The optimal grouping of commodities for indirect taxation

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Introduction

A hot public debate about taxation of restaurants in France!

Insights from the theory of indirect taxation: The Ramsey rule.

But this rule does not take into account the (institutional) constraint on the number of available rates in the UE.

The paper discusses the structure of indirect taxation under such constraint, both the levels of tax rates and the groups of goods taxed at each rate.
What the paper highlights

The role of the **purported rate**, which corresponds to the rate that would be applied to an individual commodity if it could be taxed freely, all the other rates being held fixed.

Under a **single-peakedness assumption**, a good should be taxed to one of the (typically two) actual rates that are closest to its purported rate.

**An application to the UK:** The social welfare function that rationalizes the current VAT puts most of the weight of the fourth and the fifth deciles of consumption, about 2/3 of (relevant) aggregate consumption is correctly taxed, but 'Food out', which comprises restaurants and fast foods, is too heavily taxed (some items in this group should be exempted) ...
Consumer problem

Consumer $c$ maximizes

$$\int_G u_g(x_g, c) \mu_g dg + m$$

subject to

$$\int_G (1 + t_g) x_g \mu_g dg + m \leq w_c,$$

or equivalently

$$\int_G (u_g(x_g, c) - (1 + t_g) x_g) \mu_g dg.$$ 

The solution is $x_g = \xi_g(t_g, c)$ for each $g \in G$ and the corresponding indirect utility (up to the constant $w_c$) writes

$$\int_G v_g(t_g, c) \mu_g dg.$$
Unconstrained Ramsey Problem

The social planner chooses the tax rates \((t_g)_{g \in G}\) maximizing

\[
\int_C \alpha(c) \int_G v_g(t_g, c) \mu_g \, dg \, d\nu(c) - \lambda \left( \int_C \int_G t_g \xi_g(t_g, c) \mu_g \, dg \, d\nu(c) - R \right)
\]

or equivalently

\[
\int_G \mathcal{L}_g(t_g) \mu_g \, dg,
\]

with

\[
\mathcal{L}_g(t_g) = \int_C (\alpha(c) v_g(t_g, c) \, d\nu(c) - \lambda t_g \xi_g(t_g, c)) \, d\nu(c)
\]

being the 'social contribution of good \(g\)'.

A single peaked assumption

We impose that $L_g(t_g)$ is single peaked:

1. if it is increasing with $t_g$, then good $g$ should be made infinitely expensive; if it is decreasing with $t_g$, it should be made free;

2. if it is first increasing and then decreasing, the optimal taxe rate satisfies the Ramsey rule:

$$\frac{\partial L_g(t_g)}{\partial t_g} = 0 \iff \frac{a_g(t^R_g)}{\lambda} = 1 - \frac{t^R_g}{1 + t^R_g} \varepsilon_g(t^R_g)$$

where

$$a_g(t^R_g) = \int_C \alpha(c) \frac{\xi_g(t_g, c)}{X_g(t_g)} d\nu(c)$$

is the social weight (Feldstein characteristic) of good $g$. 
A graphical representation

\[ L_g(t_g) \]
A more useful one

\[
\frac{a}{\lambda} = 1 - \frac{t_g^R}{1 + t_g^R} \varepsilon
\]
Tax rule with a finite number of rates

If there is a finite number of available tax rates $t_k$ ($k = 1, \ldots, K$). A tax rule is defined by:

1. a collection of groups $(G_k)$ of goods taxed at a common rate, with $\bigcup_k G_k = G$
2. the level of each rate $(t_k)$

An optimal tax rule maximizes

$$L(t, (G_k)) = \sum_k \int_{G_k} (V_g(t_k) - \lambda t_k X_g(t_k)) \mu_g dg.$$ 

A strategy to characterize an optimal tax rule:

→ Given the groups, the levels of tax rates maximize $L(t, \cdot)$
→ Given the tax rates, the groups maximize $L(\cdot, (G_k))$. 
Diamond tax rates

The groups are treated as aggregate commodities to which the Ramsey rule is applied:

\[
\frac{a_{G_k}(t_k)}{\lambda} = 1 - \frac{t_k}{1 + t_k} \varepsilon_{G_k}(t_k)
\]

where

\[
a_{G_k}(t_k) = \int_{G_k} a_g(t_k) \frac{X_g(t_k)}{X_{G_k}(t_k)} \mu_g dg,
\]

and

\[
\varepsilon_{G_k}(t_k) = \int_C \varepsilon_g(t_k) \frac{X_g(t_k)}{X_{G_k}(t_k)} \mu_g dg.
\]
Optimal groups

**Theorem 1.** A necessary condition for optimality is that any good $g$ be attached to a group $k$ such that $\mathcal{L}_g(t_k) \geq \mathcal{L}_g(t_h)$ for every $h = 1, \ldots, K$.

Thus, under the single peakedness assumption,

1. if $\mathcal{L}_g$ is increasing, $g$ should be taxed at $t_K$; if it is decreasing, it should be taxed at $t_1$.

2. Otherwise,
   - if $t_g^R \geq t_K$, it should be taxed at $t_K$,
   - if $t_g^R \leq t_1$, it should be taxed at $t_1$,
   - if $t_k \leq t_g^R \leq t_{k+1}$, it should be taxed at either $t_k$ or $t_{k+1}$. 
An illustration

The graph illustrates a function $L_g(t_g)$, which increases from $t_1$ to $t_2$, reaches a peak at $t_g^R$, and then decreases to $t_4$.
One more

\[ \frac{a}{\lambda} = 1 - \frac{t_2}{1 + t_2} \epsilon \]

\[ \frac{a}{\lambda} = 1 - \frac{t_g^R}{1 + t_g^R} \epsilon \]

\[ \frac{a}{\lambda} = 1 - \frac{t_3}{1 + t_3} \epsilon \]
The case of constant elasticities

Let

\[ u_g(x, c) = \begin{cases} 
A_g(c)^{1/\varepsilon_g(c)} \frac{x^{1/\varepsilon_g(c)}}{1 - 1/\varepsilon_g(c)} & \text{for } \varepsilon_g(c) > 0, \text{ and } \varepsilon_g(c) \neq 1 \\
A_g(c) \ln \frac{x}{A_g(c)} & \text{for } \varepsilon_g(c) = 1.
\end{cases} \]

We impose that \( \varepsilon_g(c) = \varepsilon_g \) for single-peakedness to be preserved under aggregation. Then, for \( t < t' \),

\[ \mathcal{L}_g(t) < \mathcal{L}_g(t') \iff \frac{a_g}{\lambda} < \phi(\varepsilon_g, t, t') \]

where \( \phi \) is convex in \( \varepsilon_g \) (but not necessarily monotonic) and asymptote to \( (1 - \varepsilon_g)t/(1 + t) \) when \( \varepsilon_g \) goes to infinity.
A complete characterization

\[ \frac{a}{\lambda} \]

Taxed at rate \( t_2 \)

Taxed at rate \( t_3 \)
Ramsey’s insights: with a zero tax rate

\[
\frac{a}{\lambda} = t_1 = 0 \quad \text{for} \quad t_2 < t_3 \quad \text{and} \quad t_2 > 0
\]
Ramsey’s insights: without a zero tax rate
An application

Is the actual tax structure far from the optimal one?

What kind of reforms, if any, should be implemented?
   → How restaurants and fast food should be taxed?

Separability assumptions made so far are very strong in practice:
   → Cross price effects
   → Income effects
A more general framework

Preferences of consumer $c$ are represented by

$$u_c(x, l)$$

and her budget constraint writes

$$\int_G (1 + t_g)x_g \mu_g dg \leq w(c)l - T(w(c)l).$$
Diamond first-order condition

\[
\int_{g \in G_k} \left( -\frac{a_g}{\lambda} + X_g \right) \mu_g \, dg
+ \int_{g \in G_k} t_k \frac{\partial X_g}{\partial t_g} + \sum_h t_h \frac{\partial X_{G_h \setminus \{g\}}}{\partial t_g}
+ \int_{C} T'(Y(c)) \frac{\partial Y_c}{\partial t_g} d\nu(c) \mu_g \, dg = 0,
\]

where \(a_g = \int_{C} \alpha(c) \rho(c)(\xi_g/X_g) \, d\nu(c)\) and \(\rho(c)\) is the marginal utility of income of consumer \(c\). This can be rewritten as

\[
\frac{a_{G_k}}{\lambda} - b_{G_k} = 1 - \frac{t_k}{1 + t_k} \varepsilon_{G_k}
\]

and, for an individual good

\[
\frac{a_g}{\lambda} - b_g = 1 - \frac{t_k}{1 + t_k} \varepsilon_g.
\]
49% of consumption is taxed at the standard rate (17.5%), 10% at the reduced rate (5%) and 27% is exempted; the remaining (tobacco, alcohol, petrol and diesel) is subject to large excise taxes.

We assume that the tax authority takes as given after-tax income and the current grouping of commodities. If tax rates are optimally chosen, the Diamond first-order must be satisfied. This gives some information about social redistributive aims and allows us to draw a fan corresponding to these weights.

The issue is whether the actual composition of the commodity groups is optimal.
A digression on uniform commodity taxation

If the Atkinson Stiglitz conditions hold (preferences are separable between commodities and labor, and the preferences for commodities are identical across individuals at the microeconomic level), and if the government can freely tax incomes in a non-linear way, all the goods should be taxed at the same rate.

But:

1. There is no general agreement on the empirical relevance of the Atkinson Stiglitz conditions. Browning and Meghir (1991) find some evidence of non-separability.

2. We work with fixed after tax incomes, so that our exercise provides information on the optimal indirect tax rates, given the current income tax schedule.
To answer this question

We must first recover the implicit social weights consistent with the current VAT.

We have three Diamond first-order conditions (standard, reduced, exempted) and a normalization condition on social weights.

We observe (1) the tax rates, (2) own and cross price elasticities for twenty categories of goods, and (2) the shares of consumption by (consumption) decile for each category.

The unknown are the ten weights $\alpha_c\rho_c$ and the marginal cost of public funds $\lambda$.

→ The problem has many solutions!
Moreover, since Diamond first-order conditions are homogenous of degree 1 in \((\alpha_c\rho_c, \lambda)\): we can only expect to recover the ten ratios \(\alpha_c\rho_c/\lambda\). Let choose the normalization

\[
\int_C \alpha_c\rho_c\,d\nu(c) = 1.
\]

Then, an equal lump-sum transfer of 1 unit (of aggregate consumption) leads to an increase of 1 unit of social welfare: social welfare is measured in tenths of aggregate consumption.

Minimizing the square of the LHS of Diamond FOC with respect to \((\alpha_c\rho_c, \lambda)\) and using the normalization condition leads to:

\[
\hat{\lambda} = 1.11,
\]

\[
\alpha_1\rho_1 = 0.03, \quad \alpha_4\rho_4 = 0.54 \quad \text{and} \quad \alpha_5\rho_5 = 0.43.
\]
Is the grouping optimal?
Results

If one excludes goods subject to excise taxes (other considerations than mere redistribution should matter in such cases), it remains 87% of aggregate consumption, and about $2/3$ of these 87% appear to be correctly taxed.

Main exceptions:
Exempted goods that should be taxed:
   → dairy products, fruits and vegetables, other non-VAT foods
Goods taxed at the standard rate that should be exempted:
   → food out and Public transport.

Impact of the redistributive stance of the government:
   → Utilitarian vs. Rawlsian?