Separability and Public Finance

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Introduction

In a first best setting, efficiency can be achieved independently of equity concerns through lump sum transfers without indirect taxes, the Samuelson rule applies to the provision of public goods, and Pigovian taxes are used to correct externalities.

In a second best environment, the social planner faces additional constraints, but second best rules may sometimes have a ‘first best’ flavor.

(Most of) These examples involve separability coupled with specific informational assumptions. This paper highlights their common underlying structure, in the spirit of Laroque (2005) and Kaplow (2006), through an argument of global nature.

Extensions to preference heterogeneity and individual production.
A non-satiation property

Agent $n$ ($n$ possibly multidimensional, with cdf $F$) buys private goods $c$, supplies labor $\ell$. Her utility is $U(c, \ell, n)$.

For a net production $\zeta$, the feasibility constraint on private goods is

$$\int_n c_n dF(n) \leq \zeta.$$

**Definition 1.** A feasible allocation which satisfies all second best constraints is **non-satiated** when an increase $dx$, $dx \geq 0$, $dx \neq 0$, in the resources of private goods, leading to the feasibility constraint

$$\int_n c_n dF(n) \leq \zeta + dx,$$

allows a Pareto improvement while satisfying the second best constraints.
Indirect taxes: Atkinson Stiglitz setup

Agent $n$ has separable utility $U(V(c), \ell, n)$ and her before tax income (‘efficient labor’) is $y = n\ell$.

The technology is linear. The feasibility constraint is

$$\int_n pc_n dF(n) \leq \int_n n\ell_n dF(n),$$

where $p$ is a fixed vector of (producer) prices.

The government observes $y = n\ell$, but neither $n$ nor $\ell$ separately. It announces an after tax income schedule $R(\cdot)$ and linear taxes $q - p$ on consumption goods.

Given $y$, an agent maximizes $V(c)$ subject to $qc = R(y)$. Her conditional demand function is $\gamma(q, R(y))$. 
The government chooses \((q, (y_n, R(y_n)))\) which maximizes a SW function subject to the feasibility constraint and the incentive constraints (IC),

\[
U \left( V(\gamma(q, R(y_n))), \frac{y_n}{n}, n \right) \geq U \left( V(\gamma(q, R(y_m))), \frac{y_m}{n}, n \right)
\]

for every \((m, n)\).

**Lemma 1.** Consider a non-satiated second best allocation in which agent \(n\) has before tax income \(y_n^*\) and consumes \(c_n^*\). Given \((y_n^*)\), \((c_n^*)\) is a first best allocation of the economy where all the agents have the same quasi-concave and increasing utility function \(V(\cdot)\) and the aggregate production set is

\[
\int_n pc_n dF(n) \leq \int_n y_n^* dF(n).
\]
The proof is simple!

1. With $V_n^* = V(c_n^*)$, (IC) rewrite

$$U \left( V_n^*, \frac{y_n^*}{n}, n \right) \geq U \left( V_m^*, \frac{y_m^*}{n}, n \right) \text{ for every } (m, n).$$

Hence, given $(y_n^*)$, any $(c_n)$ that yields $(V_n^*)$ satisfies (IC).

2. From the second welfare theorem, any first-best optimum (of the economy described in the lemma) can be decentralized with an appropriate choice of $(q, (R_n))$. Note that $R_n$ can be written as $R(y_n)$: two agents $m$ and $n$ with the same $y$ must have the same $V$ (by (IC)), and thus the same $R$.

3. If the reference allocation is not a first best optimum, then one can achieve $(V_n^*)$ with less than $\int_n y_n^* dF(n)$. There is an extra $dy$: the non-satiation property gives the desired contradiction.
Indirect taxes: heterogeneous preferences

Utility of agent \( n \) is \( U(V(c, a), \ell, a, n) \).

Indirect taxes are useful when \( a \) is private information.

Indirect taxes are useless when \( a \) is publicly observable (given or chosen). If chosen, we have:

**Lemma 2.** Suppose that the after tax income schedule can be made dependent on the observable characteristics \( a \). Consider a non-satiated second best allocation in which agent \( n \) chooses \( a_n^* \), has a before tax income \( y_n^* \) and consumes \( c_n^* \). Given \( (a_n^*, y_n^*), (c_n^*) \) is a first best allocation of the economy where, for all \( n \), agent \( n \) has preferences for goods given by the quasi-concave and increasing utility function \( V(\cdot, a_n^*) \) and the aggregate production set is

\[
\int_n pc_n dF(n) \leq \int_n y_n^* dF(n).
\]
Public good provision

Agent $n$ has utility $U(V(c, g), \ell)$, where $g$ is a public good (chosen by the government).

The aggregate feasibility constraint is

$$\int_n c_n dF(n) + g = \int_n y_n dF(n).$$

**Lemma 3.** Consider a non-satiated second best allocation $((y^*_n, c^*_n), g^*)$. Then, given $(y^*_n)$, $((c^*_n), g^*)$ is a first best allocation of the economy with utility functions $V(\cdot)$ and production set

$$\int_n c_n dF(n) + g = \int_n y^*_n dF(n).$$

(A similar result holds in the presence of externalities.)
Individual production sets

Agent \( n \) has utility \( U(c, \ell, n) \). She must work:

\[
\ell = \tilde{G}(z, n),
\]

to ‘produce’ the vector \( z = (z_+, -z_-) \) (\( z_+ \) for the vector of outputs and \( z_- \) for the inputs, both of the dimension of the number of goods).

The feasibility constraint becomes

\[
\int_n c_n dF(n) \leq \int_n [z_{n+} - z_{n-}] dF(n).
\]

The government observes \( (z_n) \) (this can be relaxed).

It sets production prices \( p(z) = (p_+(z), p_-(z)) \), consumer prices \( q \), and an after tax income \( R(p(z)z) \).
Lemma 4. Let $\tilde{G}(z, n) = G(H(z), n)$, where $G$ is increasing in $H$, and $H$ is increasing and quasi-convex. At a non-satiated second best allocation, the production profile $(z_n^*)$ is efficient: there does not exist $(z_n)$ that requires less labor input

$$H(z_n) \leq H(z_n^*) \quad \text{for all } n$$

while

$$\int_n [z_{n+} - z_{n-}]dF(n) \geq \int_n [z_{n+}^* - z_{n-}^*]dF(n),$$

with some strict inequality. Therefore $z_n^*$ is profit maximizing for some linear price vector $p^*$, with $p^*_+ = p^*_-$, subject to the constraint $H(z) \leq H(z_n^*)$.

- Producers and users should face the same price, and this price should not depend on $z$. Think of education (with education grants conditional on income), fertilizers in LCDs, etc.
- The government only has to observe the before tax income (not necessarily the whole vector $z$).
With the separable utility of Atkinson Stiglitz, one obtains:

**Corollary 1.** Assume a separable labor cost function. Assume furthermore that the preferences of the agent of characteristics $n$ are represented by $U(V(c), \ell, n)$, where $V$ is a smooth increasing quasi-concave function. Then, given $(z_n^*)$, $(c_n^*)$ is a first best allocation of the economy in which agents have preferences given by the utility function $V$ and the feasibility constraint is

$$\int_n c_n dF(n) \leq \int_n [z_{n+}^* - z_{n-}^*] dF(n).$$

The producer prices $p_{+}^*$ and $p_{-}^*$ and consumer prices $q^*$ can be taken to be equal.