Credit and Liquidity in Interbank Rates: 
a Quadratic Approach

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Abstract

In this paper, we propose a quadratic term-structure model of the EURIBOR-OIS spreads. As opposed to OIS, EURIBOR rates incorporate credit and liquidity risks. Two sorts of premia are thus distinguished for unsecured interbank loans: compensation for (a) facing default risk of debtors, and (b) possible unexpected funding needs on the lender’s side. Our approach allows us to identify credit and liquidity effects and to further decompose the whole term structure of spreads into credit- and liquidity-related parts and into an expectation part and a risk-premium. Our results shed new light on the effects of unconventional monetary policy carried out in the Eurosystem. In particular, our findings suggest that most of the recent easing in the euro interbank market is liquidity related.

JEL Codes: E43, E44, G12, G21

Key-words: Quadratic term-structure model, liquidity risk, credit risk, interbank market, unconventional monetary policy

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1 Introduction

Since the beginning of the financial crisis, the interbank market has been carefully scrutinized by commentators and policy-makers, both in Europe and in the US. This paper focuses on the spread between the rates on unsecured interbank loans (the EURIBORs in the euro area) and their risk-free counterparts, proxied by the Overnight Indexed Swap rate (OIS). This spread is considered as a crucial indicator at the very core of the financial crisis: it reveals not only banks’ concerns regarding the credit risk of their counterparts, but also their own liquidity needs.

Disentangling between those credit and liquidity effects has essential policy and central banking implications. If a rise in spreads reflects poor liquidity, policy measures should aim at improving funding facilities. On the other hand credit concerns should be treated by enhancing debtors’ solvency (see Codogno, Favero, and Missale (2003)). This question is of utmost importance in the euro area, where most of the unconventional monetary operations conducted by the European Central Bank focused on the curbing of interbank risk (see Gonzales-Paramo (2011)). Many attempts have been made to provide a credit/liquidity decomposition of the interbank risk (see next Section). Whereas most studies reckon that liquidity risk has been an important driver of interbank yields during the last 5 years, there is no consensus on the precise size of these effects: Schwarz (2010) estimates that one third of the EURIBOR-OIS 1-month spread is linked to liquidity in January 2008, whereas Filipovic and Trolle (2013) find that nearly all the spread is liquidity-related at that date. Also, some

\footnote{The ECB has changed dramatically its operational framework to counterbalance the interbank markets freeze, by conducting special refinancing operations with longer-than-usual maturity, or by establishing a fixed rate full allotment rule to provide unlimited amount of liquidity to euro commercial banks at fixed cost.}
models do not incorporate explicitly the *no-arbitrage* assumption, thus possibly leading to unrealistic behavior of modeled financial outcomes.

In this paper, we provide a new and robust technique to separate credit and liquidity risks in interbank markets. To reproduce the dynamics of the term structure of EURIBOR-OIS spreads, we use a no-arbitrage two-latent-factor quadratic term structure model (QTSM) for riskless and risky yields. We exploit the quadratic framework to preclude negative spreads, consistently with finance theory. We also emphasize that the QTSM specification is more flexible and yields more realistic results than the standard Gaussian affine term structure models: higher-order moments of spreads are better captured, and availability of closed-form spread pricing formulae ensures high tractability of our model.\(^3\) Our identification scheme and interpretation of the factors rely on credit and liquidity proxies. The estimation is performed using an innovative quadratic Kalman filter (see Monfort, Renne, and Roussellet (2013)). Our approach allows us to disentangle spread fluctuations attributable to liquidity and credit risk while authorizing causality between the factors. Besides, the EURIBOR-OIS term structure can be decomposed into an expectation part and a risk premium. All in all, a double decomposition of interbank spreads is obtained: credit/liquidity on the one hand and expectations/risk premium on the other hand. Our results suggest that risk premiums account for a significant share of IBOR-OIS spreads. Further, we find that credit components feature low-frequency fluctuations, and shows a persistent increase from August 2007 before stabilizing in August 2012. Liquidity components experience higher-frequency variations and

\(^3\)The appropriateness of the quadratic framework to model interest rates and spreads is emphasized in the next section.
have monotonously dropped since late December 2011. Eventually, the liquidity part of the spreads is economically negligible in January 2013.

We also analyze the consequences of unconventional monetary policies conducted by the ECB during the period 2007-2013 performing an event study. Our results support the claim that the recent 3-year ECB loans to euro commercial banks and the recently-announced ECB bond purchase program have helped to reduce the perception of liquidity risk and its related premium.\footnote{We refer to the 3-year ECB loans to euro commercial banks as \textit{Very Long-Term Refinancing Operations} (VLTRO) and to the ECB bond purchase program as \textit{Outright Monetary Transactions} (OMT).} However, we find little evidence that the LSAP programs in the EU (SMP 1 & 2) has had any significant impact on the interbank risk, with no particular change in any components.

The remainder of the paper is organized as follows. Section 2 presents the related literature. Section 3 details the construction of interbank rates. Section 4 develops the quadratic term-structure model. Section 5 describes the identification strategy and shows the estimation results. Section 6 performs the decomposition of EURIBOR-OIS spreads and discusses the impact of the ECB unconventional monetary policies. The last section concludes. Proofs are gathered in the Appendices.

## 2 Literature Review

In most term structure models, the authors assume that the default intensity and/or the short-term rate are affine functions of the underlying factors. A quadratic specification
however possesses several advantages over the standard affine case.\textsuperscript{5} Constantinides (1992) shows that a standard QTSM with a specific quadratic short-term interest rate can generate positive yields for all maturities and more flexibility in the term structure to fit bond data. Leippold and Wu (2002) generalize the quadratic short-rate term structure models showing that this specification provides closed-form or semi closed-form formulae for bond pricing of most fixed-income derivatives. Ahn, Dittmar, and Gallant (2002) provide further empirical evidence that QTSM often outperforms the standard affine term structure specification (ATSM). Leippold and Wu (2007) study the joint behavior of exchanges rates and bond yields using QTSM models for Japan and the US. More recently, Andreasen and Meldrum (2011) and Kim and Singleton (2012) exploit the QTSM framework to model the term structure of interest rates in a context of extremely low monetary-policy rates. Turning to the credit literature, Hoerdaehl and Tristani (2012) use a quadratic specification to model euro-area sovereign spreads, and Doshi, Jacobs, Ericsson, and Turnbull (2013) consider a quadratic intensity to price corporate CDS.

Our identification scheme follows several recent studies that model yield curves associated with different fixed-income instruments (e.g. bonds, repo, swaps). These studies usually exploit this modelling to split credit spreads or swap spreads into different components. Specifically, Liu, Longstaff, and Mandell (2006) use a five-factor affine framework to jointly model Treasury, repo and swap term structures: one factor is related to the pricing of the Treasury-securities liquidity and another one reflects default risk. Feldhutter and Lando

\textsuperscript{5}Filipovic (2002) shows that the maximal degree of an arbitrage-free polynomial term-structure model is two. The quadratic models are related to the broader class of Wishart term structure models, see e.g. Gourieroux and Sufana [2003].
(2008) develop a six-factor model for Treasury bonds, corporate bonds and swap rates: they decompose swap spreads into three components: a convenience yield from holding Treasuries, a credit-element associated with the underlying LIBOR rate, and a factor specific to the swap market. Their results indicate that the convenience yield interpreted as liquidity premium is by far the largest component of spreads. Longstaff, Mithal, and Neis (2005) use information in credit default swaps in addition to bond prices to obtain measures of the non-default components in corporate spreads. Their estimation suggests that the non-default component is strongly related to measures of bond-specific illiquidity as well as to macroeconomic measures of bond-market liquidity. Monfort and Renne (2012) show that a substantial part of euro-area sovereign spreads are driven by a liquidity component. The identification of the latter relies on the interpretation of the spreads between the bonds issued by KfW, a public German agency, and their sovereign counterparts. Since KfW bonds are fully and explicitly guaranteed by the Federal Republic of Germany, these spreads should essentially reflect liquidity-pricing effects.

The present paper is also related to the interbank spreads literature. A wide range of studies deals with the determinants of interbank spreads: Taylor and Williams (2009) claim that counterparty risk was the main driver of the LIBOR/OIS spread, Michaud and Upper (2008) and Gyntelberg and Wooldridge (2008) find that credit and liquidity factors both played a role, while the results by Schwarz (2010) and Filipovic and Trolle (2013) suggest that liquidity risk has accounted for most of the LIBOR/OIS and EURIBOR/OIS spread variations over the period 2007-2009. In comparison, Smith (2012) emphasize that most of the variation in the risk-premia of interbank spreads is explained by credit risk. Finally,
Angelini, Nobili, and Picillo (2011) highlight the main role of macro-factors – such as the aggregate risk-aversion as opposed to individual lenders’ and borrowers’ characteristics – to account for the dynamics of unsecured/secured money-market spreads. The measured impact of unconventional monetary policies is ambiguous: Taylor and Williams (2009) find no effects of the Fed’s intervention in 2008, contrary to Christensen, Lopez, and Rudebusch (2009). According to the latter, the Fed’s TAF reduced significantly the 3-month maturity interbank spread by about 70 basis points. In Europe, Angelini, Nobili, and Picillo (2011) measure a modest impact of ECB exceptional 3-month refinancing operations, in contradiction with Abbassi and Linzert (2011).

### 3 Interbank market rates and risks

#### 3.1 The unsecured interbank rates

The interbank money market is at the heart of bank funding issues. It is an over-the-counter market (OTC) where interbank loans are negotiated with maturities ranging from one day to 12 months. As banks do not possess the same characteristics and underlying risks, there is no uniqueness of interbank rates. Only the disaggregated rates are really representative of the funding issues of each institution. However, such data are not publicly available. In or-

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6 The TAF (for Term Auction Facility) enabled the US Federal Reserve to refund depository institutions against a wide range of collateral. The program aimed to provide a wider access to the Fed liquidity to financial institutions, in a period of heightened concerns regarding the liquidity needs of financial institutions.

7 See Cecioni, Ferrero, and Secchi (2011) for a review of (a) the quantitative assessment regarding the relative importance of the interbank spread drivers, and (b) of the effects of unconventional monetary policies in the USD and euro interbank market.

8 Notwithstanding, the individual contributions of panel banks are available at [http://www.euribor-ebof.eu/euribor-org](http://www.euribor-ebof.eu/euribor-org). However, given the specific question that is posed to the banks (see below), their contributions do not necessarily reveal their own lending or borrowing costs.
der to conduct an analysis on interbank risks, a more aggregated measure must be considered.

The Euro Interbank Offered Rate (EURIBOR) provides a measure of the interest rate at which banks can raise unsecured funds from other financial institutions.\(^9\) The European Banking Federation publishes a daily reference rate based on the trimmed averaged interest rates at which Eurozone banks offer to lend unsecured funds to other banks in the euro wholesale money market. There is one rate for each maturity between one week and twelve months. More precisely, a daily survey is sent to a panel of 40 to 50 banks in the Euro area. The selected banks are those with supposedly high creditworthiness. The question of the survey is what are the rates at which euro interbank term deposits are being offered within the Eurozone by one prime bank to another. Contrary to the LIBOR survey (US), the banks are not asked about their own situation. The trimmed mean erases the 15% banks of each distribution tail.

The loans that underlie the EURIBOR are unsecured, that is the lending bank does not receive collateral as protection against default by the borrowing one. Therefore, these rates carry some compensation for solvency issue that we refer to as credit risk. Furthermore, through an interbank loan, a lending bank exposes its funds during the time-to-maturity

\(^9\)Temporary cases of frauds on both LIBOR and EURIBOR declarations by individual banks have been revealed by traders’ communications disclosure. In particular, Barclay’s Bank was charged $200 million by the Commodity Futures Trading Commission, $160 million by the American Department of Justice, and $59.5 million by the Financial Services Authority. However, there is no information on both (i) how those figures have been determined, (ii) the overall bias of LIBOR declarations. Using CDS data, Mollenkamp and Whitehouse (2008) evaluate the difference between the reported rate and the actual rate ranging from 3 to 87 bps depending on the institution. Nonetheless, contrary to LIBOR, Eisl, Jankowitz, and Subrahmanyam (2013) show that EURIBOR rates are less likely to be manipulated and less exposed in size to those frauds. Considerations on the precise evaluation of misleading declarations are beyond the scope of this paper.
of the loan although those funds might be needed to cover the bank’s own shortfalls (see e.g. Taylor and Williams (2009) or Michaud and Upper (2008)). Moreover, since an unsecured interbank loan is highly specific to the identity of both counterparties, its unwinding is a costly task. Thus the liquidity risk affects the rate at which this bank is willing to lend.  

While there are no reliable data on volumes in term money markets, anecdotal evidence suggests a sharp decline in unsecured term money market volumes over the last five years (see Eisenschmidt and Tapking (2009)). In spite of this, there is evidence that EURIBOR rates remain reliable proxies for bank funding costs. Typically, EURIBOR are very close to quotations of certificates of deposits issued by banks. Moreover, using U.S. data, Kuo, Skeie, and Vickery (2012) find that public data beyond Libor are moderately informative about bank funding costs.

Figure 1 presents the evolution of the 3-month EURIBOR from August 2007 to January 2013. During the first year, the rate is stable around 500 basis points. The Lehman bankruptcy of September 2008 is followed by a sharp decline in EURIBOR of about 400 basis points, to 80 basis points. From mid-2010 onwards, the EURIBOR rises slowly to 150 basis points in September 2011 and decays to nearly 20 basis points during the recent period. Table 1 presents the descriptive statistics for 3, 6, 9, and 12-month EURIBOR maturities.

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10 This liquidity risk encompasses both market and funding liquidity issues. These are known to be difficult to assess separately. Brunnermeier and Pedersen (2009) define market liquidity as the difference between market and fundamental values of an asset, and funding liquidity as "speculator’s scarcity of capital".

11 Both volumes and yield data of certificates of deposits are available from the ECB statistical data warehouse; according to STEP data (for Short Term European Papers, see http://sdw.ecb.europa.eu/browse.do?node=7082906), a daily average of EUR1bn of certificates of deposit with maturities ranging between 101 and 200 days were issued in 2012. The average of the spread between the issuance yields for this bucket of maturities and the EURIBOR rate is lower than 3 basis points over the 2008-2012 period.
In this paper, the risk-free rates are proxied by the Overnight Indexed Swap (OIS) rates. An OIS is a fixed-for-floating interest rate swap with a floating rate leg indexed on overnight interbank rates, the EONIA in the euro-area case. OIS have become especially popular hedging and positioning vehicles in euro financial markets and grew significantly in importance during the financial turmoil of the last few years. The OIS curve is more and more seen by market participants as a proxy of the risk-free interbank yield curve (see e.g. Joyce, Lasaosa, Stevens, and Tong (2011)). As no principal is exchanged, the OIS requires nearly no immobilisation of capital. Further, due to netting and credit enhancement mechanisms (including call margins), the counterparty risk is limited in the case of a swap contract (see
The upper panel of Figure 1 displays the 3-month OIS rate from August 2007 to January 2013. While this chart shows that EURIBOR and OIS rates present strong common fluctuations, the lower panel also highlights that the spread between the two rates has undergone substantial variations over the last five years. In the next subsection, we discuss the term structure of the EURIBOR-OIS spreads.

### 3.3 Preliminary analysis of the EURIBOR-OIS spreads

Being mostly stable before August 2008, the spread abruptly increased during Lehman crisis until December 2008, the 3-month spread peaking at 200 basis points, where a slow decay begins (see Figure 1, bottom).\(^{12}\) Then, following a long stabilization period between August 2009 and 2010, a sharp rise stroke again in mid-2011. Since the beginning of 2012, the EURIBOR-OIS spreads have decreased, alternating between a linear decreasing trend and stable plateaux. Standard descriptive statistics of spreads are provided in Table 1. The OIS average for different maturities is between 50 and 90 basis points below the EURIBOR averages. It is also less volatile and the standard deviations are similar across maturities of OIS. In comparison, the volatility of EURIBOR rates decreases more deeply with maturity. The means of spreads increase with respect to maturity, from 53 to 87 basis points. This indicates a positive slope in the term structure of spreads, that is graphically illustrated by the bottom panel in Figure

\(^{12}\)For sake of comparison, before summer 2007, the EURIBOR-OIS spread was around ten basis points. Part of this deviation was accounted for by the fact that the EURIBOR is an offer rate while the OIS is a mid rate.
## Table 1: Descriptive statistics of EURIBOR and OIS rates

<table>
<thead>
<tr>
<th></th>
<th>min</th>
<th>max</th>
<th>amplitude</th>
<th>mean</th>
<th>std</th>
<th>skewness</th>
<th>excess kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>EURIBOR 3M</td>
<td>18,4</td>
<td>538,1</td>
<td>519,7</td>
<td>191,3</td>
<td>165,7</td>
<td>1,00</td>
<td>−0,69</td>
</tr>
<tr>
<td>EURIBOR 6M</td>
<td>31,6</td>
<td>543,1</td>
<td>511,5</td>
<td>211,1</td>
<td>157,7</td>
<td>1,00</td>
<td>−0,63</td>
</tr>
<tr>
<td>EURIBOR 9M</td>
<td>42,6</td>
<td>546,3</td>
<td>503,7</td>
<td>222,7</td>
<td>153,3</td>
<td>1,00</td>
<td>−0,60</td>
</tr>
<tr>
<td>EURIBOR 12M</td>
<td>53,7</td>
<td>549,3</td>
<td>495,6</td>
<td>233,5</td>
<td>149,8</td>
<td>0,99</td>
<td>−0,56</td>
</tr>
<tr>
<td>OIS 3M</td>
<td>4,5</td>
<td>434,6</td>
<td>430,1</td>
<td>138,3</td>
<td>148,5</td>
<td>1,14</td>
<td>−0,45</td>
</tr>
<tr>
<td>OIS 6M</td>
<td>2,35</td>
<td>442,85</td>
<td>440,5</td>
<td>140,0</td>
<td>147,6</td>
<td>1,15</td>
<td>−0,39</td>
</tr>
<tr>
<td>OIS 9M</td>
<td>−0,5</td>
<td>453,5</td>
<td>454</td>
<td>142,9</td>
<td>146,0</td>
<td>1,14</td>
<td>−0,35</td>
</tr>
<tr>
<td>OIS 12M</td>
<td>−1,1</td>
<td>465,3</td>
<td>466,4</td>
<td>146,5</td>
<td>144,1</td>
<td>1,13</td>
<td>−0,32</td>
</tr>
<tr>
<td>Spread 3M</td>
<td>10,4</td>
<td>206,9</td>
<td>196,5</td>
<td>53,0</td>
<td>34,4</td>
<td>1,64</td>
<td>3,44</td>
</tr>
<tr>
<td>Spread 6M</td>
<td>26,15</td>
<td>222,5</td>
<td>196,35</td>
<td>71,1</td>
<td>35,3</td>
<td>1,78</td>
<td>3,83</td>
</tr>
<tr>
<td>Spread 9M</td>
<td>37,45</td>
<td>227,9</td>
<td>190,45</td>
<td>79,8</td>
<td>37,3</td>
<td>1,72</td>
<td>3,17</td>
</tr>
<tr>
<td>Spread 12M</td>
<td>41,82</td>
<td>239</td>
<td>197,18</td>
<td>87,0</td>
<td>39,5</td>
<td>1,55</td>
<td>2,29</td>
</tr>
</tbody>
</table>

**Notes:** Those figures are computed with weekly data ranging from 31st August 2007 to 4th January 2013.

1: except at the very beginning of the sample, the 12-month spread is always larger than the 3-month spread, up to around 50 basis points in late 2011. Furthermore, the same plot shows that the slope is time-varying.

Whereas the standard deviations are respectively stable and decreasing with maturity for OIS and EURIBOR rates, the standard deviations of spreads slightly increase with maturity. Regarding higher-order moments, Table 1 indicates that both EURIBOR and OIS rates for all maturities are positively skewed and possess thin tail distributions (negative excess kurtosis). For all maturities, spreads are more positively skewed than the rates in level; also, contrary to the latter, spreads are heavy-tailed (positive excess kurtosis). The heavy-tail behavior is typically illustrated during the Lehman crisis on Figure 1, where both 3-month and 12-month spreads peak to 207 and 239 basis points, respectively. These levels are about 4 standard-deviation far from their respective sample means.
Finally, a principal component analysis performed on the four EURIBOR-OIS spreads proves that the first principal component captures most of spread fluctuations. It explains nearly 96% of the whole variance of the spreads, emphasizing the very similar patterns observed in the variations of the spreads of different maturities.

In the next section, we develop a model that is consistent with these observations.

4 The model

4.1 The intensity

At date $t$, market participants get the new information $w_t = \{r_t, X_t, d_t\}$, where $r_t$ is the short-term risk-free rate between dates $t$ and $t + 1$, $X_t = [x_{c,t}, x_{l,t}]'$ is a $(2 \times 1)$ vector whose components are respectively a credit-risk and a liquidity-risk factor, and $d_t$ is a binary variable. A switch from $\{d_{t-1} = 0\}$ to $\{d_t = 1\}$ corresponds to one of two adverse situations from the lender point of view: either (a) the borrower on the unsecured interbank market defaults at date $t$, or (b) the lender on the unsecured market would have needed the (lent) amount of liquidity for other purposes, which translates into costs for her.\(^{13}\) The state $\{d_t = 1\}$ is assumed to be absorbing. Let us denote by $\underline{w_t}$ the cumulated information up to date $t$, that is $\underline{w_t} = \{w_t, w_{t-1}, \ldots\}$. Conditionally on $(r_t, X_t, \underline{w_{t-1}})$, the probability of switching

\(^{13}\)In Appendix A.1, we provide a more general model where the two kinds of events (credit and liquidity) intervene explicitly and not in an aggregated way. However, in terms of pricing, this extended model entails the same implications than the one presented here.
from \( \{d_{t-1} = 0\} \) to \( \{d_t = 1\} \) is given by:

\[
\mathbb{P}(d_t = 1|d_{t-1} = 0, r_t, X_t, w_{t-1}) = 1 - \exp(-\lambda_t).
\]

where \( \lambda_t \) is a function of \( X_t \) that we call intensity.\(^{14}\)

### 4.2 General pricing formulae

We assume that there exists a stochastic discount factor (SDF) between \( t \) and \( t+1 \), which is denoted by \( M_{t,t+1} \). This existence implies that the variables gathered in \( w_t \) have both physical \((\mathbb{P})\) and risk-neutral \((\mathbb{Q})\) dynamics. The SDF specification is formally given in Appendix A.2.

Let us denote by \( R_{t,h}^{OIS} \) and \( R_{t,h}^{EUR} \) the OIS rate of maturity \( h \) and the EURIBOR of the same maturity, respectively.\(^{15}\) Both rates are homogeneous to zero-coupon rates.\(^{16}\) Recalling that we consider the OIS rates as risk-free yields, we have \( r_t = R_{t,1}^{OIS} \) and for longer maturities, no-arbitrage implies:

\[
R_{t,h}^{OIS} = -\frac{1}{h} \log \left( \mathbb{E}_t^{\mathbb{Q}} \left[ \exp \left\{ -r_t - \ldots - r_{t+h-1} \right\} \right] \right) \tag{1}
\]

where \( \mathbb{E}_t^{\mathbb{Q}} \) denotes the expectation under the risk-neutral measure, conditional on \( w_t \). Turning

\(^{14}\)We also assume no Granger causality from \( d_t \) to \( (r_t, X_t) \).

\(^{15}\)The pricing formulas derived in this paper implicitly feature continuously-compounded interest rates. Let \( r \) denote a market-quoted interest rate (the OIS, say). Using the fact that the money-market day-count convention is ACT/360, the corresponding continuously-compounded rate is given by \( \ln(1+d \times r/360) \times 365/d \) where \( d \) is the residual maturity of the instrument.

\(^{16}\)Whereas this is obvious in the EURIBOR case, Appendix A.3 explain why it is also the case for OIS rates.
to the EURIBOR rates, we have:¹⁷

\[ R_{t,h}^{EUR} = -\frac{1}{h} \log \left( \mathbb{E}_t^Q \left[ \exp \left\{-r_t - \lambda_{t+1} - \ldots - r_{t+h-1} - \lambda_{t+h} \right\} \right] \right). \]  

(2)

As in, e.g., Berndt, Douglas, Duffie, Ferguson, and Schranz (2005), Pan and Singleton (2008) or Longstaff, Pan, Pedersen, and Singleton (2011), we assume that the short-term risk-free interest rate and the intensity \( \lambda_t \) are independent under \( \mathbb{Q} \). Denoting by \( S(t,h) \) the EURIBOR-OIS spread of maturity \( h \), it follows that:

\[ S(t,h) = R_{t,h}^{EUR} - R_{t,h}^{OIS} = -\frac{1}{h} \log \left( \mathbb{E}_t^Q \left[ \exp \left\{-\lambda_{t+1} - \ldots - \lambda_{t+h} \right\} \right] \right). \]  

(3)

Equation (3) shows that, under these assumptions, the study of EURIBOR-OIS spreads does not require the modelling of short-term risk-free interest rate \( r_t \).

### 4.3 Intensity specification

The preliminary analysis in Section 3.3 provides evidence that one factor is sufficient to account for most of the EURIBOR-OIS term structure. Hence, we assume that the intensity depends on a single common factor denoted by \( x_t \), which is the sum of the the credit-related factor \( x_{c,t} \) and the liquidity-related one \( x_{l,t} \).

\[ x_t = x_{c,t} + x_{l,t} \]  

(4)

¹⁷See Appendix A.1 for a formal derivation of this formula.
Moreover, the intensity is a quadratic function of $x_t$:

$$
\lambda_t = \lambda_0 + \lambda_1 x_t + \lambda_2 x_t^2.
$$

(5)

To ensure that the underlying probability is constrained between 0 and 1, $\lambda_t$ has to be positive whatever the value of $x_t$. This constraint writes $\lambda_0 \geq \lambda_1^2 / 4 \lambda_2$.\(^{18}\)

Under the risk-neutral measure, $x_t$ follows a stationary AR(1):

$$
x_t = \mu^* + \varphi^* x_{t-1} + \varepsilon_t^* \text{ where } \varepsilon_t^* \sim \mathcal{IN}^Q(0, 1), \quad |\varphi^*| < 1,
$$

(6)

and the standard deviation of $x_t$’s innovations are set to one for sake of identification.

### 4.4 Dynamics of $x_{c,t}$ and $x_{l,t}$

Let us now present the physical dynamics of the credit and the liquidity factors $x_{c,t}$ and $x_{l,t}$.

As the two risks can influence each other (see e.g. Ericsson and Renault (2006)), we authorize lagged causality between the two factors. However, these factors are contemporaneously influenced by independent idiosyncratic shocks $\varepsilon_{c,t}$ and $\varepsilon_{l,t}$, that we refer to as credit shock and liquidity shock, respectively. Moreover, their joint dynamics is described by the following

\(^{18}\)In a preliminary analysis, whose results are not reported here for sake of brevity, a Gaussian ATSM was estimated (hence excluding the quadratic term in the intensity). Given that (a), in such a model, the distribution of model-implied spreads is Gaussian and that (b) consistently with the high persistence of observed spreads, the resulting model-implied variance of the spreads was large, we obtained that the model-implied unconditional probability of having negative spreads was huge and close to 50% for all maturities. This clearly illustrates the inappropriateness of Gaussian ATSM to model such spreads.
\[
\begin{pmatrix}
x_{c,t} \\
x_{l,t}
\end{pmatrix} = 
\begin{pmatrix}
\mu_c \\
\mu_l
\end{pmatrix} + 
\begin{pmatrix}
\varphi_{1,1} & \varphi_{1,2} \\
\varphi_{2,1} & \varphi_{2,2}
\end{pmatrix}
\begin{pmatrix}
x_{c,t-1} \\
x_{l,t-1}
\end{pmatrix} + 
\begin{pmatrix}
\sigma_c & 0 \\
0 & \sigma_l
\end{pmatrix}
\begin{pmatrix}
\varepsilon_{c,t} \\
\varepsilon_{l,t}
\end{pmatrix}
\]
\tag{7}
\]

where \((\varepsilon_{c,t}, \varepsilon_{l,t})' \sim \mathcal{IIN}^p(0, I_2)\), and the eigenvalues of the autoregressive matrix \(\Phi = [\varphi_{i,j}]_{i,j=(1,2)}\) are smaller than one in modulus.

Assuming that the SDF is exponential-affine in \((x_{c,t}, x_{l,t})'\), the expanded risk-neutral dynamics of these factors is of the form:\footnote{The SDF specification is provided in appendix A.2.}

\[
\begin{pmatrix}
x_{c,t} \\
x_{l,t}
\end{pmatrix} = 
\begin{pmatrix}
\mu_{c}^* \\
\mu_{l}^*
\end{pmatrix} + 
\begin{pmatrix}
\varphi_{1,1}^* & \varphi_{1,2}^* \\
\varphi_{2,1}^* & \varphi_{2,2}^*
\end{pmatrix}
\begin{pmatrix}
x_{c,t-1} \\
x_{l,t-1}
\end{pmatrix} + 
\begin{pmatrix}
\sigma_c & 0 \\
0 & \sigma_l
\end{pmatrix}
\begin{pmatrix}
\varepsilon_{c,t}^* \\
\varepsilon_{l,t}^*
\end{pmatrix}
\]
\]

where \((\varepsilon_{c,t}^*, \varepsilon_{l,t}^*)' \sim \mathcal{IIN}^q(0, I_2)\). As for the physical dynamics, the process is assumed stationary under the risk-neutral measure. Given that we want the risk-neutral dynamics of \(x_t = x_{c,t} + x_{l,t}\) to be as described by Equation (6), the parameter specifying the risk-neutral dynamics of \((x_{c,t}, x_{l,t})'\) have to satisfy:

\[
\mu^* = \mu_{c}^* + \mu_{l}^* \text{ and } \varphi_{1,1}^* + \varphi_{2,1}^* = \varphi_{1,2}^* + \varphi_{2,2}^* = \varphi^* \text{ and } \sigma_c^2 + \sigma_l^2 = 1. \tag{8}
\]

\[4.5 \text{ Recursive pricing formulae}\]

It is a well-known result that Equations (5) and (6) implies that the spreads \(S(t, h)\), defined by Equation 3, can be expressed as a quadratic function of \(x_t\).\footnote{This stems from the fact that the conditional Laplace transform of the vector \((x_{t+1}, x_{t+1}^2)\) given \(x_t\) is exponential quadratic in \((x_t, x_t^2)\), see e.g. Gouriéroux and Sufana (2011) or Cheng and Scaillet (2007).} From Equations (3), (5)
and (6), we have:

$$S(t,h) = \theta_{0,h} + \theta_{1,h}x_t + \theta_{2,h}x_t^2$$  \hspace{1cm} (9)$$

The three parameters $\theta_{0,h}$, $\theta_{1,h}$ and $\theta_{2,h}$ are maturity-dependent and are functions of $\mu^*$, $\phi^*$, $\lambda_0$, $\lambda_1$, and $\lambda_2$. It is well-known that the $\theta_{0,h}$, $\theta_{1,h}$, and $\theta_{2,h}$ loadings can be computed recursively as:

$$
\begin{align*}
-h\theta_{0,h} &= -\lambda_0 + \theta_{0,h-1} + \frac{\tilde{\theta}_{1,h-1}}{K_h} \left( \mu^* + \frac{1}{2} \tilde{\theta}_{1,h-1} \right) + (\mu^*)^2 \frac{\tilde{\theta}_{2,h-1}}{K_h} - \frac{1}{2} \log(K_h) \\
-h\theta_{1,h} &= \frac{\phi^*}{K_h} \left( 2\mu^* \tilde{\theta}_{2,h-1} + \tilde{\theta}_{1,h-1} \right) \\
-h\theta_{2,h} &= (\phi^*)^2 \frac{\tilde{\theta}_{2,h-1}}{K_h}
\end{align*}$$  \hspace{1cm} (10)$$

where $\tilde{\theta}_{h-1} = \theta_{h-1} - \lambda_1$, $\tilde{\theta}_{2,h-1} = \theta_{2,h-1} - \lambda_2$, and $K_h = (1 - 2\tilde{\theta}_{2,h-1})$. We also set $\theta_{0,0} = \theta_{1,0} = \theta_{2,0} = 0$.

## 5 Estimation procedure

### 5.1 The structural identification of credit and liquidity factors

Observations of spreads are not sufficient to separate the credit factor and the liquidity one since, as shown by Equation (9), spreads depend on $x_t$ only. We therefore introduce credit and liquidity proxies in order to identify $x_{c,t}$ and $x_{l,t}$.

The liquidity proxy is the first principal component of a set of three liquidity-related variables.

---

21 A detailed proof of the recursive formulae is provided in an online appendix.
These variables are chosen in order to capture different aspects of liquidity pricing. In particular, the first two proxies are mostly related to market liquidity whereas the last one is mostly related to funding liquidity.

- A first liquidity-pricing factor is the KfW-Bund spread. KfW is a public German agency. KfW bonds are guaranteed by the Federal Republic of Germany. Hence, they embed the same credit quality as their sovereign counterparts, the so-called Bunds. KfW bonds being less liquid than their sovereign counterpart, the KfW-Bund spread essentially reflect liquidity-pricing effects, see Schwarz (2010), Monfort and Renne (2013) or Schuster and Uhrig-Homburg (2013). In the same spirit, Longstaff (2004) computes liquidity premia based on the spread between U.S. Treasuries and bonds issued by Refcorp, that are guaranteed by the Treasury.

- A second liquidity factor is the Tbill-repo spread, computed as the yield differential between the 3-month German T-bill and the 3-month general-collateral repurchase agreement rate (repo). From an investor point of view, the credit qualities of the two instruments are comparable (as argued by Liu, Longstaff, and Mandell (2006)). The differential between the two rates corresponds to the convenience yield, that can be seen as a premium that one is willing to pay when holding highly-liquid Treasury securities.

- A third factor is based on the Bank Lending Survey conducted by the ECB on a quarterly basis. Specifically, this indicator is based on the following question: Over the

\[^{22}\] Nearly 50% of the total variance is explained by the first principal component.

\[^{23}\] The KfW bond-yield not being available for all maturities, we use the 5-year KfW-Bund spread.

\[^{24}\] This premium stems from various features of Treasury securities, such as repo specialness (see Feldhutter and Lando (2008)).

\[^{25}\] This survey addresses issues such as credit standards for approving loans as well as credit terms and
past three months, how has your bank’s liquidity position affected the credit standards as applied to the approval of loans or credit lines to enterprises?\textsuperscript{26} A weekly series is obtained by linearly interpolating the quarterly series.

The credit proxy is the first principal component of a set of 36 Euro-zone bank CDS denominated in USD.\textsuperscript{27} Eight are German, six Italian, five Spanish, four French, four Dutch, three Irish, three Portuguese, two Austrian, and one Belgian.

As a robustness exercise, alternative proxies for credit and liquidity risks were tested. For credit risk, instead of taking the first principal component, we used alternatively mean, trimmed mean of the first and the last 15%, and median of the 36 CDSs as in Filipovic and Trolle (2013). Also, the iTraxx financials was tested. For the liquidity proxy, we alternatively took the first principal component of 2 out of 3 of the aforementioned measures. The KfW-Bund spread alone and the repo-Tbill spread alone were also tested. All those specifications do not alter our results.

The time-series of the proxies are provided on Figure 2. The liquidity proxy experiences a great peak right before the Lehman crisis, whereas the credit proxy tends to increase until late November 2011. For both proxies, we observe a particularly calm period from August 2009 to April 2010, when the interbank market became less tensed. Looking at the monetary policy events, we see that VLTRO events of December 2011 and March 2012 (the

\textsuperscript{26} The respondents can answer ++, +, 0, − or −− to that question. Our indicator is computed as the proportion of − and −− as a ratio of total answers.

\textsuperscript{27} Nearly 75% of the total variance is explained by the first principal component.
Figure 2: Proxies dynamics

Credit-risk (left axis) and liquidity-risk (right axis) proxies

Notes: Time ranges from August 31, 2007 to January 4, 2013. Solid lines are observed proxies. The black vertical axes stand from left to right for: SMP program announcements (first two axis), VLTRO announcement and allotments (next three axis), and Mario Draghi’s London speech (last axis).

announcement and the two allotments, third to fifth vertical black line on Figure 2) are associated with a decrease of the proxies. The same result can be seen for Mario Draghi’s London speech of late July 2012 (last vertical black line).²⁸

5.2 Identification strategy: linking proxies and latent factors

We denote the credit and liquidity proxies by $P_{c,t}$ and $P_{l,t}$ respectively. We assume that – up to a measurement error term – the proxies are quadratic functions of the corresponding latent factors.²⁹ Therefore, using the moving average representation of the factors, the credit (resp. liquidity) proxy is a combination of past (resp. past and current) liquidity shocks, of

²⁸In a context of mounting fears of euro-area break-up, the President of the ECB, Mario Draghi, declared that "the ECB [was] ready to do whatever it takes to preserve the euro" at the Global Investment Conference in London, 26ᵗʰ of July 2012.

²⁹This relationship, of the same kind of the one relating the latent factors to modelled spreads, is consistent with the fact that several variables used in the computation of proxies are also homogeneous to interest rates.
Estimation procedure

past and current (resp. past) credit shocks and of an error $\nu_{c,t}$ (resp. $\nu_{l,t}$). Formally:

$$
\begin{align*}
P_{c,t} &= \pi_{c,0} + \pi_{c,1}x_{c,t} + \pi_{c,2}x_{c,t}^2 + \sigma_{\nu,c} \nu_{c,t} \\
P_{l,t} &= \pi_{l,0} + \pi_{l,1}x_{l,t} + \pi_{l,2}x_{l,t}^2 + \sigma_{\nu,l} \nu_{l,t}
\end{align*}
$$

(11)

where $(\nu_{c,t}, \nu_{l,t})' \sim \mathcal{IN}(0, I_2)$. We assume that there is no instantaneous causality between the two proxies. Thus, as long as $x_{c,t}$ and $x_{l,t}$ are not instantaneously correlated, the same is true for the proxies.

The state-space representation of the model is obtained by gathering: (a) the $\mathbb{P}$-dynamics of the factors $x_{c,t}$ and $x_{l,t}$ (Equation (7)), (b) the spread formulae (Equation (9)) and (c) the proxies measurement equations (Equation (11)).

Transition:

$$
\begin{pmatrix}
x_{c,t} \\
x_{l,t}
\end{pmatrix} = \begin{pmatrix}
\mu_c \\
\mu_l
\end{pmatrix} + \begin{pmatrix}
\varphi_{1,1} & \varphi_{1,2} \\
\varphi_{2,1} & \varphi_{2,2}
\end{pmatrix} \begin{pmatrix}
x_{c,t-1} \\
x_{l,t-1}
\end{pmatrix} + \begin{pmatrix}
\sigma_c & 0 \\
0 & \sigma_l
\end{pmatrix} \begin{pmatrix}
\varepsilon_{c,t} \\
\varepsilon_{l,t}
\end{pmatrix}
$$

Measurement:

$$
\begin{align*}
S_t &= \theta_0 + \theta_1(x_{c,t} + x_{l,t}) + \theta_2(x_{c,t} + x_{l,t})^2 + \sigma_\eta \eta_t \\
P_{i,t} &= \pi_{i,0} + \pi_{i,1}x_{i,t} + \pi_{i,2}x_{i,t}^2 + \sigma_{\nu,i} \nu_{i,t} \\
\forall i &= \{c, l\}
\end{align*}
$$

(12)

where $S_t$ is the vector of observed spreads, the components of the vector of pricing errors $\eta_t$ are independent Gaussian white noises with unit variance and the components of vectors $\theta_0$, $\theta_1$, and $\theta_2$ are $\theta_{0,h}$, $\theta_{1,h}$, and $\theta_{2,h}$ for the four considered maturities. All the parameters of the $\mathbb{P}$-dynamics and $\mu^*$ and $\varphi^*$ are identifiable.\textsuperscript{30} The estimation constraints on the parameters are presented in Appendix A.4.

\textsuperscript{30}See Appendix A.4.
5.3 The Quadratic Kalman Filter

We estimate the state-space model with maximum likelihood techniques accompanied with a non-linear Kalman filter. Whereas recent articles use extensively the so-called Unscented Kalman Filter (UKF, see for instance Filipovic and Trolle (2013) or Christoffersen, Dorion, Jacobs, and Karoui (2012)), we rely on the Quadratic Kalman filter (QKF) of Monfort, Renne, and Roussellet (2013) fitted to quadratic measurement equations.

The QKF is based on the fact that the measurement equations are quadratic in the latent factor $X_t = (x_{c,t}, x_{l,t})'$ but affine in the augmented vector $W_t = (X_t', 	ext{Vech}(X_tX_t'))'$. This stacked vector $W_t$ defines a new state-space representation, and new factor dynamics. In particular, the measurement equations can be transformed into:

$$
\begin{pmatrix}
S_t \\
P_{c,t} \\
P_{l,t}
\end{pmatrix} = \begin{pmatrix}
\theta_{0,h} \\
\pi_{c,0} \\
\pi_{l,0}
\end{pmatrix} + \begin{pmatrix}
\theta_1 & \theta_1 & \theta_2 & 2\theta_2 & \theta_2 \\
\pi_{c,1} & 0 & \pi_{c,2} & 0 & 0 \\
0 & \pi_{l,1} & 0 & \pi_{l,2} & 0
\end{pmatrix} W_t + \begin{pmatrix}
\sigma_\eta \eta_{t,h} \\
\sigma_{\nu_c} \nu_{c,t} \\
\sigma_{\nu_l} \nu_{l,t}
\end{pmatrix},
$$

where $W_t = \begin{pmatrix}
x_{c,t} \\
x_{l,t} \\
x_{c,t}^2 \\
x_{c,t}x_{l,t} \\
x_{l,t}^2
\end{pmatrix}'.$

Monfort, Renne, and Roussellet (2013) show that the first two moments of $W_t$ conditional on its past values are available in exact closed-form. Approximating the conditional distribution of $W_t$ given $W_{t-1}$ by a Gaussian distribution and considering an augmented state-space model based on $W_t$, a standard linear Kalman filter can be used for filtering and estimation.
purposes.\textsuperscript{31}

5.4 Estimation results

We compute the estimations on weekly data from August 31, 2007 to January 4, 2013. The EURIBOR and OIS data are extracted from Bloomberg for the following maturities: 3, 6, 9, and 12 months.

Figure 3: Factor physical dynamics

![Factor physical dynamics graph](image)

Notes: Time ranges from August 31, 2007 to January 4, 2013. The grey shaded areas are the 95% confidence intervals of the latent factors. The horizontal black line is the $\arg\min$ of the intensity function given in Equation (5).

Table 2 reports the estimates of the physical dynamics parameters of $x_{c,t}$ and $x_{l,t}$. Both processes are very persistent through time, as the diagonal coefficient are close to one. Both standard errors of the residuals are significant, and give an intuition on the size of each idiosyncratic shocks in the common factor $x_t$. The liquidity shocks have a larger impact on the variance of the innovations of $x_t$ as $\sigma_l$ is largely above $\sigma_c$. Figure 3 illustrates the higher

\textsuperscript{31}In order to get the global likelihood maximum, the estimation is achieved in two steps. The Artificial Bee Colony stochastic algorithm (see Karaboga and Basturk (2007)) is used to find the potential maxima areas of parameters. The results are then used as starting values for a usual simplex maximization algorithm and the best estimate is selected.
volatility of the liquidity component compared to credit. In particular, the liquidity component experiences a more irregular behaviour with large jumps. Such jumps are manifest in late 2008 and late 2011, when the Lehman collapse and the tensions on the European sovereign markets were associated with large positive liquidity shock.

Table 2: Factor parameter estimates

<table>
<thead>
<tr>
<th></th>
<th>intercept</th>
<th>$x_{c,t-1}$</th>
<th>$x_{l,t-1}$</th>
<th>$\varepsilon_{c,t}$</th>
<th>$\varepsilon_{l,t}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_{c,t}$</td>
<td>0.0289</td>
<td>0.9921***</td>
<td>−0.0005</td>
<td>0.1916***</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>(0.0343)</td>
<td>(0.0045)</td>
<td>(0.0024)</td>
<td>(0.0145)</td>
<td></td>
</tr>
<tr>
<td>$x_{l,t}$</td>
<td>0.5079</td>
<td>−0.0557</td>
<td>0.9576***</td>
<td>0</td>
<td>0.9815***</td>
</tr>
<tr>
<td></td>
<td>(0.4236)</td>
<td>(0.0695)</td>
<td>(0.0164)</td>
<td></td>
<td>(0.0028)</td>
</tr>
</tbody>
</table>

Notes: Robust standard errors are in parentheses. Significance code: ‘***’ for p-value < 0.01, ‘**’ for p-value < 0.05, ‘*’ for p-value < 0.1.

The remaining parameter estimates are gathered in Table 3, which shows the prevalent effect of the quadratic term in the intensity specification (Equation (5)) and in those of the proxies (Equations (11)). These results emphasize the importance of quadratic terms; in particular, $\pi_{c,2}$, $\pi_{l,2}$ and $\lambda_2$ are highly significantly different from zero, whereas $\pi_{1,c}$ and $\lambda_1$ are not. The risk-neutral parameters also show a great persistence of $x_t$ in the risk-neutral world.

Looking at the last row of Table 3, the variance estimate $\hat{\sigma}_\eta^2$ associated with the error terms in the spread equation is 0.0106, which yields an average pricing error of 10 basis points for all maturities. Therefore, the estimation illustrates the flexibility and performance of our model to simultaneously account for the dynamics of the EURIBOR-OIS spread and credit and liquidity proxies.
Table 3: Risk-neutral and measurement parameter estimates

<table>
<thead>
<tr>
<th>Equation</th>
<th>Parameter</th>
<th>Estimate</th>
<th>Parameter</th>
<th>Estimate</th>
<th>Parameter</th>
<th>Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_t$</td>
<td>$\mu^*$</td>
<td>0.2627***</td>
<td>$\varphi^*$</td>
<td>0.9962***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0387)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P_{c,t}$</td>
<td>$\pi_{c,0}$</td>
<td>$-8.9650^{***}$</td>
<td>$\pi_{c,1}$</td>
<td>$-0.000006$</td>
<td>$\pi_{c,2}$</td>
<td>0.4496***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(3.0444)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P_{l,t}$</td>
<td>$\pi_{l,0}$</td>
<td>$-1.3098^{**}$</td>
<td>$\pi_{l,1}$</td>
<td>0.1382***</td>
<td>$\pi_{l,2}$</td>
<td>0.0045***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0534)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\lambda_t$</td>
<td>$\lambda_0$</td>
<td>0.1015</td>
<td>$\lambda_1$</td>
<td>0.0003</td>
<td>$\lambda_2$</td>
<td>0.0023***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0261)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Noise</td>
<td>$\sigma^2_{\nu_c}$</td>
<td>0.0081</td>
<td>$\sigma^2_{\nu_l}$</td>
<td>0.0099</td>
<td>$\sigma^2_{\eta}$</td>
<td>0.0106***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.4206)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: Robust standard errors are in parentheses. Significance code: ‘***’ for p-value < 0.01, ‘**’ for p-value < 0.05, ‘*’ for p-value < 0.1. The ‘—’ sign indicates that the constraint on $\sigma^2_{\nu_l}$ is binding thus the parameter is not estimated.

Lastly, Table 4 reports the factor-loadings $\theta_{0,h}$, $\theta_{1,h}$, and $\theta_{2,h}$ associated with the spread specification for different maturities. It is useful to look at the derivatives of the spreads equations. Suppose that we shock the $x_t$ factor of 1 unit. The instantaneous effect on the spreads depends on the current value of $x_t$ and not only on the size of the shock $\Delta x_t$. These effects are approximately given by Equation (13). This highlights the non-linear aspect of our model. If the interbank market is already in distress, then the agents react more strongly to any modification of the underlying risks. For instance, at the peak of Lehman crisis and 2 months after the OMT announcement, the values of $x_t$ are around 28 and 6, respectively.

Table 4: Factor loadings estimates and derivatives

<table>
<thead>
<tr>
<th>Spread</th>
<th>intercept</th>
<th>$x_t$</th>
<th>$x_t^2$</th>
<th>$\partial S(t,h) / \partial x_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3M</td>
<td>0.1278</td>
<td>0.0084</td>
<td>0.0022</td>
<td>$0.0084 + 0.0022 \times x_t$, $h = 3M$</td>
</tr>
<tr>
<td>6M</td>
<td>0.1682</td>
<td>0.0154</td>
<td>0.0021</td>
<td>$0.0154 + 0.0021 \times x_t$, $h = 6M$</td>
</tr>
<tr>
<td>9M</td>
<td>0.2223</td>
<td>0.0216</td>
<td>0.0020</td>
<td>$0.0216 + 0.0020 \times x_t$, $h = 9M$</td>
</tr>
<tr>
<td>12M</td>
<td>0.2885</td>
<td>0.0272</td>
<td>0.0019</td>
<td>$0.0272 + 0.0019 \times x_t$, $h = 12M$</td>
</tr>
</tbody>
</table>

\[
\partial S(t,h) / \partial x_t = \begin{cases} 
0.0084 + 0.0022 \times x_t, & h = 3M \\
0.0154 + 0.0021 \times x_t, & h = 6M \\
0.0216 + 0.0020 \times x_t, & h = 9M \\
0.0272 + 0.0019 \times x_t, & h = 12M 
\end{cases}
\] (13)

The respective instantaneous response to a 1 standard error unit positive shock on the factor at those periods on the 3-month maturity spread would be 7 in 2008 and 2.16 bps in 2013.
6 Decomposing EURIBOR-OIS spreads

In this section, we derive a simple decomposition method for our QTSM specification separating the credit and liquidity components in EURIBOR-OIS spreads for all maturities.

6.1 The decomposition method

Rewriting the spreads equation, we get:

\[
S(t, h) = \theta_{0,h} + \theta_{1,h} x_t + \theta_{2,h} x_t^2
\]

\[
= \theta_{1,h} x_{c,t} + \theta_{2,h} x_{c,t}^2 + \theta_{1,h} x_{l,t} + \theta_{2,h} x_{l,t}^2 + 2 \theta_{2,h} x_{c,t} x_{l,t} + \theta_{0,h}
\]

The spreads can be separated in four different parts. Two first parts are the credit and liquidity components of the spreads. A third term that we call interaction represents the price effect of the joint presence of both risks in the economy. Since \( \theta_{2,h} \) is positive, the interaction becomes more positive where the two factors co-move, and more negative if they evolve in opposite directions. Interactions between credit and liquidity also stems from possible lagged Granger causality in the latent factors that is authorized in our framework. The fourth component is the intercept \( \theta_{0,h} \) which is constant through time, and cannot be attributed to any of those three parts excepted arbitrarily. As a consequence, we set it apart and focus on the time-varying terms.

Also, spreads can be split in an other dimension. Indeed, our estimation strategy provides us with both the physical and the risk-neutral dynamics of the factors. This knowledge enables
Decomposing EURIBOR-OIS spreads

us to extract risk premia from observed spreads. Generally, risk premia are defined as the differentials between observed (or model-implied) yields or spreads and the ones that would prevail if investors were risk-neutral. In the latter case (that is consistent with the expectation hypothesis), spreads would be obtained by using the physical dynamics to compute the expectation term in Equation (3). Using the estimated $\mathbb{P}$-dynamics parameters, we calculate a new set of factor loadings under the expectation hypothesis (see Equation (15) and Table 5). Those coefficients are now functions of $(\lambda_0, \lambda_1, \lambda_2, \mu_c, \mu_l, \varphi_{1,1}, \varphi_{1,2}, \varphi_{2,1}, \varphi_{2,2})$. Contrary to the risk-neutral parameters $\mu^*$ and $\varphi^*$, the physical parameters are less constrained and authorize $x_{c,t}$ and $x_{l,t}$ to have a differentiated impact on the spreads under the expectation hypothesis. The decomposition writes:

$$S^p(t,h) = \theta_{0,h}^p + (\theta_{1,h,c}^p \theta_{1,h,l}^p) \begin{pmatrix} x_{c,t} \\ x_{l,t} \end{pmatrix} + (\theta_{2,h,c}^p \theta_{2,h,l}^p) \begin{pmatrix} x_{c,t} \\ x_{l,t} \end{pmatrix} \begin{pmatrix} x_{c,t} \\ x_{l,t} \end{pmatrix}$$

$$= \theta_{0,h}^p x_{c,t} + \theta_{1,h,c}^p x_{c,t}^2 + \theta_{1,h,l}^p x_{l,t} + \theta_{2,h,c}^p x_{c,t} x_{l,t}^2 + 2\theta_{2,h,cl}^p x_{c,t} x_{l,t} + \theta_{0,h}^p$$

(15)

Table 5: Factor loadings estimates under $\mathbb{P}$-measure

| $\mathbb{P}$-measure | intercept $x_{c,t}$ $x_{l,t}$ $x_{c,t}^2$ $x_{l,t}^2$ $x_{c,t}x_{l,t}$ | $x_{c,t}$ $x_{l,t}$ $x_{c,t}^2$ $x_{l,t}^2$ $x_{c,t}x_{l,t}$ |
|----------------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Spread 3M            | 0.1536          | 0.0082          | 0.0104          | 0.0010          | 0.0013          |
| Spread 6M            | 0.2087          | 0.0067          | 0.0122          | 0.0005          | 0.0009          |
| Spread 9M            | 0.2565          | 0.0033          | 0.0117          | 0.0004          | 0.0006          |
| Spread 12M           | 0.2947          | -0.000006       | 0.0104          | 0.0003          | 0.0005          |

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6.2 Decomposition results

The decompositions of the 6-month maturity spread are represented in Figure 4. On average, the liquidity component accounts for most of the spread averages over the sample period representing more than 37% of the spreads for all maturities (see Table 6). The interaction represents between 12 and 23% of the spreads, and the credit component represents about 16% of the spreads.

Table 6: Descriptive statistics of components

<table>
<thead>
<tr>
<th></th>
<th>Spread components</th>
<th>Term premia components</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Credit</td>
<td>Liquidity</td>
</tr>
<tr>
<td>Spread 3M</td>
<td>8,08</td>
<td>21,63</td>
</tr>
<tr>
<td>Spread 6M</td>
<td>10,80</td>
<td>25,74</td>
</tr>
<tr>
<td>Spread 9M</td>
<td>13,27</td>
<td>29,44</td>
</tr>
<tr>
<td>Spread 12M</td>
<td>15,48</td>
<td>32,74</td>
</tr>
</tbody>
</table>

Interestingly, we also find a maturity-invariant percentage for the liquidity component average whereas the credit component average has an importance increasing with maturity. This indicates that credit risk plays relatively more at the long end of the term-structure. Concentrating on the top panel of Figure 4, we see however that the liquidity factor accounts for much of the high-frequency variations in the spreads, in particular during the distress period of late 2008 (after the Lehman collapse) and in end 2011 (in a period of particular strain in the Euro sovereign markets). Panel a) and b) of Figure 4 emphasize the low-frequency

---

\[ \frac{\text{spread average}}{\text{spread avg}} \]

\[ \frac{\text{credit average}}{\text{credit avg}} \]

\[ \frac{\text{liquidity average}}{\text{liquidity avg}} \]

\[ \frac{\text{interaction average}}{\text{interaction avg}} \]

\[ \frac{\text{term premia average}}{\text{term premia avg}} \]

---

The relative shares of the spreads attributed to the credit, liquidity and interaction parts being very similar to the risk-neutral decomposition depicted on Figure 4, panel (a), we do not present the related graphs for the sake of clarity.
fluctuations in the credit component, which increased almost monotonously since August 2007 before stabilizing in the summer 2012. This result is in line with the previous findings of Schwarz (2010) and Filipovic and Trolle (2013). The interaction term which is always positive, is substantial over the sample period. For the 6-month maturity, the interaction term evolves between 0 and 40 bps over 2007-2013. However, looking at panel (b) of Figure 4, we observe that its evolution is smoother than the liquidity component.

The decomposition of the spread between the term premia and the spread under physical dynamics is also computed (bottom panel of Figure 4): the term premia component and the observed spread have very similar features, and are positively correlated. Figure 5 presents decompositions of the term structure of EURIBOR-OIS spreads at different dates. In particular, the bottom row shows the share of the modelled spreads that is accounted for by term premium. The longer the maturity, the larger this share.

6.3 The impact of unconventional monetary policy on interbank risk

The main programs of unconventional monetary policies in the Eurozone can be broadly separated into three periods. The Securities Market Program (SMP) consisted in sterilized bond-buying on the secondary market. It was designed to "ensure depth and liquidity in [...] market segments that are dysfunctional" and was implemented in May 2010 and August 2011. Later, on the 8th of December 2011, the ECB disclosed the design of Very Long Term Refinancing Operations (VLTRO). 3-year maturity open market operations were proposed
Notes: Date ranges from August 31, 2007 to January 4, 2013. Units are in basis points. Panel(a) represents the stacked components of the spread: light grey constant component is the intercept of the spread equation, the other legends are given on Panel(b) graph. Panel(b) represents the – not stacked – components of the spread. The graph at the bottom represents the modelled spread and its term premia. The black vertical axes stand from left to right for: SMP program announcements (first two axes), VLTRO announcement and allotments (next three axes), and Mario Draghi’s London speech (last axis).
in the form of reverse repo. Two allotments were granted on the 21st of December 2011 and on the 29th of February 2012 of respectively 489 and 530 bln Euros to 523 and 800 banks. More recently, during August 2012, Mario Draghi announced the setting of Outright Monetary Transactions (OMT) in his London speech. Conditionally on fiscal adjustments or precautionary programs enforcement by candidate countries, the ECB is allowed to trade in secondary sovereign bond markets with "no ex ante quantitative limits". Whereas this framework has been announced it has not been applied in practice yet. In the following section, we discuss the effects of those different events.

Interestingly, the EURIBOR-OIS spreads have decreased continuously since the VLTRO announcement in December 2011. This drop has led many commentators (and central bankers) to claim that the ECB unconventional refinancing operations were successful in alleviating interbank market tensions. In particular, according to ECB officials, the non-standard VLTRO operations addressed "only the liquidity side of the [interbank market] problem". Our results seem to support this view as the liquidity component of the spreads has slowly faded away to nearly zero since the VLTRO announcement date (see Figure 4, Panel b). A further positive effect can also be attributed to the OMT announcement through liquidity.

The same pattern can be observed on Figure 5 (first and second rows of charts). After the SMP and before the VLTRO announcement, liquidity risk still accounts for most of the term structure of interbank spreads (second column of charts). However, after the VLTRO allot-

\footnote{See Mario Draghi’s interview with the Wall Street Journal, published on February 24, 2012, or the lecture by Peter Praet in February 20, 2012.}
ments, liquidity risk represents only 10 to 20 basis points across maturities (third column) and becomes negligible for all maturities after the OMT announcement (fourth column). In comparison, looking at both Figures 4 and 5, those policy measures had virtually no impact on the credit component of the spread.

Turning to the last row of Figure 5, it appears that unconventional monetary policies were followed by decreases in both the expected component and the term premia. This result contradicts somehow the findings in Angelini, Nobili, and Picillo (2011), according to whom term premium embodies much of the spreads fluctuations. It is however difficult to compare the two results due to methodological differences: contrary to our specification, Angelini, Nobili, and Picillo (2011) blend spreads with different maturities without handling the whole term structure of spreads in a coherent no-arbitrage framework.

All in all, even if the EURIBOR-OIS spreads have not really reacted to the SMP program. Our results suggest that the recent unconventional monetary policy measures undertaken within the Eurosystem have contributed to reinforce banks liquidity positions and a stabilization of the credit risk in the Eurozone.

Conclusion

We develop a no-arbitrage two-factor quadratic term structure model for the EURIBOR-OIS spreads across several maturities, from August 2007 to January 2013. To identify credit and liquidity components in the spreads, we introduce credit and liquidity proxies based on CDS prices, market liquidity and funding liquidity measures. Our decomposition
Figure 5: Decomposition of EURIBOR - OIS term structure

Notes: Units are in basis points. First row represents the stacked components of the term structure: the lightest grey component is the intercept of the spread equation, then from dark to light are the liquidity component, credit component and interaction. Second row presents the same components (except the intercept) not stacked. The graph at the bottom represents the modelled term structure and its term premia. The white dots are the observed spreads.
handles potential interdependence between credit and liquidity risks and is consistent across maturities. We find that the liquidity risk generates most of the variance of the spread over the estimation period. The credit risk is less volatile, but represents most of the spread level in late 2012. Our decompositions allows us to shed new light on the effects of unconventional monetary policy of the ECB on the interbank risk. We show that whereas the bond-purchase programs of 2010 and 2011 were not followed by decreases in any of the EURIBOR-OIS spread components, the VLTROs and the OMT announcements have had a substantial impact, mainly on the liquidity risk. At the end of the sample, the liquidity risk is negligible, and the remaining part of the spreads is only credit risk related.
Appendix

A Appendix

A.1 Interpreting the intensity $\lambda$

We provide a structural interpretation of the intensity modeling proposed in Section 4.1. Let us introduce binary variables denoted by $d_{c,t}^{(i)}$ and $d_{l,t}^{(i)}$. The former describes the credit state of bank $i$, $d_{c,t}^{(i)} = 1$ being the default state; the latter indicates whether bank $i$ has been hit by the liquidity shock at date $t$, by $d_{l,t}^{(i)} = 1$. We assume that the panel of banks is homogenous, in the sense that, conditional on $(x_{c,t}, x_{l,t})$, the probabilities of switching from the different credit and liquidity states are the same for all the banks of the EURIBOR panel.

Let us describe a stylized set up in which credit and liquidity effects induce some losses for lenders on the interbank unsecured market (market for which the EURIBOR rates are supposed to apply). Suppose that bank $i$ has lent to bank $j$ at $t^* < t$ and that at date $t$ the residual maturity of the loan is of $h$ periods.

- If bank $j$ defaults at date $t$, then bank $i$ will not obtain full repayment at $t+h$. Instead, it recovers a fraction $\theta_c < 1$ of the “market value” of the loan that would have prevailed at date $t$ without default. This “market value” corresponds to the face value of the loan discounted by the EURIBOR rate (this set up builds on the recovery at market value assumption defined e.g. by Duffie and Singleton (1999)).

- If bank $i$ is hit by a liquidity shock at date $t$ (i.e. if $d_{l,t-1}^{(i)} = 0$ and $d_{l,t}^{(i)} = 1$), then it has to find some cash in a limited period of time to meet an unexpected liquidity need. A possibility for bank $i$ is to negotiate a premature termination of the loan. It
is assumed that bank $j$ agrees to do so at a discount, such that the repayment at date $t$ is expressed as a fraction $\theta_l < 1$ of the “market value” of the loan.

Let us then consider the price at date $t$ of an interbank loan of residual maturity $h$. At date $t$, the lender (bank $i$) has not been hit by the liquidity shock and the borrower (bank $j$) has not defaulted. Using the previous descriptions of credit and liquidity events, we have:

$$B(t, h) = \mathbb{E}^Q [\exp(-r_t)B(t + 1, h - 1) \times \left(\theta_c d_{c,t+1} + \theta_l (1 - d_{c,t+1})d_{l,t+1} + (1 - d_{c,t+1})(1 - d_{l,t+1})\right) | X_t, d_t = 0].$$

Then, using the law of iterated expectations and under assumptions, we have:

$$B(t, h) = \mathbb{E}^Q \left[ \mathbb{E}^Q \left(\exp(-r_t)B(t + 1, h - 1) \times \left(\theta_c d_{c,t+1} + \theta_l (1 - d_{c,t+1})d_{l,t+1} + (1 - d_{c,t+1})(1 - d_{l,t+1})\right) | X_{t+1}, d_t = 0\right) | X_t, d_t = 0\right].$$

where the function $\lambda^Q(\bullet)$ is defined by:

$$\lambda^Q : X \mapsto \mathbb{E}^Q \left\{ \left(\theta_c d_{c,t+1} + \theta_l (1 - d_{c,t+1})d_{l,t+1} + (1 - d_{c,t+1})(1 - d_{l,t+1})\right) | X_{t+1} = X, d_t = 0 \right\}.$$  

Moreover, since the SDF $M_{t,t+1}$ is not function of $(d_{c,t+1}, d_{l,t+1})$, we can show (see Monfort and Renne (2013) or Gouriéroux, Monfort, and Renne (2013)) that the conditional distribution

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of \((d_{c,t+1}, d_{l,t+1})\) given \(X_{t+1}\) are the same under \(P\) and \(Q\), which implies \(\lambda^Q(X_{t+1}) = \lambda(X_{t+1})\). Formula 2 is obtained by recursion, with \(\lambda_t = \lambda(X_t)\). Note that the recursion requires that \(d_t\) does not \(Q\)-cause \(X_t\), which is the case as soon as (a) it the case under \(P\) and (b) the SDF does not depends on \(d_t\).

### A.2 Market prices of risk definition

The exponential-affine SDF between \(t\) and \(t + 1\), denoted \(M_{t,t+1}\) is given by:

\[
M_{t,t+1} = \exp \left[ \Gamma_t' \begin{pmatrix} \varepsilon_{c,t+1} \\ \varepsilon_{l,t+1} \end{pmatrix} - \frac{1}{2} \Gamma_t' \Gamma_t - r_t \right],
\]

where \(\Gamma_t = \Gamma_0 + \Gamma_1(x_{c,t} x_{l,t})'\) corresponds to the vector of market prices of risks. The mapping between the parameters defining the historical and the risk-neutral dynamics depends on these prices of risk:

\[
\begin{pmatrix} \mu^c_c \\ \mu^s_l \end{pmatrix} = \begin{pmatrix} \mu_c \\ \mu_l \end{pmatrix} + \begin{pmatrix} \sigma_c & 0 \\ 0 & \sigma_l \end{pmatrix} \Gamma_0
\]

\[
\begin{pmatrix} \varphi^*_{1,1} & \varphi^*_{1,2} \\ \varphi^*_{2,1} & \varphi^*_{2,2} \end{pmatrix} = \begin{pmatrix} \varphi_{1,1} & \varphi_{1,2} \\ \varphi_{2,1} & \varphi_{2,2} \end{pmatrix} + \begin{pmatrix} \sigma_c & 0 \\ 0 & \sigma_l \end{pmatrix} \Gamma_1
\]

### A.3 OIS are equivalent to zero-coupon rates

Consider a bank that has access to the overnight interbank market as well as to the EONIA swap market. Assume further that this bank wants to invest 100 euros for a period of \(h\) weeks. Then, this bank can replicate a zero-coupon investment yielding a compounded
interest rate of $R_{t,h}^{OIS}$ by (a) entering a maturity-$h$ OIS swap (where it pays the floating leg) while (b) lending, every day, the 100 euros (plus daily accrued interests) on the overnight interbank market. By definition of the OIS contract, the accrued interests earned by investing in the overnight market are going to equalize the interest accrued on the floating leg of the swap. Therefore, this strategy boils down to lending 100 euros at date $t$ and to get $100 \exp(h/52 \times R_{t,h}^{OIS})$ at date $t+h$, which demonstrates that $R_{t,h}^{OIS}$ can be interpreted as a zero-coupon yield.

### A.4 Identifiability and estimation constraints

Let us consider another Gaussian VAR(1) vector of factors $\tilde{X}_t = (\tilde{x}_{c,t}, \tilde{x}_{l,t})'$, which is an affine transformation of $X_t$.

$$\tilde{X}_t = m + MX_t$$

and let us see whether it can lead to an observationally equivalent model. As the proxies are respectively functions of only one component of $X_t$, this imposes $M$ to be diagonal. Hence the alternative factors can be written: $\tilde{x}_{i,t} = M_i x_{i,t} + m_i$ for $i = \{c,l\}$. In addition, to ensure that only $\tilde{x}_i = \tilde{x}_{c,t} + \tilde{x}_{l,t}$ enters the spread formulae, we must impose $M_c = M_l$. Finally, the conditional variance of $\tilde{x}_t$ must be equal to 1, that is

$$M_c^2 \sigma_c^2 + M_l^2 \sigma_l^2 = 1 \iff M_c^2 (\sigma_c^2 + \sigma_l^2) = 1$$

$$\iff M_c = 1, \text{ since } \sigma_c^2 + \sigma_l^2 = 1$$

Thus $M = I_2$. 
For interpretation purposes, we also impose that a large proportion of the values of latent factors are such that the intensity function and the proxies are monotonously increasing in the factors. That is to say:

$$
\mathbb{P}\left( \frac{\partial \lambda_t}{\partial x_t}(x_{i,t}) > 0 \right) = 1 - \alpha, \quad \mathbb{P}\left( \frac{\partial P_{i,t}}{\partial x_{i,t}}(x_{i,t}) > 0 \right) = 1 - \alpha \quad \forall i = \{c, l\},
$$

where $\alpha$ is typically a small number. The rationale is the following: we impose that when the latent factors are increasing, the proxies increase as well. We also want an increase in the underlying credit or liquidity risk to be translated into an increase in the spread. There must therefore be a monotonously increasing relationship between the intensity and the factors.

The previous constraint implies conditions on the mean of the latent process. Using the notation $\Phi = [\varphi_{i,j}]_{i=\{1,2\}, j=\{1,2\}}$ and $\beta_i = \max\left(\arg\min_x \lambda_t, \arg\min_x \mathbb{E}(P_{i,t})\right) \forall i = \{c, l\}$, we have

$$
\mathbb{P}[x_{i,t} < \beta_i] = \alpha \iff \mathbb{E}(x_{i,t}) = \beta_i - q_{N(0,1)}(\alpha)\sqrt{\mathbb{V}(x_{i,t})}
$$

where $\mathbb{E}(\bullet)$ and $\mathbb{V}(\bullet)$ are the unconditional expectation and variance operators, and $q_{N(0,1)}(\alpha)$ is the level $\alpha$ quantile of the normalized gaussian distribution. Since:

$$
\mathbb{E}(X_t) = (I_2 - \Phi)^{-1}(\mu_c, \mu_l)',
$$

$$
\text{Vec}[\mathbb{V}(X_t)] = (I_4 - \Phi \otimes \Phi)^{-1}\text{Vec}\left[\begin{pmatrix} \sigma_c^2 & 0 \\ 0 & \sigma_l^2 \end{pmatrix}\right] \equiv \text{Vec}\left[\begin{pmatrix} v_c & v_{cl} \\ v_{cl} & v_l \end{pmatrix}\right],
$$
we get the condition:

\[
\begin{pmatrix}
\mu_c \\
\mu_l
\end{pmatrix} = (I_2 - \Phi) \begin{pmatrix}
\beta_c \\
\beta_l
\end{pmatrix} - q_{N(0,1)}(\alpha) \begin{pmatrix}
\sqrt{\upsilon_c} \\
\sqrt{\upsilon_l}
\end{pmatrix},
\]

where the \(\sqrt{\upsilon_i}\) are the unconditional standard errors of the \(x_{i,t}\). These additional constraints on the intercepts \(\mu_c\) and \(\mu_l\) imply \(m = 0\) and therefore \(\tilde{X}_t = X_t\).

In the estimation, we set \(\alpha = 0.025\). We also control the accuracy of the fit of the proxies, and impose that both \(\sigma_{\nu_c}^2\) and \(\sigma_{\nu_l}^2\) are below 0.1.
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