A Theory of Wages and Labor Demand with Intrafirm Bargaining and Matching Frictions

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April 2007*

Abstract

Firms are the field of several strategic interactions that both standard neo-classical analysis and search-matching approaches often ignore. Such strategic considerations concern relations between capital owner and labor, relations between marginal employees and incumbents and, more generally, all relations between different groups within the firm with different bargaining positions. This paper provides a synthetic model of the labor market equilibrium with search frictions in a dynamic framework where wage bargaining is influenced by within-firm strategic interactions, with explicit closed form solutions.

Accounting for the heterogeneity of labor and different bargaining power of workers drastically change the results compared to the homogeneous labor case. With heterogeneous labor, higher relative bargaining power for some groups may lead to overemployment relative to other groups, such other groups being underemployed if they have a lower relative bargaining power. The overemployment results do not necessarily hold at the macroeconomic level. Finally, the hold-up problem between capital owners and employees does not necessarily lead to underinvestment in physical capital as is usually the case. Actually, strategic overemployment can induce over-investment when employees substitutable to capital have strong bargaining power.

*We thank Jean-Michel Grandmont, Christian Haefke, Michael Keane, Guy Laroque, Andreu Mas Colell and Richard Rogerson for their discussions and comments. This paper was presented at INSEE-CREST, Université Laval in Quebec, Université de Montreal, Labour Seminar at UQAM, Pompeu Fabra and Arizona State University. We also thank three referees for their very helpful comments.

Beyond the neo-classical analysis, firms are the field of several strategic interactions between capital owners and labor, between marginal employees and incumbents and more generally between different groups with various bargaining positions within the firm. It is usually considered that the bargaining power of workers entails an employment level that is lower than in a competitive labor market, and any increase in the bargaining power of workers raises unemployment. These results are true for a wide class of models, and notably in the now standard framework to analyze unemployment, namely search and matching models of Pissarides (2000). However, Stole and Zwiebel (1996a and b) show that the above results are totally changed in the *intra-firm bargaining model*, where employees engage in individual wage negotiation with the firm. In fact, under the assumption that contract incompleteness does not enable either party to commit to future wages and employment decisions, they show that intra-firm bargaining yields no rent for employees and gives rise to overemployment compared to a competitive labor market. The basic idea of Stole and Zwiebel is that a firm with a diminishing marginal productivity of labor can decrease the bargained wage, which increases in the marginal productivity of labor, by raising employment. In Stole and Zwiebel's setting, firms over-employ strategically, up to the point where workers get their reservation wage.

Our paper attempts to resolve the conflict between these various theories, by providing a simple synthetic framework that combines the intra-firm bargaining assumption with the search and matching model. At least two reasons motivate our approach. First, in the Pissarides model, the wage determination relies on rather special assumptions, including homogeneity of labor and instantaneous capital adjustment. Second, the Stole and Zwiebel model is a static, partial equilibrium model and a dynamic general equilibrium approach is needed to determine the implications of their overemployment result. In fact it seems that the Pisssarides model is actually the ideal general equilibrium framework for intra-firm bargaining, because it provides naturally an environment in which firms cannot immediately replace a worker after a breakdown in individual bargaining, which is the central assumption for the Stole and Zwiebel model. We thus provide a synthetic model of the labor market with search frictions in a dynamic framework, where the wage bargaining is influenced by within firm strategic interactions[1] between employers and heterogeneous workers. The derivation of wages in this context cannot be the usual bargaining solution derived in the one-worker firm standard version of the Pissarides model. Wages must solve, in partial equilibrium, a system of linear differential equations. We exhibit a general strategy for solving the system by using spherical coordinates, and obtain closed form solutions that can be directly exploited by both labor economists and macroeconomists in their quest for a theory of wages in large firms in the presence of search frictions. We also show the existence and uniqueness of the solutions for wages and employment in general equilibrium.

We then continue exploring the implications of our general wage determination theory along several dimensions. First, in line with the research agenda in the literature, we examine the solidity of the Stole and Zwiebel analysis with respect to the introduction of the labor market equilibrium feedback in a dynamic framework.[2] This focus is particularly relevant as it allows making a synthesis between the macroeconomic analysis with search frictions and the new insights generated by Stole and Zwiebel in the industrial organization literature and organizational design. Along these lines, we show that intra-firm bargaining sheds new light on the macroeconomic consequences of conflicts between employers and employees. Indeed, when employers manipulate wages through their employment policies, an increase in the bargaining power of

[1] We thank a referee for the formulation of the previous sentences. Note also that throughout the paper, within-firm strategic interactions are limited to wage determination. We exclude the possibility that different workers act strategically with respect to each others with respect to employment determination or job search.

[2] Recent papers have questioned the robustness of the basic mechanism put forward by Stole and Zwiebel according to which firms could employ strategically up to the point where workers get their reservation wage. Stole and Zwiebel's static analysis as been enriched by Wolinsky (2000) in a dynamic partial equilibrium setting. Wolinsky shows that multiple equilibria occur if employers can use complex dynamic strategies. Namely, Wolinsky shows that there exists, among others, an equilibrium, sustained by trigger strategies, where employment is efficient (i.e. the marginal productivity of labor equates the reservation wage) and the wage exhibits a mark-up over the reservation wage. However, Wolinsky (2000, p. 875) stresses that his analysis "confirms earlier results derived by Stole and Zwiebel (1996a, b) in the context of a static model and shows that they are very strong". In contrast, de Fontenay and Gans (2003) have argued that, allowing firms to replace insiders by block by outsiders would undo the main result, in the sense that even if the outsiders received the Stole and Zwiebel wage, this wage would be in the limit equal to workers' reservation wage.

workers does not necessarily decrease equilibrium employment. It actually induces firms to raise employment to reduce the marginal product of workers, in order to attenuate the wage increase. We show that, in theory, this phenomenon can imply that increases in the bargaining power of workers raise employment.

Second, by generalizing the analysis to several labor factors, we find different, richer and somewhat unanticipated interactions between the firm and employees and between heterogeneous employees themselves. Notably, changes in the bargaining power of an employee entail changes in the whole wage and employment structure of the firm. It is shown that these changes are related to the distribution of bargaining power across employees and to the properties of the firm's technology. It turns out that an increase in the bargaining power for a type of employees can decrease some wages and increase some others, in both partial and general equilibrium. In some cases, a rise in the bargaining power of some employees leads to overall employment increases. Interestingly, we show that the overemployment result in general equilibrium is more pervasive in the multi-labor extension of the model, when labor inputs have different and independent bargaining power. We notably show that a larger bargaining power for a given group of workers raises employment in this group if firms can easily substitute these workers to other workers with low bargaining power.

Third, modelling the demand for physical capital generates a number of additional results. When the stock of capital is predetermined with a standard constant return to scale technology, we obtain results that we would not obtain with a standard wage determination process, as e.g. in Pissarides (2000) in the large firm matching model with a perfect second hand market for physical capital—or equivalently, rented capital.[3] When capital is predetermined, one en-

[3] In Cahuc and Wasmer (2001), we recovered the solutions of Pissarides when intra-firm bargaining was allowed and clarified the partial equilibrium properties of wage determination in this context. In the Pissarides case of constant returns to scale in production, the solution for the wage is the "static" bargaining solution, i.e. the weighted average of the reservation wage and the marginal product. For this, capital must not be a predetermined variable, e.g. it has to be rented. Smith (1999) and Cahuc and Wasmer (2002) studied some general equilibrium properties of the matching model with intra-firm bargaining in the single factor case–homogenous labor. The present paper nests these papers and provides a generalization of their results to multi-dimensional cases.

counters, without much surprise, a typical hold-up problem: workers appropriate part of the rent of employment, which discourages firms from investing in capital. However, when labor is heterogeneous and workers have different bargaining powers, the hold-up problem can give rise to over-investment in physical capital: this is a new but not totally unexpected result. In fact, the intuition of overemployment still carries through the demand for physical capital. For instance, when workers substitutable to capital have a strong bargaining power with respect to those who are complementary to capital, firms wish to over-employ them, which raises the demand for capital. An implication of our paper, left unexplored here, is a potential explanation for why the hold-up problem, at the heart of the macroeconomic literature inspired by Grout (1984), fails to explain the fact that capital-labor ratio are still pretty high in Europe compared to the US.

We finally raise the question of which of the various ingredients, namely intra-firm bargaining à la Stole and Zwiebel or bilateral monopoly à la Pissarides, provide the most relevant model for macroeconomists. We argue that both approaches are complementary, as they illustrate different mechanisms at work in employment relationships. The approach of Stole and Zwiebel yields a good account of the strategic interactions related to wage formation within the firm, but it is static and does not go beyond the analysis of the firm's equilibrium. The Pissarides approach is explicitly dynamic, offers an elegant analysis of labor market equilibrium in the presence of search frictions that give rise to rent sharing problems, but does not account for any strategic interactions within the firm. Our synthesis incorporates these ingredients and may help to overcome some theoretical difficulties encountered by several authors.[4] These

[4] The earliest paper acknowledging this result was Bertola and Caballero (1994), more recent models with such feature being Marimon and Zilibotti (2000), Smith (1999), Bertola and Garibaldi (2001) and Ebell and Hafke (2003). Even in the absence of search frictions, individual bargaining generates additional insights about labor hoarding and cyclical movements in real wages and consumption. See for instance the paper by Rotemberg (1998). The search-RBC literature (Merz 1995, Andolfaltto 1996) has typically excluded the strategic interactions between firms and workers emphasized by Stole and Zwiebel. A typical way to overcome the difficulty in presence of heterogeneous labor (a case typically implying decreasing returns in each labor factor) has been to introduce several sectors (a final good sector and a sector for intermediate goods, specific to each type of labor) in order to generate a separate and standard wage and employment determination for each type of labor, at the cost of

difficulties prevent the use of a large firm matching model with decreasing returns to scale in labor inputs because, in the absence of well-known solutions, they lead to inconsistencies in wage determination.

The paper is organized as follows. In Section 1, we expose the setup and the environment of the firm. In section 2, we derive partial equilibrium results. In Section 3, we determine the general equilibrium, which is extended to capital in Section 4. Section 5 examines various implications of the model, and Section 6 concludes.

1 A general model

1.1 Setup

We consider an economy with a numeraire good produced thanks to $n \geq 1$ labor types. Each type of labor, $i = 1, .., n$, is supplied by a continuum of infinitely lived workers of size normalized to one. Each worker supplies one unit of labor. The number of firm is exogenous and infinite, so that each firm is atomistic. This number is however countable, so that the employment level of each firm can still be a continuous variable and the law of large numbers will apply later on at the firm level (see e.g. Judd 1985), simplifying the derivation of steady-state equations. In each firm, production of the numeraire good is obtained thanks to a concave production function denoted $F(N_1, N_2, ...)$, where $N_i \geq 0$, $i = 1, ...n$, stands for employment of type i. $\mathbf{N} = (N_1, .., N_n)$ denotes the vector of N_i. In our framework, employment $\mathbf{N}$ is a state variable that cannot be increased instantaneously. To recruit, the firm has to post vacancies. It incurs a group-specific hiring cost γ_i per unit of time and per vacancy posted. Vacancies are matched to the pool of unemployed workers according to a technology h_i determining the mass of aggregate contacts between the mass of vacancies, denoted by V_i, and unemployed $u_i = 1 - N_i$ where the population of workers is normalized to one. Functions $h_i(u_i, V_i)$ are assumed to be constant

increasing the complexity of models. See e.g. Acemoglu (2001). Another route is to use the large firm matching model and let the firm be wage taker (see e.g. Merz and Yashiv, 2003, page 7).

returns to scale, increasing and concave in each argument. $\theta_i = V_i/u_i$ denotes the group-specific ratio of vacancies to unemployed workers (the tightness of the labor market). The probability to fill a vacant slot per unit of time is given by $h_i(u_i, V_i)/V_i = q_i(\theta_i)$ with $q_i'(\theta_i) < 0$, $q_i(0) = +\infty$, while $p_i = h_i(u_i, V_i)/u_i = \theta_i q_i(\theta_i)$ with $d\left[\theta_i q_i(\theta_i)\right]/d\theta_i > 0$. Note that θ_i is exogenous to the firms' decisions. Finally, we assume an exogenous job destruction rate s_i for each group i.

As the wage is continuously negotiated, it is potentially a function of employment. We denote by $w_i(\mathbf{N})$ the wage of the type-i workers which potentially depends on employment of all types of labor. At this stage, it is assumed that this function is continuous and differentiable. It will be shown later on that this assumption is fulfilled. Note that the wage w_i is *common to the group i of workers by symmetry but nevertheless it is individually bargained.*[5]

1.2 Labor demand

Denoting employment of group i at date t by N_i, and employment at date $t + dt$ by N_i^+, the value function of the firm, $\Pi(\mathbf{N})$, solves the Bellman equation:

$$\Pi(\mathbf{N}) = \underset{\mathbf{V}}{\text{Max}} \left(\frac{1}{1+rdt}\right) \left\{ \left[F(\mathbf{N}) - \sum_{j=1}^{n} w_j(\mathbf{N}) N_j - \gamma_j V_j \right] dt + \Pi(\mathbf{N}^+) \right\}, \tag{1}$$

$$\text{subject to } N_i^+ = N_i(1 - s_i dt) + V_i q_i dt \tag{2}$$

where the constraint (2) is the law of motion of jobs and where $\mathbf{V} = (V_1, .., V_n)$ denotes the vector of vacancies, the control variable of firms. We denote by $J_i(\mathbf{N})$ the marginal value of a additional worker of type i, i.e. $J_i(\mathbf{N}) = \frac{\partial \Pi(\mathbf{N})}{\partial N_i}$. The first-order and the envelope conditions for an optimal choice of V_i are derived in Appendix A in equations (A1) and (A2). We exclusively focus on stationary solutions where $\mathbf{N} = \mathbf{N}^+$. One finds that the steady state employment level, $\mathbf{N}^*$, satisfies:

$$J_i(\mathbf{N}^*) = \frac{\gamma_i}{q_i} \tag{3}$$

[5] We follow the approach of Stole and Zwiebel (1996a, 1996b) who provided a simple strategic bargaining game where identical workers get the same wage through invididual bargaining.

while marginal profit of employment N_i is obtained as

$$J_i(\mathbf{N}) = \frac{\frac{\partial F(\mathbf{N})}{\partial N_i} - w_i(\mathbf{N}) - \sum_{j=1}^{n} N_j \frac{\partial w_j(\mathbf{N})}{\partial N_i}}{r + s_i} \tag{4}$$

The marginal profit appears above as the discounted marginal product, net of the individual wage and net of the effect of the marginal hire on the wage bill of all employees. Equation (3) states that the marginal product at the optimal level of employment is precisely equal to the expected recruitment costs of workers.

One can then re-express the marginal product of labor at the firm's optimal employment in combining the two equations (3) and (4). The marginal value of a job hinges not only on its marginal productivity and wage: there is in addition the potential impact of an extra-unit of labor N_i on wages of all groups, as the third term in equation (5) indicates.

$$F_i(\mathbf{N}) = \underbrace{w_i(\mathbf{N})}_{\text{Wage}} + \underbrace{\frac{\gamma_i(r + s_i)}{q_i}}_{\text{Turnover costs}} + \underbrace{\sum_{j=1}^{n} N_j \frac{\partial w_j(\mathbf{N})}{\partial N_i}}_{\text{Employment effect on wages}} \tag{5}$$

We thus obtain a relation between the marginal productivity and the labor cost for each category of job. Only the first two terms are found in the large firm analysis in Pissarides (2000): in Cahuc and Wasmer (2001), we showed that the last term can be ignored in the case with instantaneous capital adjustment and constant returns to scale in total factors, as already assumed in Pissarides (1990). Let us now explore the determination of the wage $w_i(\mathbf{N})$.

1.3 Wage determination

Wages are continuously and instantaneously negotiated. Since firms need time to hire workers, employment is considered as a state variable during the negotiation process. The spirit of wage determination in this at-will firm is that the individual wage in a group i is determined through a split of the surplus in shares $\beta_i \in (0, 1)$, where β_i is an index of the bargaining power of workers *specific* to their type. The surplus of a type$-i$ worker is $E_i - U_i$ where E_i and U_i denote the expected discounted utility of an employed and an unemployed type$-i$ worker respectively. At steady state, E_i solves:

$$rE_i = w_i(\mathbf{N}) + s_i(U_i - E_i). \tag{6}$$

The surplus of the worker only depends on its own individual wage w_i: using (6), we have $E_i - U_i = \frac{w_i(\mathbf{N}) - rU_i}{r+s_i}$.

As in the intrafirm bargaining model of Stole and Zwiebel, the threat point of the firm is to layoff the worker and renegotiate individual wages for all remaining workers. Note that this marginal decision only involves an infinitly small unit of labor, since employment is a continuous variable. In order to embed the same sharing rule as in the static model of Stole and Zwiebel, it is necessary to have an explicit bargaining within our dynamic framework. For that we make the assumption that renegotiations with the remaining workers occur very quickly so that nobody losses his job for exogenous cause during the renegotiation process.[6] Then, the usual Nash-sharing rule

$$\beta_i J_i(\mathbf{N}) = (1 - \beta_i)(E_i - U_i), \tag{7}$$

yields a non-linear system of partial first order differential equations of dimension n :[7]

$$w_i(\mathbf{N}) = (1 - \beta_i) rU_i + \beta_i \left(\frac{\partial F(\mathbf{N})}{\partial N_i} - \sum_{j=1}^{n} N_j \frac{\partial w_j(\mathbf{N})}{\partial N_i} \right), \text{ for } i = 1, .., n. \tag{8}$$

where the summation over j reflects that one additional unit of labor i has potential consequences on the wages of all the groups $j = 1, .., n$. In partial equilibrium, U_i is treated as a parameter. It can be noticed that the wage of labor i depends not only on its own level of employment N_i and the effect of an additional unit of N_i on the wage bill of labor i, but more generally on the effect of an additional unit of N_i on the wage of all labor inputs j, $j = 1, ...n$. As it will be shown, this mechanism is *a priori* dependent of the structure of substitutability/ complementarity of

[6] See Cahuc et al. (2006) for an explicit derivation of bargaining solutions with this type of assumption.

[7] To our knowledge, this system, which was derived by Stole and Zwiebel (1996), has not been solved in the case $n > 1$. Several authors, willing to deal with multiple labor in matching model and large firms, have thus to adopt a multi-sector model with intermediate and final goods, such as Acemoglu (2001). As we will show, once calculated, the solutions of our model are quite simple and allow us to account for strategic interactions that are usually neglected.

the different labor inputs in production.

2 Partial equilibrium

Let us now describe the partial equilibrium of the search and matching model with intrafirm bargaining. By definition, in partial equilibrium, the reservation wage rU_i and the labor market tightness θ_i are exogenous variables. In this context, we can obtain a solution for wages and labor demand. Both are parametrized by a quantity reflecting the extent to which the marginal product of labor differs from labor costs. This quantity, depending on the properties of the production function and on the bargaining power of workers, captures the extent to which there is over-employment.

2.1 Solutions

The system of differential equations (8) is solved in appendices B.1 to B.4. One obtains the following expression for wages:

$$w_i(\mathbf{N}) = (1-\beta_i)rU_i + \int_0^1 z^{\frac{1-\beta_i}{\beta_i}} F_i(\mathbf{NA}_i(z))dz \tag{9}$$

where F_i stands for the partial derivative of F with respect to the argument i and the term $\mathbf{A}_i(z)$ stands for a diagonal matrix defined in appendix B.4.[8] It reflects the distortion of labor inputs in the outcome of wage bargaining and is such that the vector $\mathbf{NA}_i(z)$ reads

$$\mathbf{NA}_i(z) = (N_1 z^{\frac{\beta_1}{1-\beta_1}\frac{1-\beta_i}{\beta_i}}, N_2 z^{\frac{\beta_2}{1-\beta_2}\frac{1-\beta_i}{\beta_i}}, ..., N_n z^{\frac{\beta_n}{1-\beta_n}\frac{1-\beta_i}{\beta_i}}). \tag{10}$$

The wage equation generalizes Stole and Zwiebel (1996 a and b). It implies that the solution for wages is a weighted average of rU_i and of an additional term in which infra-marginal returns show up with weights depending on β_i and other β_j. For convenience, the exact intuition of this second term is described later on, first in more specific cases and then in general cases.

[8] See Appendix B for further details on the existence of the integral and the solution of the wage differential equation.

Determining the value of $\sum_{j=1}^{n} N_j \frac{\partial w_j(\mathbf{N})}{\partial N_i}$ from equation (8) and replacing it into (5), one obtains straight away the following expression for labor demand

$$F_i(\mathbf{N}) = \underbrace{w_i(\mathbf{N}) + \frac{\gamma_i(r+s_i)}{q_i}}_{\text{Labor costs}} + \underbrace{\sum_{j=1}^{n} N_j \int_0^1 z^{\frac{1-\beta_j}{\beta_j}(\frac{\beta_i}{1-\beta_i}+1)} F_{ji}(\mathbf{N}\mathbf{A}_j(z))dz}_{\text{Empl. effect on wages}} \tag{11}$$

where it can be seen that over-employment, defined here as a marginal product below the labor costs, can arise when the last term above is negative, i.e. when the negative effect of F_{ii} dominate over the other, possible positive effects, of F_{ij}.

We can simplify the exposition of the two main equations (9) and (11) of partial equilibrium, namely the wage equation and labor demand, after an integration by part in equation (11), as follows:

$$OE_i F_i(\mathbf{N}) = w_i(\mathbf{N}) + \frac{\gamma_i(r+s_i)}{q_i} \tag{12}$$

$$w_i(\mathbf{N}) = (1-\beta_i) r U_i + OE_i \beta_i F_i(\mathbf{N}), \text{ for } i = 1,..,n. \tag{13}$$

where both equations are parametrized by OE_i defined as an over-employment factor equal to

$$OE_i = \frac{\int_0^1 \frac{1}{\beta_i} z^{\frac{1-\beta_i}{\beta_i}} F_i(\mathbf{N}\mathbf{A}_i(z))dz}{F_i(\mathbf{N})} > 0 \text{ and } \lesseqgtr 1 \tag{14}$$

Indeed, it appears that, since the right hand-side in equation (12) is again the labor cost of group i, including the turnover cost, the marginal product of labor i is below its labor cost and there is over-employment if and only if $OE_i > 1$. If $OE_i < 1$, the marginal product is above the labor cost and there is instead under-employment of labor i.

Equations (12) and (13) fully characterize the partial equilibrium and wage determination at the firm's level. To get a sense of the intuitions of the various concepts introduced here, notably OE_i and $\mathbf{A}_i(z)$, let us first study specific cases.

2.2 Specific cases

2.2.1 The single labor case

The wage equations (9) can be reduced to:

$$w(N) = (1-\beta)rU + \int_0^1 z^{\frac{1-\beta}{\beta}} F'(Nz)dz. \tag{15}$$

while labor demand (11) is

$$F'(N) = w(N) + \frac{\gamma(r+s)}{q} + N\int_0^1 z^{\frac{1-\beta}{\beta}+1} F''(Nz)dz$$

When returns to labor are constant, i.e. when $F'(N) = y$ for all N and thus $F'' = 0$, one obtains the standard value of wages and labor demand in the matching literature. The wage is a weighted average of the reservation wage rU and of the marginal product of labor, the latter being now independent of N, $w = (1-\beta)rU + \beta y$. Labor demand is such that the marginal product of labor is equal to the labor cost.

When there are strictly decreasing returns to scale, i.e. as soon as $F'' \leq 0$ with strict inequality over some sub-part of $(0, N)$, things are different. The first equation above shows, as discussed above and in Stole and Zwiebel, that *infra-marginal* products of labor contribute to wages with a weight represented by the factor $z^{(1-\beta)/\beta}$ in the integrals which is decreasing with the distance to N, i.e. distance to the margin. The second equation above has a strictly negative integral. This clearly means that *firms exploit decreasing returns to scale to reduce workers' wages in expanding employment.* This case was first analyzed in Smith (1999) who focussed more on normative results, notably of the efficiency of the job creation margin. As we will show now, once the complications of the analytical solution are behind, the heterogenous cases bring many more insights.

2.2.2 Homogeneous production functions with identical bargaining power

Let us first start with the case in which all groups have the same bargaining power. In such a case, equation (10) shows that $\mathbf{NA}_i(z)$ is proportional to $\mathbf{N}$ and actually equal to $z\mathbf{N}$. If in addition the production function is homogeneous of degree $1-\lambda$, $0 \leq \lambda < 1$, the OE_i term writes

$$OE_i = (1-\beta\lambda)^{-1} \geq 1 \tag{16}$$

The marginal productivity is always below the marginal cost of labor when labor has decreasing returns to scale ($\lambda > 0$). When $\lambda = 0$, the OE factor goes to one and one is back to the standard matching analysis with no over-employment.

The value of the OE term in equation (16) in fact indicates that workers' bargaining power and returns to scale interact complementarily with each other. The further away from a constant returns to scale world, the larger the impact of the bargaining power of workers on the propensity of firms to raise employment. A partial and quite surprising conclusion here is that over-employment is a feature of high bargaining power.

This result and the one obtained with a single type of labor seem to be a good illustration of Stole and Zwiebel's convincing story that intra-firm bargaining leads to over-employment. In those cases, our model as Stole and Zwiebel's model always predict over-employment as OE is larger than 1. However, the generalization we offer with several labor inputs and different bargaining power shows that the single factor case or the identical bargaining power cases can be misleading. When the bargaining power parameters are different, the terms OE_i can be either larger or smaller than one.

2.2.3 Homogeneous production functions with different bargaining power

When β_i differ across groups, there is a *geometric* interpretation of $\mathbf{NA}_i$: different bargaining power distort the metric of the space of employment, and the distortion is such that groups with higher bargaining power than group i count more in infra-marginal products of equation (9), as they are given an exponent larger than 1. The converse holds for groups with a smaller bargaining power than i. In the limit, a group j with no bargaining power is not given any weight in the integral. In wage determination, the *distortion factor* can be measured by the quantity

$$\chi_{ij} = \frac{1-\beta_i}{\beta_i}\frac{\beta_j}{1-\beta_j} \tag{17}$$

with $\chi_{ii} = 1$ and $\chi_{ij} = \chi_{ji}^{-1}$. This distortion factor will show up in the value of OE_i: with a Cobb-Douglas technology $F(\mathbf{N}) = \prod_i N_i^{\alpha_i}$, and denoting by $\lambda = 1 - \Sigma_j \alpha_j$ to keep consistent notations throughout, one obtains:

$$OE_i = [1 - \beta_i\lambda + \beta_i \Sigma_{j\neq i}\alpha_j(\chi_{ij} - 1)]^{-1} \tag{18}$$

The first interesting new finding here is that the distortion factor χ_{ij} interacts with the individual returns to scale specific to each group. Notably, for a given level of β_i, a higher β_j implies a higher χ_{ij} which means a lower OE_i, i.e. a lower over-employment of group i. This means that over-employment of a group can be reduced if other groups have a high bargaining power, leading even to a situation of under-employment, in which the marginal productivity of type$-i$ workers is higher than their marginal cost ($OE_i < 1$). The intuition is that decreasing the employment of type$-i$ workers allows the firm to decrease the marginal productivity of type$-j$ workers (because F_{ij} is positive here), and then their wages. Therefore, the firm faces a trade-off: recruiting less type$-i$ workers increases their marginal productivity and then their wage, but decreases the wages of the other workers. The latter effect dominates if the bargaining power of the other workers is relatively strong. This trade-off explains why workers with strong bargaining powers can be over-employed whereas workers with weak bargaining power can be under-employed. It should be noticed that this result cannot hold if $F_{ij} < 0$, for all (i, j) because, in that case, there is necessarily over-employment for all labor types, as shown by equation (12).

The second interesting effect is that, in absence of decreasing returns to scale, i.e. when $\lambda = 0$ or $\Sigma_j\alpha_j = 1$, OE_i can still be greater or smaller than 1. The distortion factor χ_{ij} on over-employment has still the same effect on over-employment and, if larger than 1, leads to under-employment. OE_i is also influenced by the returns to scale of the other group α_j. The direction of this effect depends on whether χ_{ij} is greater or lower than 1: indeed, a group with stronger decreasing returns to scale (lower α_j) will lead to over-employment of group i only if $\beta_j > \beta_i$. Take for instance the two factors case with $\beta_1 = 1/3$ and $\beta_2 = 0.5$, we have the distortion parameter $\chi_{12} = 2$ and $\chi_{21} = 1/2$, and thus

$$OE_1 = (1 + 2\alpha_2/3)^{-1} < 1 \ ; OE_2 = (1 - \alpha_1/4)^{-1} > 1$$

This example suggests that increases in the bargaining power of one type of worker can raise their employment level and might have ambiguous effects on aggregate employment. Before discussing this, we will extend the model to physical capital and come back on it in Section 5.

2.3 Partial equilibrium results on wages

A change in β_j has a direct impact on *all* bargained wages. Consider the differentiation of equation (9), for a given level of $\mathbf{N}$, with respect to β_j. One obtains, still at constant rU_i:

$$\frac{\partial w_i(\mathbf{N})}{\partial \beta_j} = \int_0^1 \frac{z^{\frac{1-\beta_i}{\beta_i}} \log(z)}{(1-\beta_j)^2} N_j \frac{1-\beta_i}{\beta_i} z^{\frac{1-\beta_i}{\beta_i}\frac{\beta_j}{1-\beta_j}} F_{ij}(\mathbf{N}\mathbf{A}_i(z))dz \text{ if } i \neq j$$

Given that $\ln z < 0$ for all z between 0 and 1, this equation indicates that an increase in β_j entails a drop of the wages of workers i who are complement to type-j workers (such that $F_{ij} > 0$) but an increase in the wage of substitute workers (such that $F_{ij} < 0$). It is worth noting that this mechanism can lead to conclusions that are very different from those obtained from a standard wage equation, that posits $w_i = (1 - \beta_i)rU_i + \beta_i F_i(\mathbf{N})$. This effect relies on the assumption that wages are continuously renegotiated. Indeed, in case of disagreement with a type-i worker, the worker leaves the firm and the other wages are renegotiated. Let us suppose that $F_{ij} < 0$. In this case, the departure of a type-i worker leads to an increase in the marginal productivity of type-j workers who will get a wage increase that will be larger the larger is their bargaining power β_j. Therefore, the loss of the firm in case of disagreement with a type-i worker is larger, the larger the bargaining power of type-j workers, which implies that the wage of substitutable type-i workers increases with β_j.

This result has the consequence that substitutable workers might support together more than complementary workers. It is reminiscent of the result of Horn and Wolinsky (1988) who analyzed the relation between the pattern of unionization between two unions, each union representing one type of labor, and the degree of substitutability between the two types of labor. Horn and Wolinsky argued that when the two types of labor are close substitute (the marginal revenue product of one type is decreasing in the quantity of the other), then the equilibrium form of unionization is an encompassing union. When the two types of labor are sufficiently strong complement, the two types are likely to be organized in two separate unions. Obviously, our result holds true only for a given level of $\mathbf{N}$, i.e. among the workers who are within the firm. The attitude of employees towards potential entrants might be very different, as insiders might be opposed to the recruitment of workers substitutable to them but support the hiring of workers who instead increase their own marginal productivity.[9]

3 Labor market equilibrium

So far, the analysis has been focused on partial equilibrium, where the values of the reservation wage and of the labor market tightness are exogenous. Let us analyze now labor market equilibrium, where the reservation wage, rU_i, and the labor market tightness, θ_i, are endogenous variables. The equilibrium value of rU_i is given by

$$rU_i = b_i + \theta_i q_i(\theta_i)(E_i - U_i) = b_i + \gamma\theta_i \frac{\beta_i}{1-\beta_i}$$

where b_i stands for the income flow of unemployed type-i workers. The first equality is the arbitrage equation, in which b_i is the flow income of unemployed workers of group i and $\theta_i q_i(\theta_i) =$

[9] This point shows up in calculating $\partial w_i/\partial\beta_i$ from equation (9). First, there is a standard positive term: workers get a share of their marginal product $F_i(\mathbf{NA}_i(z))$ that increases with their bargaining power. However, the interesting new terms here are the cross derivatives F_{ij}. It turns out that workers who are substitutes to i reduce the positive impact of β_i on wages. When the firm has more possibility to substitute some workers to the type-i worker, the loss of the firm in case of disagreement with the type-i worker is smaller. Accordingly, for every category of worker, the presence of other, closed substitute workers limits their possibility to take advantage of their own bargaining power.

$h(u_i, V_i)/u_i$ is their exit rate from unemployment, and the second equality combines the first one with Nash-bargaining (7) and the optimal value of marginal employment in (3).

Now, eliminating wages from (12) and (13), one obtains n relations between $\theta = (\theta_1, \theta_2, ...)$ and $\mathbf{N}$ which are vacancy curves denoted by (VC)

$$OE_i F_i(\mathbf{N}) = \frac{1}{\beta_i}\int_0^1 z^{\frac{1-\beta_i}{\beta_i}} F_i(\mathbf{N}\mathbf{A}_i(z))dz_i = b_i + \frac{\beta_i}{1-\beta_i}\gamma_i\theta_i + \frac{1}{1-\beta_i}\frac{\gamma_i(r+s_i)}{q_i(\theta_i)} \qquad \text{(VC)}$$

The right-hand side is without ambiguity increasing in θ_i. Therefore, this equation provides a unique θ_i as a function of all $\mathbf{N}$.[10] On the other hand, the (VC) curve does not characterize the equilibrium in each single segment: all demand functions are interdependent, as labor market tightness depends on other groups' employment.

The steady-state condition on flows and stocks on each labor market $i = 1, .., n$, implies that $(1 - N_i)\theta_i q_i(\theta_i)$ exits from unemployment and $N_i s_i$ entry into unemployment per unit of time compensate each other. Accordingly, the flow equilibrium implies in steady-state that:

$$N_i s_i = (1 - N_i)\theta_i q_i(\theta_i) \qquad \text{(SS)}$$

This equation implicitly defines N_i as an upwards sloping function of θ_i. Inverting $\theta_i(N_i)$ from equation (SS) and plugging into (VC), we have a system of n equations and n unknowns $\mathbf{N}$.

We can prove that if the production function is Cobb-Douglas and has constant or decreasing returns to scale, an equilibrium $\mathbf{N}$ exists and is unique. The proof is in Appendix C. Uniqueness can be shown in the Cobb-Douglas case because the OE_i factors are constant and depend only on β_i and exponents of the Cobb-Douglas function.[11]

Interestingly, the strategy of the proof remains valid in imposing $OE_i \equiv 1$ for all i, i.e, neglecting all strategic interactions at the firm's level. This result means that, in the large firm

[10] In the space (θ_i, N_i), equation (SS) implies that N_i has limits zero (resp. one) as θ_i goes to zero (resp. infinity). Thus, if there exists a θ_i when $N_i = 1$ for a given set of N_j, $j \neq i$, which corresponds to the usual viability assumption on the sub-market i, there is a single intersection between the two curves.

[11] It may be possible to prove that this uniqueness results generalizes to production functions which are locally more substitute than the Cobb–Douglas (i.e generalized CES), using the strategy derived in the general equilibrium literature. See e.g. Mas Colell (1991).

matching model *à la* Pissarides, there is also existence and uniqueness which, to our knowledge, had never been established except in the case $n = 1$. When the production function is not Cobb-Douglas but still exhibits constant or decreasing returns to scale, we have never been able to find numerical examples of multiple solutions. It seems that the multiplicity of solutions would require very strong and rapid variations of the sign of F_{ij} which CES function generalizing Cobb-Douglas functions do not have.

We also have some general results for the existence of a solution. Indeed, we can show that a solution always exists when the production is not Cobb-Douglas under some conditions on the value of unemployment benefits. If we define I_n the hypercube $(0,1) \times (0,1)...(0,1)$ to which a solution $\mathbf{N}$, must belong, and $\underline{I_n}$ the hypercube minus its surface (thus excluding 0 and 1 for all components of $\mathbf{N}$) and also $\mu_i = \inf_{\mathbf{N} \in I_n} \frac{\partial F}{\partial N_i}(\mathbf{N})$, then a solution always exists if

$$\inf_i \mu_i \geq \sup_i b_i$$

which generalizes the usual viability rule stating that the marginal product is larger than unemployment benefits. Further, it is interior, i.e. solutions belong to $\underline{I_n}$. Note however that for a Cobb-Douglas production functions, $\mu_i = 0$ which would impose $b_i = 0$ which is too restrictive an assumption.

4 Extension to capital

Pissarides (1990, 2000) stressed that the large firm model is equivalent to the one-firm one-job model even in the presence of capital and labor. However, this result relies on the assumption of *instantaneous adjustment of the stock of capital*, as Pissarides discusses and as we emphasized in Cahuc and Wasmer (2001). Since it is conventionally believed that firms actually face severe adjustment costs adopting new capital or selling old capital, a general model of intrafirm bargaining in presence of such adjustments costs is necessary to have a better understanding of the effect of distortions due to search frictions and wage bargaining on employment and real wages. From this point of view, it is interesting to deal with the case where physical capital is a predetermined variable, that cannot change instantaneously when workers quit the firm.

4.1 Firm's program

Let us extend the previous analysis by the integration of capital into the model. We assume that the production function now reads $F(\mathbf{N}, K)$, where K denotes the stock of capital and $\mathbf{N}$ the vector of labor inputs. We denote by $w(\mathbf{N}, K)$ the wage, that is potentially a function of the capital and the vector of jobs. A straightforward extension of our previous analysis indicates that the expression of the negotiated wage will be similar to the one found before, as the presence of capital, which is a fixed input during the bargaining process, does not modify the previous analysis. Accordingly, one can write

$$w_i(\mathbf{N}, K) = (1-\beta_i)rU_i + \int_0^1 z^{\frac{1-\beta_i}{\beta_i}} F_i(\mathbf{N}\mathbf{A}_i(z), K)dz, \; i = 1, .., n. \tag{19}$$

where $F_i(\mathbf{N}\mathbf{A}_i(z), K)$ denotes the partial derivative of F with respect to the ith coordinate of the vector $\mathbf{N}\mathbf{A}_i(z)$.

Denoting by I the investment, by δ the depreciation rate of capital, the value function of the firm, $\Pi(\mathbf{N}, K)$, solves the Bellman equation:

$$\Pi(\mathbf{N}, K) = \underset{\{V_i, I\}}{\text{Max}} \left(\frac{1}{1+rdt}\right) \left\{ \left[F(\mathbf{N}, K) - \sum_{j=1}^{n} w_j(\mathbf{N}, K)N_j - \gamma_j V_j - I \right] dt + \Pi(\mathbf{N}^+, K^+) \right\}, \tag{20}$$

subject to $K^+ = K(1-\delta dt) + Idt$; $N^+ = N(1-sdt) + Vq(\theta)dt$

The first-order and the envelope conditions for an optimal choice of V_i and I are presented in appendix. We obtain an identical first order condition on employment (see Equation (D25) in Appendix D) and the following first order condition on capital:

$$F_K(\mathbf{N}, K) = r + \delta + \underbrace{\sum_{i=1}^{n} N_i \frac{\partial w_i(\mathbf{N}, K)}{\partial K}}_{\text{Cap. effect on wages}} \tag{21}$$

It can be seen that there is a term that does not show up usually in the capital demand equation (21): the marginal product of capital in equilibrium has to be equal to the sum of discount factors plus a "capital effect on wages" term reflecting that raising capital affects the marginal product of labor, which will thus affect the wage bill, and so the optimal stock of capital has to incorporate these effects.

4.2 Capital and labor demand

Using the wage equation (19), we can rewrite the capital and labor demand as:

$$F_i(\mathbf{N}, K) = \underbrace{w_i(\mathbf{N}, K) + \frac{\gamma_i(r + s_i)}{q_i(\theta_i)}}_{\text{Labor costs}} + \underbrace{\sum_{j=1}^{n} N_j \int_0^1 z^{\frac{1-\beta_j}{\beta_j}\left(\frac{\beta_i}{1-\beta_i}+1\right)} F_{ji}(\mathbf{N}\mathbf{A}_j(z), K)dz}_{\text{Empl. effect on wages}} \tag{22}$$

$$F_K(\mathbf{N}, K) = r + \delta + \underbrace{\int_0^1 \sum_{i=1}^{k} N_i z^{\frac{1-\beta_i}{\beta_i}} F_{iK}(\mathbf{N}\mathbf{A}_i(z), K)dz}_{\text{Cap. effect on wages}} \tag{23}$$

The employment equation (22) has the same features as in the absence of capital in equation (11): the marginal productivity of labor is different than the labor cost. Equation (23) shows that the sign of the impact of an increase in the capital stock on the wage bill depends on the sign of the cross-derivatives of capital with the labor inputs and the bargaining power of every group of workers. Let us remark that this term is necessarily positive if there is only one type of labor input, because the concavity of the production function together with the constant returns assumption implies that the cross derivative between capital and labor is necessarily positive.[12] In this case, strategic interactions between capital and labor necessarily imply under-investment. The bargaining power allows the workers to take a share of the returns of capital and then diminishes the returns of investment to the firm. This is the standard hold-up problem, stressed by Grout (1984) and many others, which entails under-investment.

This properties show up clearly with a Cobb-Douglas technology, $F(\mathbf{N}, K) = N^{1-\alpha}K^{\alpha}$, if

[12] If $F(N, K)$ if homogeneous of degree one, Euler's theorem implies that $F_N(N, K)$ is homogeneous of degree zero and thus $NF_{NN}(K, N) = -KF_{KN}(K, N)$. Since the concavity of F implies that $F_{NN} < 0$, one gets $F_{KN} > 0$.

one re-introduces the convenient notation for the over-employment factor $OE = (1 - \alpha\beta)^{-1}$, which yields the capital and labor demand:

$$(1-\beta)OE \cdot F_K(\mathbf{N}, K) = r + \delta \tag{24}$$

$$OE \cdot F_N(\mathbf{N}, K) = w(N, K) + \frac{\gamma(r+s)}{q} \tag{25}$$

where, using the notation $k = K/N$, marginal products are equal to $F_K(\mathbf{N}, K) = \alpha k^{\alpha-1}$ and $F_N(\mathbf{N}, K) = (1-\alpha)k^{\alpha}$. Equation (24) shows that the marginal product of capital differs from its cost $r + \delta$ by two terms, $(1-\beta)$ and OE. On the one hand, $(1-\beta)$ characterizes the hold-up problem, in the sense that firms obtain only a share $1-\beta$ of marginal profits. On the other hand, over-employment (aiming at reducing the wage bill) raises the marginal product of labor and partly compensate under-investment. One can easily see that under-investment dominates as $OE(1-\beta)$ is smaller than one.

However, the "capital effect on wages" term can be either positive or negative when there are multiple labor inputs. It is obviously negative if $F_{iK} < 0$ for all i. More generally, it is negative if the bargaining power of the workers who are substitutable to capital is large, relatively to the workers who are complement to capital, because workers with larger β_i have larger weight in the term[13] $\int_0^1 \sum_{i=1}^n N_i z^{\frac{1-\beta_i}{\beta_i}} F_{iK}(\mathbf{N}\mathbf{A}_i(z), K)dz$. If this term is negative, strategic interactions between capital and multi-labor inputs entail over-investment. The firm over-invests in order to lower the wage of the workers who are substitutable to capital.

5 Implications

The theory of labor demand and wages, extended to capital investments, has many quantitative implications explored in detail in the discussion paper version of this text. We only summarize some of them.

[13] Because the derivative of $z^{\frac{1-\beta_i}{\beta_i}}$ with respect to β_i is $-(1/\beta_i^2)z^{\frac{1-\beta_i}{\beta_i}} \ln z > 0$.

A first result is about the relative position of wages with respect to the reservation wage rU_i and the marginal product of labor. Equations (12) and (13) indicate that the wage is in the general case above rU_i.[14] It is interesting to remark that rU_i is a reservation wage: indeed, from (6), it is clear that workers are indifferent between employment and unemployment i.e. $E_i = U_i$ if and only if $w_i = rU_i$. Thus, as long as turnover costs γ_i are positive, workers obtain a rent over their reservation wage. This result is also in contrast with the analysis of Stole and Zwiebel who look at the case in which there is no hiring cost ($\gamma_i = 0$), which in fact implies that the employed workers are paid their reservation wage in their setup as well as in our setup.

One can show on simulations that when the turnover costs γ_i are small but positive, we have $F_i(\mathbf{N}) < rU_i < w_i$, i.e. turnover costs are sufficiently small for firms to over-employ and reduce the marginal product of labor below the reservation wages. As γ_i increase, this over-employment possibility becomes more costly, and the marginal product of labor becomes larger than the reservation wage but smaller thant the wage: $rU_i < F_i(\mathbf{N}) < w_i$. Eventually, for sufficiently high values of the cost of vacant jobs, one can get the standard situation, $rU_i < w_i < F_i(\mathbf{N})$, met in matching models where the wage is a convex combination of the marginal product of labor and of the workers' reservation wage.

A second result concerns the level of employment. We can show that whether or not employment increases or decreases when intrafirm bargaining is introduced crucially depends on capital adjustment. Indeed, compared to an economy with endogenous capital and where wages correspond to the standard wage solution in Pissarides (2000), we find that the introduction of intrafirm bargaining reduces employment if capital is fixed at the benchmark level; but that it raises employment if capital adjusts.

A third result concerns the effect of the bargaining power β_i on wages and employment.

[14] Eliminating $OE_iF_i(\mathbf{N})$ from (12) and (13) one gets

$$w_i(N) = rU_i + \frac{\beta_i}{1-\beta_i}\frac{\gamma_i(r+s_i)}{q_i}.$$

Ignoring capital first, we can investigate these two effects together (still in partial equilibrium) in eliminating wages from equations (12) and (13) to get:

$$OE_iF_i(\mathbf{N},K) = \underbrace{rU_i + \frac{1}{1-\beta_i}\frac{\gamma_i(r+s_i)}{q_i}}_{\text{Labor costs}}$$

A larger β_i raises labor costs in the last term above which tends to reduce employment, but also raises incentives to over invest with a larger OE_i (defined in equation (18)). These two effects that play in opposite directions suggest that employment and labor market tightness θ_i may have non monotonic variations with respect to the index of bargaining power of worker β_i. The positive employment effect of increases in the bargaining power of worker does not dominate in the single labor case for relevant values of the parameters. Unreported simulations show that it is only when the returns to scale become very decreasing that the bargaining power of workers reduces unemployment.

A very different conclusion is reached with two types of labor and two distinct bargaining powers. Assume that β_2 increases. The employment of the type-1 workers can increase because their bargaining power diminishes with β_2 as it has been shown previously. The employment level of the type-2 workers increases because the over-employment margin is raised by their bargaining power. These two effects push the employment of both type of workers upwards for sufficiently low values of β_2. However, the strong bargaining power of the type-2 workers exerts a negative impact on profits, which is detrimental to labor demand and it turns out that there is a non-monotonous relation between employment and β_2 for each group of worker.

We can also investigate the role of the bargaining power of workers on physical capital. If capital is predetermined, as it is assumed in Section 4, bargaining power increases diminish the capital-labor ratio, that goes to zero when the bargaining power of workers goes to one. This implies that the unemployment rate is much more sensitive to changes in the bargaining power than in the economy without adjustment cost of capital, in which the capital-labor ratio is not influenced by β.

Can we conclude that intra-firm bargaining is always associated with under-investment in capital which always lead under-employment? The answer is in fact no. With two types of labor and capital sufficiently substitutes to labor type 2, it is easy to observe that increases in β_2 can increase physical investment. The firm over-invests in capital in order to decrease the marginal productivity of labor 2 which limits their wage increase. The total impact of changes in β_2 on the employment of type-2 workers stems from the composition of three effects: first, over-employment of type-2 workers; second, over-investment; these two effects, which play positively on employment, aim at decreasing the marginal productivity of type-2 workers in order to exert a downward pressure on their wage when β_2 increases. But, as β_2 increases, there is also less surplus accruing to the firm; this third effect leads to less job creation.

6 Concluding comments

We showed that the consequence of intra-firm bargaining in a model with labor market search frictions is precisely the existence of overemployment, as firms indeed recruit up to a point where the marginal productivity of labor is smaller than marginal labor cost. However, the presence of search frictions and of physical investment decisions implies that this phenomenon does not necessarily give rise to a positive impact of the bargaining power of workers on employment in the single labor case. Actually, quantitative exercises suggest that the opposite holds true when labor is considered as a homogeneous input: the strategic interactions involved in intra-firm bargaining contribute to increase unemployment. Moreover, increases in the bargaining power of workers are more detrimental to employment when firms use employment and physical investment than employment only as strategic instruments to manipulate wages. From this point of view, it would seem that the overemployment phenomenon put forth by Stole and Zwiebel (1996a, b) does not play an important role at the macroeconomic level.

However, this conclusion has been reached only in a framework with homogeneous labor. Now, the analysis of the multi-labor inputs case allows highlighting some non-trivial mechanisms that mitigate this conclusion. First, it turns out that the firm can underemploy some low bargaining power workers in order to over-employ the high-bargaining power workers. Second, this phenomenon implies that increases in the bargaining power of one type of workers can lead to increases in overall employment and physical investment for relevant set of values of the parameters of the model. Our contribution is the first step to a better understanding of the consequences of intra-firm bargaining in labor markets with frictions.

From this point of view, many extensions would be necessary. First, the continuous renegotiation of wages is a strong assumption. It would be interesting to look at other cases where the wages would not be renegotiated continuously. Second, our approach neglects job to job mobility. Introducing job to job mobility would allow us to account for the strategic interactions within the firm and between different firms (as it is the case in Cahuc et al., 2006, and Shimer, 2004). Third, our analysis of the interactions between heterogeneous workers could be a promising way to study the emergence of collective versus individual bargaining.

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Appendix

A Demand for factors

First-order conditions to the problem of the firm in equation (1) subject to constraint (2) are

$$-\gamma_i + q_i J_i(\mathbf{N}^+) = 0, \tag{A1}$$

$$\left[\frac{\partial F(\mathbf{N})}{\partial N_i} - \sum_{j=1}^{n} N_j \frac{\partial w_j(\mathbf{N})}{\partial N_i} - w_i(\mathbf{N})\right] dt + (1 - s_i dt) J_i(\mathbf{N}^+) = J_i(\mathbf{N})(1 + r dt). \tag{A2}$$

and lead, in a steady-state where $N^+ = N$, to (3) and (4).

B Wage determination with multiple labor inputs

We now solve the system of differential equations (8) by first considering specific cases and then, excluding those cases, proceed to the general solution.

B.1 Case $\beta = 0$

In this case, $w_i(\mathbf{N}) = rU_i$: the wage is equal to the equity value of unemployment, i.e. the reservation wage of workers. We now assume $\beta > 0$.

B.2 Case $n = 1$

With one type of labor, equation (8) becomes

$$w(N) = (1-\beta)rU + \beta\left(\frac{\partial F(N)}{\partial N} - N\frac{\partial w(N)}{\partial N}\right) \tag{B3}$$

which reads, without the constant $(1-\beta)rU$:

$$\frac{dw}{dN} + \frac{w}{\beta N} - \frac{F'(N)}{N} = 0 \tag{B4}$$

The solution of the homogeneous equation, $\frac{dw}{dN} + \frac{w}{\beta N} = 0$, reads

$$w(N) = CN^{-1/\beta} \tag{B5}$$

where C is a constant of integration of the homogeneous equation. Assuming that C is a function of N and deriving (B5) with respect to N, one gets

$$\frac{dw}{dN} = \frac{dC}{dN}N^{-1/\beta} - \frac{1}{\beta}CN^{-1-\left(\frac{1}{\beta}\right)} \tag{B6}$$

Substituting (B5) and (B6) into (B4) yields

$$\frac{dC}{dN} = N^{\frac{1-\beta}{\beta}}F'(N)$$

or, by integration

$$C = \int_0^N z^{\frac{1-\beta}{\beta}}F'(z)dz + D$$

where D is a constant of integration. This last equation implies that the solution of equation (B4) satisfies

$$w(N) = \int_0^1 z^{\frac{1-\beta}{\beta}} F'(Nz)dz + DN^{-1/\beta}$$

or equivalently:

$$w(N) = \frac{1}{N}\int_0^1 z^{(1/\beta)-2} zNF'(zN)dz + DN^{-1/\beta} \tag{B7}$$

Since $1/\beta - 2 > -1$ for all $\beta < 1$, it is sufficient to show that $NF'(N)$ is continuous in zero to obtain the convergence of the integral $\int_0^1 z^{\frac{1-\beta}{\beta}} F'(Nz)dz$.

Assumption 1: Assume that $NF'(N)$ is continuous in zero and more generally, $N^p F^{(p)}(N)$ is continuous in zero where $F^{(p)}(.)$ is the p-th derivative of F.

As a consequence of this assumption, the integral $\int_0^1 z^{\frac{1-\beta}{\beta}} F'(Nz)dz$ is defined for all positive N and is actually smooth and can be differentiated at all orders for all strictly positive N.

At this stage, an additional assumption is needed to determine the constant of integration Din equation (B7). Henceforth, it will be assumed that $\lim_{N\to 0} Nw(N) = 0$.Thus, equation (B7) implies that $D = 0$.

Accordingly, the solution of equation (B3) is:

$$w(N) = (1-\beta)rU + N^{-1/\beta}\int_0^N z^{\frac{1-\beta}{\beta}} F'(z)dx. \tag{B8}$$

B.3 Case $n \geq 1$ and $\beta_i \equiv 1$

Equation (8) is now:

$$w_i(\mathbf{N}) = \frac{\partial F(\mathbf{N})}{\partial N_i} - \sum_{j=1}^n N_j \frac{\partial w_j(\mathbf{N})}{\partial N_i}, \text{ for } i = 1, .., n. \tag{B9}$$

Remarking that

$$w_i(\mathbf{N}) + \sum_{j=1}^n N_j \frac{\partial w_j(\mathbf{N})}{\partial N_i} = \frac{\partial \sum_{j=1}^n N_j w_j(\mathbf{N})}{\partial N_i},$$

the system is described by

$$\frac{\partial(\sum_{j=1}^n N_j w_j(\mathbf{N}) - F(\mathbf{N}))}{\partial N_i} = 0,$$

for all $i = 1, 2, ...n$. This implies that $\sum_{j=1}^n N_j w_j(\mathbf{N}) - F(\mathbf{N})$ is a constant of $\mathbf{N}$, i.e. of N_i for all i, that is:

$$\sum_{j=1}^n N_j w_j(\mathbf{N}) = F(\mathbf{N}) \tag{B10}$$

under the assumption that

$$\lim_{N_i\to 0} w_i(\mathbf{N})N_i = 0$$
$$F(0) = 0$$

Equation (B10) does not provides the individual wage of each labor input, but tells us that profits are equal to zero. Since firms have to pay a vacancy cost to hire workers, no firm wishes to open a vacancy and thus, there is no general equilibrium solution with $N_i > 0$ to this problem.

B.4 General Case ($n \geq 1$ and $0 < \beta_i < 1$), but with different β's

To solve for this system, we need to take the partial derivative of equation (8) for a given i with respect to a labor input l, $i \neq l$. We obtain,

$$\frac{\partial w_i}{\partial N_l} + \beta_i \frac{\partial w_l}{\partial N_i} = \beta_i \left(\frac{\partial^2 F(\mathbf{N})}{\partial N_i \partial N_l} - \sum_{j=1}^{n} N_j \frac{\partial^2 w_j}{\partial N_i \partial N_l} \right)$$

which rewrites, denoting by $(E_i)'_l$ the second-order differential equation :

$$\frac{\partial w_i}{\partial N_l}(1-\beta_i) = \beta_i \frac{\partial^2}{\partial N_i \partial N_l} \left(F(\mathbf{N}) - \sum_{j=1}^{n} N_j w_j \right) \tag{$(E_i)'_l$}$$

for all $i, l = 1, ...n$. One can then note that the difference between $\beta_l (E_i)'_l$ and $\beta_i (E_l)'_i$ eliminates the symmetric terms, which, given that $0 < \beta_i < 1$, leads to

$$\frac{\partial w_l}{\partial N_i} = \frac{\beta_l}{1-\beta_l} \frac{1-\beta_i}{\beta_i} \frac{\partial w_i}{\partial N_l} \tag{B11}$$

for all $i, l = 1, ...n$. Let us denote by

$$\chi_{il} = \frac{\beta_l}{1-\beta_l} \frac{1-\beta_i}{\beta_i} \tag{B12}$$

It implies that

$$\sum_{j=1}^{n} N_j \frac{\partial w_j(\mathbf{N})}{\partial N_i} = \sum_{j=1}^{n} \chi_{ij} N_j \frac{\partial w_i(\mathbf{N})}{\partial N_j} \tag{B13}$$

which allows to conveniently rewrite (8) as

$$w_i(\mathbf{N}) = (1-\beta_i) r U_i + \beta_i \left(\frac{\partial F(\mathbf{N})}{\partial N_i} - \sum_{j=1}^{n} \chi_{ij} N_j \frac{\partial w_i(\mathbf{N})}{\partial N_j} \right) \tag{B14}$$

Let us first assume identical β_i's: $\beta_i \equiv \beta$. Then, $\chi_{ij} \equiv 1$. To proceed with the system (B14) for $i = 1, ...n$, , one can remark that $\sum_{j=1}^{n} N_j \frac{\partial w_i(\mathbf{N})}{\partial N_j}$ has a simple expression in another system of coordinates. Indeed, using the generalized spherical coordinates ρ, ϕ_1,...,ϕ_{n-1}, where ρ is the distance to the origin such that $\sum_{j=1}^{n} N_j^2 = \rho^2$ and ϕ_i are the angles of projection in different sub-planes, one can write:

$$\begin{aligned}
N_1 &= \rho \cos\phi_1 ... \cos\phi_{n-2} \cos\phi_{n-1} \\
N_2 &= \rho \cos\phi_1 ... \cos\phi_{n-3} \sin\phi_{n-2} \\
N_2 &= \rho \cos\phi_1 ... \cos\phi_{n-2} \sin\phi_{n-3} \\
&\vdots \\
N_{n-1} &= \rho \cos\phi_1 \sin\phi_2 \\
N_n &= \rho \sin\phi_1
\end{aligned}$$

Then, we have that, using the notation $\phi = (\phi_1, ..., \phi_{n-1})$ for convenience,

$$\sum_{j=1}^{k} N_j \frac{\partial w_i(\mathbf{N})}{\partial N_j} = \rho \frac{\partial w_i(\rho, \phi)}{\partial \rho}.$$

Note that the economic interpretation of ρ is the scale of the use of labor inputs, while ϕ reflects the proportions in which different labor inputs are used. For instance, $\phi = (0, 0, ..0)$ means that the firm employes only labor of type 1. Then, (B14) in this system of coordinates is simplified to

$$\beta\frac{\partial w_i(\rho,\phi)}{\partial\rho} + w(\rho,\phi) = (1-\beta)rU_i + \beta\frac{\partial F(\rho,\phi)}{\partial N_i} \tag{B15}$$

which is similar to equation (B3) and follows the same resolution. Therefore, one gets:

$$w_i(\rho,\phi) = (1-\beta)rU_i + \rho^{-1/\beta}\left(\kappa_i(\phi) + \int_0^\rho z^{\frac{1-\beta}{\beta}}\frac{\partial F(z,\phi)}{\partial N_i}dz\right). \tag{B16}$$

where $\kappa_i(\phi)$ is a constant which is function of ρ. Using again that $\rho w_i(\rho,\phi)$ goes to zero when ρ goes to zero, we have that the constant $\kappa_i(\phi)$ is identically equal to zero. Further, noticing that if $\mathbf{N} = (\rho,\phi)$, then $(z\rho,\phi) = (zN_1, zN_2, .., zN_n) = z\mathbf{N}$, one can eliminate the spherical coordinates by rewriting this solution as

$$w_i(\mathbf{N}) = (1-\beta)rU_i + \int_0^1 z^{\frac{1-\beta}{\beta}}F_i(z\mathbf{N})dz, \tag{B17}$$

where $F_i$stands for the derivative of the function $F$with respect to its argument $i = 1, .., n$.

Now, in the case β_i different from each other, one simply need to introduce a new variable $\mathbf{M}_i = (M_{i1}, M_{i2}, ...M_{in})$ such that

$$\sum_{j=1}^n M_{ij}\frac{\partial v_j(\mathbf{M}_i)}{\partial M_{ij}} = \sum_{j=1}^n \chi_{ij}N_j\frac{\partial w_i(\mathbf{N})}{\partial N_j}$$

with $v_i(\mathbf{M}_i) = w_i(\mathbf{N})$. Note that this new notation $\mathbf{M}_i$ is indexed on i since variable change needs to be done for each equation (E_i). Denote further $G(\mathbf{M}_i) = F(\mathbf{N})$ the production function. To find the right $\mathbf{M}_i$ as a function of $\mathbf{N}$,denoted by $\mathbf{M}_i(\mathbf{N})$, let us assume a simple form: assume that $M_{il} = M_{il}(N_l)$, then, we only need to have

$$M_{il}\frac{\partial v_j(\mathbf{M}_i)}{\partial M_{ij}} = \chi_{ij}N_j\frac{\partial w_i(\mathbf{N})}{\partial N_j}$$

Since by definition,

$$\frac{\partial w_i(\mathbf{N})}{\partial N_j} = \frac{\partial v_j(\mathbf{M}_i)}{\partial M_{ij}}\frac{dM_{ij}}{dN_j}$$

one obtains a differential equation for M_{ij} which is

$$M_{ij} = \chi_{ij}N_j\frac{dM_{ij}}{dN_j}$$

One needs one solution only, the simplest being

$$M_{ij} = N_j^{1/\chi_{ij}} = N_j^{\chi_{ji}},$$

remarking that $1/\chi_{ij} = \chi_{ji}$. Then, using $\partial F/\partial N_j = \chi_{ji}N_j^{\chi_{ji}-1}\partial G/\partial M_{ij}$ and notably $\partial F/\partial N_i = \chi_{ii}N_j^{\chi_{ii}-1}\partial G/\partial M_{ii} = \partial G/\partial M_{ii}$ since $\chi_{ii} = 1$, the system (8) can be rewritten as

$$v_i(\mathbf{M}_i) = (1-\beta_i)rU_i + \beta_i\left(\frac{\partial G(\mathbf{M}_i)}{\partial M_{ii}} - \sum_{j=1}^n M_j\frac{\partial v_i(\mathbf{M}_i)}{\partial M_{ij}}\right)$$

which is formally equivalent to (B14). Hence, we now a solution for v_i which is

$$v_i(\mathbf{M}_i) = (1-\beta_i)rU_i + \int_0^1 z^{\frac{1-\beta_i}{\beta_i}}G_i(z\mathbf{M}_i)dz,$$

where $G_i$stands for the derivative of the function $G$with respect to its argument $i = 1, .., n$. Coming back to the initial notations, we thus obtain the equation (13):

$$w_i(\mathbf{N}) = (1 - \beta_i) r U_i + \int_0^1 z^{\frac{1-\beta_i}{\beta_i}} F_i(\mathbf{N}\mathbf{A}_i(z)) dz, \ i = 1, .., n. \tag{B18}$$

with

$$\mathbf{A}_i(z) = \begin{pmatrix} z^{\frac{\beta_1}{1-\beta_1} \frac{1-\beta_i}{\beta_i}} & 0 & 0 & 0 \\ 0 & \ddots & 0 & 0 \\ 0 & 0 & z^{\frac{\beta_j}{1-\beta_j} \frac{1-\beta_i}{\beta_i}} & 0 \\ 0 & 0 & 0 & \ddots \end{pmatrix}$$

is a diagonal matrix with $z^{\frac{\beta_j}{1-\beta_j} \frac{1-\beta_i}{\beta_i}}$, $j = 1, .., n$, on its main diagonal.

It should be noticed that, like in the single labor input case, equation (B18) and the existence of $w_i(\mathbf{N})$ rely on two assumptions:

- First: $\lim_{N_i \to 0} w_i(\mathbf{N}) N_i = 0$
- Second: $F(\mathbf{N})$ is continuous for all $N_i \geq 0$ and infinitely differentiable for all $N_i > 0$. Assume in addition that $N_i \frac{\partial F}{\partial N_i}(\mathbf{N})$ is continuous in zero and further, that the quantity $N_1^{m_1} ... N_n^{m_n} \frac{\partial^{\overline{m}} F}{\partial N_1^{m_1} ... \partial N_n^{m_n}}$(denoted for simplicity by $\mathbf{N}^m F^{(m)}(\mathbf{N})$) with $\overline{m} = \sum m_i$ is continuous in zero.

These assumptions imply that $w_i(\mathbf{N})$ exists and is smooth for all $N_i > 0$.

C Existence and uniqueness of the decentralized equilibrium

C.1 Single factor: existence and uniqueness

With a single factor ($n = 1$) and $0 < \beta < 1$, the system writes

$$\frac{1}{\beta} \int_0^1 z^{\frac{1}{\beta} - 1} F'(zN) dz = b + \frac{\beta}{1 - \beta} \gamma \theta + \frac{\gamma (r + s)}{1 - \beta} \frac{1}{q(\theta)}$$

or with simplifying notations,

$$a(N) = v(\theta)$$

with $v' > 0$ and goes from b to $+\infty$. Further, $a(N) \geq 0$ and $a' \leq 0$ since $F'' \leq 0$. Given that $F'(N)$ is decreasing, $a(N) \geq \frac{1}{\beta} \int_0^1 z^{\frac{1}{\beta} - 1} dz \times F'(1) = F'(1)$ so that a sufficient condition for the existence and uniqueness of $\theta(N)$is $F'(1) \geq b$. The other equation (SS) indicates that $N = \frac{p(\theta)}{s + p(\theta)}$ which is strictly increasing with $N = 0$ for $\theta = 0$ and $N = 1$ for $\theta = +\infty$. By continuity, there is then a unique decentralized equilibrium (θ^*, N^*).

C.2 Existence with $n \geq 1$

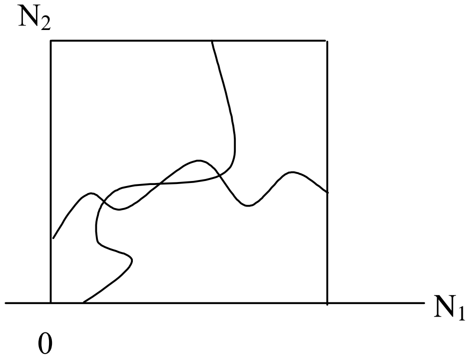
C.2.1 General case

In the case $n \geq 1$ et $0 < \beta_i < 1$, equation (VC) writes with similar notations

$$a_i(N_i) = v_i(\theta_i)$$

We denote by $I_n = (0, 1)^n$ and

$$\mu_i = \inf_{\mathbf{N} \in I_n} \frac{\partial F}{\partial N_i}(N) \geq 0$$



Simple calculations indicates that $a_i(N) \geq \mu_i$. Further, $v_i(\theta_i)$ increases with θ_i with $v_i(0) = b_i$ and $v_i(+\infty) = +\infty$. As before, existence and uniqueness of $\theta_i = \theta_i(N) \geq 0$ is obtained when $\mu_i \geq b_i$. A sufficient condition for the existence of all θ_i is

$$\inf_i \mu_i \geq \sup_i b_i \tag{C19}$$

which is a generalization of the viability condition of labor markets, as μ_i is the marginal product of workers of type i at zero employment. Equations (SS$_i$) $N_i = \frac{p_i(\theta_i)}{s_i + p_i(\theta_i)}$ generate a strictly increasing link between N_i and θ_i for all i. Let us denote by

$$\hat{N}_i = (N_1, \ldots, N_{i-1}, N_{i+1}, \ldots, N_n)$$

the $(n-1)$ array of employment levels of all labor inputs but i. One can eliminate N_i from (VC$_i$) in combining it with (SS$_i$) and create a set of solutions denoted with a superscript $^\times$, so that

$$\theta_i^\times = \theta_i^\times(\hat{N}_i)$$

and then using (SS$_i$) again,

$$N_i^\times(\hat{N}_i) = \frac{p_i(\theta_i^\times(\hat{N}_i))}{s_i + p_i(\theta_i^\times(\hat{N}_i))}$$

The couple $(\theta_i^\times,\ N_i^\times)$ is uniquely obtained from $\hat{N}_i$. Given that N belongs to I_n, the system $N_i^\times(\hat{N}_i)$ implies n hypersurfaces in the hypercube I_n of dimension n.

To obtain an intuition, let us consider the case $n = 2$. We have

$$N_1^\times(N_2) = \frac{p_1(\theta_1^\times(N_2))}{s_1 + p_1(\theta_1^\times(N_2))}$$

and similarly, $N_2^\times(N_1)$. In the square I_2, the equation $N_1^\times(N_2)$ defines a curve going continuously from the bottom of the square to its top. In the same way, $N_2^\times(N_1)$ defines a curve going continuously from the left part of the square to its right part. There is thus at least one intersection $N^* = (N_1^*, N_2^*)$ and thus $\theta^* = (\theta_1^*, \theta_2^*)$. See the figure.

As it indicates, in the general case, there is no reason for having a unique interection to the system of n hypersurfaces.

C.2.2 Cobb-Douglas case

Note that condition (C19) is very restrictive and is not suitable to the Cobb-Douglas case as $\mu_i = 0$. However, in the Cobb-Douglas case, we can exhibit a less restrictive condition. Indeed, when $F(N) =$

$\Pi N_i^{\alpha_i}$, equation (VC$_i$) is in the Cobb-Douglas case

$$OE_i\alpha_i\frac{F}{N_i} = v_i(\theta_i) = b_i + a_i\theta_i + c_i\frac{1}{q_i(\theta_i)}$$

For a given $\hat{N}_i = (N_1, \ldots, N_{i-1}, N_{i+1}, \ldots, N_n)$ with $N_j > 0$, $i \neq j$, a sufficiently small N_i makes the left-hand side sufficiently large. In other words, this equation always define a $\theta_i(N)$ given that $v_i(\theta_i)$ is strictly increasing in θ_i.

C.3 Uniqueness with $n \geq 1$ in the Cobb-Douglas case

When $F(N) = \Pi N_i^{\alpha_i}$, equation (VC$_i$) is in the Cobb-Douglas case

$$OE_i\alpha_i\frac{F}{N_i} = b_i + a_i\theta_i + c_i\frac{1}{q_i(\theta_i)}$$

with OE_i a constant defined in (18) and $a_i = \frac{\beta_i}{1-\beta_i}\gamma_i$ and $c_i = \frac{\gamma_i(r+s_i)}{1-\beta_i}$. Let us define $t_i = \ln N_i$. Equation (VC$_i$) is then

$$\sum_j \alpha_j t_j = t_i + g_i(\theta_i) - \ln(OE_i\alpha_i) \tag{C20}$$

where

$$g_i(\theta_i) = \ln\left(b_i + a_i\theta_i + c_i\frac{1}{q_i(\theta_i)}\right)$$

Note that $g_i()$ is strictly increasing in θ_i since $a_i, c_i > 0$ and $q_i(\theta_i)$ is increasing. Further, given (SS), we have

$$p_i(\theta_i) = \frac{s_i N_i}{1 - N_i}$$

with $N_i = e^{t_i}$ so that one can define n functions $\theta_i = \theta_i(t_i)$ that are strictly increasing in t_i. It follows that in the right hand-side of equation (C20), one has a function $H_i(t_i)$ with

$$H_i(t_i) = t_i + g_i(\theta_i(t_i)) - \ln(OE_i\alpha_i)$$

which is strictly increasing in t_i. Define $t_i^{\sup} = \sup_{N_i^\times} t_i \in (-\infty; 0)$ and $J_n = \prod_i) - \infty; t_i^{\sup})$ on which functions $H_i(t_i)$ are defined simultaneously. In such a space J_n, we thus have, remarking that the left-hand side of (C20) is independent of i, for all i,

$$H_i(t_i) = H_1(t_1)$$

which implies that, given the fact that $H_i$are strictly increasing, there is one and only one solution $t_i(t_1)$ denoted by $h_i(t_i)$, such that

$$t_i = h_i(t_1) \text{ with } \frac{dh_i}{dt_1}(t_1) = \frac{H_1'(t_1)}{H_i'(t_i)} > 0$$

where the derivative using the theorem of implicit functions. Summing up all $t_i$in the left hand-side of (C20), there is an equation defining t_1 which is

$$K(t_1) = \alpha_1 t_1 + \sum_{i>1} \alpha_i h_i(t_1) - H_1(t_1) = 0$$

One can now easily verify that $K(t_1) = 0$ defines a unique solution. Indeed,

$$\begin{aligned} K'(t_1) &= \alpha_1 + \sum_{i>1} \alpha_i \frac{H_1'(t_1)}{H_i'(t_i)} - H_1'(t_1) \\ &= \sum_{i=1}^{n} \alpha_i \frac{H_1'(t_1)}{H_i'(t_i)} - H_1'(t_1) \\ H_1'(t_1) &\left[\sum_{i=1}^{n} \frac{\alpha_i}{H_i'(t_i)} - 1 \right] \end{aligned}$$

Now, $H_i^{'}(t_i) = 1 + g_i'(\theta_i)\frac{\partial \theta_i}{\partial t_i} > 1$ because g_i and θ_i are strictly increasing. Thus $K'(t_1) < H_1'(t_1)\left[\Sigma\alpha_i - 1\right] < 0$. There is thus at most one solution in t_1. Since there is necessarily one solution in Nas shown in Appendix B.2.2, there is at least one solution for t_1 and thus t_1 is unique. This implies uniqueness of all $t_i$and thus of N_i. The decentralized equilibrium is thus unique.

D Physical capital

The resolution of the program of the firm leads to the first order conditions:

$$\left[F_i(\mathbf{N}, K) - \sum_{j=1}^{n} N_j \frac{\partial w_j(\mathbf{N}, K)}{\partial N_i} - w_i(\mathbf{N}, K) \right] dt + (1 - s_i dt) \frac{\partial \Pi(\mathbf{N}^+, K^+)}{\partial N_i} = \frac{\partial \Pi(\mathbf{N}^+, K^+)}{\partial N_i} (1 + rdt), \tag{D21}$$

$$\left[F_K(\mathbf{N}, K) - \sum_{j=1}^{n} N_j \frac{\partial w_j(\mathbf{N}, K)}{\partial K} \right] dt + (1 - \delta dt) \frac{\partial \Pi(\mathbf{N}^+, K^+)}{\partial K} = \frac{\partial \Pi(\mathbf{N}, K)}{\partial K} (1 + rdt) \tag{D22}$$

where $F(\mathbf{N}, K)$ denotes the partial derivative of F with respect to the ith coordinate of the vector N.The envelop conditions write as

$$-\gamma_i + q_i \frac{\partial \Pi(\mathbf{N}^+, K^+)}{\partial N_i} = 0, \tag{D23}$$

$$-1 + \frac{\partial \Pi(\mathbf{N}^+, K^+)}{\partial K} = 0 \tag{D24}$$

Looking for the steady state solution and eliminating $\partial\Pi(\mathbf{N}, K)/\partial N_i$ (that is henceforth denoted as $J_i(\mathbf{N}, K)$) and $\partial\Pi(\mathbf{N}, K)/\partial K$ from these four equations, one obtains

$$J_i(\mathbf{N}, K) = \gamma_i / q_i, \text{with } J_i(\mathbf{N}, K) = \frac{F_i(\mathbf{N}, K) - w_i(\mathbf{N}, K) - \sum_{j=1}^{n} N_j \frac{\partial w_j(\mathbf{N}, K)}{\partial N_i}}{r + s_i} \tag{D25}$$

and equation (21) in the text.