Transparency Matters: Price Formation in Presence of Order Preferencing

Abstract

Using a market making inventory model, we analyze the impact of order preferencing on dealers’ quoting behavior by changing the degree of quote disclosure. We find that preferred orders raise the inventory holding costs of preferred dealers, making them less able to post attractive quotes. In turn, competitors choose less aggressive prices, but still attract more likely public orders. Price competition is smoothed and expected market spreads widen. Promoting competition might be, however, enforced by (i) fine tuning through the degree of market transparency; (ii) favoring the entry of unpreferred dealers, or (iii) requiring preferred market-makers to have more funding capital.

Keywords: Preferencing arrangements, Inventory management, Transparency, Bid-ask spread.

JEL Classification: D43, L21
1 Introduction

Preferencing and one of its various forms, internalization, refer to practices that allow brokers and institutional investors to have prearranged transactions with a preferred dealer — a “quote-matcher” — avoiding search costs of the best price. In the equity and options-listed markets, the volume of orders which are preferred is economically sizeable. In the U.S., 20-30% of NASDAQ order flow, and 20-40% of the retail options orders are subject to preferencing arrangements.\(^1\) Currently, these practices - pervasive in the U.S. and the U.K. - are expected to extend worldwide. In Europe, the “Markets in Financial Instruments Directive” (MiFID) allows internalization. In Australia, AXE, an ECN enabling internalization and preferencing, is currently awaiting the licence to trade.

Historically, preferencing arrangements constituted the market-makers’ response to increased competition for order flow. Market-makers bypass direct price-competition by offering economic inducements to brokers, who, in return, agree to route their order flow to them.\(^2\) Not surprisingly, these methods have raised concerns. First, these practices yield to order flow fragmentation, which is known to mechanically enlarge spreads. Second, these routing techniques are based on quote-matching, which is predicted to yield to anti-competitive outcomes (Salop, 1986). In addition, quote-matching is suspected to discourage market-makers to post aggressive prices since price-competition is not rewarded. Finally, at the regulatory level, there are divergences regarding the transparency level to adopt: MiFID requires to “make public current bid and offer prices and the depth of trading interests at these prices” for liquid securities. In contrast, the AXE ECN does not require any pre-trade exposure.

The first contribution of this paper is to analyze the impact of quote disclosure on the quoting behavior of dealers in a market in which trades are prearranged. Second, our paper attempts to contribute to the literature by studying theoretically the impact on market quality of preferencing, which is complex. Empirical evidence on whether these practices are deleterious for market competition is mixed at best. On one hand, Klock and McCormick (2002) find that, even under preferencing arrangements, NASDAQ deal-

\(^1\) Before decimalization, 60-80% of NASDAQ order flow, and 75% of the retail options orders were estimated to be preferred.

\(^2\) The economic inducements may take the form of direct cash payment (the so-called payment for order flow), or indirect services (soft-dollar arrangements).
ers have still incentives to be at the inside, just as the London Stock Exchange dealers (Hansch et al., 1999), or as specialists in NYSE-listed stocks (Bessembinder, 2003). On the other hand, Chung et al. (2004) show that preferencing discourages quote competition and yields to wider spreads. In addition, even if preferred or internalized trades generate rents for dealers (otherwise they would have stopped these practices), it might make inventory management riskier for them. Under preferencing, dealers must provide liquidity, regardless of their own inventory position. Posting unaggressive quotes does not ensure dealers that they will not execute the next incoming order. Controlling inventory through quotes may, thus, be more complex, as suggested by Reiss and Werner (1998) or Kandel and Marx (1999).

In light of these issues, we analyze the impact of preferencing on price formation in a market in which two dealers differ by their inventory position and by their preferencing arrangements. A part of the order flow sent by risk-neutral liquidity demanders is preferenced off-market to one dealer (the quote-matcher), while the other part is public and routed to the best-quoting dealer. Competition to be at the inside to attract public orders is, thus, maintained among dealers.

We first cast our analysis in a transparent market in which dealers’ quotes are publicly displayed. We find that the commitment to execute preferenced trades at the best price increases the cost of providing liquidity for the preferenced dealer. Posting attractive quotes is then less feasible for her. Quotes chosen by that dealer are, thus, less competitive than in the case in which the preferenced order flow would be routed to the primary market. In a transparent market, the quoting behavior of the preferenced dealer is perfectly anticipated by the unpreferenced competitor who, in turn, quotes less aggressively while retaining, in average, a greater chance to win the public order flow. All dealers have less incentives to offer competitive execution. As a result, preferencing softens the price competition and widens market spreads to the detriment of risk-neutral investors.

We then turn to the analysis of an opaque market à la Biais (1993), in which deal-

---

3Several empirical studies show that inventory control through price quotes adjustment plays an important role in the price formation of dealers markets and even of hybrid, and limit order markets. For instance, Declerck (2007) analyzes inventory management of limit order traders using Euronext data. See also, among others, Hansch et al. (1998) for London Stock Exchange data; Lyons (1995) or Bjønnes and Rime (2005) for foreign exchange markets. For NYSE specialist data, see, e.g., Madhavan and Smitd (1993), Madhavan and Sofianos (1998), Hendershott and Seasholes (2007), or Panayidès (2007).
ers cannot observe the quotes of the competitors and must execute prearranged trades at the expected best market price. Despite the simplicity of the economic problem, the equilibrium pricing strategies of dealers are complex. The selling quotes correspond to those arising in an asymmetric first price auction. We completely characterize the Pareto dominant equilibrium for dealers. Then, we adopt a numerical approach to investigate dealers’ quoting behavior for the cases in which we cannot find analytical solutions. We find that, in an opaque market as well, preferencing generally discourages aggressive quoting strategies. However, in some cases, the preferred dealer quotes very aggressively, even more than in the benchmark case.

Dealers’ price competitiveness is, thus, not unambiguously enforced by a higher degree of transparency. In particular, when the volume of preferred orders is large, the preferred dealer can no longer post competitive quotes, which is perfectly observed by the unpreferred dealer in a transparent market. In that case, the fully pre-trade transparency helps that dealer to quote even less aggressive prices since he is less uncertain about the quotes chosen by the rival. As a result, expected market spreads are wider than those prevailing in the opaque market. Our results concerning the ambiguous role of transparency support the experimental findings of Kluger and Wyatt (2002), who find that market transparency may be used for implicit collusion in a multi-dealer market with preferencing. Our results also suggest that implementing identical rules for pre-trade transparency across platforms may not be optimal, depending on the volume of prearranged trades.

Finally, we show that the harmful effect of order preferencing might be alleviated by promoting competition among dealers. First, the degree of market transparency plays a role by enforcing less implicit collusion and more price aggressiveness. Second, we show that requiring preferred market-makers to have more funding capital is also a way to introduce more competition. Finally we also confirm that favoring the entry of multiple unpreferred dealers enhances price competitiveness, in accord with the experimental result of Bloomfield and O’Hara (1998).

The novel finding of our paper is to show that preferencing has a negative effect on the cost of providing liquidity. The increase in the inventory risk discourages aggressive quoting strategies, thereby impeding transaction costs. Does quote disclosure matter
when orders may be prearranged off-market? Our paper suggests it can, by enforcing more price aggressiveness among dealers. The degree of pre-trade transparency to adopt is, however, sensitive to the magnitude of preferencing.

Several theoretical papers examine the effects of order preferencing. None of them focus, however, on the role played by market transparency. In an infinitely repeated game, Dutta and Madhavan (1997) study preferencing and show that it facilitates sustainable collusive equilibria at which market-makers earn positive profits. Using also an infinite-period game, Parlour and Rajan (2003) study a model with competition between market-makers, brokers, and payment for order flow, often associated with preferencing.\footnote{However, as Parlour and Rajan (2003) point out, they do not model preferencing itself. In their model, all market-makers are symmetric and all pay equally brokers for order flow.} They find that payment for order flow raises transaction costs for market order submitters. In equilibria, market-making spreads are also higher. Our paper complements these findings since we get that, even in a one-shot game, preferencing causes higher spreads, showing, thus, that observable outcomes may be contracted on.

This paper is organized as follows. Section 2 describes the model and introduces the benchmark. Section 3 investigates the price formation in a transparent market in which dealers may be preferenced. Section 4 analyzes the quoting strategies of those dealers when quotes are not publicly displayed. Section 5 discusses two factors mitigating the effects of preferencing. Section 6 outlines the regulatory implications of our results and concludes. Main proofs are in the Appendix.

2 The Model

2.1 The Basic Setting

Consider the market for a risky asset, whose final cash flow is a normal random variable $\tilde{\nu}$ characterized by an expected value $\mu$ and a variance $\sigma^2_{\tilde{\nu}}$. There are two types of market participants: (i) investors who demand liquidity and (ii) dealers who supply liquidity. Liquidity is demanded by investors who are affected by liquidity shocks. These liquidity traders send market orders to any brokers. Liquidity is supplied by risk-averse dealers
who stand ready to execute incoming market orders at their bid or ask quote against their own inventory.

**Dealers’ reservation price and inventory holding cost.** For ease of exposition, we suppose that liquidity traders are affected by a negative shock and send a buy order flow to brokers. We therefore focus on the sell side of the market and on the behavior of two strategic dealers who compete to post the lowest selling price (or ask price) so as to execute an incoming buy order flow. Dealers 1 and 2 have the following common CARA utility function:

$$u(w) = -\exp(-\rho w)$$

where $\rho$ is the risk aversion, and $w$ the terminal wealth of a dealer.

Dealers differ in their (random) inventory position of the risky asset that we denote $\tilde{I}_i$. Accordingly, dealers have heterogeneous reservation selling prices, defined by equating utilities $EU(q, r_i) = EU(0, r_i)$, where $U(q, r_i)$ indicates that dealer in position $I_i$ trades $q$ shares at price $r_i$. It follows that:

$$r_i(q) \equiv r(q; I_i) = \mu + \frac{\rho \sigma^2}{2} (q - 2I_i), i = 1, 2$$

where $q$ is the incoming buy order to accommodate, and $I_i$ is dealer $i$’s initial inventory. $I_i$ is a realization of the random variable $\tilde{I}_i$ uniformly distributed on $[I_d, I_u]$.

Note that the reservation price is a one-to-one decreasing function of the inventory position, which yields two remarks. First, the longer the inventory position, the lower the ask quote. Second, we can, equivalently, consider that the reservation price $r_i(q)$ is a uniform random variable defined on $[r_u(q), r_d(q)]$. All random variables are independent and their distributions are common knowledge. Note that, in our setting, there is no asymmetric information on either the value of the risky asset or the inventory position of dealers.

---

5 This result follows Ho and Stoll’s (1983) Proposition 1.

6 The uniform distribution has been adopted for tractability reasons. Besides, to our knowledge, there exists no empirical paper that characterizes the stationary empirical distribution of dealers’ inventory that could help us to specify another distribution. In the veil of ignorance the uniform distribution plays a central role, especially in Bayesian statistics, and in the experimental literature (see, for instance, the paper of Lamoureux and Schnitzlein (2004) that suppose that agents’ inventory positions are drawn from a uniform distribution). In addition, the uniform distribution has bounded support, which fits real dealers’ inventory position.

7 Order preferencing is often used to cream-skim uninformed orders, as several empirical studies point out (see, e.g., Easley et al., 1996 or Chung et al., 2006). In the present paper, we do not deal with such
Given that future price movements are uncertain, accumulating inventory increases risks that entail costs for which that dealer has to be compensated. Thus, the reservation price may be interpreted as the cost to provide liquidity. The dealer must post ask quotes strictly above the reservation price in order to profitably trade.

**Fragmentation of the order flow.** We suppose now that part of the buy order flow routed by brokers to dealers is preferenced to dealer 2. In contrast dealer 1 has no preferencing arrangements.\(^8\) In particular, the model separates: (i) the preferenced order flow, denoted \(\kappa > 0\), which is pre-assigned to dealer 2, and (ii) the remaining unpreferenced part of the order flow, denoted \(Q > 0\), which is exposed to the whole market. This public order flow \(Q\) is accommodated by the best-quoting dealer (dealer 1 or 2). The lowest ask price, also termed best offer, is defined by \(a \equiv \min (a_1, a_2)\) where \(a_1\) (resp. \(a_2\)) is the ask price posted by dealer 1 (resp. dealer 2).

**Obligation of execution and preferencing.** We suppose that dealer 2 has guaranteed her affiliated brokers to execute the preferenced order flow \(\kappa\) at the best offer determined in the primary market, irrespective of her quotes. (Time priority is supposed not to be enforced as, for instance, on NASDAQ). In practice, under preferencing dealer 2 is not obliged to take preferenced orders. However, she must still pay her retail broker for receiving this captive order flow.\(^9\) Moreover, by refusing preferred orders, dealer 2 entails the risk to lose the business relationship with the affiliated broker. As the 2001 NASDAQ report points out, preferred dealers “rarely act in an agency capacity”. In our paper, we do not model the business relationship between the broker and the dealer, we assume that the potential cost for the dealer to act as an agent is higher than the cost to act as a principal. Consequently, under preferencing, dealer 2 commits to execute

---

\(^8\)Our model focuses on an asymmetric allocation of preferenced orders among dealers in the same vein as Dutta and Madhavan’s (1997) hypothesis, or Bloomfield and O’Hara’s (1998) set up. First, it allows to analyze the behavior of unpreferenced dealers who post the best price and who are not rewarded with increased order flow. Their incentives to be competitive are indeed an issue. Second, it fits stylized facts since not all dealers are preferenced in the equity or options-listed markets. For instance, in the NASDAQ, He et al. (2006) report that only 3 dealers - Madoff, Knight and Schwab - have a sizeable portion of prearranged trades.

\(^9\)Payment is usually variable, depending on spread, size, and the type of security; e.g., payments for the 4th Quarter of 2007 from UBS Capital Markets L.P. to Ridge Clearing, Inc were less than $0.0001 per share for NASDAQ-listed stocks, and $0.16 per option-listed contract (extracted from the SEC rule 606 Report). Still, some other non-cash compensations (as research services) exist.
preferred orders by matching the best price before trading, and does not re-route orders when her inventory position makes preferred transactions undesirable. This hypothesis is also adopted by Bloomfield and O'Hara (1998), or Kandel and Marx (1997).

**No trade-through rule.** Dealer 2 guarantees to execute preferred orders at the best prevailing market price (i.e., the best offer in our model) even when she does not quote it.

**The timing of the game and dealers’ payoffs.** At date 1 dealer \( i \) is endowed with an initial inventory position \( I_i \). At date 2 liquidity traders send the order flow \( Q+\kappa \) to brokers. Part of it, \( \kappa \), is preferred to dealer 2. Dealer 1 knows that dealer 2 has committed to accommodate the preferred transaction \( \kappa \). At date 3 dealers post simultaneously their ask quotes in order to win the public trade \( Q \). The dealer who posts the lowest ask price executes \( Q \). Besides dealer 2 executes the transaction \( \kappa \) at the best price whatever her own quote.\(^{10}\)

As in Biais (1993), the utility function of dealers given in (1) is linearized.\(^{11}\) Optimal prices depend however on risk aversion since reservation prices depend on risk aversion. Then, dealer 1’s trading profit is simply given by:

\[
\pi_1 (a_1, a_2) \equiv \pi (a_1, a_2; I_1) = \begin{cases} 
(a_1 - r_1 (Q)) Q & \text{if } a_1 < a_2 \\
0 & \text{if } a_1 > a_2
\end{cases}
\]  

(3)

Dealer 2’s trading profit differs from dealer 1’s. When she posts the best market price \( (a_2 < a_1) \), she attracts the public order \( Q \). In addition, she has to accommodate the preferred order flow \( \kappa \) whether she is the best-quoting dealer or not. Naturally the cost to produce liquidity is different whether she posts the best price or not. Indeed, her

\(^{10}\)Note that the transaction price of the buy order flow \( \kappa \) results from the outcome of the price competition on the public buy order flow \( Q \). Thus, \( Q \) and \( \kappa \) have necessarily the same direction in our model.

\(^{11}\)For tractability reasons, the direct effect of risk aversion on preferences is removed by using the first order linear approximation proposed by Biais (1993) and used by Rhodes-Kropf (2005), or Yin (2005). See de Frutos and Manzano (2002) for an analysis of the impact of risk-aversion in the inventory model of Biais (1993).
trading profit is given by:

$$\pi_2 (a_2, a_1) \equiv \pi (a_2, a_1; I_2) = \begin{cases} (a_2 - r_2 (Q + \kappa)) (Q + \kappa) & \text{if } a_2 < a_1 \\ (a_1 - r_2 (\kappa)) \kappa & \text{if } a_2 > a_1 \end{cases}$$  

(4) 

First, observe that the preferred dealer might incur losses whenever she has to match a best price lower than her own cost to produce liquidity ($a_1 < r_2 (\kappa)$), which is consistent with the remark of Kandel and Marx (1999): “ [...] under preferred arrangements, a dealer has less control over the trades she has to accommodate because she cannot withdraw from the market by adjusting quotes”. Then, note that the preferred dealer faces a tradeoff: either she chooses to accommodate the unique captive order flow, either she posts an aggressive price to attract the public order but she faces a higher inventory-holding cost ($r_2 (Q + \kappa) > r_2 (\kappa)$). Let us introduce a specific reservation price, $k_2$, termed as the cutoff price below which dealer 2 is not willing to trade the public order $Q$, in addition of the captive order flow $\kappa$.

**Definition 1** Let $k_2$ be the value of the ask price at which the preferred dealer is indifferent between trading $\kappa$ or trading $(Q + \kappa)$, i.e. $EU(\kappa, k_2 (Q + \kappa)) = EU(Q + \kappa, k_2 (Q + \kappa))$. That cutoff price $k_2$ writes

$$k_2 (Q + \kappa) \equiv k (Q + \kappa; I_2) = \mu + \rho \sigma_v^2 (Q - 2I_2)/2 + \rho \sigma_v^2 \kappa.$$

The preferred dealer has no incentive to quote below the cutoff price, $k_2 (Q + \kappa)$. That fee compensates her for the impact of the public transaction, $Q$, and for the additional inventory risk of trading the required preferred order flow $\kappa$. This cutoff price is the effective reservation price of dealer 2.\(^{12}\) We can indeed re-write dealer 2’s trading profit function as follows:

$$\pi_2 (a_2, a_1) = \frac{\rho \sigma_v^2}{2} \kappa (Q + \kappa) + \begin{cases} (a_2 - k_2 (Q + \kappa)) (Q + \kappa) & \text{if } a_2 < a_1, \\ (a - k_2 (Q + \kappa)) \kappa & \text{if } a_2 > a_1 = a. \end{cases}$$  

(5)

\(^{12}\)In the remaining part of the paper we use the term cutoff price or reservation price for dealer 2 interchangeably.
Note that

\[ k_2(Q + \kappa) > r_2(Q + \kappa) > r_2(\kappa) \], for each \( \kappa \).

The ranking is consistent with the monopolistic situation of the preferred dealer facing the (inelastic) demand \( \kappa \). The reservation price \( k_2(Q + \kappa) \), which is the lowest ask price at which dealer 2 would like to trade \( Q \), is strictly greater than the costs to produce liquidity whenever she supplies liquidity only for \( \kappa \) or for the total order flow \( (Q + \kappa) \).

In our setting the preferred dealer commits to execute the preferred order flow \( \kappa \) before trading, regardless of her quotes. Thus at the reception time of \( \kappa \), dealer 2 knows that the transaction \( \kappa \) will necessarily shorten her inventory position \( I_2 \). She raises her reservation price to reflect the adjustment of her effective position \( (I_2 - \kappa) \). Accordingly, we can rewrite the reservation price of dealer 2 as

\[ k_2(Q + \kappa) = \mu + \rho \sigma^2_v (Q - 2(I_2 - \kappa))/2 = r(Q; I_2 - \kappa). \]

In sum, preferencing leads to higher inventory costs for preferred dealers, which is consistent with the empirical results of Menyah and Paudyal (2000). The authors argue that the inventory holding cost component is larger on the London Stock Exchange (23% of the quoted bid-ask spread) due to the presence of order preferencing; in addition they find that this cost becomes substantially higher in illiquid stocks (27% of the quoted bid-ask spread) since positions cannot be unwound quickly and order preferencing is riskier.

### 2.2 The Benchmark: Market-Making Without Preferencing

To examine how preferencing influences dealer behavior, we first need to analyze a market in which preferencing is not authorized. We refer to that competitive market as the benchmark case, in which orders must be consolidated and cannot be preferred. Thus, the total quantity to accommodate by the best-quoting dealer is \( (Q + \kappa) \). Accordingly, dealers’ reservation prices are equal to \( r_i(Q + \kappa) \) \( (i = 1, 2) \), and are independently and uniformly distributed on \([r_u(Q + \kappa), r_d(Q + \kappa)]\). Dealers’ trading profit therefore writes as follows:

\[
\pi_i^c(a_i, a_{-i}) = \begin{cases} 
(a_i - r_i(Q + \kappa))(Q + \kappa) & \text{if } a_i < a_{-i} \\
0 & \text{if } a_i > a_{-i}.
\end{cases}
\] (6)
Observe that the distribution of reservation prices and the trading profit function $\pi_i^e$ are the same for all dealers. This situation involves symmetric dealers. Therefore, we focus on symmetric equilibria. Then, equilibrium pricing strategies differ across market structures. Fully transparent markets, in which dealers observe competitors’ quotes (e.g., on a centralized structure, such as the NYSE), were first studied by Ho and Stoll (1983). In contrast, Biais (1993) deals with optimal pricing strategies in opaque markets (e.g., fragmented markets, such as the NASDAQ or the bond market), in which dealers cannot observe competitors’ quotes.

**Proposition 1** (i) (Ho and Stoll, 1983). In a transparent market, optimal quotes are strategically equivalent to optimal prices in an English auction:

$$a_i^{HS} = \begin{cases} 
 r_{-i}(Q + \kappa) - \varepsilon & \text{if } r_i(Q + \kappa) < r_{-i}(Q + \kappa) \\
 r_i(Q + \kappa) & \text{otherwise},
\end{cases}$$

(7) where $\varepsilon$ corresponds to the minimum tick size.

(ii) (Biais, 1993). In an opaque market, optimal quotes are strategically equivalent to optimal prices in a Dutch auction:

$$a_i^B = \frac{r_i(Q + \kappa) + r_d(Q + \kappa)}{2}.$$  

(8)

When quotes are fully disclosed, Ho and Stoll (1983) show that the dealer endowed with the longest inventory position undercuts its competitors by posting an ask price just below the competitors’ lowest reservation price. Note that the ask price of the best-quoter will usually be strictly upper his own reservation price. Thus, in such transparent markets, the (Nash) equilibrium strategy results in setting the best offer equal to the second-best reservation price, or equivalently, $a_i^{HS} = \max(r_1(Q + \kappa), r_2(Q + \kappa))$. In opaque markets, Biais (1993) shows that all dealers post ask quotes higher that their own private valuation for the asset. This result is similar to Dutch, or sealed first-price auction. The degree of pre-trade transparency has, however, no impact on the average bid-ask spreads. Biais proves indeed that the expected best offer is equal in both transparent and opaque markets, i.e. $E(a_i^{HS}) = E(a_i^B)$, where $a_i^B = \min(a_1^B, a_2^B)$.

In the following sections we analyze how preferencing affects dealer quoting strategy,
by varying the degree of quote disclosure in the market.

## 3 Preferencing in a Transparent Market

This section assumes that dealers perfectly observe competitors’ quotes.

### 3.1 Optimal Quotes

Our paper puts forward that preferenced orders increase the cost of providing liquidity to public orders. Therefore dealer 2 is less willing to quote competitive ask prices. Since dealer 1 is assumed to observe the volume of preferenced shares, he anticipates correctly how dealer 2 will adjust her quotes upward.

**Theorem 1** The dealer with the lowest reservation price \( \min(r_1(Q), k_2(Q + \kappa)) \) posts an ask quote just below the second lowest reservation price. In other words, the Nash equilibrium consists of each dealer using the following pure strategy:

\[
\begin{align*}
    a_1 &= \begin{cases} 
    k_2(Q + \kappa) - \varepsilon & \text{if } r_1(Q) < k_2(Q + \kappa) \\
    r_1(Q) & \text{otherwise,}
    \end{cases} \\
    a_2 &= \begin{cases} 
    r_1(Q) - \varepsilon & \text{if } k_2(Q + \kappa) < r_1(Q) \\
    k_2(Q + \kappa) & \text{otherwise,}
    \end{cases}
\end{align*}
\]

where \( \varepsilon \) corresponds to the minimum tick size.

If preferencing is not authorized, dealer 1 executes the total order flow \((Q + \kappa)\) if and only if he is endowed with the most extreme inventory position (see Proposition 1). In contrast, under preferencing, it exists cases in which dealer 1 in a shorter position \((I_1 < I_2)\) wins the public trade because his cost to provide liquidity is still lower than dealer 2’s inventory holding cost: \(r_1(Q) < k_2(Q + \kappa)\), or \(I_2 > I_1 \geq I_2 - \kappa\). Dealer 2 is indeed not induced to undercut her opponent in that case, letting him quoting the best price. Thus, preferencing softens dealers competition by reducing their capacity to supply liquidity to public investors.
Let us now analyze more deeply how order preferencing distorts incentives to compete aggressively. The commitment to execute preferenced orders makes dealer 2 more reluctant to accumulate more trades in the same direction in the primary market. She displays quotes that are a direct function of her inventory adjusted from preferenced trades. Preferencing therefore makes her less likely to be the best-quoting dealer ex ante, as the following corollary stresses it.

**Corollary 1** The ex ante probability that the preferenced dealer wins the public order flow is smaller than the ex ante probability that the non preferenced dealer wins the public order flow.

Opponents of preferencing argue that it reduces incentives to cut prices, because unpreferenced market-makers who undercut other market-makers cannot attract their captive order flows. In our model, it is actually the preferenced dealer who is less induced to post attractive prices. Facing a higher inventory holding cost, she is less able to attract public trades. In contrast, the unpreferenced dealer keeps incentives to post the best price. Dealer 1 has actually a greater chance to win the public order flow since he is more likely to have a low reservation price, compared to his rival. But, he posts, in average, higher ask prices since he perfectly anticipates that preferencing makes his rival less able to compete aggressively.

In sum, consistently with concerns of opponents to preferencing, we find that preferencing reduces dealers’ incentives to offer competitive execution. This result supports the findings of Chung et al. (2004). The authors indeed observe that a dealer is less likely to post the best price if the portion of his volume that is preferenced is high. In addition, they show that the dealer is likely to enlarge his quoted spreads, consistently with higher inventory holding costs supported in the presence of prearranged trades.

### 3.2 Market Performance

This section analyzes the impact of preferenced trades on the overall market performance by using the benchmark described in Section 2.2.
Theorem 1 shows that the best-quoting dealer posts an ask price just below the reservation price of his rival, strictly higher than his own’s. Thus, the best offer is equal to the highest reservation price, i.e. $a = \max(r_1(Q), k_2(Q + \kappa))$. Using the expression of the cutoff price, we rewrite the best offer as follows: $a = \max(r_1(Q), r_2(Q) + \rho\sigma^2\kappa)$. It is then direct to show that when the volume of preferencing increases, the expected best offer goes up, i.e. $\partial E(a)/\partial \kappa > 0$. This result corroborates the findings of Chung et al. (2004) who show that bid-ask spreads in NASDAQ stocks are positively related to the proportion of their preferred trades.

The next corollary compares the expected best offer under two regimes: when $\kappa$ is preferred and executed off the primary market or when it is directly routed to it (Benchmark).

**Corollary 2** The expected best offer is higher in a market with preferencing than in a market without preferencing, ceteris paribus; that is, $E(a) > E(a^{HS})$.

This corollary suggests two comments. First, it supports the point of view of Huang and Stoll (1996) and He et al. (2006). Both argue that execution costs are larger on NASDAQ relative to the NYSE, because of preferencing which is much more prevalent on NASDAQ. In particular, Huang and Stoll use CRSP data in 1991 to measure and compare quoted, effective, realized, and Roll implied spreads of a paired sample of NYSE-listed and NASDAQ stocks. They find that execution cost is systematically higher on NASDAQ by every measure. That surprisingly large difference survives all of their robustness checks (including the absence of odd-eighths suspected to facilitate implicit collusion on NASDAQ). According to the authors, it is mainly explained by the limited price competition due to preferencing and internalization. Regarding the findings of He et al. (2006), the authors use different data (SEC 11Acl-5 reports in Q1 and Q2 2004). They find that, for NASDAQ stocks, realized spreads, which they show to be a better measure of market quality in the presence of preferencing, exceed by as much as $0.025$ per share in average those for comparable NYSE stocks. This poorer execution quality is also shown to be related to the degree of preferencing.

Second, we can deduce from this corollary that liquidity traders face execution costs generated by market spreads that are minimized in markets in which preferencing is
not allowed. The competitive market is therefore the best outcome for market order submitters. Turning to the case of dealers, these agents will prefer the market structure where their expected profit is larger.

**Corollary 3** (a) The unpreferenced dealer may expect a higher profit in presence of order preferencing, i.e. \( E(\Pi_1) \geq E(\Pi_{1H}) \), if \( \kappa \geq \kappa(Q) \), where \( \kappa(Q) \) is detailed in the Appendix.
(b) The preferenced dealer earns in average a higher profit when preferencing is authorized, i.e. \( E(\Pi_2) > E(\Pi_{2H}) \).

Preferencing increases the expected profit of the preferenced dealer even if she is supposed to have less control on the price execution of preferenced transactions. Actually, in this transparent two-dealer market, there is no risk in price execution: In case dealer 2 does not post the best price, the best offer to match is equal to her cutoff price, which is strictly greater than her cost to supply liquidity for the transaction \( \kappa \), i.e., \( k_2(Q + \kappa) > r_2(\kappa) \). From the unpreferenced dealer’s perspective, two opposite effects interplay: On one hand, relative to the competitive case, dealer 1 loses the potential trading profit on the order flow \( \kappa \) which he cannot attract. On the other hand, he benefits from the softness of price-competition since, ex ante, he posts less aggressive prices while keeping a greater chance to win the public order flow. There exist cases in which the benefits outweighs the costs of preferencing. Thus, preferencing may enrich all dealers, which corroborates the results of He et al. (2006). Market-making profits appear indeed higher in NASDAQ stocks than in comparable NYSE stocks: NASDAQ dealers would get additional profits of up to 55% of the effective spreads on average, according to He et al. (2006).

## 4 Preferencing in an Opaque Market

In this section, we vary the degree of pre-trade transparency and deals with price formation when information on dealer quotes is not publicly disclosed. It is well-known that the degree of pre-trade transparency has an effect on the quoting behavior of dealers (see, e.g., Flood et al., 1999 or Simaan et al., 2003). In our model, the preferenced dealer can only form an expectation of the best price at which she could be constrained to execute the
preferred transaction in case she does not post the best price. Does market opaqueness smoothen or strengthen price competition among dealers when part of the total order flow is already prearranged?

Dealer 1 executes the public transaction when his ask price is lower than his competitor’s ask price, as described by the profit function \( \pi_1 \) defined in equation (3). In an opaque market, dealer 1 has to compute the probability of this event that we denote \( \Pr (a_1 < a_2) \). Thus, the expected profit of dealer 1 is:

\[
\Pi_1 (a_1, r_1(Q)) = \Pr (a_1 < a_2) (a_1 - r_1(Q)) Q. \tag{11}
\]

Dealer 2 must determine her ask price given (i) her probability to win the public transaction, denoted \( \Pr (a_2 < a_1) \) and (ii) the obligation to execute the preferenced transaction even when she does not post the best price, which happens with probability \( \Pr (a_2 > a_1) \). Thus, given the trading profit function \( \pi_2 \) described by equation (4), dealer 2 expects:

\[
\Pi_2 (a_2, k_2(Q + \kappa)) = \Pr (a_2 < a_1) (a_2 - k_2(Q + \kappa)) (Q + \kappa) + \Pr (a_2 > a_1) (E (a_1 \mid a_2 > a_1) - k_2(Q + \kappa)) \kappa + \frac{\rho \sigma_v^2}{2} \kappa (Q + \kappa). \tag{12}
\]

Order preferencing generates two types of asymmetry among dealers. First, dealers’ expected profit functions are not symmetric due to the different trading profit functions \( \pi_1 \) and \( \pi_2 \) described in equations (3) and (4). Second, reservation prices of dealers are not defined on the same distribution support, since dealer 2 will not quote under the cutoff price \( k_2(Q + \kappa) \). Therefore, we make a distinction between \( F \), the uniform cumulative distribution function (cdf) of the reservation price of dealer 1 on the support \( [r_u(Q), r_d(Q)] \), and \( F_\kappa \), the uniform cdf of the reservation price of dealer 2 on \( [k_u(Q + \kappa), k_d(Q + \kappa)] \). In particular, the distribution support of the reservation price of dealer 2 is “shifted” to the right compared with the distribution support of dealer 1’s reservation price: \([k_u(Q + \kappa), k_d(Q + \kappa)] = [r_u(Q) + \rho \sigma_v^2 \kappa, r_d(Q) + \rho \sigma_v^2 \kappa]\). Dealer 1 is thus “stronger” in the sense that he cannot draw high reservation prices (in \([r_d(Q), r_d(Q) + \rho \sigma_v^2 \kappa]\)) and may have low reservation prices (in \([r_u(Q), r_u(Q) + \rho \sigma_v^2 \kappa]\)).
Due to this double asymmetry, the equilibrium quote-setting strategies are obtained in closed-form solutions only in some special cases. Numerical methods are required in the alternative cases.

4.1 Optimal Quotes in an Opaque Market

This section provides the detailed analysis of the equilibrium that consists of a pair of ask functions: $a_1 : [r_u(Q), r_d(Q)] \rightarrow \mathbb{R}$ and $a_2 : [k_u(Q + \kappa), k_d(Q + \kappa)] \rightarrow \mathbb{R}$. The ask strategies are increasing functions (see, e.g., Lebrun (1999) for formal proofs). We then can define the inverse - increasing - quoting functions, which are more convenient to analyze. Consequently, we denote $v_1(y)$ and $v_2(y)$ the reservation prices drawn respectively by dealer 1 and dealer 2, that lead them to quote $y$. Observe that the probability that dealer 1 (resp. dealer 2) wins writes $\Pr(y < a_2) = \Pr(v_2(y) < k_2(Q + \kappa)) = 1 - F_\kappa(v_2(y)) \equiv \bar{F}_\kappa(v_2(y))$ (resp. $1 - F(v_1(y)) \equiv \bar{F}(v_1(y))$).

The (Bayes-Nash) equilibrium is defined by the couple of inverse functions $(v_1, v_2)$ such that dealer 1’s maximization problem is rewritten as

$$(i) \quad v_1^{(-1)}(r_1(Q)) \in \arg \max_y \Pi_1(y, r_1(Q))$$

with

$$\Pi_1(y, r_1(Q)) = \bar{F}_\kappa(v_2(y)) (y - r_1(Q)) \times Q$$

(13)

Similarly $a_2(k_2(Q + \kappa)) = v_2^{(-1)}(k_2(Q + \kappa))$ solves

$$(ii) \quad v_2^{(-1)}(k_2(Q + \kappa)) \in \arg \max_y \Pi_2(y, k_2(Q + \kappa))$$

with

$$\Pi_2(y, k_2(Q + \kappa)) = \bar{F}(v_1(y)) (y - k_2(Q + \kappa)) (Q + \kappa) + (1 - \bar{F}(v_1(y))) (E(a_1 | y > a_1) - k_2(Q + \kappa)) \kappa + \frac{\rho \sigma_v^2}{2} \kappa (Q + \kappa).$$

(14)

Technically, without preferencing, the Bayes-Nash equilibrium exists and is unique.
Dealers’ prices correspond to those arising in a Dutch auction, which is strategically equivalent to a sealed-bid first-price auction, as described in Proposition 1(ii). Order preferencing, however, leads to asymmetries. Previous work on asymmetric first-price auction show that equilibria do not necessarily exist, and, when they do, asymmetries generally preclude explicit solution (see, e.g., Lebrun, 1999, Maskin and Riley, 2000b, or Cantillon, 2008). In our setting, we show that the equilibrium exists and we use a numerical approach based on Marshall et al. (1994) to derive equilibrium quoting strategies when we are not able to characterize analytical equilibrium strategies.

When preferencing is small, i.e. $\kappa < 2 (I_u - I_d)$, Dealers’ ask strategies have the same domain of values $[a^{\inf}, a^{\sup}]$. In this range, both dealers have a positive probability to execute public orders.

The lower bound $a^{\inf}$ is the lowest possible ask price quoted by a dealer and it is defined such that $\bar{F}_\kappa (v_2 (a^{\inf})) \bar{F} (v_1 (a^{\inf})) = 1$. Intuitively, if dealer 1 should post a lower price than dealer 2 ($a^{\inf}_1 < a^{\inf}_2$), then he could quote any price $a_1 \in [a^{\inf}_1, a^{\inf}_2]$ and be sure to post the best price. However, this strategy is strictly dominated by, for instance, $(a_1 + a^{\inf}_2)/2$. Hence, it cannot be an equilibrium by elimination of iterated dominated strategies (the same holds in case $a^{\inf}_1 > a^{\inf}_2$). As a result, dealers’ optimal quotes must have the same lower bound $a^{\inf}$.

The upper bound $a^{\sup}$ is the largest possible ask price quoted by a dealer who has a positive probability to execute the public order flow. This upper bound is defined such that $\bar{F}_\kappa (v_2 (a^{\sup})) \bar{F} (v_1 (a^{\sup})) = 0$. Using the same argument as before, we conclude that dealers must quote no more than the largest possible ask price to keep a chance to execute public orders.

**Proposition 2** Assume that preferencing is small, i.e. $\kappa < 2 (I_u - I_d)$. Then

(i) The equilibrium inverse ask functions $v_1$ and $v_2$ are solutions to the following pair of non-linear ordinary differential equations:

$$\frac{F'_\kappa (v_2 (y))}{F_k (v_2 (y))} v'_2 (y) = - \frac{1}{y - v_1 (y)}, \quad (15)$$

$$\frac{F'_\kappa (v_1 (y))}{F (v_1 (y))} v'_1 (y) = - \frac{1 + \kappa/Q}{y - v_2 (y)}. \quad (16)$$

13The cut-off ($\kappa < 2 (I_u - I_d)$ or $\kappa \geq 2 (I_u - I_d)$) will be economically justified below.
(ii) If \( r_d(Q) \leq a^{\text{sup}} \leq (r_u(Q) + k_d(Q + \kappa))/2 \), there exists an equilibrium.

The equilibrium is not necessarily unique and, with very few exceptions (see Appendix B of Marshall et al., 1994), the lower bound \( a^{\text{inf}} \) cannot be found analytically. It is endogenously determined by the upper bound \( a^{\text{sup}} \) and by the condition \( \bar{F}_u (v_2 (a^{\text{inf}})) \bar{F} (v_1 (a^{\text{inf}})) = 1 \). In our model, \( a^{\text{inf}} \) is numerically determined, but it is, analytically, proved to be increasing with the upper bound \( a^{\text{sup}} \) (see Lemma 1 in the Appendix).

Note also that when the reservation price drawn by dealer 2 is greater than the largest possible ask price \( (k_2(Q + \kappa) > a^{\text{sup}}) \), she cannot win the public order \( Q \). In that case, we assume that (in equilibrium) dealer 2, with a zero probability of winning, quotes her reservation price: \( a_2 = k_2(Q + \kappa) \).\(^{14}\) Furthermore, in case when the lowest reservation price possibly drawn by dealer 2 is still larger than the largest possible ask price, i.e.,

\[
k_u(Q + \kappa) > (r_u(Q) + k_d(Q + \kappa))/2 \text{ or, equivalently, } \kappa \geq 2 (I_u - I_d)
\]

she can never profitably execute the public order flow by posting the lowest ask price and the equilibrium described in Proposition 2 degenerates.\(^{15}\) (See Proposition 3 below).

The following theorem determines the equilibrium which is more likely to be selected by dealers among the multiplicity of equilibria described in Proposition 2.

**Theorem 2** The unique Pareto dominant equilibrium, from the point of view of both dealers, is obtained when \( a^{\text{sup}} \) is equal to \((r_d(Q) + k_d(Q + \kappa))/2\).

The Pareto dominant equilibrium is the equilibrium that maximizes both dealers’ expected profit. As described earlier, we have not been able to obtain an analytical solution to this asymmetric equilibrium and we thus adopt a numerical approach.

It is worth noting that the system of differential equations coincides with dealers drawing reservation prices from different power distributions as in Marshall et al. (1994).\(^{16}\) The asymmetry in trading profits thus corresponds to an asymmetry in distribution functions. In addition, as mentioned above, dealers 1 and 2 do not share the same distribution

\(^{14}\)This assumption is also used by Kaplan and Zamir (2007).

\(^{15}\)Observe that, if \( \kappa = 2 (I_u - I_d) \), then \( a^{\text{inf}} = a^{\text{sup}} \).

\(^{16}\)In particular, \( H = F^{\bar{F} \bar{F} \bar{F} \bar{F}} \) is the power distribution that would lead to a similar first order condition for dealer 2.
support for reservation prices, as in Maskin and Riley (2000a), (2000b) or Kaplan and Zamir (2007). Order preferencing therefore generates asymmetries both in distribution functions and in support, which rules out standard assumptions in the existing theoretical literature on asymmetric auctions. In particular, distributions cannot be ordered according to the first order stochastic dominance, which does not hold in this game. This means that quoting strategies of the dealers cannot be ordered so that one is consistently more aggressive than the other. Quoting strategies may thus intersect several times. Related cases have been reported in Kaplan and Zamir (2007) or Kierkegaard (2009).

Figure 1 depicts this case. It combines dealers’ quoting strategies in the presence of order preferencing and without preferencing (the Biais case). First, it shows several crossings of dealers’ quoting strategies. Second, we observe that the unpreferenced dealer posts less aggressive prices than in the benchmark, which is consistent with a larger chance to draw a low reservation price (Maskin and Riley, 2000a). Dealer 2’s aggressiveness depends on the level of her inventory: For some extremely short inventory positions for which she is weaker (in the sense she cannot have low reservation prices), she posts more aggressive prices than in the benchmark. In some other regions, the guarantee to attract captive order flow makes her post less aggressive prices (like collusive bidders in the model proposed by Marshall et al., 1994). The combination of both support and distribution asymmetries creates two opposite quoting forces which generate the equivocal price aggressiveness of the preferenced dealer. Overall, order preferencing generally makes dealers to compete less aggressively than in the Biais case.

When preferencing is large, i.e. $\kappa \geq 2 (I_u - I_d)$. In that case, Proposition 2 fails, but we are able to characterize the equilibrium in closed form, as the next proposition shows.

**Proposition 3** Assume that preferencing is large, i.e. $\kappa \geq 2 (I_u - I_d)$. Then, dealer 2 quotes an ask price equal to her reservation price: $a_2 = k_2 (Q + \kappa)$, and dealer 1 posts $a_1 = k_u (Q + \kappa)$.

Note that the competitive symmetric case determinated by Biais (1993) is a special case of our model, in which identical inverse ask functions are used to solve the equilibrium, i.e., $v_1 = v_2 = v^B$, with $v^B = (a^B)^{(-1)}$. The analytical solution is unique and linear in reservation prices (see equation (8)).
When the portion of the preferenced order flow is large \((\kappa \geq 2(I_u - I_d))\), it precludes any price competition between dealers. Dealer 1 is indeed sure to post the best price and to execute public trades by choosing a quote slightly below the lowest possible reservation price drawn by the preferenced dealer. Moreover dealer 2 posts her reservation price since she cannot profitably win the public order flow anyway.

### 4.2 Preferencing, Market Performance, and Transparency

This section analyzes how the degree of quote disclosure alters market spreads and dealers’ profits regarding preferencing.

**The expected best offer.** To investigate the impact of transparency on the level of the best ask in a market under order preferencing, we conduct both numerical and analytical analysis. Figure 2 illustrates our findings. It depicts the expected best ask evolving with the size of the preferenced order flow \(\kappa\), under four regimes (transparent versus opaque market; \(\kappa\) being public versus being preferenced to dealer 2). Remember that, when \(\kappa\) is public, the best ask is identical whether the market is transparent or opaque. First, Fig. 2 shows that any best ask is increasing with the size of \(\kappa\), consistent with an increasing cost to provide immediacy for dealers. Second, Fig. 2 shows that the degree of market transparency has a varying impact on the expected best offer, depending on the magnitude of order preferencing. We analyze these points in more detail.

First, regardless of the degree of pre-trade transparency, the expected best offer is found to be higher in presence of preferencing than in a competitive market (Benchmark), which is consistent with the previous remarks on the negative impact of preferencing on dealers’ price competitiveness.

Second, quotation transparency has an ambiguous impact on the expected best offer, depending on the magnitude of preferencing. When preferencing is small, numerical analysis shows that the transparent market may offer a lower expected best offer than the opaque market (see Figure 2). The reverse is (analytically) true when preferencing is large \((\kappa \geq 2(I_u - I_d))\). Recall that, in that case, the preferenced dealer can never profitably execute the public order flow, and her strategy is to quote her reservation price both in the transparent market and in the opaque market. However, transparency plays a
non negligible role since it helps dealer 1 to quote an even less competitive ask price in a transparent market compared to an opaque market. In particular, in the opaque market, he does not quote above the lowest reservation price of his rival \((k_u(Q + \kappa))\), while he chooses a quote price greater or equal to that lower bound in the transparent market (see the optimal quotes of Theorem 1).

A fully pre-trade transparent market removes dealer 1’s uncertainty about the price quoted by the competitor. He responds then by quoting the least possible aggressively, which raises transaction costs for liquidity traders (as measured by the bid-ask spread). Kluger and Wyatt (2002) also find that, when a non-competitive outcome occurs among preferred and unpreferred dealers, expected spreads are wider in a transparent market than in a more opaque market. This result suggests that regulating preferring or internalization by enforcing identical rules on transparency across markets may have opposite effects on transaction costs, depending on the magnitude of prearranged trades in the different markets.

Finally, these results highlight that expected best offers in the transparent and opaque market are not equal, in contrast to Biais’s (1993) irrelevance proposition of market structures. In the symmetric setting of the benchmark, the “revenue-equivalence theorem” in auction holds. Therefore, any opaque or transparent market structures yields the same expected best offer. In our model, preferencing creates asymmetries, which are known to prevent the revenue-equivalence theorem to prevail (see, e.g., Maskin and Riley, 2000a). That is also a reason why market structures, that is the level of transparency, matter in presence of order preferencing.

Dealers’ expected profit. Direct algebra shows that when preferencing is large \((\kappa \geq 2(I_u - I_d))\), dealer 1 and 2 prefer the transparent market since they compete less aggressively in that case and enjoy higher expected profits. When preferencing is small, unreported numerical results show that dealer 1 prefers the transparent market (consistent with results obtained in auction theory by Maskin and Riley, 2000a). In contrast, the expected profit of dealer 2 may be higher in the opaque market, depending on the magnitude of the preferred order flow.
What is worth stressing is that dealer 2 may incur losses in executing her preferenced transaction in the opaque market. For some levels of her inventory, she matches the best price posted by her opponent which may be lower than her cutoff price. This result is consistent with the empirical evidence of Hansch et al. (1999) who find that preferenced dealers on the LSE make zero profits over all trades. Note that (i) losses are not enough to outweigh the ex ante preference of dealer 2 for the opaque market in some cases; (ii) losses could even be bigger if we now assume that dealer 1 cannot observe whether a preferenced order flow is received or not by dealer 2, since the best price to match would be even more competitive.\footnote{Numerical results are available upon request.}

5 Discussion

In this section, we are interested in assessing the impact of relaxing some simplifying hypotheses of the model. We look at two alternative formulations. First, we generalize the model to incorporate more dealers. Second, we investigate the impact of allocating more risk capital to market-makers who commit to execute preferenced orders. Due to the complexity of the opaque game, we focus on the transparent market à la Ho and Stoll (1983) developed in Section 3.

5.1 More than two dealers

To obtain more intuition on the role of preferencing in multiple-dealers markets, we extend the model to $N$ dealers. We denote the reservation price of dealer $i$ as $k_i(Q + \kappa_i) = r_i(Q) + \rho \sigma^2 \kappa_i$ ($i = 1, \ldots, N$). Note that when $\kappa_i = 0$, the dealer is not preferenced and $k_i(Q + \kappa_i) = r_i(Q)$.

**Proposition 4** In a transparent market with order preferencing, the dealer with the lowest reservation price, denoted by $T$, posts the best price. At equilibrium, the best- quoting dealer undercuts the second-lowest reservation price and the $(N - 1)$ other dealers quote
their own reservation price, i.e.

\[
\begin{align*}
 a_T &= \min_{i \in [1;N] \setminus T} k_i(Q + \kappa_i) - \varepsilon \\
 a_i &= k_i(Q + \kappa_i), \text{ for } i \in [1;N] \setminus \{T\}
\end{align*}
\]

where \(\varepsilon\) corresponds to the minimum tick size.

The ranking of dealers’ effective inventory positions \((I_i - \kappa_i)_{i \in [1;N]}\) determines the ranking of dealers’ reservation prices \((k_i(Q + \kappa_i))_{i \in [1;N]}\), which, in turn, yields the outcome of the quote-competition between dealers at date 3. Note that, under preferencing, the market mechanism may fail to allocate efficiently the order flow, since the dealer with the extreme inventory position is not necessarily the dealer who posts the best price, which is in contrast to Ho and Stoll (1983).

A Numerical Example. In order to determine whether the number of preferenced vs. unpreferenced dealers matters, we consider a three-dealer market and two settings. In the first one, there is only one dealer receiving preferenced orders \((\kappa)\) and in the second there are two preferenced dealers receiving \(\kappa/2\).\(^{19}\) Other parameters are as follows: \(\rho = 1\), \(\mu = \$99.75\), \(\sigma^2 = 1/10,000\), \(Q = 5,000\), \(I_d = 0\), and \(I_u = 20,000\).

Figure 3 displays how the expected best offer evolves with the size of the preferenced order in the three-dealer-market settings, compared to the two-dealer market.\(^{20}\) First, numerical results confirm that increasing the size of the preferenced order increases the price impact of the trade, which naturally pushes upward the expected best offer, irrespective of the total number of dealers. Second, we observe that increasing the number of dealers (from \(N = 2\) to \(N = 3\)) strengthens competition and reduces the expected best offer, as predicted by Biais (1993). Third, Fig. 3 shows that, in the three-dealer market, the expected best offer is lower when only one dealer receives a preferenced order than when two dealers receive it. This result suggests that preferencing is less detrimental in markets with multiple unpreferenced dealers competing for public orders. It is consistent with the experimental findings of Bloomfield and O’Hara (1998). The authors find that preferencing has no effect when more than one dealer do not receive preferenced orders.

\(^{19}\)We keep the scope of preferencing fixed to focus on the impact of varying the number of preferenced dealers.

\(^{20}\)The expected best offer in the three-dealer market has been calculated analytically (in both settings). Results are available upon request.
Empirically, Peterson and Sirri (2003) also fail to find any significant detrimental effect of preferencing on the market quality of preferred regional U.S. equity exchanges. We suggest that it might be due to the high competitive pressure faced by preferred dealers who compete with multiple unpreferenced liquidity suppliers, like the specialist on the NYSE, or limit order traders.\textsuperscript{21}

Testable relation. An additional point of the previous extension is to highlight a new testable link between preferencing, inventories and quotes. From Proposition 4, we deduce the following relation:

\[ a_i - a = \mathcal{F} ((I_i - \kappa_i) - (I_T - \kappa_T)) \quad (18) \]

where \( \kappa_i \) and \( \kappa_T \) are respectively the preferenced trades executed by dealer \( i \) and by the best-quoting dealer \( T \). We observe that, contrary to public trades, the volume of preferred orders enters directly into the testable relation between the position of dealers’ quote and the relative level of their inventory position. Hansh et al. (1999) did not take into account preferred orders, and find that, on the London Stock Exchange (LSE), dealers with the most extreme positions execute only 59\% of public trades. The authors claim that the invalidation of the prediction of Ho and Stoll (1983) is due to preferencing. Our result (i) provides support to their claim, and (ii) suggest how to deal with it.

5.2 Amount of capital at risk

So far we have studied the impact of preferencing arrangements on liquidity provision, by assuming the risk capital of market-makers to be exogenous. This section discusses how our results are sensitive to this hypothesis.

We have showed that price-matching arrangements increase the risk of providing immediacy. In this regard, the preferred market-maker could be allocated a larger amount of risk capital for trading, compared to unpreferenced market-makers. The risk-bearing

\textsuperscript{21}On NASDAQ, prearranged trades (by the way of internalization or payment for order flow) dropped as SuperSoes started in July 2001. Price competition from limit order traders became indeed more aggressive; this quoting aggressiveness has been reinforced with penny trading in US equities (Q1 2001).
capacity of the preferred market-maker would be larger, which translates in our model by lowering risk-aversion (D’Souza and Lai, 2006).

To illustrate the impact of allocating more capital to market-making with preferencing and to obtain new comparative static results, we consider a numerical example in which dealer 2’s risk aversion decreases with the volume of preferred orders, whereas the risk aversion parameter of the unpreferenced dealer remains unchanged. In particular, \( \rho_2 \) decreases exponentially with \( \kappa \) from 1 to a constant value (0.7, 0.5 or 0.2). Other parameters are equal to those of the previous section.

Figure 4 illustrates this case. In a clockwise manner, we depict dealers’ risk aversion, dealer 2’s expected profit, dealer 1’s expected profit, and the expected best offer. We observe that dealer 1’s profit shrinks as dealer 2’s risk aversion gets smaller. When \( \rho_2 \) is lower than 0.7, dealer 1’s expected profit is even lower than in the benchmark case. The decreasing parameter of risk aversion makes dealer 2 more able to draw a lower reservation price. Dealer 1 responds by quoting more aggressively, which reduces his expected profit. The expected profit of dealer 2 is also lower when she is getting less risk-averse, although it remains higher than in the Biais case unless \( \rho_2 \) becomes very small (around 0.2). Finally, the expected best offer is less negatively impacted by order preferencing as more risk capital is allocated to dealer 2. It is even more competitive than the Benchmark for \( \rho_2 \) lower than 0.5

In sum, allocating more funding capital to preferred market-makers may be a way to introduce more competition and to limit the detrimental impact of preferencing on bid-ask spreads.

[INSERT FIGURE 4 AROUND HERE]

6 Conclusion

This paper theoretically investigates whether quotes disclosure matters in a market-making model with inventories in which trades may be prearranged and executed off-
market. This question is of interest because transparency is, in general, under researched, and, also, because recent changes in regulation in Europe now allows broker-dealers to preference or internalize their order flow.

A novel finding of the paper is that preferencing has an effect on the cost of providing liquidity. We show that order preferencing endogenously raises inventory risks for the preferenced dealer, who responds by requiring an additional premium in compensation that pushes up her inventory-holding costs. As a consequence, the preferenced dealer has less incentives to offer competitive executions, which is perfectly anticipated by the competitor in a transparent setting. The latter, in turn, quotes less competitive prices while retaining a greater chance to attract the public order flow. Price-competition is then softened, and market spreads enlarge.

When quotes are not publicly displayed, dealers still post less competitive prices in presence of order preferencing. The best prices are, thus, negatively impacted. However, in some cases, the uncertainty of the opaque market forces dealers to post more aggressive prices than in the transparent setting, which results in smaller expected market spreads. We, thus, show that fostering competition among dealers is not unambiguously enforced by a higher degree of transparency.

This paper has some regulatory implications. First, we provide further evidence to supporting the claim that preferencing might be a detrimental feature of equities or options markets. In particular, it discourages aggressive quoting strategies by increasing the cost of providing liquidity, thereby impeding transaction costs. Second, we show that these harmful effects may be alleviated by promoting competition among dealers. While favoring the entry of multiple unpreferenced dealers is a result pointed out by the literature, our paper also suggests new ways to enhance competitiveness in presence of order preferencing: (i) fine tuning competition through pre-trade transparency, and (ii) requiring preferenced market-makers to have more funding capital.

This paper suggests that market transparency is a sensitive issue for market liquidity when orders may be prearranged. Whether it matters also for market efficiency is another challenging question. While this is an issue outside the scope of this paper, we believe it is an interesting topic for future research.
References


Appendix

Preliminary Remarks. (i) Recall that $F$ (resp. $F_\kappa$) is the uniform distribution function of the r.v. $r_1(Q)$ (resp. $k_2(Q + \kappa)$), on the support $S \equiv [r_u(Q), r_d(Q)]$ (resp. $S_\kappa \equiv [k_u(Q + \kappa), k_d(Q + \kappa)]$). We denote by $f$ (resp. $f_\kappa$) the respective density function. (ii) If $\kappa > (I_u - I_d)$, or equivalently, $k_u(Q + \kappa) > r_d(Q)$, $S$ and $S_\kappa$ are separated.


Proof of Theorem 1: Consider two cases separately.

CASE 1: $\kappa \leq (I_u - I_d)$. Let us show that $a_2$ given by equation (10) is a best reply to dealer 1’s strategy $a_1$.

(i) Suppose $k_2(Q + \kappa) < r_1(Q)$. In case dealer 2 chooses an ask price strictly lower than dealer 1’s quote ($a_2 < a_1 = r_1(Q)$), she wins the public order flow with probability 1. In that case, it is optimal to choose the ask price which is the closest than dealer 1’s quote. Thus, we deduce that the best reply is $a_2 = r_1(Q) - \varepsilon$, where $\varepsilon$ may be interpreted as the minimum regulatory tick size in the market. In case dealer 2 chooses an ask price equal or larger than dealer 1’s quote ($a_2 \geq a_1 = r_1(Q)$), she is not any more sure to win the public order flow, which is not optimal.

(ii) Suppose $k_2(Q + \kappa) \geq r_1(Q)$. Dealer 1’s strategy is to post: $a_1 = k_2(Q + \kappa) - \varepsilon$. If dealer 2 chooses an ask price strictly lower than $a_1$, then she must execute the public order flow with probability 1. Her trading profit is, however, lower than the trading profit obtained by avoiding any deviation: $(a_1 - \varepsilon - k_2(Q + \kappa)) (Q + \kappa) < (a_1 - r_2(\kappa)) \kappa$. As a consequence, it is optimal for dealer 2 to quote her cutoff price: $a_2 = k_2(Q + \kappa)$.

Similarly, we can prove that $a_1$ given by equation (9) is a best reply to dealer 2’s strategy $a_2$.

CASE 2: $\kappa > (I_u - I_d)$. Dealer 1 posts the best price with probability 1 and quotes $a_1 = k_2(Q + \kappa) - \varepsilon$, whereas it is optimal for dealer 2 to quote her cutoff price $a_2 = k_2(Q + \kappa)$.

Proof of Corollary 1: Consider two cases.

CASE 1: $\kappa \leq (I_u - I_d)$. The ex ante probability that dealer 1 posts the lowest ask price
is:

\[
\Pr(a_1 < a_2) = \int_{r_u(Q)}^{k_u(Q+\kappa)} 1 \times f(x)dx + \int_{k_u(Q+\kappa)}^{r_d(Q)} \frac{k_d(Q+\kappa) - x}{r_d(Q) - r_u(Q)} f(x)dx
\]

\[
= \frac{1}{2} + \frac{\kappa}{(I_u - I_d)} - \frac{\kappa^2}{2(I_u - I_d)^2} > \frac{1}{2},
\]

(19)

whereas dealer 2 is the best-quoting dealer with the following ex ante probability \( \Pr(a_2 < a_1) = 1 - \Pr(a_1 < a_2) < 1/2. \)

CASE 2: \( \kappa > (I_u - I_d). \) \( \Pr(a_1 < a_2) = 1 \) and \( \Pr(a_2 < a_1) = 0. \) ■

Proof of Corollary 2: Let us compare \( E(a) \) with \( E(a^{HS}) \) which prevails in the benchmark. From Proposition 1(i), we deduce that \( E(a^{HS}) = (2r_d(Q + \kappa) + r_u(Q + \kappa))/3. \) Let us consider two cases separately.

CASE 1: \( \kappa \leq (I_u - I_d). \) Let \( \psi(\kappa) = E(a) - E(a^{HS}). \) After straightforward computations, we get

\[
\psi(\kappa) = \frac{\rho \sigma_v^2}{2} \left( \frac{I_u - I_d}{3} - \left( 1 - \frac{\kappa}{(I_u - I_d)} \right)^3 - 1 \right) + \kappa, \quad \text{(20)}
\]

\[
\psi'(\kappa) = \frac{\rho \sigma_v^2}{2} \left( 1 - \left( 1 - \frac{\kappa}{(I_u - I_d)} \right)^2 \right) \geq 0. \quad \text{(21)}
\]

Observe that \( \psi(0) = 0 \) and \( \psi(I_u - I_d) > 0. \) We then deduce that \( \psi(\kappa) > 0 \) for each \( \kappa \leq (I_u - I_d). \) It follows that \( E(a) > E(a^{HS}). \)

CASE 2: \( \kappa > (I_u - I_d). \) It is direct to show that \( E(a) = (k_d(Q + \kappa) + k_u(Q + \kappa))/2 > E(a^{HS}). \) ■

Proof of Corollary 3. From Proposition 1(i), dealers' expected profits are symmetric and we deduce that: \( E(\Pi^{HS}) = \rho \sigma_v^2 (I_u - I_d) (Q + \kappa)/6. \) In order to determine whether dealers prefer a market with or without preferencing, we first compute dealer \( i \)'s expected profit in a market with preferencing (Step \( i.1 \)) and, then, we turn to the comparison with the benchmark (Step \( i.2), i = 1, 2. \)

STEP 1.1: Dealer 1’s expected profit in a market with preferencing.
CASE 1: $\kappa \leq (I_u - I_d)$. Dealer 1 expects the following profit:

$$\Pi_1(r_1(Q)) = \Pr(k_2(Q + \kappa) > r_1(Q)) (E(k_2(Q + \kappa) \mid k_2(Q + \kappa) > r_1(Q)) - r_1(Q)) Q.$$  \hspace{1cm} (22)

Given that $\Pr(k_2(Q + \kappa) > r_1(Q)) = \bar{F}_\kappa(r_1(Q))$, the latter expression rewrites:

$$\Pi_1(r_1(Q)) = (\frac{(k_d(Q + \kappa) - r_1(Q))^2}{2(r_d(Q) - r_u(Q))}1_{k_u(Q + \kappa) \leq r_1(Q)}Q$$

$$+ (\frac{k_d(Q + \kappa) + k_u(Q + \kappa)}{2} - r_1(Q))1_{k_u(Q + \kappa) > r_1(Q)}Q.$$  \hspace{1cm} (23)

At date 1, routine calculation then shows that dealer 1’s ex ante trading profit is:

$$E(\Pi_1) = \rho\sigma^2_v Q (\frac{(I_u - I_d)}{6}) - \frac{1}{6(I_u - I_d)^2} + \rho\sigma^2_v (\frac{(I_u - I_d + \kappa)}{2(I_u - I_d - \kappa)}).$$  \hspace{1cm} (24)

CASE 2: $\kappa > (I_u - I_d)$. Dealer 1 posts the best price with probability 1, and his trading profit is

$$\Pi_1(r_1(Q)) = (E(k_2(Q + \kappa)) - r_1(Q)) Q = \frac{k_d(Q + \kappa) + k_u(Q + \kappa)}{2} - r_1(Q)) Q.$$  \hspace{1cm} (25)

At date 1, dealer 1 then expects the following profit:

$$E(\Pi_1) = \left(\int_{r_u(Q)}^{r_d(Q)} \frac{k_d(Q + \kappa) + k_u(Q + \kappa)}{2} - x) f(x) dx \right) Q = \rho\sigma^2_v \kappa Q.$$  \hspace{1cm} (26)

STEP 1.2: Comparison of dealer 1’s expected profits in the benchmark and in the transparent market with preferencing. We get

$$E(\Pi_1) - E(\Pi_1^{HS}) = \rho\sigma^2_v \kappa Q g_Q(\kappa),$$

where $g_Q$ is the positive function defined by:

$$g_Q(\kappa) = \frac{1}{6} \left(-\frac{\kappa^2}{(I_u - I_d)^2} + \frac{\kappa}{(I_u - I_d)} + 3 - \frac{(I_u - I_d)}{Q} \right)1_{\kappa \leq (I_u - I_d)}$$

$$+ \left(1 - \frac{I_u - I_d}{6Q}(1 + \frac{Q}{\kappa}) \right)1_{\kappa > (I_u - I_d)}.$$  \hspace{1cm} (27)
with the following values at bounds $g_Q(0) = (3 - (I_u - I_d)/Q)/6$ and $g_Q(\kappa) \to 1 - (I_u - I_d)/6Q$ as $\kappa \to \infty$. Now, note that $g_Q(I_u - I_d) = (5 - (I_u - I_d)/Q)/6$. Then, $g_Q$ is increasing since

$$g_Q'(\kappa) = \frac{1}{6(I_u - I_d)} \left( -\frac{2\kappa}{(I_u - I_d)} + 3 \right) \mathbb{1}_{\kappa \leq (I_u - I_d)} + \frac{(I_u - I_d)}{6\kappa^2} \mathbb{1}_{\kappa > (I_u - I_d)} > 0.$$ 

Thus, we deduce that the sign $g_Q(\kappa)$ depends on the value of the parameter $Q$.

- If $Q \geq (I_u - I_d)/3$, then, given that $g_Q(0) > 0$, $g_Q(\kappa)$ is increasing from 0, and we deduce that $E(\Pi_1) > E(\Pi_1^{HS})$;

- If $Q < (I_u - I_d)/3$, straightforward computations show that we must consider three different cases: (i) if $(I_u - I_d)/5 \leq Q < (I_u - I_d)/3$, then $E(\Pi_1) \geq E(\Pi_1^{HS})$ for $\kappa \geq \kappa^*(Q)$, where $\kappa^*(Q) = (I_u - I_d)/2 \left( 3 - \sqrt{21 - 4(I_u - I_d)/Q} \right)$; (ii) if $(I_u - I_d)/6 < Q < (I_u - I_d)/5$, then $E(\Pi_1) \geq E(\Pi_1^{HS})$ if $\kappa \geq \kappa^{**}(Q)$, where and $\kappa^{**}(Q) = (I_u - I_d)/Q(6Q - (I_u - I_d))$; (iii) $Q < (I_u - I_d)/6$, $E(\Pi_1) < E(\Pi_1^{HS})$.

To sum up, $E(\Pi_1) > E(\Pi_1^{HS})$ for $\kappa \geq \kappa(Q)$, where

$$\kappa(Q) = \kappa^*(Q) \mathbb{1}_{(\frac{I_u - I_d}{3}) \leq Q < (\frac{I_u - I_d}{5})} + \kappa^{**}(Q) \mathbb{1}_{(\frac{I_u - I_d}{6}) \leq Q < (\frac{I_u - I_d}{5})}.$$ (27)

**STEP 2.1: Dealer 2’s expected profit in a market with preferencing.** Following Theorem 1, we consider two cases separately:

**CASE 1: $\kappa \leq (I_u - I_d)$.** We still must consider two subcases:

- When $r_d(Q) < k_2(Q + \kappa)$, dealer 2’s trading profit writes:

  $$\Pi_2(k_2(Q + \kappa)) = (k_2(Q + \kappa) - r_2(\kappa)) \kappa = \frac{\rho \sigma^2_v}{2} (Q + \kappa) \kappa.$$ (28)

- When $k_2(Q + \kappa) \leq r_d(Q)$, dealer 2 expects the following trading profit:

  $$\Pi_2(k_2(Q + \kappa)) = \bar{F}(k_2(Q + \kappa)) \left( E(r_1(Q) | r_1(Q) > k_2(Q + \kappa)) - k_2(Q + \kappa) \right) (Q + \kappa) + \frac{\rho \sigma^2_v}{2} (Q + \kappa) \kappa.$$
The latter expression rewrites,

\[
\Pi_2 (k_2 (Q + \kappa)) = \left( \frac{(r_d (Q) - k_2 (Q + \kappa))^2}{r_d (Q) - r_u (Q)} + \rho \sigma_v^2 \kappa \right) \frac{(Q + \kappa)}{2}.
\] (29)

Then, combining equations (28) and (29), dealer 2 expects, at date 1, the following profit:

\[
E (\Pi_2) = \int_{r_d (Q)}^{r_u (Q + \kappa)} \left( \frac{(r_d (Q) - x)^2}{r_d (Q) - r_u (Q)} + \rho \sigma_v^2 \kappa \right) \frac{(Q + \kappa)}{2} f_\kappa (x) \, dx + \int_{r(Q)}^{k_d (Q + \kappa)} \frac{\rho \sigma_v^2}{2} (Q + \kappa) \kappa f_\kappa (x) \, dx
\]

\[
= \frac{(Q + \kappa)}{2} \left( \frac{(r_d (Q) - r_u (Q) - \rho \sigma_v^2 \kappa)^3}{3 (r_d (Q) - r_u (Q))^2} + \rho \sigma_v^2 \kappa \right).
\] (30)

CASE 2: \( \kappa > (I_u - I_d) \). Dealer 2 quotes her cutoff price, her expected trading payoff is:

\[
\Pi_2 (k_2 (Q + \kappa)) = (k_2 (Q + \kappa) - r_2 (\kappa)) \kappa = \frac{\rho \sigma_v^2}{2} (Q + \kappa) \kappa,
\] (31)

**STEP 2.2: Comparison of dealer 2’s expected profits**

CASE 1: \( \kappa \leq (I_u - I_d) \). Then,

\[
E (\Pi_2) - E (\Pi_2^{HS}) = \frac{\rho \sigma_v^2 (Q + \kappa)}{2} \left( \frac{((I_u - I_d) - \kappa)^3}{3 (I_u - I_d)^2} + \left( \kappa - \frac{(I_u - I_d)}{3} \right) \right) > 0.
\] (32)

CASE 2: \( \kappa > (I_u - I_d) \). Then,

\[
E (\Pi_2) - E (\Pi_2^{HS}) = \frac{\rho \sigma_v^2 (Q + \kappa)}{2} \left( \kappa - \frac{(I_u - I_d)}{3} \right) > 0.
\] (33)

\[\blacksquare\]

*Proof of Proposition 4.* Similar to that of Theorem 1.\[\blacksquare\]

*Proof of Proposition 2.* We proceed by steps.

**STEP (i): Determination of the ordinary differential equations system.** Given the best reply of dealer 2, dealer 1 chooses \( y \) so as to maximize his profit,

\[
\Pi_1 (y, r_1 (Q)) = \bar{F}_\kappa (v_2 (y)) (y - r_1 (Q)) Q.
\]
Then the first order condition (FOC) yields

$$\bar{F}_\kappa (v_2 (y)) + v_2' (y) \bar{F}'_\kappa (v_2 (y)) (y - r_1(Q)) = 0. \quad (34)$$

At equilibrium, if \(a_1\) is the optimal strategy \((a_1(r_1(Q)) = y)\), then \(v_1 (y)\) must verify the FOC such that for each \(y\):

$$\bar{F}_\kappa (v_2 (y)) + v_2' (y) \bar{F}'_\kappa (v_2 (y)) (y - v_1 (y)) = 0. \quad (35)$$

Now, given that dealer 1 quotes \(a_1 = (v_1)^{-1}\), then dealer 2 chooses \(y\) so as to maximize her profit \(\Pi_2\), where

$$\Pi_2 (y, k_2(Q + \kappa)) = \bar{F} (v_1 (y)) (y - k_2(Q + \kappa)) (Q + \kappa)$$

$$+ \left(1 - \bar{F} (v_1 (y))\right) \left(E (a_1 (r_1(Q)) | y > a_1 (r_1(Q))) - k_2(Q + \kappa)\right) \kappa$$

$$+ \frac{\rho \sigma^2}{2} \kappa (Q + \kappa).$$

Then the FOC yields:

$$\bar{F} (v_1 (y)) (1 + \kappa/Q) + v_1' (y) \bar{F}' (v_1 (y)) (y - k_2(Q + \kappa)) = 0. \quad (36)$$

Now, at equilibrium, if \(a_2\) is the optimal strategy, then \(v_2 (y)\) must verify the first order condition of dealer 2 such that for each \(y\):

$$\bar{F} (v_1 (y)) (1 + \kappa/Q) + v_1' (y) \bar{F}' (v_1 (y)) (y - v_2 (y)) = 0. \quad (37)$$

Equations (35) and (37) give the following system of ordinary differential equations (ODE):

$$\frac{\bar{F}'_\kappa (v_2 (y))}{\bar{F}_\kappa (v_2 (y))} v_2' (y) = - \frac{1}{y - v_1 (y)},$$

$$\frac{\bar{F}' (v_1 (y))}{\bar{F} (v_1 (y))} v_1' (y) = - \frac{(1 + \kappa/Q)}{y - v_2 (y)}.$$

**STEP (ii): Existence of an equilibrium.** Given the definition of \(\bar{F}\) and \(\bar{F}_\kappa\), the ODE
system writes also:

\[ v'_1(y) = \frac{(r_d(Q) - v_1(y))(1 + \kappa/Q)}{y - v_2(y)}, \]
\[ v'_2(y) = \frac{k_d(Q + \kappa) - v_2(y)}{y - v_1(y)}. \]  

Following Theorem 3 of Griesmer et al. (1967), since \((r_d(Q) + k_d(Q + \kappa))/2 > k_u(Q + \kappa)\), we can prove that there exists a multiplicity of equilibria parameterized only by \(a^{\text{sup}}\). In such an equilibrium:

(i) \(\max (r_d(Q), k_u(Q + \kappa)) \leq a^{\text{sup}} \leq (r_d(Q) + k_d(Q + \kappa))/2\),

(ii) \(v_2(a^{\text{sup}}) = a^{\text{sup}}, v_1(a^{\text{sup}}) = r_d(Q)\),

(iii) \(a^{\text{inf}}\) is such that \(v_1(a^{\text{inf}}) = r_u(Q)\) and \(v_2(a^{\text{inf}}) = k_u(Q + \kappa)\). ■

Proof of Theorem 2. The equilibrium denoted by the subscript \(^{(2)}\) is Pareto dominant under the parameter \(a^{\text{sup}(2)}\) if for each equilibrium denoted by the subscript \(^{(1)}\) under other parameters \(a^{\text{sup}(1)}\), both following inequalities hold:

\[
\Pi_1(a_1, r_1; a^{\text{sup}(1)}) \leq \Pi_1(a_1, r_1 : a^{\text{sup}(2)}) \text{ for each } r_1(Q) \in [r_u(Q), r_d(Q)],
\]
\[
\Pi_2(a_2, k_2; a^{\text{sup}(1)}) \leq \Pi_2(a_2, k_2; a^{\text{sup}(2)}) \text{ for each } k_2(Q + \kappa) \in [k_u(Q + \kappa), k_d(Q + \kappa)].
\]

We proceed by proving 5 successive lemmas (1 to 5). First, we prove that the inverse ask functions \(v_1\) and \(v_2\) are uniformly decreasing with the parameter \(a^{\text{sup}}\) (Lemma 1). Then, by using this result, we compare dealers’ expected profit according to the parameter \(a^{\text{sup}}\), where \(a^{\text{sup}} \in [\max (r_d(Q), k_u(Q + \kappa)), (r_d(Q) + k_d(Q + \kappa))/2]\) (see Lemmas 3 and 4). Then we deduce that a Pareto dominant equilibrium is obtained when \(a^{\text{sup}} = (r_d(Q) + k_d(Q + \kappa))/2\). Finally, we show that this Pareto dominant equilibrium is unique (Lemma 5).

Preliminary remark: Let us define \(v_j^{(i)} = v_j(y; a^{\text{sup(i)}}, j = 1, 2\). Note that (i) the lower bound \(a^{\text{inf(i)}}\) is a function of the initial condition on the upper bound \(a^{\text{sup(i)}}\) and, that (ii) at bounds, \(v_2^{(i)}(a^{\text{sup(i)}}) = a^{\text{sup(i)}}, v_2^{(i)}(a^{\text{inf(i)}}) = k_u(Q + \kappa), v_1^{(i)}(a^{\text{sup(i)}}) = r_d(Q)\) and \(v_1^{(i)}(a^{\text{inf(i)}}) = r_u(Q)\).

**Lemma 1** The lower bound \(a^{\text{inf}}\) is increasing with the upper bound \(a^{\text{sup}}\).
Proof. Let us assume by way of contradiction that \( v^\inf \) is decreasing with \( v^\sup \). In that case, if \( v^\sup(1) < v^\sup(2) \), then \( v^\inf(1) > v^\inf(2) \). Then there exists \( y_0 \) such that \( v_2(y_0; v^\sup(1)) = v_2(y_0; v^\sup(2)) \). Since \( v_2(y; v^\sup(1)) \) crosses \( v_2(y; v^\sup(2)) \) from below, it must be the case that \( v_2(y_0; v^\sup(1)) > v_2(y_0; v^\sup(2)) \). Using this inequality and equation (39), it is simple to show that \( v_1(y_0; v^\sup(1)) > v_1(y_0; v^\sup(2)) \). Necessarily, there exists \( y_1 \) such that \( y_1 < y_0 \) where \( v_1(y; v^\sup(1)) \) crosses \( v_1(y; v^\sup(2)) \) from below: \( v_1(y_1; v^\sup(1)) = v_1(y_1; v^\sup(2)) \) and \( v_1'(y_1; v^\sup(1)) > v_1'(y_1; v^\sup(2)) \). Using equation (38), we deduce that \( v_2(y_1; v^\sup(1)) > v_2(y_1; v^\sup(2)) \), which contradicts that \( y_0 \) is the first place where \( v_2(y; v^\sup(1)) \) crosses \( v_2(y; v^\sup(2)) \). \( \text{QED} \)

Lemma 2 Suppose that \( v^\sup(1) < v^\sup(2) \). Then, \( v_j(y; v^\sup(1)) > v_j(y; v^\sup(2)) \) for all \( y \in [v^\inf(2), v^\sup(1)] \), \( j = 1, 2 \).

Proof : The proof is obtained by way of contradiction. Suppose that the inverse strategy of dealer 2 \( v_2(y; v^\sup(i)) \) does not decrease uniformly with the boundary condition \( v^\sup(i) \). Then there exists a smallest value of \( y_0 \) such that \( v_2(y; v^\sup(1)) \) crosses \( v_2(y; v^\sup(2)) \) from below. It must be the case that \( v_2'(y_0; v^\sup(1)) > v_2'(y_0; v^\sup(2)) \). Then, by adopting the same reasoning as in the proof of Lemma 1, we finally get that there necessarily exists \( y_1 \) such that \( y_1 < y_0 \) and such that \( v_2(y_1; v^\sup(1)) < v_2(y_1; v^\sup(2)) \), which contradicts that \( y_0 \) is the first place where \( v_2(y; v^\sup(1)) \) crosses \( v_2(y; v^\sup(2)) \).

Similar arguments apply for proving that \( v_1(y; v^\sup(1)) > v_1(y; v^\sup(2)) \) for \( y \in [v^\inf(2), v^\sup(1)] \).

QED

Corollary 4 When \( v^\sup(1) < v^\sup(2) \), then \( a_1(z; v^\sup(1)) < a_1(z; v^\sup(2)) \) for each \( z \in [r_u(Q), r_d(Q)] \) and \( a_2(z; v^\sup(1)) < a_2(z; v^\sup(2)) \) for each \( z \in [k_u(Q + \kappa), v^\sup(1)] \).

Proof : Straightforward. \( \text{QED} \)

Let us use define the following notations

\[
\Pi_{1}^{(i)}(z) \equiv \Pi_1(a_1(z; v^\sup(i)), z) \text{ for } z \in [r_u(Q), r_d(Q)], \\
\Pi_{2}^{(i)}(z) \equiv \Pi_2(a_2(z; v^\sup(i)), z) \text{ for } z \in [k_u(Q + \kappa), k_d(Q + \kappa)].
\]

Lemma 3 When \( v^\sup(1) < v^\sup(2) \), then \( \Pi_{1}^{(1)}(z) < \Pi_{1}^{(2)}(z) \) for each \( z \in [r_u(Q), r_d(Q)] \).
Proof: Let us first characterize the expected profit of dealer 1 at bounds under the initial condition \(a^{sup(i)}\) (Step 1), and then we analyze the variation in his expected profit due to a change in the initial condition \(a^{sup(i)}\) (Step 2).

**STEP 1.** On the lower bound \(r_u(Q)\), dealer 1’s profit is equal to \(\Pi_1^{(i)}(r_u(Q)) = (a^{inf(i)} - r_u(Q)) Q\). Suppose \(a^{sup(1)} < a^{sup(2)}\), then, by using Lemma 1, we get \(\Pi_1^{(1)}(r_u(Q)) < \Pi_1^{(2)}(r_u(Q))\). On the upper bound \(r_d(Q)\), the expected profit is \(\Pi_1^{(i)}(r_d(Q)) = F_\kappa(a^{sup(i)}) (a^{sup(i)} - r_d(Q)) Q\). Given that the function \(y \rightarrow F_\kappa(y)(y - r_d(Q)) Q\) is increasing on \([r_d(Q), (r_d(Q) + k_d(Q + \kappa))/2]\), we deduce that \(\Pi_1^{(1)}(r_d(Q)) < \Pi_1^{(2)}(r_d(Q))\).

**STEP 2.** Suppose, by way of contradiction, that the expected profit of dealer 1 is not uniformly increasing with the upper bound \(a^{sup(i)}\). Then there exists \(z_0\) such that: (i) \(\Pi_1^{(1)}(z_0) = \Pi_1^{(2)}(z_0)\) and (ii) \(\Pi_1^{(1)}(z) < \Pi_1^{(2)}(z)\) for each \(r_u(Q) < z < z_0\). Then it must be the case that

\[
\frac{d\Pi_1^{(1)}}{dz}(z_0) > \frac{d\Pi_1^{(2)}}{dz}(z_0).
\]

Using equation (35), we rearrange the latter expression:

\[
\frac{d\Pi_1^{(1)}}{dz}(z) = -F_\kappa(v_2^{(i)}(a_1(z; a^{sup(i)}))) Q
+ a_1'(z; a^{sup(i)}) \left( v_2^{(i)r}(a_1(z; a^{sup(i)})) F_\kappa'(v_2^{(i)}(a_1(z; a^{sup(i)}))) (a_1(z; a^{sup(i)}) - z) 
+ F_\kappa(v_2^{(i)}(a_1(z; a^{sup(i)}))) \right) Q.
\]

Now let us differentiate the expected profit of dealer 1 under the initial condition \(a^{sup(i)}\):

\[
\frac{d\Pi_1^{(1)}}{dz}(z) = -F_\kappa(v_2^{(i)}(a_1(z; a^{sup(i)}))) Q.
\]

Now, by using expression (41) and inequality (40), we deduce that \(\Pi_1^{(1)}(z_0) = \Pi_1^{(2)}(z_0)\), or, equivalently

\[
F_\kappa(v_2^{(i)}(a_1(z_0; a^{sup(1)}))) (a_1(z_0; a^{sup(1)}) - z_0) Q = F_\kappa(v_2^{(i)}(a_1(z_0; a^{sup(2)}))) (a_1(z_0; a^{sup(2)}) - z_0) Q,
\]

this equality and the previous inequality imply that \(a_1(z_0; a^{sup(1)}) > a_1(z_0; a^{sup(2)})\). However this inequality violates Corollary 4 and a contradiction is therefore obtained. QED

We deduce from the proof of Lemma 3 that, under the highest initial condition \(a^{sup(1)} =
the expected profit of dealer 1 is uniformly larger than any other profits determined under other lower initial conditions: \( a^{\sup(i)} < \frac{r_d(Q) + k_d(Q + \kappa)}{2} \).

**Lemma 4** When \( a^{\sup(1)} < a^{\sup(2)} \), then \( \Pi_2^{(1)}(z) < \Pi_2^{(2)}(z) \) for each \( z \in [k_u(Q + \kappa), k_d(Q + \kappa)] \).

**Proof:** We adopt a similar reasoning for dealer 2. QED

By using Lemmas 3 and 4, we deduce that there exists a Pareto dominant equilibrium when \( a^{\sup} = \frac{r_d(Q) + k_d(Q + \kappa)}{2} \).

**Lemma 5** The Pareto dominant equilibrium is unique.

**Proof.** Let us prove that the ask price quoted by each dealer is a global maximum on \([a^{\inf}, a^{\sup}]\), where \( a^{\sup} = \frac{r_d(Q) + k_d(Q + \kappa)}{2} \).

**Uniqueness of a maximum sell price for dealer 1.** Let us define first the function \( g_1 \) by

\[
g_1(y) = \Pi_1^*(y, r_1(Q)) (I_u - I_d) = (k_d(Q + \kappa) - v_2(y)) - (y - r_1(Q)) v_2'(y) \quad \text{for} \quad y \in \left[ \max\left(a^{\inf}, r_1(Q)\right), \infty \right]
\]

Notice that

\[
g_1\left(\max\left(a^{\inf}, r_1(Q)\right)\right) = k_d(Q + \kappa) - v_2\left(\max\left(a^{\inf}, r_1(Q)\right)\right) > k_d(Q + \kappa) - v_2\left(a^{\sup}\right)
\]

\[
eq \frac{\rho \sigma^2 \kappa}{2} > 0,
\]

\[
g_1\left(a^{\sup}\right) = k_d(Q + \kappa) - a^{\sup} - (a^{\sup} - r_1(Q)) = (r_1(Q) - r_d(Q)) < 0.
\]

We observe that \( g_1'(y) = -2v_2''(y) - (y - r_1(Q)) v_2''(y) \). Then, by differentiating equation (39), we obtain

\[
g_1'(y) = -v_2'(y) \left(2 - (y - r_1(Q)) \frac{2 - v_1'(y)}{(y - v_1(y))}\right).
\]
Now, let us proceed by way of contradiction. Suppose that there exist two maxima \( y_0 \) and \( y_1 \), i.e. \( g_1(y_1) = g_1(y_0) = 0 \) with \( g'_1(y_1) > 0 \) and \( g'_1(y_0) < 0 \) (which is equivalent to assume that conditions are only local). Since \( v'_2(y_0) > 0 \) (see equation (39)), then

\[
2 - \frac{(y_0 - r_1(Q)) (2 - v'_1(y_0))}{(y_0 - v_1(y_0))} < 0
\]

and

\[
2 - \frac{(y_0 - r_1(Q)) (2 - v'_1(y_0))}{(k_d(Q + \kappa) - v_2(y_0))} v'_2(y_0) < 0 \quad \text{(using equation (39))}
\]

Given that \( g_1(y_0) = 0 \) i.e. \( (k_d(Q + \kappa) - v_2(y_0)) - (y_0 - r_1(Q)) v'_2(y_0) = 0 \),

\[
2 - \frac{(y_0 - r_1(Q)) (2 - v'_1(y_0))}{(k_d(Q + \kappa) - v_2(y_0))} v'_2(y_0) = v'_1(y_0) < 0,
\]

which is in contradiction with equation (38).

**Uniqueness of a maximum sell price for dealer 2.** We adopt a similar reasoning to show the uniqueness of a maximum sell price for dealer 2.$^\text{■}$

**Proof of Proposition 3.** Suppose that dealer 2 quotes an ask price equal to her reservation price \( a_2 = k_2(Q + \kappa) \) (we will prove ultimately that this reply is the best one). When \( a_1 \geq k_u(Q + \kappa) \), dealer 1 chooses a selling quote that maximizes his profit,

\[
\Pi_1(r_1(Q)) = \Pr(a_1 < a_2) (a_1 - r_1(Q)) Q
= \Pr(a_1 < k_2(Q + \kappa)) (a_1 - r_1(Q)) Q = \bar{F}_\kappa(a_1) (a_1 - r_1(Q)) Q.
\]

The FOC yields to

\[
a_1 = \frac{k_d(Q + \kappa) + r_1(Q)}{2}.
\]

\( a_1 \) is increasing in \( r_1(Q) \leq r_d(Q) \). Setting \( a_1 = (k_d(Q + \kappa) + r_d(Q))/2 = a^{\sup} \leq k_u(Q + \kappa) \) gives dealer 1 an equal probability to post the best price. However dealer 1 maximizes his profit when he quotes \( a_1 = k_u(Q + \kappa) \). Given the dealer 1’s best reply, dealer 2 has no chance to win the unpreferenced order flow and quotes \( a_2 = k_2(Q + \kappa) \) (since it is not optimal for dealer 2 to quote a price below her cutoff price).$^\text{■}$
Figure 1. Reservation ask quotes and optimal quotes. In an opaque market without preferencing, a dealer with a reservation price \( r_i(Q + \kappa) \) has an ask price denoted \( a^B(r_i(Q + \kappa)) \). In a market with preferencing, a dealer follows ask strategy \( a_2(k_2(Q + \kappa)) \) if preferred or \( a_1(r_1(Q)) \) if not preferred.
Figure 2. Expected best ask, the volume of preferred order flow varying. This figure illustrates the impact of preferencing on the expected best offer in case of a transparent market and an opaque market, relative to the level of the expected best offer in the Benchmark case.
Figure 3. **Expected best ask, the number of preferred dealers varying.** This figure illustrates whether the number of preferred $M$ vs. unpreferred $(N - M)$ dealers matters by analyzing the expected best offers in 3 different cases: (i) $N = 2$ and $M = 1$ (ii) $N = 3$ and $M = 1$ and (iii) $N = 3$ and $M = 2$. 
Figure 4. Impact on the market performance of a varying risk aversion parameter for the preferred dealer (as a function of the volume of the preferred order flow. In a clockwise manner, Fig. 4 depicts dealer 2’s risk aversion, dealer 1’s expected profit, dealer 2’s expected profit, and the expected best offer.

- - - - $\rho_2$ is decreasing to 0.7; · · · $\rho_2$ is decreasing to 0.5; --- $\rho_2$ is decreasing to 0.2; ••• is the Benchmark case.