Annals of Economics and Statistics, Number 131, September 2018

ON THE LINK BETWEEN VOLATILITIES, REGIME SWITCHING PROBABILITIES AND CORRELATION DYNAMICS

JEAN-DAVID FERMANIAN[a] AND HASSAN MALONGO[b]

When markets are stressed, volatilities and correlations tend to increase jointly, and volatilities often react quicker than correlations. Based on this intuition, we extend the Dynamic Conditional Correlation model (Engle, 2002) in order to check whether the individual volatilities and/or the probabilities that some assets belong to a high/low volatility regime influence their correlation dynamics. We evaluate potential asymmetrical leverage effects too. We apply our methodology to MSCI Developed Markets indexes that cover twenty-three countries. The new models provide better in-sample fits and forecasts of the portfolio return distributions. Therefore, they are valuable frameworks for portfolio allocation and financial risk management.

JEL Codes: G11, G15, G17.
Keywords: Dynamic Correlations, Multivariate GARCH Models, Regime-Switching, Volatility Regimes.

1. INTRODUCTION

Nowadays, modeling the joint behavior of several financial assets has become a key challenge for academics and practitioners. It is not so easy to build a realistic model that is statistically relevant and consistent with some well-known stylized features of financial asset returns (fat tails, volatility clustering, autocorrelation of absolute returns, etc) simultaneously. A usual key part of such models is given by the process of the conditional variance-covariance matrices that are associated with the underlying assets. Indeed, accurate estimations of volatility and correlation risks are crucial for risk management, asset pricing and portfolio management purpose. In practical terms, a classical hurdle is related to the so-called "curse of dimensionality" as the specification of a general multivariate dynamic model often induces an explosion of the number of parameters, inducing practical problems of inference and statistical uncertainties (overfitting).

In the literature, many specifications for discrete-time multivariate dynamic models have been proposed. Broadly speaking, most of them belong to the multivariate GARCH family or to the multivariate stochastic volatility (MSV) family: see the surveys of Bauwens, Laurent, and Rombouts (2006) and Asai, McAleer, and Yu (2006), respectively. By specifying the dynamics of the first two conditional moments of the underlying distributions on one side, and the law of the innovations on the other side, such models are easy to simulate and to forecast (at least one-period ahead). Convenient estimation techniques have been proposed, notably QML for GARCH-type models (see Francq and Zakoian (2011), e.g.) or simulation-based methods for MSV (Harvey, Ruiz, and Shephard, 1994). Among numerous contributions of GARCH-type models, some important sub-families are the VEC

We thank M. Brière, M. Fengler and E. Otranto for fruitful remarks and discussions. Moreover, two anonymous reviewers have added interesting comments and ideas for potential extensions. Finally, the authors thank the Labex "Ecodec" for its financial support.

[a]Ensae-Crest, 5 avenue Henry Le Chatelier, TSA 26644, 91764 Palaiseau cedex, France, `jean-david.fermanian@ensae.fr` (corresponding author)
[b]Amundi Asset Management, 90 Boulevard Pasteur, 75015 Paris, France, `hassan.malongo@amundi.com`

(Bollerslev, Engle, and Wooldridge, 1988) and the BEKK (Engle and Kroner, 1995) specifications. In the case of MSV models, we can cite Harvey, Ruiz, and Shephard (1994) and Wishart processes (Gouriéroux, Jasiak, and Sufana, 2009), among others.

Although the analysis of volatility dynamics is a well-established topic in the literature, an increasing interest has emerged around correlation dynamics for twenty years, particularly in the multivariate GARCH framework. Instead of specifying some processes of variance-covariance matrices directly, some two-step procedures have been proposed, to tackle volatility and correlation dynamics separately: first, evaluate individual volatility processes (assuming no cross-effects), and deduce the vectors of standardized asset returns. As a second step, evaluate the asset correlations "knowing" the conditional volatilities. Therefore, at the cost of a small loss of efficiency, the task of finding a complex multivariate process has been divided into two simpler problems, circumventing the curse of dimensionality and inducing simpler optimization schemes. This fruitful idea has been put in place in the Constant Conditional Correlation (CCC) model (Bollerslev, 1990), extended by Engle (2002) with the so-called Dynamic Conditional Correlation (DCC) model. The latter framework has generated numerous variations and empirical applications. In the same spirit, Tse and Tsui (2002) considered weighted averages of several correlation matrices directly. Switching regime versions of the previous models have been proposed by Pelletier (2006), Billio and Caporin (2005) and Lee (2010).

In this stream of the literature, the analysis of the comovements between volatilities and correlations appeared as a kind of "new frontier". As written in the influential paper of Longin and Solnik (2001), "the conclusion that international correlation is much higher in periods of volatile markets (large absolute returns) has indeed become part of the accepted wisdom among practitioners and the financial press". Indeed, when markets are stressed, it has been observed by practitioners that (instantaneous or realized) volatilities and correlations tend to increase, even if volatilities tend to react quicker than correlations[1]. Several papers have tried to solve this volatility/correlation puzzle, particularly inside the framework of DCC models that have become the benchmarks in this stream of the literature. The usual two-step approach above splits the treatment of volatility and correlation dynamics, the former influencing the latter only through standardized cross-returns. Even practical, one of the dangers of this methodology is to undervalue the regimes of high/low market volatility, or even the individual volatility patterns, to explain correlation moves. This was partly the intuition of the DCC-MIDAS model (Colacito, Engle, and Ghysels, 2011), where the long-term correlation matrix may depend on realized volatilities, or other macro-economic variables. Recently, Bauwens and Otranto (2016) have added a VIX component, an aggregated measure of the "market volatility", to Engle (2002) DCC model. Through a binary threshold structure in a DCC model, Kasch and Caporin (2013) have shown the relevance of individual volatilities to explain correlation dynamics. Following the same intuition as the latter authors, we propose several multivariate DCC-type models that seek to measure to what extent individual volatilities may explain asset correlations, with a leverage effect possibly. Actually, the same idea could be applied replacing these volatilities by some probabilities of being in a high/low volatility regime, as given by an underlying Markov-switching model. Indeed, intuitively and for any asset, a high probability of being in a stressed state often means belonging to a

[1] This may be a consequence of the existence of very liquid vanilla option markets, when multi-name derivatives are typically traded less frequently.

high volatility regime. Hereafter, we will check if the probabilities of being in high/low volatility regimes influence the conditional correlations directly.

In section 2, we introduce our specifications of new DCC-type models. The updating equations of correlations now include some additional terms that depend on individual instantaneous volatilities (approximated by moving averages of squared returns) and current probabilities of a high regime, possibly with asymmetries. Section 3 presents our database of stock returns, composed of twenty-three country indices (MSCI Developed Markets), and we provide in-sample estimation results. Section 4 examines the out-of-sample performances of these models using two test procedures and evaluate historical portfolio performances. Our main findings are: (i) our new specifications allow to capture explicitly the dependence between the return correlations and the likelihood of high/low regimes, asymmetries and volatility levels; (ii) some of these models are preferred to the classical DCC model as they provide better (in sample) fits and predictive ability. Finally, Section 5 provides some concluding comments and outlines areas for future research.

2. NEW DCC-TYPE MODELS BASED ON INDIVIDUAL VOLATILITIES AND/OR SWITCHING REGIME PROBABILITIES

2.1. *The framework*

Let $(r_t)_{t=1,\dots,T}$ be a series of $(n \times 1)$ vectors of asset returns. To remove their conditional means, we have filtered this time series out through a simple ARMA(1,1) process [2]. Let H_t be the covariance matrix of r_t conditionally on its past values: $r_t|\Omega_{t-1} \sim H_t^{1/2} z_t$, where Ω_{t-1} represents the information set (filtration induced by the observations) at time $t-1$, i.e. $\Omega_{t-1} = \sigma(r_{t-l}, l \geq 1)$ and $(z_t)_{t\geqslant 1}$ is an i.i.d. white noise: $\mathbb{E}(z_t) = 0$ and $Var(z_t) = I_n$. Note that we do not assume normally distributed conditional returns, even if we will rely on a Gaussian QMLE, for inference purpose. We decompose the conditional covariance matrix H_t as $H_t := D_t R_t D_t$, where D_t is the $n \times n$ diagonal matrix of the time-varying conditional standard deviations $\sqrt{h_{i,t}}$, $i = 1, \dots, n$, that will be obtained from univariate GARCH-type models: $h_{i,t} = \mathbb{E}[r_{i,t}^2|\Omega_{t-1}]$. And $R_t := [\rho_{ij,t}]$ is the $n \times n$ time-varying conditional correlation matrix of r_t.

As noticed by Engle (2002), the Gaussian QMLE allows to rewrite the total loglikelihood as the sum of two terms:

(i) the first one is the sum of log-likelihoods associated to the individual asset return series.
(ii) the second term is related to the correlation dynamics, i.e. allows the estimation of the (R_t) process, once the (D_t) processes have been fitted.

This feature allows to independently specify the volatility dynamics on one side, and the correlation dynamics on the other side. Moreover, a two-step inference procedure is now feasible and greatly simplifies numerical optimizations. We will work inside this usual "DCC-type" two-step way of working, but the novelty of our approach is related to the second stage (new time-varying correlation processes).

For the moment, let us assume as given an underlying model for very univariate series

[2] Even if richer models could be proposed at this stage, the modeling of the asset returns means is not the main focus of this paper. Therefore, we consider an usual ARMA(1,1) should yield a fair job. Alternatively, the trimmed means methodology of Krause and Paolella (2014) may be invoked when asset returns exhibit strong fat tails, what is not really the case here.

$(r_{i,t})_{t=1,\dots,T}$, $i = 1,\dots,n$. Moreover, these models induce time-varying individual variances $(h_{i,t})_{t=1,\dots,T}$ and time-varying probabilities of being in a "high volatility" regime $(p_{i,t})_{t=1,\dots,T}$. Typically, the former ones are outputs of GARCH-type models and the latter ones may be deduced from a two-state Markov switching model, but it is not necessary to be more specific at this stage. Given such time-varying quantities, we are now able to propose new correlation dynamics.

2.2. *Specifications of conditional correlation models*

The assumption that conditional correlations are constant in time seems to be denied in many empirical studies (see Longin and Solnik (1995) for instance). That has motivated Engle (2002) to propose the so-called "dynamic conditional correlation" models, that we recall now. Typically, in the usual DCC scalar form, the conditional correlation process (R_t) satisfies

(1) $$R_t = (Q_t^*)^{-1/2} Q_t (Q_t^*)^{-1/2},$$

(2) $$Q_t = (1-\theta_1-\theta_2)S + \theta_1 \varepsilon_{t-1}\varepsilon'_{t-1} + \theta_2 Q_{t-1},$$

(3) $$Q_t^* = diag(q_{11,t}, \dots, q_{nn,t}),$$

where $\varepsilon_{i,t} = \dfrac{r_{i,t}}{\sqrt{h_{i,t}}}$ denotes the i-th standardized residual, S is an unknown positive definite matrix, θ_1 and θ_2 are unknown scalars and $Q_t = [q_{ij,t}]$ is an intermediate variance-covariance matrix. To ensure that Q_t is positive definite (and hence that R_t is really a correlation matrix), it is assumed that $\theta_1, \theta_2 \geq 0$, $\theta_1 + \theta_2 < 1$. Engle (2002) has suggested to estimate the matrix S by the empirical covariance matrix of the standardized residuals $\varepsilon_{i,t}$. Recently, Aielli (2013) has shown that this rule generates inconsistent estimates of S, because $S = \mathbb{E}[\varepsilon_t\varepsilon'_t]$ if and only if $\mathbb{E}[\varepsilon_t\varepsilon'_t] = \mathbb{E}[Q_t]$. Apart from the case of constant conditional correlation model ($\theta_1 = \theta_2 = 0$), this argument is not valid because $\mathbb{E}[\varepsilon_t\varepsilon'_t] = \mathbb{E}[\mathbb{E}_{t-1}[\varepsilon_t\varepsilon'_t]] = \mathbb{E}[R_t] = \mathbb{E}[Q_t^{*-1/2} Q_t Q_t^{*-1/2}] \neq \mathbb{E}[Q_t]$. To restore consistency, Aielli (2013) introduced the cDCC model (*corrected* DCC) which replaces the previous correlation driving process by

(4) $$Q_t = (1-\theta_1-\theta_2)S + \theta_1\{Q_{t-1}^{*1/2}\varepsilon_{t-1}\varepsilon'_{t-1}Q_{t-1}^{*1/2}\} + \theta_2 Q_{t-1},$$

where S is now the covariance matrix of $\varepsilon_t^* = Q_t^{*1/2}\varepsilon_t$, that can be estimated by "variance-targeting". From now on, we keep the cDCC specification.

Now, we extend the previous specification by introducing additional "individual" effects to explain the dynamics of correlations. These effects will come as additive terms in Q_t's updating equations. The natural candidates are the individual probabilities of being in high/low volatility regimes, as deduced from the basic Markov-switching models (see Eq. (17)). The time-varying conditional variances $h_{i,t}$ obtained from the MN-GARCH model of Haas, Mittnik, and Paolella (2004b) are candidates too, and they are already used to standardize the asset returns. But we have found it is more relevant empirically to introduce realized individual volatilities instead. Some of these variables will integrate asymmetries, to capture a leverage effects: Equation (2) will be replaced by

(5) $$Q_t = (1-\theta_1-\theta_2)S + \theta_1\varepsilon^*_{t-1}\varepsilon^{*\prime}_{t-1} + \theta_2 Q_{t-1} + g(\eta_{t-1}\eta'_{t-1} - N),$$

where $N = E[\eta_t \eta_t']$ is typically estimated by $\sum_{t=1}^{T} \eta_t \eta_t'/T$. The addition of $N = E[\eta_t \eta_t']$ in Equation (5) is relatively standard in the literature (see the AG-DCC model of Cappiello, Engle, and Sheppard (2006), for instance). By centering the "matrix of covariates" $\eta_{t-1}\eta_{t-1}'$, simple interpretations of the g values are allowed. Moreover, by taking the unconditional expectation of (5) and invoking the same arguments as Aielli (2013), we get $E[\varepsilon_{t-1}^* \varepsilon_{t-1}^{*\prime}] = E[Q_t]$ and

$$E[Q_t] = (1 - \theta_1 - \theta_2)S + \theta_1 E[\varepsilon_{t-1}^* \varepsilon_{t-1}^{*\prime}] + \theta_2 E[Q_t] + 0.$$

This implies $S = E[\varepsilon_{t-1}^* \varepsilon_{t-1}^{*\prime}]$, as in the usual cDCC.

The new variables η_t are defined as:

(6) **Specification 1 (cDCC-ASYM 1)** : $\eta_{i,t} = \mathbf{1}[\varepsilon_{i,t}^* < 0] \cdot \varepsilon_{i,t}^*,$

(7) **Specification 2 (cDCC-ASYM 2)** : $\eta_{i,t} = \mathbf{1}[\varepsilon_{i,t}^* > 0] \cdot \varepsilon_{i,t}^*,$

(8) **Specification 3 (cDCC-VRI 1)** : $\eta_{i,t} = \mathbf{1}[p_{i,t}^H > 0.5] \cdot \varepsilon_{i,t}^*,$

(9) **Specification 4 (cDCC-VRI 2)** : $\eta_{i,t} = \mathbf{1}[1 - p_{i,t}^H > 0.5] \cdot \varepsilon_{i,t}^*,$

(10) **Specification 5 (cDCC-PRI)** : $\eta_{i,t} = p_{i,t}^H,$

(11) **Specification 6 (cDCC-RVE 1)** : $\eta_{i,t} = \dfrac{\sigma_{i,t}^{RV}(p)}{\bar{\sigma}_i^{RV}(p)},$

(12) **Specification 7 (cDCC-RVE 2)** : $\eta_{i,t} = \mathbf{1}[p_{i,t}^H > 0.5] \cdot \dfrac{\sigma_{i,t}^{RV}(p)}{\bar{\sigma}_i^{RV}(p)},$

(13) **Specification 8 (cDCC-ARE 1)** : $\eta_{i,t} = \dfrac{|r_{i,t}|}{\bar{r}_i},$

(14) **Specification 9 (cDCC-ARE 2)** : $\eta_{i,t} = \mathbf{1}[p_{i,t}^H > 0.5] \cdot \dfrac{|r_{i,t}|}{\bar{r}_i},$

where $p_{i,t}^H = Pr(s_{i,t} = 2|\Omega_{t-1})$ represents the conditional probability that the asset i lies in the high volatility regime at date t. For this asset, $\sigma_{i,t}^{RV}(p) = \sqrt{\dfrac{\sum_{s=1}^{p} r_{i,t-s}^2}{p}}$ is a "rolling-window" estimated standard deviation and $\bar{\sigma}_i^{RV}(p) = \sum_{t=1}^{T} \sigma_{i,t}^{RV}(p)/T$ estimates the unconditional mean of $\sigma_{i,t}^{RV}(p)$. To get reactive assets volatilities, we have chosen $p = 20$ as this period corresponds to one month of daily data, even if we could consider p as an additional tuning parameter [3]. Moreover, $|r_{i,t}|$ denotes the absolute t-value of the i'th return and $\bar{r}_i = \sum_{t=1}^{T} |r_{i,t}|/T$ estimates of the unconditional mean of $|r_{i,t}|$. To ensure that Q_t is positive definite, we impose the constraints $g \geq 0$ and $\theta_1 + \theta_2 + \delta g < 1$ where δ denotes the maximum eigenvalue of $S^{-1/2}NS^{-1/2}$, assuming S is invertible [4].

The introduction of $N = E[\eta_t \eta']$ and the imposed positive definiteness of $(1 - \theta_1 - \theta_2)S - gN$ are model restrictions, compared to the usual DCC (or cDCC). For instance, model specifications as $Q_t = S + g(\eta_{t-1}\eta_{t-1}' - N)$ are not allowed in our framework

[3]Other choices of p as $p = 10$ and/or $p = 30$ have provided comparable results.

[4]Since $Q_t = \{(1 - \theta_1 - \theta_2)S - gN\} + \theta_1 \varepsilon_{t-1}^* \varepsilon_{t-1}^{*\prime} + \theta_2 Q_{t-1} + g\eta_{t-1}\eta_{t-1}'$, the positive definiteness of Q_t is obtained if $(1 - \theta_1 - \theta_2)S - gN$ is positive definite. The latter condition is equivalent to the positive definiteness of $(1 - \theta_1 - \theta_2)I - gS^{-1/2}NS^{-1/2}$, i.e. the positiveness of all its eigenvalues.

when the norm of S is "small", or when δg is larger than one. Nonetheless, in practice S is far from negligible and g is small. Therefore, in our opinion, such a reduction of the scope of possible models is minor and mainly of theoretical interest.

The first specification (6) corresponds to the *asymmetric DCC* model of Cappiello, Engle, and Sheppard (2006) with the correction of Aielli (2013). Moreover, we have considered the symmetrical model (6), replacing negative return shocks by positive ones in the new additive term. We call them *cDCC with asymmetric shocks* (**cDCC-ASYM**). Concerning the other cDCC-type models, we follow our initial intuition: high individual volatility levels and/or high probabilities of being in a "stressed" regime should have a direct impact on the correlations between any couple of asset returns. Therefore, in Equation (8) (resp. (9)), an extra correlation bump is induced on the (i, j)-th correlation if both of these assets belong to a high (resp. low) volatility regime: *cDCC with volatility regime impact* (**cDCC-VRI**). Instead of indicator functions, one can argue that continuous transitions may be more relevant. This is the idea of Equations (10) where the levels of the individual transition probabilities are considered directly: *cDCC with probability regime impact* (**cDCC-PRI**) [5]. In the case of *cDCC with realized volatility effect* (**cDCC-RVE**), the correlation between two assets will be bumped upwards when their realized volatilities will be high relative to their long term level $\bar{\sigma}_i$, possibly when both of them are in a stressed regime: see Equations (11) and (12). The models *cDCC with absolute return effect* (**cDCC-ARE**) (see (13) and (14)) share the same intuitions, but dealing with instantaneous absolute returns instead of realized volatilities. The term $\eta_{i,t}$ in Equation (13) differs from $\varepsilon_{i,t}$. When the latter is a (standardized) return, the former is comparable to a (standardized) volatility, but estimated on the shortest possible time interval, one day in our case (when p days are necessary to define $\sigma_{i,t}^{RV}(p)$).

A lot of alternative specifications could be evaluated, following the same ideas. For instance, volatilities in (11) could be replaced by the instantaneous volatilities $h_{i,t}$ themselves, possibly through a threshold structure (Kasch and Caporin, 2013). In Bauwens and Otranto (2016) (DCC-AVE), they are the same for all the names and are equal to the current VIX index. They could be replaced even by other measures of asset volatilities based on high/low intraday/intra week range data of asset prices: see Garman and Klass (1980), Rogers, Satchell, and Yoon (1994). An idea would be to introduce leverage effects through some volatility threshold. Nonetheless, we have found that working with $p_{i,t}$ instead was simpler (no need for normalization) and provides better results. Actually, after many attempts, we have kept only the nine models above because they convey the main ideas, intuitions and convincing empirical results. Nonetheless, it is likely there is some space for other models of the type $\eta_{i,t} = \psi_i(p_{i,t}^H, \varepsilon_{i,t}^*, \sigma_{i,t}^{RV}(p), h_{i,t}, ...)$, with some (non-linear) functions ψ_i.

These additional impacts on correlations will be positive by assumption ($g \geq 0$). It makes sense with homogeneous portfolios, but it is questionable in the case of anti-correlated asset returns, as it may happen in diversified portfolios. Therefore, these specifications should be modified to generate stronger negative correlations during periods of stress. To be specific, assume that the underlying assets can be classified into two subsets A and B: under a typical period of market stress, all the couples of assets inside A or

[5] Note the difference between our cDCC-PRI model and the DCC-ARE model of Bauwens and Otranto (2016). There, a unique scalar probability is used instead our $\eta_{t-1}\eta_{t-1}'$ matrices, inducing the same jumps for all the components of the Q_t matrix (a rather strong assumption).

inside B are positively correlated. Conversely, the couples (i, j), $i \in A$ and $j \in B$, are negatively correlated. For instance, this is the case with sub-portfolios of stocks (A) or bonds (B) (through their yields). Then, the new models apply after the multiplication of $\eta_{i,t}$ by an indicator function $I_i = \mathbf{1}(i \in A)$ in Equations (10), (11), (12), (13) and (14).

Like Engle (2002) and Tse and Tsui (2002), we have estimated all these specifications above using a two-stage procedure. In a first stage, we fit independently a univariate GARCH-type model for each asset return series. Therefore, we get the conditional standard deviations $\sqrt{h_{i,t}}$ and we standardize r_{it}. In a second stage, we use these standardized residuals to estimate the parameters that govern the dynamics of correlations, following the methodology of the cDCC estimator proposed by Aielli (2013). The parameters are calibrated by quasi-maximum likelihood estimates (QMLE), whose theory is outlined in Engle and Sheppard (2001).

2.3. *The individual asset dynamics: some guidelines*

Now, let us provide some guidelines to obtain convenient time-varying variances and switching probabilities. To induce some well known stylized facts like volatility clustering, leverage effects, regime shifts of our asset return dynamics, several families of conditional variance models are available in the academic literature. Since the seminal paper of Bollerslev (1986), GARCH-type models are still the most commonly used ones in empirical studies, through many variations. Beside the usual $GARCH(p, q)$, a lot of alternatives exist: see Francq and Zakoian (2011), for instance. Recently, the Markov-switching GARCH (MS-GARCH) models of Gray (1996), Klaassen (2002) and Haas, Mittnik, and Paolella (2004a) have also gained in popularity. They enable to capture leptokurtic, skewed and multimodal characteristics of assets return distributions. Moreover, the volatility shift persistence, as measured by usual GARCH models, may be due to shocks/jumps or parameter changes. Then, the omission of switching parameters can cause biased estimates of the persistence parameters, that can be reflected on volatility forecast, especially during a high volatility period: see Hamilton and Susmel (1994), Cai (1994), e.g.

The three Markov-switching models cited above provide some ways of estimating the conditional variances $h_{i,t}$ and the probabilities of being in high/low volatility regimes together. Nonetheless, the volatility regimes empirically obtained with these models are often not sufficiently persistent. When applied to our financial asset returns, the time spent in the"high-regime" seems to be too short, typically: the series of instantaneous probabilities show a too spiky behavior, with a lot of "one or two days" high regimes.

Beside the MS-GARCH models we have previously discussed, the MixN-GARCH-LOG and MixN-GARCH-LIK models proposed by Haas, Krause, Paolella, and Steude (2013) may be cited. Nonetheless, the former specification links instantaneous probabilities and the last squared return by a rather strong parametric assumption. And the inference of latter model relies on EM algorithms, making the practical task significantly more complex.

Mixture-normal GARCH (MN-GARCH) models often yield simple and nice alternatives. In their simplest version, it is assumed that the series $(r_{i,t})_t$ follow the two-component MN-GARCH model of Haas, Mittnik, and Paolella (2004b) or Alexander and Lazar (2006): for any asset i,

$$r_{i,t}|\Omega_{t-1} \sim N(\mu_{i,k}, h_{i,k,t}), \text{ with probability } p_{i,k}, \;\; k = 1, 2, \tag{15}$$

where $p_{i,1} + p_{i,2} = 1$ and $p_{i,1}\mu_{i,1} + p_{i,2}\mu_{i,2} = 0$. The time-varying volatility dynamics of each mixture component is a GARCH(1,1) process are

$$(16) \qquad h_{i,k,t} = \omega_{i,k} + \alpha_{i,k} r_{i,t-1}^2 + \beta_{i,k} h_{i,k,t-1}, \quad k = 1, 2,$$

where $\omega_{i,k} > 0$, $\alpha_{i,k}$ and $\beta_{i,k} \geq 0$. A necessary and sufficient condition for the existence of the unconditional variance is $\sum_{k=1}^{2} \frac{p_k}{1-\beta_k}\alpha_k < 1$ (see (Wirjanto and Xu, 2013)). The conditional variance $h_{i,t}$ is simply $p_{i,1}h_{i,1,t} + p_{i,2}h_{i,2,t}$. It has been noticed that MN-GARCH models tend to provide better *out-of-sample* forecasts when compared with MS-GARCH models: see Alexander and Lazar (2006), Paolella and Taschini (2008).

The simplest way of obtaining time-varying switching probabilities is probably to refer to the usual Markov-switching model of Hamilton (1989), with two regimes. In this "MS-Basic" model, each asset return follows the process

$$(17) \qquad r_{i,t} = \mu_{i,s_t} + \sigma_{i,s_t}\epsilon_{i,t}, \quad i = 1, \ldots, n; \; t = 1, \ldots, T,$$

where $(\epsilon_{i,t})_t$ is an i.i.d. white noise. The parameters μ_{i,s_t} and σ_{i,s_t} switch across different regimes (or *states* of the world) according to a hidden Markov chain (s_t) of order one, with a constant transition probability matrix. Denote $p_{ij} = P(s_t = j | s_{t-1} = i)$, $i, j = 1, 2$. The state 1 is called the "low volatility" regime and the state 2 is the "high volatility" regime. Since the latter is less persistent than the former, we expect $p_{12} < p_{21}$ and $\sigma_1 < \sigma_2$. As the state variable is unobservable, the regime probabilities $p_{i,t}^H$ of the MS-Basic model may be estimated by using the filter of Hamilton (1989). Figure 1 highlights the fact that a MS-Basic model provides greater regime persistence than a MS-GARCH model, especially during stressed periods. This empirical feature is inline with intuition and what has been observed by financial markets practitioners.

3. EMPIRICAL RESULTS

We consider a dataset of daily log-returns related to the twenty-three country indices that compose the MSCI Developed Markets universe. These MSCI series are free float-adjusted market capitalization weighted indices, that are designed to measure the equity market performance of developed markets. All time series were downloaded from Bloomberg in total return and expressed in US dollars. The sample spans the period of time between January 1999 and August 2013, yielding more than 3670 daily observations.

First, we estimate the basic regime switching model, to obtain the $p_{i,t}^H$ probabilities: see Table VII. We ordered the regimes with respect to the declining long run probability, that is $\pi_1 = Pr(s_t = 1) > \pi_2 = Pr(s_t = 2)$ conventionally. In the low volatility regimes, all time series have positive unconditional means and small conditional volatilities (in average), whereas in high volatility states, they have negative unconditional means and large conditional volatilities ($\mu_1 > 0, \mu_2 < 0, \sigma_1 < \sigma_2$). The average time spent in the low (resp. high) volatility regime, estimated by $1/(1 - p_{ii})$, $i = 1, 2$, stays within the interval $[38; 142]$ (resp. $[10; 62]$) days. Therefore, the MS-Basic is able to provide persistent low/high regime probabilities $Pr(s_t = i|\Omega_{t-1})$, at least with this dataset and contrary to other MS-GARCH models. That is why we have decided to use a MS-Basic model to get the low/high regime probabilities, rather than a MS-GARCH model [6].

Second, we fit a two-component MN-GARCH(1,1) model for every individual return series, assuming normal distribution (QMLE). The estimation results are reported in Table I. We have used Bollerslev and Wooldridge (1992) robust standard errors to estimate the t-stats. In line with the literature (Haas and Paolella (2012), Wirjanto and Xu (2013) for instance), we found that: (i) the small variances in the first component $\hat{\sigma}_1^2$ tend to be accompanied by positive means μ_1 and larger values of the mixing weight parameter p; (ii) the larger variances in the second component $\hat{\sigma}_2^2$ tend to be accompanied by negative means μ_2 and lower value of $1 - p$. This highlights the fact that MN-GARCH models are able to distinguish clearly two intuitive volatility regimes among MCSI developed markets indices: usual market conditions versus abnormal market conditions, with higher volatilities and stock price decreases in average. The impact of a shock on volatilities is higher during a high volatility state than during a low volatility state ($\alpha_2 > \alpha_1$), but this impact generally dies out soon since it displays a shorter persistence ($\beta_2 < \beta_1$). This indicates a tendency to overreact wrt news in abnormal market conditions.

It may be seen strange to use two different non-nested models to obtain instantaneous volatilities on one side and instantaneous transition probabilities on the other side. In an ideal world, it would be better to exhibit an encompassing multivariate "meta-model" that would provide all these quantities and correlations dynamics together and consistently. Actually, the candidates (see Subsection 2.3) have not provided convincing empirical results concerning our new correlation dynamics (very few significant coefficients g). Therefore, we have chosen simple and standard univariate models (MN-GARCH and MS-Basic) rather than complex univariate dynamics. Such models have to be seen as instrumentary, providing us "reasonable" instantaneous volatilities and transition probabilities, the main objective of the paper being correlation dynamics. In other words, the sequences (p_{it}) are covariables, to study conditional correlation dynamics. Since they cannot be observed, they will be replaced by simple and (hopefully) robust proxies.

Third, we use the resulting conditional variances to calculate the standardized residuals $\varepsilon_{i,t}$, and we evaluate the "correlation-part" of the models by Gaussian QMLE. For any of the nine models we introduced above, the parameter estimates, the likelihood, AIC criteria and their respective ranking are provided in Table II. For all alternative cDCC-type models, the g coefficients are statistically significant. Our new specifications are associated with lower AIC than the usual cDCC and than the versions that do not involve individual volatilities and/or probabilities. This justifies our intuition a posteriori. Interestingly, by mixing both types of information, we get better results. Indeed, the asymmetrical versions of our models provide a better fit than the symmetrical versions: compare **cDCC-RVE 2** (resp. **cDCC-ARE 2**) with **cDCC-RVE 1** (resp. **cDCC-ARE 1**).

We also found that the specifications with asymmetric positive or negative shocks (**cDCC-ASYM**, **cDCC-VRI**) provide both statistically significant g coefficients. This means the correlations between equity asset returns tend to increase when most basket returns are (strongly) positive or negative, respectively. The latter case has been observed in the literature (see Cappiello, Engle, and Sheppard (2006) for instance): in a stressed global market,

[6]Ex post, this choice was the right one. Indeed, working with the instantaneous probabilities provided by a MS-GARCH model has empirically provided mostly insignificant parameters, for the additional parameters that involved those probabilities: the cDCC-VRI 1, cDCC-VRI 2, cDCC-PRI, cDCC-RVE 2 and cDCC-ARE 2 specifications.

correlations tend to strengthen. But the former effect seems to be new, to the best of our knowledge. This rather strong effect of increasing correlations under positive asymmetric return shocks cannot be explained easily with economic arguments. This looks like a type of mimicry effect, when markets are frankly bullish.

All of the new specifications introduced above provide a better fit than the usual cDCC model. Therefore, taking into account asymmetries, instantaneous volatility or information on current volatility regimes improves the modeling of correlation dynamics. In the next section, we try to evaluate to what extent such improvements provide valuable insights/forecasts w.r.t simpler and more usual approaches.

4. EVALUATION OF THE ALTERNATIVE CDCC-TYPE MODELS

Several methods and tests have been proposed in the literature, to evaluate the absolute and/or the relative performances of such Garch-type models. These evaluations are usually done in a direct or an indirect way : see Patton and Sheppard (2009) or Caporin and McAleer (2010), e.g. The direct evaluation method consists in testing formally whether one or a subset of the models provide better volatility forecasts of a portfolio. The indirect way is an out-of-sample analysis of the risk-return performances of some "optimal" portfolios, as generated by the different models.

In this study, we choose both ways of evaluating our competing correlation models. We focus on out-of-sample forecasts, one-period ahead between March 2008 and August 2013. We re-estimate our cDCC-type models monthly using a rolling window of nine years of daily data. We use the parameters' estimates to produce daily one-step-ahead variance forecasts from March 2008 to August 2013. These models are compared using the evaluation methods described below and the results are reported in Tables III–VI. For the sake of comparison, we also add the same results generated by the usual Exponentially Weighted Moving Average model[7] (EWMA) model introduced by RiskMetrics.

To perform a pairwise comparison of covariance matrix estimators, we employ the methodology proposed by Engle and Colacito (2006) and Clements, Doolan, Hurn, Becker, et al. (2009). To briefly summarize their approach, let us consider an investor who chooses the optimal portfolio weights associated to n securities in order to minimize the variance of his/her portfolio at any time t. The asset allocation problem is :

$$(18) \qquad \widehat{w}_t = \arg\min_{w_t} \; w_t' H_{t|t-1} w_t, \;\; \text{subject to } w_t'\iota = 1,$$

where w_t is the time t vector of portfolio weights, but chosen at time $t-1$, ι denotes a $n \times 1$ vector of ones and $H_{t|t-1}$ is the one-day forecasted conditional covariance matrix of the vector of asset returns. The solution of the latter problem is given by $w_t = H_{t|t-1}^{-1}\iota / \iota' H_{t|t-1}^{-1}\iota$. Engle and Colacito (2006) have shown that the conditional variance of every portfolio will not be smaller than the conditional variance of the portfolio that has been optimized under the true covariance matrix, whatever an assumed constant vector of expected returns. Broadly speaking, if one constructs optimal weights using two different sequences of covariance matrices $(H^i_{t|t-1})_t$ and $(H^j_{t|t-1})_t$, the strategy providing the smallest covariance will be the best strategy. See the discussion in Patton and Sheppard (2009)

[7] The EWMA model defines the covariance matrix process as $H_t = (1-\lambda)\varepsilon_{t-1}\varepsilon_{t-1}' + \lambda H_{t-1}$. Among practitioners, the parameter λ is generally equal to 0.94, when dealing with daily data.

too. To test the equality of two competing models, we consider a sequence of minimum variance portfolio weights based on each series of covariance matrices. Let i and j denote two different models. A convenient metric comparing their outputs can be based on the difference of the squared returns of the two portfolios, i.e. on

$$(19) \qquad u_{ij,t} = [(\widehat{w}_t^i)' r_t]^2 - [(\widehat{w}_t^j)' r_t]^2, \;\; i \neq j.$$

If the two models provide similar forecasts, the corresponding portfolio variances are not significantly different. Therefore, the null hypothesis of equal variance is simply $\mathcal{H}_0 : \mathbb{E}(u_{ij,t}) = 0$. This can be tested by using the Diebold and Mariano (2002) procedure, based on a least squares regression of $u_{ij,t}$ on a constant, and applying heteroskedasticity and autocorrelation consistent standard errors. If the mean of $u_{ij,t}$ is positive (resp. negative) then the forecasts given by the covariance matrices of model j (resp. i) are the best ones.

The empirical findings of the Diebold-Mariano (DM) test over the whole out-of-sample period (2008-2013), when applied to the global minimum variance portfolio (GMV), are presented in Table III. Defining the loss functions as above, and applying the DM test to pairwise alternative cDCC models, we found globally that: (i) we do not reject the null hypothesis of equal predictive accuracy between **cDCC**, **cDCC-ASYM** and **cDCC-VRI** models; (ii) the null hypothesis of equal predictive accuracy with the other models is rejected for **cDCC-RVE**, **cDCC-ARE** and **cDCC-PRI** (less strongly); (iii) when compared to cDCC-type models, the **EWMA** model has the poorest predictive ability. Broadly speaking, we found that most of the models that take into account individual volatilities and probabilities of being in high volatility regimes produce better forecasts than classical cDCC models. They are preferred to the asymmetric cDCC model too. These results can be explained by the fact that our new models take into account re-correlation phenomena when they occur in times of abnormal market behaviors. An analysis of the evolution of the maximum eigenvalue of the forecasted correlation matrices over the period 2008-2013 (available under request) confirms this explanation.

In order to shed some light on these results, we recomputed the DM test over the subsample period around Lehman's collapse (March 2008 - December 2009), characterized by very high volatilities in the equity markets: see Table IV. Globally, our previous findings are confirmed. Notably, the predictive accuracy of **cDCC-RVE**, **cDCC-ARE** is better than the other models, as expected, particularly for their asymmetrical versions. The poor performance of **cDCC-RVE1** could be explained by the fact the realized volatilities stay at high levels permanently during this period of time. Similarly, **cDCC-PRI** is less performing than previously, probably because the $p_{i,t}$ probabilities are very often close to one around this crisis episode.

To complete the picture, we applied this DM test on the covariance matrices forecasts generated between January 2010 and December 2011 (the European debt crisis), and between January 2012 and August 2013 (recovery, i.e. expansion cycles in most developed countries). The results are available under request. We get similar conclusions, even if the last period is less favorable to our new specifications.

However, the conclusions based on the Diebold-Mariano test are limited as they represent pairwise comparisons, so that it is not possible to ensure that an optimal test is clearly identified. To resolve this issue, Hansen, Lunde, and Nason (2003, 2011) proposed the Model Confidence Set (MCS) approach, which constitutes a testing framework for the null hypothesis of equivalence across subsets of models. The idea is to start with a full set of "candidate models" and then sequentially to trim the elements of this set, thereby reducing the number of viable models. To be specific, this approach performs an iterative selection procedure testing the null hypothesis of equal predicting ability among all models included in a set $\mathcal{M}$ (the starting set contains all models) under a given loss function. The null hypothesis of MCS is $\mathcal{H}_0 : \mathbb{E}(u_{ij,t}) = 0,\ i > j$, for all $i, j \in \mathcal{M}$. Following Hansen, Lunde, and Nason (2003), the null hypothesis can be tested by means of two statistics, namely :

$$(20) \qquad t_R = \max_{i,j\in\mathcal{M}} |\frac{\bar{u}_{ij}}{\sqrt{\widehat{Var}[\bar{u}_{ij}]}}|, \text{ and } t_{SQ} = \sum_{i,j\in M, j>i} \left(\frac{\bar{u}_{ij}}{\sqrt{\widehat{Var}[\bar{u}_{ij}]}}\right)^2,$$

where $\bar{u}_{ij} = \sum_{t=1}^{h} u_{ij,t}/h$ with h representing the number of one-period ahead forecasts, and $\widehat{Var}[\bar{u}_{ij}]$ is a bootstrap estimate of the variance of $\bar{u}_{ij}$. If the null hypothesis is rejected, the worst performing model is excluded from the set $\mathcal{M}$ using the following rule :

$$(21) \qquad j = \arg\max_{j\in\mathcal{M}} (\sum_{i\in M, i\neq j} \bar{u}_{ij})(\widehat{Var}[\sum_{i\in M, i\neq j} \bar{u}_{ij}])^{-1/2},$$

where both the variance and the rejection region are again determined by bootstrap.

Table V reports the MCS results, when applied to the loss function $u_{ij,t}$ over different out-of-sample forecast periods. We have used the t_{SQ} statistic to calculate the p-values that provide information about the included or excluded models at different confidence levels (5%, 10% and 20%). When a p-value is higher than the fixed confidence level, then the corresponding model is included in the MCS test of "statistically equivalent" models. The higher is the p-value, the better is the corresponding model in terms of forecast accuracy. For the period 2008-2013, we found that: (i) the EWMA model is always excluded whatever the considered confidence level and it represents the worst performing model; (ii) the cDCC, cDCC-ASYM and cDCC-VRI models are also excluded; (iii) the cDCC-PRI, cDCC-RVE and cDCC-ARE models are included whatever the confidence level we consider, and the cDCC-ARE 2 model is the best performing model globally. Looking at the results by sub-periods, the comparative advantage of cDCC-ARE specifications is strengthened when markets are stressed (Lehman's collapse or the Euro-debt crisis). At the opposite, during periods of recovery and growth (2012-2013), it becomes difficult to discriminate between all these models, except EWMA, that is always put aside. This is in line with what we found previously with the DM test.

To deal with the indirect evaluation of these models, we consider an asset allocation framework and compare the impact of model choice by contrasting the out-of-sample performances of optimal GMV portfolios. We report the results in Table VI. Whatever the considered sample period, the covariance forecasts from cDCC-RVE and cDCC-ARE family models provide optimal weights that generate the portfolios with the lowest volatilities. The EWMA model always produced the highest volatility portfolios, but with the highest returns in general (except during the 2008-2009 period where it suffered a lot).

5. CONCLUSION

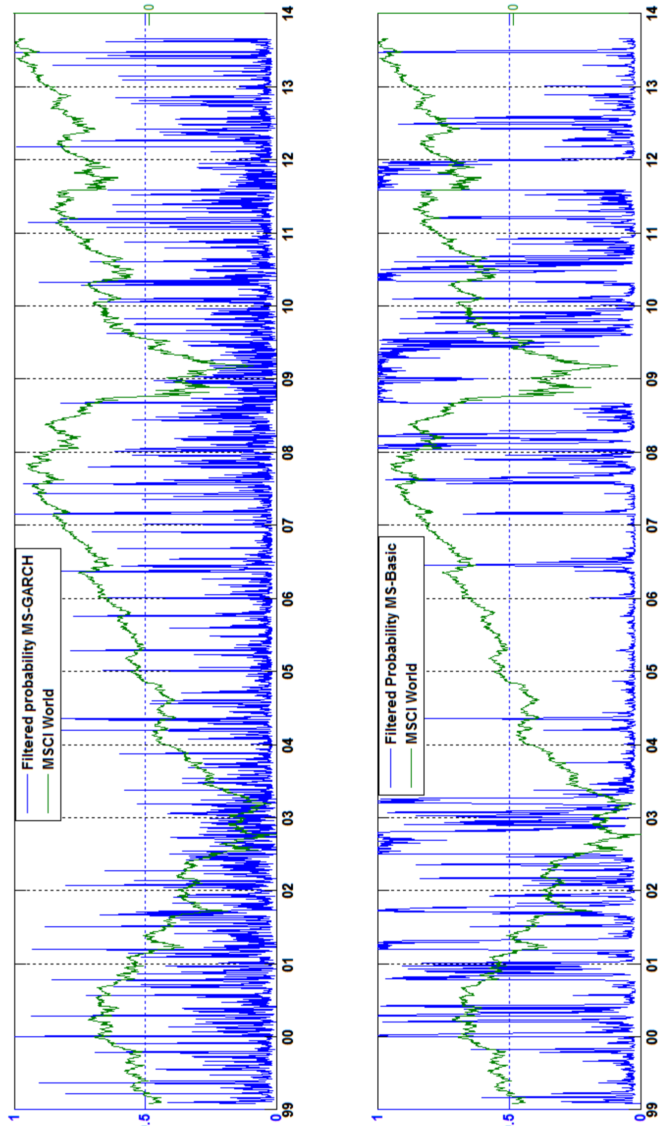
We have introduced new dynamical models of conditional correlations between asset returns. These specifications are extensions of the DCC model of Engle (2002) with the correction of Aielli (2013). They provide a way of taking into account individual volatility levels and switching regime probabilities to predict correlations. The relative forecasting performances of these models have been evaluated out-of-sample by the Diebold-Mariano and the Model Confidence Set test procedures, with a focus on global minimum variance portfolios.

The models that include the effects of individual volatilities and regimes additively are most often preferred to the EWMA, the standard DCC and even the asymmetric DCC, in terms of in-sample fit. They provide more accurate forecasts of minimum variance portfolios. This is particularly true when a leverage effect based on the $p^H_{i,t}$ is introduced. These new models are able to capture simultaneously the volatility clustering effect, fat tailed distributions and the re-correlation phenomenon that occurred in the equity market recently.

This paper opens the door towards many variations, extensions and empirical works. For instance, an interesting avenue for future research would be to reconsider our choice of scalar DCCs. Indeed, to capture different correlation effects per asset class in a diversified portfolio (different asset classes, sectorial/geographical discrepancies), the Flexible DCC approach proposed by Billio, Caporin, and Gobbo (2006) could be invoked. Moreover, it should be valuable to evaluate the impact of our additional correlation drivers on the long-term correlation matrix, possibly through the introduction of switching matrices S, and possibly in a multiplicative way. We could add some macro-economic variables, that may be relevant drivers of conditional correlations. And it would be possible to consider more than two switching regimes, for example those that correspond to “quiet”, ”normal” and ’stressed” markets. This would be at the cost of a more complex inference and (probably too) numerous possible model specifications, due to the increasing number of instantaneous transition probabilities.

Finally, we have chosen the DCC framework in this study because it is the most usual one, but other multivariate GARCH models exist and similar ideas could be extended there. For instance, the RSDC model of Pelletier (2006), the hyperspherical GAS model of Creal, Koopman, and Lucas (2011), the vine-GARCH model Poignard and Fermanian (2018), etc. Such extension are left for further studies.

Figure 1: **Filtered probabilities of a high volatility regime (MS-GARCH vs MS-Basic), 2008-2013**



Note: in-sample filtered probabilities $p_{i,t}^{H}$ of being in a high volatility regime (under the MS-GARCH and the MS-Basic models) and the MSCI World stock price, between January 1999 and August 2013.

TABLE I

MIXTURE-NORMAL GARCH ESTIMATES, 1999-2013

	μ_1	ω_1	α_1	β_1	μ_2	ω_2	α_2	β_2	p	$\hat{\sigma}^2$	$\hat{\sigma}_1^2$	$\hat{\sigma}_2^2$	log L
Australia	0.051^b	0.009	0.041^c	0.938^c	-0.242	0.310^b	0.377^c	0.758^c	0.827^a	1.387	1.191	2.066	-6108.287
Austria	0.040	0.006	0.021	0.955^c	-0.061	0.043^b	0.145^c	0.905^c	0.603^c	1.555	1.134	2.033	-6435.071
Belgium	0.038^a	0.011	0.062^a	0.911^c	-0.233	0.161	0.290	0.856^c	0.860^b	1.530	1.327	2.414	-6162.508
Canada	0.043	0.015^c	0.059^c	0.921^c	-0.438	0.004	0.118^b	0.948^c	0.911^c	1.300	1.204	1.980	-5970.901
Denmark	0.076^c	0.011	0.067^a	0.897^c	-0.226	0.070	0.095	0.929^c	0.749^a	1.394	1.174	1.887	-6100.533
Finland	0.022	0.010	0.038^c	0.950^c	-0.368	0.020	0.078	0.982^c	0.945^a	2.340	2.090	4.923	-7687.670
France	0.131	0.006	0.019	0.952^c	-0.103	0.034^b	0.125^b	0.902^c	0.880^c	1.536	1.035	1.830	-6416.298
Germany	0.080	0.003	0.022	0.953^c	-0.063	0.047	0.135^b	0.893^c	0.880^c	1.766	1.230	2.091	-6575.889
Greece	0.003	0.044^b	0.081^c	0.894^c	-0.078	0.000	0.347	0.937^c	0.958^c	3.172	2.843	7.440	-7522.692
Hong Kong	0.003	0.012^b	0.065^c	0.935^c	-0.044^c	0.000	0.000	0.132	0.946^c	1.752	0.027	1.801	-5851.208
Ireland	0.032	0.024^b	0.058^c	0.910^c	-0.176	0.080	0.207^a	0.906^c	0.846^c	1.732	1.479	2.725	-6675.182
Italy	0.051^a	0.010	0.039^c	0.934^c	-0.096^a	0.020	0.170^c	0.894^c	0.652^c	1.771	1.422	2.282	-6346.572
Japan	0.048	0.214	0.740^a	0.787^c	-0.003	0.034^c	0.050^c	0.922^c	0.949^b	1.473	1.352	2.921	-6270.829
Netherlands	0.098^c	0.008	0.030^b	0.940^c	-0.128	0.036^b	0.154^c	0.891^c	0.568^b	1.479	1.101	1.854	-6171.137
New Zealand	0.049	0.007	0.017	0.969^c	-0.174	0.078	0.192	0.893^c	0.780^b	1.403	1.143	2.069	-6157.225
Norway	0.095^b	0.007	0.038	0.937^c	-0.205	0.112	0.133^c	0.900^c	0.682^c	1.631	1.297	2.166	-6802.877
Portugal	0.013	0.009	0.039	0.940^c	-0.075	0.126	0.291	0.865^c	0.857^c	1.379	1.173	2.242	-5932.101
Singapore	0.053	0.005	0.028	0.945^c	-0.088	0.032^a	0.199	0.882^c	0.624^a	1.885	1.379	2.505	-5905.160
Spain	0.015	0.007	0.043	0.941^c	-0.076	0.192	0.316	0.821^c	0.840^a	1.607	1.414	2.375	-6607.961
Sweden	0.088	0.001	0.028	0.956^c	-0.196	0.139	0.169	0.880^c	0.690^c	1.798	1.448	2.388	-7140.297
Switzerland	0.014	0.015	0.027	0.941^c	-0.033	0.039	0.214^b	0.871^c	0.700^c	1.217	0.964	1.662	-5584.092
United Kingdom	0.100^c	0.001	0.107^c	0.889^c	-0.420	0.002	0.002	0.997^c	0.808^c	1.160	1.140	1.147	-5756.622
USA	0.176^c	0.000	0.026	0.899^c	-0.075	0.018^c	0.102^c	0.913^c	0.700^b	1.249	0.634	1.428	-5348.847

Note: parameter estimates of the Mixture-Normal GARCH model (see Eq. (15)-(16)) fitted on 23 MSCI Developed Markets stock indices, over the period January 1999 to August 2013. $\hat{\sigma}^2$ is the overall unconditional variance. $\hat{\sigma}_1^2$ and $\hat{\sigma}_2^2$ are the unconditional variance of each individual mixture component. ^{a}, ^{b} and ^{c}: Statistical significance at 10%, 5% and 1% respectively.

TABLE II

cDCC-MN-GARCH MODELS ESTIMATES, 1999-2013

	θ_1	θ_2	g	log L	AIC	R
cDCC	$6.082e-03^c$	0.991^c		-108004	59.279	10
cDCC-ASYM 1	$5.77e-03^c$	0.991^c	$8.40e-04^a$	-108002	59.278	9
cDCC-ASYM 2	$5.25e-03^c$	0.991^c	$2.64e-03^c$	-107992	59.273	8
cDCC-VRI 1	$\mathbf{5.78e-03^c}$	$\mathbf{0.990^c}$	$\mathbf{1.33e-03^c}$	**-107937**	**59.242**	**3**
cDCC-VRI 2	$5.70e-03^c$	0.991^c	$9.36e-04^c$	-107954	59.252	6
cDCC-PRI	$\mathbf{6.15e-03^c}$	$\mathbf{0.990^c}$	$\mathbf{1.79e-03^c}$	**-107942**	**59.245**	**4**
cDCC-RVE 1	$6.09e-03^c$	0.991^c	$2.53e-04^c$	-107986	59.270	7
cDCC-RVE 2	$\mathbf{5.91e-03^c}$	$\mathbf{0.990^c}$	$\mathbf{5.77e-04^c}$	**-107911**	**59.228**	**2**
cDCC-ARE 1	$6.00e-03^c$	0.990^c	$5.76e-04^c$	-107946	59.247	5
cDCC-ARE 2	$\mathbf{5.78e-03^c}$	$\mathbf{0.990^c}$	$\mathbf{6.29e-04^c}$	**-107884**	**59.213**	**1**

Note: estimated parameters of the cDCC models (Eq. (5)-(14)) fitted on 23 MSCI Developed Markets indices, between January 1999 and August 2013 (see Section 2.2 for the acronyms). AIC: Akaike information criteria. The "R" column provides the ranks of models according to the AIC criterion (the models in bold are the best ones). log L: value of the maximized log-likelihood function. ^{a}, ^{b} and ^{c}: statistical significance at 10% , 5% and 1% respectively.

TABLE III

DIEBOLD-MARIANO T-STATS FOR cDCC-MN-GARCH MODELS, 2008-2013

	cDCC	cDCC-ASYM		cDCC-VRI		cDCC-PRI	cDCC-RVE		cDCC-ARE		EWMA
		ASYM 1	ASYM 2	VRI 1	VRI 2		RVE 1	RVE 2	ARE 1	ARE 2	
cDCC		$\mathbf{-1.951^b}$	-0.787	0.046	$\mathbf{-1.519^a}$	$\mathbf{1.336^a}$	$\mathbf{2.527^c}$	$\mathbf{1.931^b}$	$\mathbf{2.570^c}$	$\mathbf{1.768^b}$	$\mathbf{-2.886^c}$
cDCC-ASYM 1	$\mathbf{1.951^b}$		-0.558	0.301	-1.213	$\mathbf{1.622^a}$	$\mathbf{2.947^c}$	$\mathbf{2.069^b}$	$\mathbf{2.788^c}$	$\mathbf{1.817^b}$	$\mathbf{-2.880^c}$
cDCC-ASYM 2	0.787	0.558		0.690	-0.258	$\mathbf{1.464^a}$	$\mathbf{1.643^b}$	$\mathbf{2.058^b}$	$\mathbf{2.380^c}$	$\mathbf{1.898^b}$	$\mathbf{-2.920^c}$
cDCC-VRI 1	-0.046	-0.301	-0.690		-0.916	$\mathbf{1.589^a}$	0.988	$\mathbf{2.987^c}$	$\mathbf{2.153^b}$	$\mathbf{2.069^b}$	$\mathbf{-2.879^c}$
cDCC-VRI 2	$\mathbf{1.519^a}$	1.213	0.258	0.916		$\mathbf{1.814^b}$	$\mathbf{2.470^c}$	$\mathbf{2.081^b}$	$\mathbf{2.549^c}$	$\mathbf{1.851^b}$	$\mathbf{-2.832^c}$
cDCC-PRI	$\mathbf{-1.336^a}$	$\mathbf{-1.622^a}$	$\mathbf{-1.464^a}$	$\mathbf{-1.589^a}$	$\mathbf{-1.814^b}$		-0.001	$\mathbf{1.887^b}$	$\mathbf{1.483^a}$	$\mathbf{1.731^b}$	$\mathbf{-2.906^c}$
cDCC-RVE 1	$\mathbf{-2.527^c}$	$\mathbf{-2.947^c}$	$\mathbf{-1.643^b}$	-0.988	$\mathbf{-2.470^c}$	0.001		$\mathbf{1.324^a}$	$\mathbf{1.807^b}$	$\mathbf{1.555^a}$	$\mathbf{-2.916^c}$
cDCC-RVE 2	$\mathbf{-1.931^b}$	$\mathbf{-2.069^b}$	$\mathbf{-2.058^b}$	$\mathbf{-2.987^c}$	$\mathbf{-2.081^b}$	$\mathbf{-1.887^b}$	$\mathbf{-1.324^a}$		-0.470	$\mathbf{1.570^a}$	$\mathbf{-2.950^c}$
cDCC-ARE 1	$\mathbf{-2.570^c}$	$\mathbf{-2.788^c}$	$\mathbf{-2.380^c}$	$\mathbf{-2.153^b}$	$\mathbf{-2.549^c}$	$\mathbf{-1.483^a}$	$\mathbf{-1.807^b}$	0.470		$\mathbf{1.340^a}$	$\mathbf{-2.955^c}$
cDCC-ARE 2	$\mathbf{-1.768^b}$	$\mathbf{-1.817^b}$	$\mathbf{-1.898^b}$	$\mathbf{-2.069^b}$	$\mathbf{-1.851^b}$	$\mathbf{-1.731^b}$	$\mathbf{-1.555^a}$	$\mathbf{-1.570^a}$	$\mathbf{-1.340^a}$		$\mathbf{-2.970^c}$
EWMA	$\mathbf{2.886^c}$	$\mathbf{2.880^c}$	$\mathbf{2.920^c}$	$\mathbf{2.879^c}$	$\mathbf{2.832^c}$	$\mathbf{2.906^c}$	$\mathbf{2.916^c}$	$\mathbf{2.950^c}$	$\mathbf{2.955^c}$	$\mathbf{2.970^c}$	

Note: this table reports the out-of-sample t-statistics of the Diebold-Mariano test that checks the equality between covariance matrix forecasts using the loss function $u_{ij,t}$ defined in (19), over the period March 2008 - August 2013. This loss function is constructed as the difference of squared realized returns of alternative cDCC models indicated in row i and column j. When the null hypothesis of equal predictive accuracy is rejected, a positive number is evidence in favour of the model in the column. ^{a}, ^{b} and ^{c} : rejection of the null hypothesis at 10%, 5% and 1% respectively.

TABLE IV

DIEBOLD-MARIANO T-STATS FOR cDCC-MN-GARCH MODELS, 2008-2009

	cDCC	cDCC-ASYM		cDCC-VRI		cDCC-PRI	cDCC-RVE		cDCC-ARE		EWMA
		ASYM 1	ASYM 2	VRI 1	VRI 2		RVE 1	RVE 2	ARE 1	ARE 2	
cDCC		$\mathbf{-1.952^b}$	-0.775	0.168	$\mathbf{-1.335^a}$	1.126	0.771	$\mathbf{1.724^b}$	$\mathbf{1.495^a}$	$\mathbf{1.856^b}$	$\mathbf{-2.336^c}$
cDCC-ASYM 1	$\mathbf{1.952^b}$		-0.756	0.193	$\mathbf{-1.312^a}$	1.167	0.849	$\mathbf{1.739^b}$	$\mathbf{1.521^a}$	$\mathbf{1.861^b}$	$\mathbf{-2.336^c}$
cDCC-ASYM 2	0.775	0.756		0.835	-0.190	$\mathbf{1.311^a}$	0.938	$\mathbf{1.965^b}$	$\mathbf{1.606^a}$	$\mathbf{2.061^b}$	$\mathbf{-2.358^c}$
cDCC-VRI 1	-0.168	-0.193	-0.835		-0.942	0.945	0.091	$\mathbf{2.504^c}$	1.187	$\mathbf{2.162^b}$	$\mathbf{-2.343^c}$
cDCC-VRI 2	$\mathbf{1.335^a}$	$\mathbf{1.312^a}$	0.190	0.942		$\mathbf{1.491^a}$	$\mathbf{1.507^a}$	$\mathbf{1.850^b}$	$\mathbf{1.737^b}$	$\mathbf{1.900^b}$	$\mathbf{-2.272^b}$
cDCC-PRI	-1.126	-1.167	$\mathbf{-1.311^a}$	-0.945	$\mathbf{-1.491^a}$		-0.751	$\mathbf{1.904^b}$	0.963	$\mathbf{1.912^b}$	$\mathbf{-2.354^c}$
cDCC-RVE 1	-0.771	-0.849	-0.938	-0.091	$\mathbf{-1.507^a}$	0.751		$\mathbf{1.646^b}$	$\mathbf{1.436^a}$	$\mathbf{1.813^b}$	$\mathbf{-2.344^c}$
cDCC-RVE 2	$\mathbf{-1.724^b}$	$\mathbf{-1.739^b}$	$\mathbf{-1.965^b}$	$\mathbf{-2.504^c}$	$\mathbf{-1.850^b}$	$\mathbf{-1.904^b}$	$\mathbf{-1.646^b}$		-1.005	$\mathbf{1.765^b}$	$\mathbf{-2.417^c}$
cDCC-ARE 1	$\mathbf{-1.495^a}$	$\mathbf{-1.521^a}$	$\mathbf{-1.606^a}$	-1.187	$\mathbf{-1.737^b}$	-0.963	$\mathbf{-1.436^a}$	1.005		$\mathbf{1.664^b}$	$\mathbf{-2.383^c}$
cDCC-ARE 2	$\mathbf{-1.856^b}$	$\mathbf{-1.861^b}$	$\mathbf{-2.061^b}$	$\mathbf{-2.162^b}$	$\mathbf{-1.900^b}$	$\mathbf{-1.912^b}$	$\mathbf{-1.813^b}$	$\mathbf{-1.765^b}$	$\mathbf{-1.664^b}$		$\mathbf{-2.520^c}$
EWMA	$\mathbf{2.336^c}$	$\mathbf{2.336^c}$	$\mathbf{2.358^c}$	$\mathbf{2.343^c}$	$\mathbf{2.272^b}$	$\mathbf{2.354^c}$	$\mathbf{2.344^c}$	$\mathbf{2.417^c}$	$\mathbf{2.383^c}$	$\mathbf{2.520^c}$	

Note: this table reports the out-of-sample t-statistics of the Diebold-Mariano test that checks the equality between covariance matrix forecasts using the loss function $u_{ij,t}$ defined in (19), over the period March 2008 - December 2009. This loss function is constructed as the difference of squared realized returns of alternative cDCC models indicated in row i and column j. When the null hypothesis of equal predictive accuracy is rejected, a positive number is evidence in favour of the model in the column. ^{a}, ^{b} and ^{c} : rejection of the null hypothesis at 10%, 5% and 1% respectively.

TABLE V

MODEL CONFIDENCE SET TEST FOR cDCC-MN-GARCH MODELS

(a) **MCS *p*-value, 2008-2013**

Confidence level	5%	10%	20%
cDCC	0.038	0.047	0.041
cDCC-ASYM 1	0.029	0.036	0.030
cDCC-ASYM 2	0.050	0.055	0.046
cDCC-VRI 1	0.031	0.040	0.034
cDCC-VRI 2	0.029	0.036	0.030
cDCC-PRI	**0.094**	**0.118**	0.102
cDCC-RVE 1	**0.103**	**0.130**	0.122
cDCC-RVE 2	**0.179**	**0.203**	0.190
cDCC-ARE 1	**0.179**	**0.203**	0.190
cDCC-ARE 2	**1.000**	**1.000**	**1.000**
EWMA	0.011	0.017	0.013

(c) **MCS *p*-value, 2010-2011**

Confidence level	5%	10%	20%
cDCC	0.049	0.054	0.044
cDCC-ASYM 1	0.036	0.045	0.037
cDCC-ASYM 2	0.036	0.045	0.037
cDCC-VRI 1	0.036	0.045	0.037
cDCC-VRI 2	0.036	0.045	0.037
cDCC-PRI	**0.090**	0.088	0.085
cDCC-RVE 1	**0.090**	0.088	0.085
cDCC-RVE 2	**0.090**	0.088	0.085
cDCC-ARE 1	**1.000**	**1.000**	**1.000**
cDCC-ARE 2	**0.253**	**0.270**	**0.264**
EWMA	0.004	0.003	0.001

(b) **MCS *p*-value, 2008-2009**

Confidence level	5%	10%	20%
cDCC	**0.092**	0.083	0.101
cDCC-ASYM 1	**0.096**	0.083	0.102
cDCC-ASYM 2	**0.092**	0.083	0.101
cDCC-VRI 1	**0.092**	0.083	0.101
cDCC-VRI 2	**0.092**	0.080	0.101
cDCC-PRI	**0.092**	0.080	0.101
cDCC-RVE 1	**0.101**	0.086	0.103
cDCC-RVE 2	**0.131**	**0.119**	0.122
cDCC-ARE 1	**0.131**	**0.119**	0.122
cDCC-ARE 2	**1.000**	**1.000**	**1.000**
EWMA	**0.051**	0.049	0.049

(d) **MCS *p*-value, 2012-2013**

Confidence level	5%	10%	20%
cDCC	**0.420**	**0.432**	**0.434**
cDCC-ASYM 1	**0.446**	**0.455**	**0.447**
cDCC-ASYM 2	**0.451**	**0.456**	**0.450**
cDCC-VRI 1	**0.235**	**0.250**	**0.240**
cDCC-VRI 2	**0.446**	**0.455**	**0.447**
cDCC-PRI	**1.000**	**1.000**	**1.000**
cDCC-RVE 1	**0.451**	**0.456**	**0.450**
cDCC-RVE 2	**0.777**	**0.776**	**0.796**
cDCC-ARE 1	**0.451**	**0.456**	**0.450**
cDCC-ARE 2	**0.451**	**0.456**	**0.450**
EWMA	0.000	0.000	0.001

Note: model confidence set (MCS) results using the loss function $u_{ij,t}$ defined in (19) and the t_{SQ} statistics at 5%, 10% and 20% confidence level. Bold values denote the models that are included in the confidence set (i.e they are statistically equivalent in terms of forecast accuracy), at the corresponding confidence level. In each column, the lowest *p*-value represents the first model removed and the highest represents the best performing model. Panels (a), (b), (c) and (d) provides the results for the period March 2008 to August 2013, March 2008 to December 2009, January 2010 to December 2011 and January 2012 to August 2013 respectively.

TABLE VI

GLOBAL MINIMUM VARIANCE PORTFOLIOS PERFORMANCES OF CDCC-MN-GARCH MODELS

(a) **GMV portfolios performances, 2008-2013**

	Mean	St. Dev	R
cDCC	1.896%	15.639%	7
cDCC-ASYM 1	1.844%	15.647%	8
cDCC-ASYM 2	2.067%	15.669%	9
cDCC-VRI 1	1.507%	15.638%	6
cDCC-VRI 2	1.912%	15.683%	10
cDCC-PRI	1.600%	15.603%	5
cDCC-RVE 1	**2.133%**	**15.602%**	**4**
cDCC-RVE 2	**1.489%**	**15.526%**	**2**
cDCC-ARE 1	**1.830%**	**15.550%**	**3**
cDCC-ARE 2	**1.333%**	**15.332%**	**1**
EWMA	3.496%	18.915%	11

(c) **GMV portfolios performances, 2010-2011**

	Mean	St. Dev	R
cDCC	2.571%	11.522%	6
cDCC-ASYM 1	2.418%	11.530%	9
cDCC-ASYM 2	2.818%	11.523%	7
cDCC-VRI 1	1.419%	11.529%	8
cDCC-VRI 2	2.103%	11.533%	10
cDCC-PRI	0.838%	11.486%	5
cDCC-RVE 1	**2.986%**	**11.456%**	**3**
cDCC-RVE 2	**1.527%**	**11.480%**	**4**
cDCC-ARE 1	**3.018%**	**11.385%**	**1**
cDCC-ARE 2	**1.325%**	**11.446%**	**2**
EWMA	5.103%	13.430%	11

(b) **GMV portfolios performances, 2008-2009**

	Mean	St. Dev	R
cDCC	-7.977%	22.855%	7
cDCC-ASYM 1	-7.971%	22.857%	8
cDCC-ASYM 2	-7.725%	22.921%	9
cDCC-VRI 1	-8.284%	22.845%	6
cDCC-VRI 2	-7.455%	22.945%	10
cDCC-PRI	**-8.286%**	**22.810%**	**4**
cDCC-RVE 1	-8.261%	22.839%	5
cDCC-RVE 2	**-8.660%**	**22.660%**	**2**
cDCC-ARE 1	**-9.160%**	**22.752%**	**3**
cDCC-ARE 2	**-8.915%**	**22.256%**	**1**
EWMA	-16.006%	27.865%	11

(d) **GMV portfolios performances, 2012-2013**

	Mean	St. Dev	R
cDCC	5.423%	12.999%	7
cDCC-ASYM 1	5.484%	12.994%	5
cDCC-ASYM 2	5.550%	13.003%	8
cDCC-VRI 1	5.352%	13.010%	9
cDCC-VRI 2	5.381%	12.996%	6
cDCC-PRI	4.855%	13.042%	10
cDCC-RVE 1	**6.115%**	**12.822%**	**2**
cDCC-RVE 2	**4.910%**	**12.941%**	**3**
cDCC-ARE 1	**6.056%**	**12.755%**	**1**
cDCC-ARE 2	**5.089%**	**12.945%**	**4**
EWMA	9.404%	15.921%	11

Note: performance measures of the unconstrained Global Minimum Variance (GMV) (see Eq. (18)) portfolios. Panels (a), (b), (c) and (d) provides the results for the period March 2008 to August 2013, March 2008 to December 2009, January 2010 to December 2011 and January 2012 to August 2013 respectively. The "R" columns provide the ranks of any model according to the minimum volatility criteria.

TABLE VII

BASIC MARKOV-SWITCHING MODEL ESTIMATES, 1999-2013

	μ_1	μ_2	σ_1	σ_2	p_{11}	p_{22}	π_1	π_2
Australia	0.042	-0.168	1.060	2.900	0.991	0.962	0.803	0.197
Austria	0.080	-0.245	1.095	3.239	0.986	0.954	0.761	0.239
Belgium	0.078	-0.184	0.997	2.447	0.984	0.961	0.711	0.289
Canada	0.043	-0.170	1.028	2.656	0.992	0.970	0.790	0.210
Denmark	0.064	-0.233	1.060	2.608	0.983	0.935	0.792	0.208
Finland	0.089	-0.136	1.396	3.416	0.989	0.983	0.606	0.394
France	0.086	-0.163	1.047	2.504	0.983	0.967	0.665	0.335
Germany	0.060	-0.157	1.177	2.753	0.991	0.974	0.734	0.266
Greece	0.104	-0.152	1.268	3.373	0.983	0.975	0.597	0.403
Hong Kong	0.016	-0.024	0.939	2.094	0.991	0.984	0.634	0.366
Ireland	0.068	-0.205	1.169	3.163	0.986	0.959	0.751	0.249
Italy	0.068	-0.133	1.037	2.645	0.988	0.975	0.684	0.316
Japan	0.026	-0.152	1.194	2.527	0.989	0.936	0.853	0.147
Netherlands	0.069	-0.192	1.040	2.646	0.988	0.963	0.751	0.249
New Zealand	0.062	-0.163	1.031	2.189	0.978	0.947	0.710	0.290
Norway	0.083	-0.310	1.266	3.496	0.989	0.959	0.793	0.207
Portugal	0.064	-0.235	0.991	2.479	0.974	0.904	0.786	0.214
Singapore	0.046	-0.096	0.927	2.143	0.982	0.963	0.670	0.330
Spain	0.052	-0.104	1.131	2.684	0.987	0.972	0.677	0.323
Sweden	0.103	-0.161	1.231	2.892	0.985	0.976	0.616	0.384
Switzerland	0.052	-0.243	0.950	2.377	0.993	0.966	0.834	0.166
United Kingdom	0.054	-0.200	0.983	2.530	0.991	0.965	0.795	0.205
USA	0.058	-0.125	0.804	1.997	0.989	0.977	0.672	0.328

Note: parameter estimates of the MS-Basic model (see Eq. (17)), fitted on the 23 MSCI Developed Markets indices over the period January 1999 to August 2013. μ_i and σ_i are the mean and the standard deviation in regime $i = 1, 2$. p_{11} and p_{22} are the diagonal elements of the transition matrix, and $\pi_1 = P(s_t = 1)$ and $\pi_2 = P(s_t = 2)$ are the unconditional regime probabilities.

REFERENCES

AIELLI, G. P. (2013): "Dynamic conditional correlation: on properties and estimation," *Journal of Business & Economic Statistics*, 31(3), 282–299.

ALEXANDER, C., AND E. LAZAR (2006): "Normal mixture GARCH (1, 1): Applications to exchange rate modelling," *Journal of Applied Econometrics*, 21(3), 307–336.

ASAI, M., M. MCALEER, AND J. YU (2006): "Multivariate stochastic volatility: a review," *Econometric Reviews*, 25(2-3), 145–175.

BAUWENS, L., S. LAURENT, AND J. V. ROMBOUTS (2006): "Multivariate GARCH models: a survey," *Journal of applied econometrics*, 21(1), 79–109.

BAUWENS, L., AND E. OTRANTO (2016): "Modeling the dependence of conditional correlations on market volatility," *Journal of Business & Economic Statistics*, 34(2), 254–268.

BILLIO, M., AND M. CAPORIN (2005): "Multivariate Markov switching dynamic conditional correlation GARCH representations for contagion analysis," *Statistical methods and applications*, 14(2), 145–161.

BILLIO, M., M. CAPORIN, AND M. GOBBO (2006): "Flexible dynamic conditional correlation multivariate garch models for asset allocation," *Applied Financial Economics Letters*, 2(02), 123–130.

BOLLERSLEV, T. (1986): "Generalized autoregressive conditional heteroskedasticity," *Journal of econometrics*, 31(3), 307–327.

——— (1990): "Modelling the coherence in short-run nominal exchange rates: a multivariate generalized ARCH model," *The review of economics and statistics*, pp. 498–505.

BOLLERSLEV, T., R. F. ENGLE, AND J. M. WOOLDRIDGE (1988): "A capital asset pricing model with time-varying covariances," *Journal of political Economy*, 96(1), 116–131.

BOLLERSLEV, T., AND J. M. WOOLDRIDGE (1992): "Quasi-maximum likelihood estimation and inference in dynamic models with time-varying covariances," *Econometric reviews*, 11(2), 143–172.

CAI, J. (1994): "A Markov model of switching-regime ARCH," *Journal of Business & Economic Statistics*, 12(3), 309–316.

CAPORIN, M., AND M. MCALEER (2010): "Ranking multivariate GARCH models by problem dimension," .

CAPPIELLO, L., R. F. ENGLE, AND K. SHEPPARD (2006): "Asymmetric dynamics in the correlations of global equity and bond returns," *Journal of Financial econometrics*, 4(4), 537–572.

CLEMENTS, A., M. DOOLAN, S. HURN, R. BECKER, ET AL. (2009): "Evaluating multivariate volatility forecasts," Discussion paper, National Centre for Econometric Research.

COLACITO, R., R. F. ENGLE, AND E. GHYSELS (2011): "A component model for dynamic correlations," *Journal of Econometrics*, 164(1), 45–59.

CREAL, D., S. J. KOOPMAN, AND A. LUCAS (2011): "A dynamic multivariate heavy-tailed model for time-varying volatilities and correlations," *Journal of Business & Economic Statistics*, 29(4), 552–563.

DIEBOLD, F. X., AND R. S. MARIANO (2002): "Comparing predictive accuracy," *Journal of Business & economic statistics*, 20(1), 134–144.

ENGLE, R. (2002): "Dynamic conditional correlation: A simple class of multivariate generalized autoregressive conditional heteroskedasticity models," *Journal of Business & Economic Statistics*, 20(3), 339–350.

ENGLE, R., AND R. COLACITO (2006): "Testing and valuing dynamic correlations for asset allocation," *Journal of Business & Economic Statistics*, 24(2), 238–253.

ENGLE, R. F., AND K. F. KRONER (1995): "Multivariate simultaneous generalized ARCH," *Econometric theory*, 11(1), 122–150.

ENGLE, R. F., AND K. SHEPPARD (2001): "Theoretical and empirical properties of dynamic conditional correlation multivariate GARCH," Discussion paper, National Bureau of Economic Research.

FRANCQ, C., AND J.-M. ZAKOIAN (2011): *GARCH models: structure, statistical inference and financial applications*. John Wiley & Sons.

GARMAN, M. B., AND M. J. KLASS (1980): "On the estimation of security price volatilities from historical data," *Journal of business*, pp. 67–78.

GOURIÉROUX, C., J. JASIAK, AND R. SUFANA (2009): "The Wishart autoregressive process of multivariate stochastic volatility," *Journal of Econometrics*, 150(2), 167–181.

GRAY, S. F. (1996): "Modeling the conditional distribution of interest rates as a regime-switching process," *Journal of Financial Economics*, 42(1), 27–62.

HAAS, M., J. KRAUSE, M. S. PAOLELLA, AND S. C. STEUDE (2013): "Time-varying mixture GARCH models and asymmetric volatility," *The North American Journal of Economics and Finance*, 26, 602–

623.

HAAS, M., S. MITTNIK, AND M. S. PAOLELLA (2004a): "Mixed normal conditional heteroskedasticity," *Journal of Financial Econometrics*, 2(2), 211–250.

——— (2004b): "A new approach to Markov-switching GARCH models," *Journal of Financial Econometrics*, 2(4), 493–530.

HAAS, M., AND M. S. PAOLELLA (2012): "Mixture and regime-switching GARCH models," pp. 71–102.

HAMILTON, J. D. (1989): "A new approach to the economic analysis of nonstationary time series and the business cycle," *Econometrica: Journal of the Econometric Society*, pp. 357–384.

HAMILTON, J. D., AND R. SUSMEL (1994): "Autoregressive conditional heteroskedasticity and changes in regime," *Journal of econometrics*, 64(1-2), 307–333.

HANSEN, P. R., A. LUNDE, AND J. M. NASON (2003): "Choosing the best volatility models: the model confidence set approach," *Oxford Bulletin of Economics and Statistics*, 65, 839–861.

HANSEN, P. R., A. LUNDE, AND J. M. NASON (2011): "The model confidence set," *Econometrica*, 79(2), 453–497.

HARVEY, A., E. RUIZ, AND N. SHEPHARD (1994): "Multivariate stochastic variance models," *The Review of Economic Studies*, 61(2), 247–264.

KASCH, M., AND M. CAPORIN (2013): "Volatility threshold dynamic conditional correlations: An international analysis," *Journal of Financial Econometrics*, 11(4), 706–742.

KLAASSEN, F. (2002): "Improving GARCH volatility forecasts with regime-switching GARCH," in *Advances in Markov-Switching Models*, pp. 223–254. Springer.

KRAUSE, J., AND M. S. PAOLELLA (2014): "A fast, accurate method for value-at-risk and expected shortfall," *Econometrics*, 2(2), 98–122.

LEE, H.-T. (2010): "Regime switching correlation hedging," *Journal of Banking & Finance*, 34(11), 2728–2741.

LONGIN, F., AND B. SOLNIK (1995): "Is the correlation in international equity returns constant: 1960–1990?," *Journal of international money and finance*, 14(1), 3–26.

——— (2001): "Extreme correlation of international equity markets," *The journal of finance*, 56(2), 649–676.

PAOLELLA, M. S., AND L. TASCHINI (2008): "An econometric analysis of emission allowance prices," *Journal of Banking & Finance*, 32(10), 2022–2032.

PATTON, A. J., AND K. SHEPPARD (2009): "Evaluating volatility and correlation forecasts," in *Handbook of financial time series*, pp. 801–838. Springer.

PELLETIER, D. (2006): "Regime switching for dynamic correlations," *Journal of econometrics*, 131(1-2), 445–473.

POIGNARD, B., AND J.-D. FERMANIAN (2018): "Dynamic Asset Correlations Based on Vines," *Econometric Theory*, pp. 1–31.

ROGERS, L. C., S. E. SATCHELL, AND Y. YOON (1994): "Estimating the volatility of stock prices: a comparison of methods that use high and low prices," *Applied Financial Economics*, 4(3), 241–247.

TSE, Y. K., AND A. K. C. TSUI (2002): "A multivariate generalized autoregressive conditional heteroscedasticity model with time-varying correlations," *Journal of Business & Economic Statistics*, 20(3), 351–362.

WIRJANTO, T., AND D. XU (2013): "A Mixture-of-Normal Distribution Modeling Approach in Financial Econometrics: A Selected Review," *SSRN*, 2242925.