicitely that this valuation exists and is unique. This is not compatible with incomplete markets.

TP.1.2. "The value of technical provisions should be equal to the sum of a best estimate and a risk margin"... "Under certain conditions that relate to the replicability of the cash flows underlying the (rc) insurance obligations, best estimate and risk margin should not be valued separately, but technical provisions should be calibrated as a whole".

When the cash flows are not replicable, too large flexibility has been left to the definition of the risk margin. Standard values have been proposed [see e.g. CEIOPS (2010), 2.92], but can significantly differ from the values derived by a more elaborate method. For instance in Norway, the IORPs calculated a risk margin of less than 2% of the best estimate, whereas the simplification assumed a fixed 8 % [EIOPA '2013), 4.14]

TP.2.1. "The best estimate should correspond to the probability weighted average of future cash-flows taking into account of the time value of money".

Under the assumption of replicability, the value of the technical provision is :

$$TP_t = \sum_{h=1}^{H} E_t^Q[\exp(-r_{t+1}, \ldots - r_{t+h-1})C_{t+h}],$$

where C_{t+h}, h *varying, denotes the sequence of future cash-flows.*

The best estimate is :

$$BE_t = \sum_{h=1}^{H} E_t[\exp(-r_{t+1} \ldots - r_{t+h-1})C_{t+h}].$$

They differ by the probability. which is used in the computation, that is the historical probability for the best estimate, a risk-neutral probability for the value of technical provision, respectively. In this case the risk margin is deduced as the difference : $TP_t - BE_t$.

Usual terminology :	*"present value"*	*instead of best estimate,*
	"market value"	*instead of technical provision.*

We have given above the "modern" formula of the best estimate, i.e. of the present value. However this notion and its use in accounting have changed over time. The first definition was assuming a constant interest rate :

$$BE_t^* = \sum_{h=1}^{H} \left[\frac{1}{(1+r)^h} E_t(C_{t+h}) \right].$$

Then a discounting has been proposed based on market prices of riskfree bonds :

$$BE_t^{**} = \sum_{h=1}^{H} [B(t,h) E_t(C_{t+h})].$$

This after formula corresponds to the "modern" definition assuming pay-offs independent of interest rate under the historical probability and us risk premia on interest rates.

The first formula BE_t^* is a technical standard in which the "fixed" interest rate has to be set in a coordinated way. The second formula BE_t^{**} is already market consistent since it is using bond prices. It was used in traditional life insurance valuation technique (see TP. 2.50). It has two drawbacks :

i) the long term zero-coupon bonds are not actively traded;

ii) this formula does not apply for derivatives written on interest rates, such as indexed mortgages or indexed life insurance contracts.

iii) Scenarios and stress

TP.2.3. "The best estimate is the average of the outcome of all possible scenarios".

Thus it is proposed to compute numerically the expectations involved in the formula of the best estimate by simulation. The scenarios are historical scenarios, in this approach.

Usual terminology : "simulation" instead of scenario.

iv) Information and calibration

TP.2.94. "The information includes(non exhaustive list) :

- risk-free interest rate term structures,
- currency exchange rates
- inflation rates
- economic scenario files (ESF)"

TP.2.96. "A model for future projection of market parameters (market consistent asset model, e.g. an economic scenario file) should :

- generate asset prices consistent with deep, liquid and transparent financial markets.
- assume no arbitrage opportunity."

There is a confusion between the notions of parameter, model and scenario, which are three different notions. Nevertheless this article tries to redefine "a market consistent modelling". (see Section 2 of this paper).

TP.2.97 c) "The asset model should be calibrated to a properly calibrated volatility measure. [The comparative merits of implied and historic volatilities are discussed in an Annex]".

This article makes a confusion between the historical and risk-neutral probabilities, leaving the choice of the volatility to the insurance company.

v) Technical standards

On the recovery rate

TP.2.135 "If the recoverables towards a counterparty correspond to deterministic payments,... we assume that the counterparty will only be able to make 40% of the further payments in case of default".

TP.2.155 "No rate higher than 50% should be used".

"These recovery rates can be adjusted for the rating AAA = 50%, AA = 45%, A=40%, BBB=35%, BB=20%." (p48).

on correlations between risk factors :

SCR.8.13

	Mortality	Longevity	Disability/ Morbidity	Lapse
Mortality	1			
Longevity	−0.25	1		
Disability/ morbidity	0.25	0	1	
Lapse	0	0.25	0	1

It is not said if these correlations are historical, or risk-neutral. Clearly these round numbers are norms, not really related with the observed historical correlations. Moreover lapse and mortality are competing risks for the end of the contract. This important feature is not taken into account.

on correlations between business lines :

	Market	Default	Life	Health	Non-Life
Market	1				
Default	0.25	1			
Life	0.25	0.25	1		
Health	0.25	0.25	0.25	1	
Non-life	0.25	0.5	0	0	1

vi) The solvency capital requirement

SCR.1.5. "The capital requirement is determined as the impact of a specified scenario on the net asset value of the undertaking (NAV)".

The SCR is a VaR, which has to be derived by averaging over a large number of scenarios, and not computed from a single specified scenario.

SCR.1.6. "The net asset value is defined as the difference between assets and liabilities"... "The liabilities should not include the risk margin of technical provisions".

There is a clear lack of coherency. The asset components are valued at the market price whereas the liabilities are valued at the best estimate.

SCR.1.9. "The SCR should correspond to the Value-at-Risk of the insurance or reinsurance undertaking subject to a confidence level of 99.5% over a one-year period".

The selected risk level 0.5% *is too small to allow for an accurate estimation of the VaR from historical data. Especially for unfrequent events, as catastrophic events (see SCR 9.45). This will imply rather inaccurate results and erratic evolution of the SCR over time.*

standard terminology : "quantile" or "percentile", instead of Value-at-Risk

Appendix 2

Acronyms

ACPR : Autorité de Controle Prudentiel et de Résolution.

CEIOPS : Committee of European Insurance and Occupational Pension Supervision.

DLT : Deep, Liquid, Transparent.

EIOPA : European Insurance and Occupational Pension Authority.

ESG : Economic Scenarios Generator.

IFRS : International Financial Reporting Standards.

IORP : Institution for Occupational Retirement Provision.

KPMG : Klynveld, Peat, Marwick, Goerdeler.

MCR : Minimum Capital Ratio Requirement.

QIS : Quantitative Impact Study .

SCR : Solvency Capital Requirement.

s.d.f. : stochastic discount factor.