Anti-Competitive Effects of Resale-Below-Cost Laws: Appendix

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This document presents a more detailed version of the proofs omitted in the paper.

1 No restriction on resale prices: proof of lemma 1

We solve the game by backward induction. We look only for symmetric equilibria where the four goods are sold.

1.1 Stage 3

Consider the stage-3 subgame, where net transfers t_{Ki}, $K \in \{A, B\}$, $i \in \{1, 2\}$, and both wholesale prices w_A and w_B are fixed. Retailer i knows the public values of the wholesale prices w_A and w_B set by producers in stage 1 and the outcome of her negotiations in stage 2, that is the true values of the unit transfers t_{Ai} and t_{Bi} if her negotiations with both suppliers have succeeded. However, she does not know the outcome of the negotiations of her competitor j with both suppliers.

Assume the four negotiations succeeded in stage 2. In stage 3, retailer i's profit is concave and can be written as:

$$\Pi_i = (p_{Ai} - t_{Ai})D_{Ai}(p_{Ai}, p_{Aj}, p_{Bi}, p_{Bj}) + (p_{Bi} - t_{Bi})D_{Bi}(p_{Bi}, p_{Bj}, p_{Ai}, p_{Aj})$$

The two best response final prices of each retailer i are, for $\{K, L\} = \{A, B\}, \{i, j\} = \{1, 2\}$[1]:

$$p_{Ki}^{BR}(t_{Ki}, t_{Li}, p_{Kj}, p_{Lj}) = \tfrac{1+t_{Ki}-b(1-p_{Kj})}{2} \tag{1}$$

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[1] Note that p_{Ki}^{BR} depends only on the final price of the same brand sold by the other retailer, p_{Kj} : all the effect of interbrand competition is absorbed by the price p_{Li}^{BR}, set simultaneously by retailer i. This stems from the linearity of demand and the assumption $c = a.b$.

The intersection of the best responses (assuming correct estimation by each retailer of the transfers paid by its competitor) gives the subgame equilibrium prices denoted $p^e_{Ki}(t_{Ki}, t_{Kj}, t_{Li}, t_{Lj})$:

$$p^e_{Ki}(t_{Ki}, t_{Kj}, t_{Li}, t_{Lj}) = \frac{2(1+t_{Ki}) + bt_{Kj} - b(1+b)}{4-b^2} \tag{2}$$

Note that final price p^e_{Ki} increases in t_{Ki} and t_{Kj} and is independent of t_{Li} and t_{Lj}. In the subgame equilibrium, final profits are:

$$\begin{aligned}
\Pi_i &= (p_{Ai} - t_{Ai})D_{Ai}(p^e_{Ai}, p^e_{Aj}, p^e_{Bi}, p^e_{Bj}) + (p^e_{Bi} - t_{Bi})D_{Bi}(p^e_{Bi}, p^e_{Bj}, p^e_{Ai}, p^e_{Aj}) \\
\Pi_K &= t_{K1}D_{K1}(p^e_{K1}, p^e_{K2}, p^e_{L1}, p^e_{L2}) + t_{K2}D_{K2}(p^e_{K2}, p^e_{K1}, p^e_{L2}, p^e_{L1})
\end{aligned}$$

1.2 stage 2

The second stage of the game is the Nash-bargaining over the net transfers. Consider the negotiation between producer K and retailer i. The associated Nash program is:

$$\underset{t_{Ki}}{Max}(\Pi^a_K - \Pi^{3a}_K)^\alpha(\Pi^a_i - \Pi^{3a}_i)^{1-\alpha} \tag{3}$$

where Π^a_K (resp. Π^a_i) is the anticipated profit of producer K (resp. retailer i) and Π^{3a}_K (resp. Π^{3a}_i) is the anticipated status-quo profit earned by producer K (resp. retailer i) if the negotiation breaks, *i.e.* if producer K only deals with retailer j (resp. retailer i only deals with producer L), all other negotiations being successful. We denote by p^a_{Kj} and p^a_{Lj} the value of the retail prices for products Kj and Lj anticipated by retailer i and producer K in stage 2, and t^a_{Kj} their anticipation of the transfer agreed between retailer j and supplier K. Note that, according to our contract equilibrium framework (with delegated agents and passive beliefs), producer K and retailer i keep constant anticipations over the outcomes of the three other pairs' negotiations while negotiating in stage 2. In equilibrium, these anticipations will be the equilibrium outcomes. More precisely, the anticipated profits are:

$$\begin{aligned}
\Pi^a_K &= t_{Ki}D_{Ki}(p^{BR}_{Ki}(X^a), p^a_{Kj}, p^{BR}_{Li}(X^a), p^a_{Lj}) + \\
&\quad t_{Kj}D_{Kj}(p^a_{Kj}, p^{BR}_{Ki}(X^a), p^a_{Lj}, p^{BR}_{Li}(X^a)) \\
\Pi^a_i &= (p^{BR}_{Ki}(X^a) - t_{Ki})D_{Ki}(p^{BR}_{Ki}(X^a), p^a_{Kj}, p^{BR}_{Li}(X^a), p^a_{Lj}) + \\
&\quad (p^{BR}_{Li}(X^a) - t_{Li})D_{Li}(p^{BR}_{Li}(X^a), p^a_{Lj}, p^{BR}_{Ki}(X^a), p^a_{Kj})
\end{aligned}$$

where $X^a = (t_{Ki}, t^a_{Li}, p^a_{Kj}, p^a_{Lj})$.

The status quo profits are defined by a break in negotiation between the producer and the retailer. Consider for instance the negotiation between producer A and retailer 1, and assume they fail. As the outcome of the negotiations is not observable ex-post, *i.e* between stages 2 and 3, firms B and 2 ignore this failure and behave according to their anticipations. Therefore while negotiating, firms 1 and A anticipate that the negotiation between producer B and retailer 2 is not affected and will lead to the equilibrium value t^*_{B2}. Furthermore, the contract equilibrium framework implies that firms 1 and A anticipate that their own negotiations with their other partners will not be affected and will lead to the equilibrium transfers t^*_{A2} and t^*_{B1}, and that, as retailer 2 will have no information about the failure in negotiation, she will set the equilibrium prices $p^a_{A2} = p^*_{A2}$ and $p^a_{B2} = p^*_{B2}$ in stage 3. However, firms 1 and A also anticipate that, at the beginning of stage 3, retailer 1 will be aware of the absence of product A on her shelves, and that she will thus set the optimal price p^3_{B1} anticipating the real final demand when good $A1$ is not distributed[2], denoted $D^3(p_{A2}, p_{B1}, p_{B2})$:

$$\begin{aligned} D^3_{B1}(p_{A2}, p_{B1}, p_{B2}) &= \tfrac{1-b-p_{B1}+bp_{B2}}{1-b^2} \\ D^3_{A2}(p_{A2}, p_{B1}, p_{B2}) &= \tfrac{1-a-p_{A2}+ap_{B2}}{1-a^2} \end{aligned}$$

The optimal price for good $B1$, set by retailer 1 in stage 3, would thus be $p^3_{B1} = \frac{1-b(1-p^a_{B2})+t^a_{B1}}{2}$. Note that $p^3_{Ki} = p^{BR}_{B1}$: given the prices chosen by retailer j, the optimal price for product Ki is the same whether i sells L or not. This property holds for any linear demand function with symmetric cross-price derivatives.[3] The status-quo profits anticipated by the negotiating firms are finally:

$$\begin{aligned} \Pi^3_A &= t^a_{A2} D^3_{A2}(p^3_{B1}, p^a_{A2}, p^a_{B2}) = t^a_{A2}(\frac{1-a-p^a_{A2}+ap^a_{B2}}{1-a^2} \\ \Pi^3_1 &= (p^3_{B1} - t^a_{B1}) D^3_{B1}(p^3_{B1}, p^a_{A2}, p^a_{B2}) = \frac{(1-b+bp^a_{B2}-t^a_{B1})^2}{4(1-b^2)}. \end{aligned}$$

The anticipated profits are, with $p^{BR}_{Ki} = \frac{1-b+bp^a_{Kj}+t_{Ki}}{2}$ and $p^{BR}_{Li} = \frac{1-b+bp^a_{Lj}+t^a_{Li}}{2}$:

$$\Pi^a_K = t_{Ki}(\tfrac{(1-a)(1-b)-p^{BR}_{Ki}+ap^{BR}_{Li}+bp^a_{Kj}-abp^a_{Lj}}{(1-a^2)(1-b^2)}) + t_{Kj}(\tfrac{(1-a)(1-b)+bp^{BR}_{Ki}-abp^{BR}_{Li}-p^a_{Kj}+ap^a_{Lj}}{(1-a^2)(1-b^2)})$$

Finally, the reduced Nash condition resulting from the resolution of the Nash programme for the negotiation between producer K and retailer i is:

$$\tfrac{[\alpha(1-b(1-p_{Kj}-t_{Kj})-2t_{Ki}+2bt_{Kj}-a(1-b(1-p_{Lj})-t_{Li}))][(1-a)(1-b)+bp_{Kj}-t_{Ki}-ap_{Lj}+at_{Li}]^2}{8(1-a^2)^2(1-b^2)^2} = 0$$

The subgame equilibrium outcome of the negotiations is given by the resolution of the four Nash programs under the condition that the anticipated retail

[2]In case of a failure in the bargaining between A and 1, the consumers are aware that only three goods are available on the market to purchase, and therefore the demand for $A1$ is divided between the three other goods. We determine the inverse demand for the three other goods by setting $q_{A1} = 0$ and invert them in order to derive the demands in that case.

[3]This property also holds for general demand functions if cross-price derivatives are symmetric and $\varepsilon^3_{AA} = \varepsilon_{AA} + \varepsilon_{AB}$ where ε^3_{AA} is the direct-price elasticity of the demand for product A when only A is sold, and ε_{AA} and ε_{AB} respectively the direct-price and cross-price elasticities of the demand when both products are sold. A change in the demand function could raise a difference between p^3_{B1} and p^{BR}_{B1} but this would not change qualitatively our results.

prices are the stage 3 subgame equilibrium prices $p^a_{Ki} = p^e_{Ki}(t_{Ki}, t_{Li}, t_{Kj}, t_{Lj})$. There exists a unique symmetric solution, irrespective of the wholesale prices w_K:

$$t_{Ki} = t^* = \tfrac{2\alpha(1-a)}{4-2a\alpha-b(2-\alpha)} \tag{4}$$

The equilibrium transfers increase in α and in b, and decrease in a.

1.3 Stage 1

In the first stage, the expected profits of the firms do not depend on the wholesale prices: any pair of wholesale prices (w_K, w_L) may be chosen in equilibrium as long as the rebates negotiated in stage 2 lead to the net transfers t^*. Therefore there exists a continuum of equilibria: First, notice that retail prices chosen in stage 3 do not depend on the wholesale prices w_K set in stage 1, but only on the net transfers decided in stage 2. Second, the wholesale prices w_K do not affect the negotiations over the net transfers t_{Ki}.[4] Therefore the wholesale prices chosen in stage 1 are immaterial to the net transfers negotiated in stage 2 and ultimately to the retail prices chosen in stage 3. In that respect, these wholesale prices have no commitment value. A direct implication is that while equilibrium net transfers and retail prices are unique, they correspond to a continuum of wholesale prices. Among this continuum of equilibrium wholesale prices, some involve loss-leading from the retailers, according to the legal definition ($w_K \geq p^*$), and others do not ($w_K \leq p^*$). Here, the practice of loss-leading is neutral with respect to prices, profit sharing and consumers' surplus.

Injecting (4) in (2) yields the symmetric equilibrium final prices (for the four goods):

$$p^* = \tfrac{2(1-b)+\alpha(1-2a+b)}{4-2a\alpha-(2-\alpha)b}$$

This equilibrium retail price increases in α and decreases in a and b.

2 No restriction on resale prices: Proof of proposition 1

From the proof of lemma 1, we can derive equilibrium profits:

$$\begin{aligned} \Pi^*_K &= \tfrac{4\alpha(1-a)(2-\alpha)}{(1+a)(1+b)(4-2a\alpha-(2-\alpha)b)^2} \\ \Pi^*_i &= \tfrac{2(1-b)(2-\alpha)^2}{(1+a)(1+b)(4-2a\alpha-(2-\alpha)b)^2} \end{aligned} \tag{5}$$

Π^*_i decreases in α and in b. Π^*_K decreases in a. Furthermore, Π^*_K is not always monotonous in α : if a is large enough relatively to b ($b \leq 2a$), Π^*_K increases in α, but otherwise Π^*_K increases in α for small values of α ($\alpha \leq \frac{4-2b}{4-2a-b}$) and decreases

[4] Note that the legal definition of rebates implies that they are deducted from the wholesale price, so that $t_{Ki} < w_K$: However allowing $w_K < t_{Ki}$ would not change our results.

after.[5] Finally, the total profit of the industry is not always monotonous in α. If a and b are large $(2(1-a) \leq b \leq 2a)$ total profit increases in α. Otherwise total profit increases in α for $\alpha \leq \frac{2b}{2-2a+b}$ and decreases after this threshold.

3 Discussion: robustness of the result to changes in the demand function

Due to the coexistence of four different products, resulting from intra- and inter-brands differentiation, solving the model with a general demand function is tedious (see for instance a discussion in Shaffer, 1991, with only two products). However, we claim that proposition 2 would hold under fairly standard assumptions. We provide some intuitions.

Assume that both inverse and direct demand functions are symmetric, continuous, invertible, twice differentiable, bounded, and downward sloping ($\frac{\partial p_{Ki}}{\partial q_{Ki}} < 0$ for inverse demand). Assume also that the four goods are imperfect substitutes, and that direct price-elasticity is larger in absolute terms that cross price elasticities:

$$\frac{\partial p_{Ki}}{\partial q_{Ki}} < \frac{\partial p_{Ki}}{\partial q_{Kj}} < \frac{\partial p_{Ki}}{\partial q_{Lj}} < 0$$
$$\frac{\partial p_{Ki}}{\partial q_{Ki}} < \frac{\partial p_{Ki}}{\partial q_{Li}} < \frac{\partial p_{Ki}}{\partial q_{Lj}} < 0$$

First, each retail price p_{Ki}^{BR} increases with the four net transfers t_{Kj} under fairly standard assumptions : by totally differentiating the stage-3 retailers' first-order conditions, one can determine $\frac{\partial p_{Ki}^{BR}}{\partial t_{Ki}}$ and $\frac{\partial p_{Ki}^{BR}}{\partial t_{Li}}$. For instance

$$\frac{\partial p_{A1}^{BR}}{\partial t_{A1}} = \frac{\frac{\partial^2 \Pi_1}{\partial p_{B1}^2} \frac{\partial D_{A1}}{\partial p_{A1}} - \frac{\partial^2 \Pi_1}{\partial p_{A1} \partial p_{B1}} \frac{\partial D_{A1}}{\partial p_{B1}}}{\frac{\partial^2 \Pi_1}{\partial p_{A1}^2} \frac{\partial^2 \Pi_1}{\partial p_{B1}^2} - \left(\frac{\partial^2 \Pi_1}{\partial p_{A1} \partial p_{B1}}\right)^2} \tag{6}$$

where the denominator of (6) is positive if the retailer's profit function is strictly concave. Assuming the positivity of the numerator is then sufficient to ensure that the final price p_{A1}^{BR} increases in t_{A1}.

[5] That producers' profits may decrease in α is a consequence of the assumption that producers behave here as schizophrenic negotiators, and has no consequence on the main results. This effect is irrespective of upstream competition, and the following example gives the main insight. Consider a monopolist producer with $\alpha = 1$. Let $(\overline{t_1}, \overline{t_2})$ be the net transfers that would maximize the producer's profit. Assume that the producer sets the net transfer $\overline{t_2}$ with retailer 2. When he bargains with retailer 1 over t_1, he considers that $\overline{t_2}$ is fixed. Increasing t_1 does not change his status-quo profit but brings about three anticipated effects: (i) an increase of his margin on product 1 (raising his profit), (ii) an increase of the retail price of product 1 leading to a decrease in demand (reducing his profit), and (iii) an increase in the demand for his product at the other retailer's (raising his profit). If $\overline{t_2}$ is fixed, the third effect partially compensates for the second, the compensation being more efficient when retailers' competition increases (b increases), and the producer increases t_1 above $\overline{t_1}$ to raise his profit. As he simultaneously does the same with the other retailer, and a joint increase of both net transfers suppresses the third effect of demand substitution: The global effect on the producer's profit is negative.

Each net transfer is also increasing with the producer's bargaining power α for a broad range of demand functions. The negotiation between K and i leads to the net transfers maximizing the following Nash condition:

$$\underset{t_{Ki}}{Max}(\Pi_K - \Pi_K^3)^{\alpha}(\Pi_i - \Pi_i^3)^{1-\alpha}$$

Assuming the concavity of the Nash condition, the sufficient first-order condition is as follows:

$$CPO(t_{Ki}) = \alpha\frac{\partial(\Pi_K - \Pi_K^3)}{\partial t_{Ki}}(\Pi_i - \Pi_i^3) + (1-\alpha)(\Pi_K - \Pi_K^3)\frac{\partial(\Pi_i - \Pi_i^3)}{\partial t_{Ki}} = 0$$

where $\frac{\partial CPO}{\partial t_{Ki}} \leq 0$ by concavity. We have:

$$\frac{\partial t_{Ki}}{\partial \alpha} = -\frac{\frac{\partial CPO}{\partial \alpha}}{\frac{\partial CPO}{\partial t_{Ki}}}$$

A sufficient condition for each net transfer to increase with the producer's bargaining power α is that given the three other net transfers, in the neighborhood of the equilibrium net transfer, the additional profit gained by a producer if his negotiation with a retailer succeeds (e.g. $\Pi_K - \Pi_K^3$) increases in the net transfer t_{Ki}, whereas the additional profit for the retailer decreases in this net transfer. In that case, $\frac{\partial CPO}{\partial \alpha} = \frac{\partial(\Pi_K - \Pi_K^3)}{\partial t_{Ki}}(\Pi_i - \Pi_i^3) - (\Pi_K - \Pi_K^3)\frac{\partial(\Pi_i - \Pi_i^3)}{\partial t_{Ki}} \geq 0$, thus $\frac{\partial t_{Ki}}{\partial \alpha} > 0$.

Finally, under these conditions, as retail prices increase in the net transfers, and as long as direct effects dominate indirect effects, Proposition 1 will hold with a broad range of standard demand functions.[6]

4 The RPM equilibrium: proof of lemma 2

If producers impose RPM contracts, the retailers have to set prices $p_{Ki} = w_K$ in stage 3 irrespective of the market structure, even in case of a breach in stage 2 negotiation. The status-quo profits are different from the *no restriction* case. Consider the negotiation between A and 1. In case of a failure, the two negotiators anticipate now that the price of product $B1$ set in stage 3 will be constrained by the RPM: $p_{B1} = w_B$. The status-quo profits are thus:

$$\Pi_A^3 = t_{A2}D_{A2}^3(w_A, w_B, w_B) = \frac{t_{A2}(1-a-w_A+aw_B)}{(1-a^2)} \quad (7)$$

$$\Pi_1^3 = (w_B - t_{B1})D_{B1}^3(w_A, w_B, w_B) = \frac{(1-w_B)(w_B-t_{B1})}{(1+b)} \quad (8)$$

[6] Dobson and Waterson (2007) discuss more extensively in their Corollaries 2 and 3 the necessary properties of the demand function for this result to hold.

Similarly, producer A and retailer 1 anticipate the following third-stage profits in case of success of the negotiation:

$$\Pi_A(t_{A1}) = t_{A1} D_{A1}(w_A, w_A, w_B, w_B) + t_{A2} D_{A2}(w_A, w_A, w_B, w_B)$$
$$= (t_{A1} + t_{A2}) \frac{(1-a-w_A+aw_B)}{(1-a^2)(1+b)}$$
$$\Pi_1(t_{A1}) = (p_{A1} - t_{A1}) D_{A1}(w_A, w_A, w_B, w_B) + (p_{A2} - t_{A2}) D_{A2}(w_A, w_A, w_B, w_B)$$
$$= (w_A - t_{A1}) \frac{(1-a-w_A+aw_B)}{(1-a^2)(1+b)} + (w_B - t_{B1}) \frac{(1-a-w_B+aw_A)}{(1-a^2)(1+b)}$$

Finally, the Nash condition for the negotiation between A and 1 is:

$$\underset{t_{A1}}{Max} (\Pi_A(t_{A1}) - \Pi_A^3)^{\alpha} (\Pi_1(t_{A1}) - \Pi_1^3)^{1-\alpha} \tag{9}$$

The resolution of the four Nash conditions gives the following optimal net transfers[7]:

$$\widetilde{t}_{Ki}(w_K, w_L) = \alpha \frac{(1-a^2\alpha-(1-\alpha)b)w_K - a(1-\alpha)(1-b)w_L}{(1-(1-\alpha)b)^2 - a^2\alpha^2} \tag{10}$$

Note that, given w_L, $\frac{\partial \widetilde{t}_{Ki}}{\partial w_K} = \alpha \frac{(1-a^2\alpha-(1-\alpha)b)}{(1-(1-\alpha)b)^2-a^2\alpha^2} \geq 0$: producer $K's$ margin now increases in w_K.

In stage 1, producer K sets the wholesale price that maximizes his profit:

$$\underset{w_K}{Max}\ \widetilde{t}_{Ki}(w_K, w_L).(D_{K1} + D_{K2})(w_K, w_K, w_L, w_L)$$

The optimal wholesale (and final) prices are then:

$$w_A = w_B = \widetilde{w} = \frac{1-a^2\alpha-b+\alpha b}{2+a\alpha-a^2\alpha-2(1-\alpha)b} \tag{11}$$

Reinjecting (11) in (10) yields the equilibrium net transfers under RPM:

$$\widetilde{t}_{Ki} = \widetilde{t} = \frac{\alpha(1-a)(1-a^2\alpha-(1-\alpha)b)}{(\alpha((1-a)a+2b)+2(1-b))(1-b+\alpha(b-a))}$$

Proposition 2, Corollaries 1 and 2 are direct consequences of lemma 1. Note for instance that for $\alpha = 0, \widetilde{w} = \frac{1-b}{2-2b} = \frac{1}{2}$, while for $\alpha = 1, \widetilde{w} = \frac{1-a^2}{2+a-a^2} = \frac{1-a}{2-a}$.

Upstream profits are:

$$\widetilde{\Pi}_K = \frac{2\alpha(1-a)(1+a\alpha-(1-\alpha)b)(1-a^2\alpha-(1-\alpha)b)}{(1+a)(1+b)\left[2+a\alpha(1-a)-2(1-\alpha)b\right]^2(1-b+\alpha(b-a))}$$

The comparison of the retail prices $\widetilde{w}$ to the equilibrium prices in the unconstrained case p^* is straightforward: $\widetilde{w} \geq p^* \Leftrightarrow \alpha \leq \alpha_I$ where

$$\alpha_I = \frac{2(1+a^2)-b-3b^2-\sqrt{4a^2+8a(1-b)^2b+(2-b+b^2)^2+4a^2(2-7b+3b^2)}}{6a^2-2a(1-b)-2b(2+b)}.$$

[7] Defined by continuity, for $\alpha = 1$ and $a = 1$: $t_{Ki}(w_K, w_L) = w_K$; and for $\alpha = 0$ and $b = 1$, $t_{Ki}(w_K, w_L) = 0$.

5 Welfare comparisons

Consumer surplus is $S(q) = U(q) - \sum\limits_{K,i} p_{Ki} q_{Ki}$. As the firms' costs are normalized to zero, total welfare is thus $W = U(q) = \sum\limits_{K,i} q_{Ki} - \frac{1}{2} \sum\limits_{K,i} q_{Ki}^2 - a \sum\limits_{i} q_{Ai} q_{Bi} - b \sum\limits_{K} q_{K1} q_{K2} - c \sum\limits_{K} q_{K1} q_{L2}$: if the four prices are equal, it increases in the total quantity sold[8], *i.e.* decreases in the retail price. Welfare comparisons across different symmetric equilibria are thus straightforward and boil down to comparing retail prices.

6 Price-floor equilibria

We assume now that the producers cannot impose final prices but are able to impose industrywide price-floors to their retailers. We look for the symmetric equilibria of the game where the four goods are sold. We first identify situations where a price-floor is sufficient to implement the RPM equilibrium (section 6.1). Then we look for the producers optimal strategies when this is not the case (sections 6.2 and 6.3)

6.1 The price-floor implements the RPM equilibrium: proof of lemma 3

We check that no deviation from the RPM equilibrium occurs if the producers set the wholesale price $\widetilde{w}$ in the first stage.

6.1.1 No deviation by a retailer in stage 3

Assume that the producers have set the price-floor $\widetilde{w}$ in stage 1, and that the stage 2 negotiations have determined the RPM equilibrium transfers $\widetilde{t}$. In stage 3, if retailer i anticipates that her rival sets price $p_{Kj} = \widetilde{w}$, she can still set a price p_{Ki} above the price-floor $\widetilde{w}$. The best response price of retailer i given by (1) is thus: $p_{Ki}^{BR}(\widetilde{t}, \widetilde{w}) = \frac{1+\widetilde{t}-b(1-\widetilde{w})}{2}$. No deviation occurs in stage 3 if $p_{Ki}^{BR}(\widetilde{t}, \widetilde{w}) \leq \widetilde{w}$, or:

$$\widetilde{t} \leq \widetilde{w}(2-b) - 1 + b \tag{12}$$

This condition is monotonous in α, and defines a threshold $\widetilde{\alpha}$ such that retail prices will indeed be constrained by a price-floor if and only if:

$$\alpha \leq \widetilde{\alpha} = \frac{1+a^2-2b^2-\sqrt{1+2a^2+a^4-8a^2b+4a^2b^2}}{2(2a^2-b-b^2)} \tag{13}$$

Note that $\widetilde{\alpha} \leq \alpha_I$ for all $\{a, b\}$ in $(0,1)^2$. For any α larger than this threshold, the RPM equilibrium no longer holds because there are profitable deviations in stage 3 at least.

[8] As long as the total quantity is less than $\frac{1}{(1+a)(1+b)}$, that is the maximum quantity sold when the four prices are zero.

6.1.2 No deviation in stage 2

Assume that the producers have set the price-floor $\widetilde{w}$ in stage 1, and that $\alpha \leq \widetilde{\alpha}$. Consider the negotiation between producer A and retailer 1 in stage 2, and assume that the three other pairs negotiate the transfer $\widetilde{t}$.

First, if the negotiation fails, the downstream price p^3_{B1} may not remain constrained by the price-floor. Retailer 1's optimal price is $p^{BR}_{B1} = \frac{1-b(1-\widetilde{w})+t_{B1}}{2}$, with $p^{BR}_{B1} \leq \widetilde{w}$ if and only if $t_{B1} \leq \widetilde{w}(2-b)-1+b$: yet (12) ensures that $t_{B1} = \widetilde{t}$ satisfies this condition. Therefore even if a negotiation fails, all final prices remain constrained in the continuation equilibrium, and the status-quo profits remain defined by (7) and (8).

Second, A and 1 may deviate by negotiating a higher transfer t^d_{A1} such that the retail price p_{A1} will be unconstrained in stage 3. Note that in such a case, (1) and (12) ensure that p_{B1} remains constrained and equal to $\widetilde{w}$. Besides, retailer 2 will be unaware of the deviation in stage 3, so that its retail prices will be unchanged. As the three other retail prices are $\widetilde{w}$, the price that maximizes retailer 1' profit is

$$p^d_{A1} = \frac{1}{2}((1-a)(1-b(1-\widetilde{w})) + t^d_{A1} - a(2w-\widetilde{t}))$$

We have $p^d_{A1} \geq \widetilde{w} \Longleftrightarrow t^d_{A1} \geq t_c = a\widetilde{t} - (1-a)(1-b(1-\widetilde{w}) - 2\widetilde{w})$.

Under price-floors, the Nash condition is defined by segments: for $t^d_{A1} \geq t_c$, both firms' profits correspond to the unconstrained price p^d_{A1}, and for $t^d_{A1} \leq t_c$ both firms' profits correspond to the constrained price $p_{A1} = \widetilde{w}$. If $\alpha \leq \widetilde{\alpha}$, the maximum of the Nash condition is in $t_{A1} = \widetilde{t} \leq t_c$, therefore the negotiated tariffs lead to constrained retail prices.

6.1.3 No deviation in stage 1

Consider now possible deviations in stage 1. Assume that producer B sets the wholesale price $\widetilde{w}$. First, it is obvious that producer A would not deviate by increasing his wholesale price: as p_{A1} and p_{A2} would remain constrained the deviation would not be profitable, whether retail prices for B were constrained or not. But it could be profitable for A to deviate by setting in stage 1 a wholesale price w_A sufficiently low to relax the stage-3 constraint and allow the retailers to set p_{Ai} above w_A but below $\widetilde{w}$.

Consider that producer A chooses such a wholesale price w_A. As the outcome of stage 1 is public, all the firms are aware of producer $A's$ deviation and adapt their strategies in stages 2 and 3. As the wholesale price in stage 1 is not committing, due to the renegotiations in stage 2, we can assume that $w_A = 0$ (t_{Ai} positive). The aim of this strategy is to reduce the retail prices of product A : thus both retailers wish to reduce the price of brand B, which thus remains constrained: $p_{Bi} = \widetilde{w}$. We consider two *scenarii* in turn. First, if p_{Ai} is not too low, there may still be a positive demand for B. Second, if p_{Ai} is low enough, product B may be excluded from the market.

Deviation of producer A without exclusion of product B Assume that product B still faces a positive demand: the four goods are carried, $p_{Bi} = \widetilde{w}$ and retailers' best response prices for brand A are :

$$p_{Ai}^{BR} = \frac{(1-a)(1-b)+t_{Ai}+a(2-b)\widetilde{w}-at_{Bi}+bp_{Aj}}{2}.$$

(i) $t_B^d \leq t_{Bdem}^d$: Assuming symmetry across the retailers[9] and denoting t_K^d the net transfers agreed with producer K, final demand for good B is indeed positive iff $t_B^d \leq t_{Bdem}^d$, with

$$t_{Bdem}^d = \frac{4(1-a)+a\alpha(6-2a(2+a^2)+a\alpha(1-a)^2)-(1-a)b[6-5\alpha+a(2+\alpha(3+2a-(3+a)\alpha))]+(1-a^2)(2-\alpha)(1-\alpha)b^2}{a^2(2-\alpha)(\alpha((1-a)a+2b)+2(1-b)} \tag{14}$$

(ii) $t_B^d \geq t_{Bpro}^d$: Given t_B^d, stage 2 negotiations with A lead to the optimal transfers $t_A^d = \frac{2\alpha(1-a(1-t_B^d))}{4-(2-\alpha)b}$. Producer A gets a deviation profit $\Pi_A^D = \frac{4\alpha(2-\alpha)\left(1-a\left(1-t_B^d\right)\right)^2}{(1-a^2)(1+b)(4-(2-\alpha)b)^2}$ where $\frac{d\Pi_A^D}{dt_B^d} > 0$ locally. The deviation is profitable, *i.e.* $\Pi_A^D \geq \widetilde{\Pi}_A$, iff t_B^d is higher than a threshold t_{Bpro}^d with

$$t_{Bpro}^d = 1 - \frac{1}{a} + \frac{(1-a)^2(4-(2-\alpha)b)^2\sqrt{(1-b+\alpha(a+b))(1-b+\alpha(b-a^2))}}{(\alpha(a(1-a)+2b)+2(1-b))(4-(2-\alpha)b)(1-a)a\sqrt{2(2-\alpha)(1-b+\alpha(b-a))}} \tag{15}$$

Finally, we have to check that, given the transfers negotiated in stage 2, each retailer does not deviate by stopping selling brand B in stage 3. This implies a condition on the transfers (iii) :

- if $b \geq \frac{2}{2+\sqrt{1-a^2}}$ and $\alpha \leq \alpha_1$, $t_{BPC1}(\alpha) \leq t_{Bi}^d \leq t_{BPC2}(\alpha)$
- if $b \geq \frac{2}{2+\sqrt{1-a^2}}$ and $\alpha \geq \alpha_1$, $t_{BPC2}(\alpha) \leq t_{Bi}^d \leq t_{BPC1}(\alpha)$ with

$$\alpha_1 = \frac{2a(3-a(2-a))+b(5-8a+2a^2-3b)-\sqrt{[(2a(3-a(2-a))+b(5-8a+2a^2-3b)]^2-8(1-b)(2-2a-b)(a^2(1-2b-3a)+3ab+b^2)}}{6a^3-6ab-2b^2-a^2(2-4b)}$$

and

$$t_{BPC1} = \frac{A_1+\alpha B_1+\alpha^2 C_1}{a^2[2(1-b(1-\alpha))+a\alpha(1-a)][2(4-8b+b^2(3+a^2))-\alpha(4(1-\sqrt{1-a^2})-b(6+2a^2-4\sqrt{1-a^2})+b^2(3+a^2))]}$$

$$t_{BPC2} = \frac{A_2+\alpha B_2+\alpha^2 C_2}{a^2[2(1-b(1-\alpha))+a\alpha(1-a)][2(4-8b+b^2(3+a^2))-\alpha(4(1+\sqrt{1-a^2})-b(6+2a^2+4\sqrt{1-a^2})+b^2(3+a^2))]}$$

with $A_1 = 2(1-b)(2(1-\sqrt{(1-a^2)}(2-b)(1-b)(2(1-a)-b)-a^2(2-b)b^2+4a^3(1-b)b+a^4b^3)$

[9] The asymmetric case is more tedious but can be studied by the same method.

$A_2 = 2(1-b)(2(1+\sqrt{(1-a^2)}(2-b)(1-b)(2(1-a)-b) - a^2(2-b)b^2 + 4a^3(1-b)b + a^4b^3)$

$B_1 = 2(1-\sqrt{1-a^2})(1-b)\left[-2a^2(4-3b)+2a(2-b)(3-4b)+b(2-b)(5-3b)\right]$
$-2a^6b^2 - 2a^5b(2-b-b^2) + a^4(-8+b(3+b)(8-b(8-3b)))$
$+2a^3(1-b)(5b^2) - 2a^3(1-b)(4-2b)\sqrt{1-a^2} - a^2[(5-3b)b^3]$

$B_2 = 2(1+\sqrt{1-a^2})(1-b)\left[-2a^2(4-3b)+2a(2-b)(3-4b)+b(2-b)(5-3b)\right]$
$-2a^6b^2 - 2a^5b(2-b-b^2) + a^4(-8+b(3+b)(8-b(8-3b)))$
$+2a^3(1-b)(5b^2) + 2a^3(1-b)(4-2b)\sqrt{1-a^2} - a^2[(5-3b)b^3]$

$C_1 = -a^6(2-b)b + a^5(1-b)b(4+b) + a^4\left[4(1-\sqrt{1-a^2}) - b(8-4\sqrt{1-a^2}) + b^2(9-b(2+b))\right]$
$-(1-b)a^3[8(1-\sqrt{1-a^2}) + 3b^2 + 2\sqrt{1-a^2}b]$
$+a^2\left[(1-\sqrt{1-a^2})(4-14b+10b^2) - b^4\right]$
$+6ab(2-b(3-b))(1-\sqrt{1-a^2}) + 2(1-\sqrt{1-a^2})b^2(2-(3-b)b)$

$C_2 = -a^6(2-b)b + a^5(1-b)b(4+b) + a^4\left[4(1+\sqrt{1-a^2}) - b(8+4\sqrt{1-a^2}) + b^2(9-b(2+b))\right]$
$-(1-b)a^3[8(1+\sqrt{1-a^2}) + 3b^2 - 2\sqrt{1-a^2}b]$
$+a^2\left[(1+\sqrt{1-a^2})(4-14b+10b^2) - b^4\right]$
$+6ab(2-b(3-b))(1+\sqrt{1-a^2}) + 2(1+\sqrt{1-a^2})b^2(2-(3-b)b)$

(1) Note first that if $b \geq \frac{2}{2+\sqrt{1-a^2}}$ and $\alpha \leq \alpha_1$, the constraints (i), (ii) and (iii) are incompatible:

For $\alpha = \alpha_1$, $t_{BPC1} = t_{BPC2} = t_D$. Yet for all $\alpha \leq \alpha_1$, $t_{BPC1} \leq t_{BPC2}$ and t_{BPC1} increases in α; furthermore t_D decreases in α, therefore for all $\alpha \leq \alpha_1$ $t_{BPC1} \leq t_D$ thus $t_{B1} \leq t_D$ is incompatible with $t_1 \geq t_{BPC1}$. This means that in that case, there are no equilibria of the subgame with the four products sold if producer A sets the anticipated x^d.

QED

(2) Otherwise, *i.e.* if $b \geq \frac{2}{2+\sqrt{1-a^2}}$ and $\alpha \geq \alpha_1$ (with α_1 such that $t_{BPC1}(\alpha_1) = t_{BPC2}(\alpha_1)$) or $b < \frac{2}{2+\sqrt{1-a^2}}$, these conditions are compatible, but the Nash condition of the negotiation between B and i is maximum for a transfer less than $\widetilde{t_{Bi}}$, therefore the transfer t_B^d negotiated in the subgame equilibrium following the deviation of A is less $\widetilde{t_{Bi}}$. Yet we show easily that $\widetilde{t_{Bi}} \leq t_{Bpro}^d$, and therefore this deviation is not profitable. This rules out any deviation without exclusion of the rival brand.

Deviation of producer A with exclusion of product B Note first that if producer B anticipates that his product is going to face zero demand, he fights back by negotiating in stage 2 the lowest possible margin $t_{Bi} = 0$.

Consider stage 3. Anticipating zero demand for product B, retailers set the following optimal prices: $p^2_{Ai} = \frac{2-b-b^2+2t_{Ai}+bt_{Aj}}{4-b^2}$.

Demand for product B is indeed zero at both retailers' if and only if:

$$\frac{-1+b+p_{A2}}{b} + \frac{(1-b)(1-\widetilde{w})}{ab} \leq p_{A1} \leq 1 - b(1-p_{A2}) - \frac{(1-b)(1-\widetilde{w})}{a}. \tag{16}$$

Besides, no retailer wishes to deviate by selling product B as well iff

$$t_{Ai} \leq t^D(t_{Aj}) \tag{17}$$

In other words, the only way for producer A to induce an equilibrium with exclusion of B is to set a unit price sufficiently low for both conditions (16) and (17) to hold. Whenever these two conditions hold, there exists a downstream subgame equilibrium with exclusion of product B. If the deviation by A led to this subgame equilibrium, it would be profitable for A, for some values of the parameters, and suppress the RPM equilibrium in that zone.[10] Yet whenever this subgame equilibrium exists (*i.e.* whenever conditions (16) and (17) hold), there exists also another continuation equilibrium in stage 3 without exclusion of product B, where both retailers sell both goods and set the following prices for product A: $p^4_{Ai} = \frac{(1-a)(1-b)(2+b)+2t_{Ai}+bt_{Aj}+a(4-b^2)\widetilde{w}}{4-b^2}$; in this equilibrium product B is sold by both retailers at a price $p_{Bi} = \widetilde{w}$.[11] In that case, we have shown in the previous section that producer $A's$ profit is less than in the RPM equilibrium.

Formally, we have proved that the following strategies and beliefs form a symmetric contract equilibrium for $\alpha \leq \widetilde{\alpha}$: in stage 1, both producers set the unit wholesale price $\widetilde{w}$; in stage 2, the four pairs negotiate the transfers $\widetilde{t}$; in stage 3, both retailers set retail price $\widetilde{w}$ for both products; all firms believe that in any subgame where one producer (say B) has chosen the unit wholesale price $\widetilde{w}$ and the other one (say A) has deviated to $w_A = 0$, and the issue of the negotiations in stage 2 leads to net transfers $t_{Bi} = 0$ and t_{Ai} such that $t_{Ai} \leq t^D(t_{Aj})$ where $\{i, j\} = \{1, 2\}$, then each retailer will choose to sell both products with prices p^4_{Ai} and $p_{Bi} = \widetilde{w}$.

6.2 The corner price-floor equilibrium: proof of lemma 4

For $\alpha \geq \widetilde{\alpha}$, the producers have to increase the price-floors above $\widetilde{w}$ in order to saturate the constraint (12). As long as this leads to the optimal negotiated transfers (10), the minimum symmetric wholesale price which satisfies this

[10] Tedious calculations show that the deviation is profitable for α less than a threshold lower than $\alpha_d(a, b)$.

[11] Comparing the retailers' profits in the two subgame equilibria shows that both retailers are better off if the four products are carried than if B is excluded. The subgame equilibrium with exclusion of B is therefore Pareto-dominated by the one without exclusion of B.

constraint is $\overline{w} = \frac{1-a\alpha-b(1-\alpha)}{2-\alpha(1+a-b)-b}$. We show that setting this price in stage 1 sustains an equilibrium for $\widetilde{\alpha} \leq \alpha \leq \overline{\alpha} = \frac{2-b}{3-b}$, with producers' equilibrium profits $\overline{\Pi_K} = \frac{2(1-a)\alpha(1-\alpha)}{(1+a)(1+b)(2-\alpha(1+a-b)-b)^2}$.

Assume that B sets $w_B = \overline{w}$ and A deviates by setting $w_A \leq \overline{w}$. If the transfers are such that $t_{Ki} \leq t_{Kil} = (2-b)w_K - 1 + b$, all retail prices are constrained in stage 3, even in case of a failure in one negotiation, and producer A anticipates the profit $\Pi_A^d = \frac{2(1-a(1-\overline{w})-w_A)((2-b)w_A-1+b)}{(1-a^2)(1+b)}$, which is maximum for $w_A^d = 1 - \frac{1}{2(2-b)} - \frac{a(1-\alpha)}{2(2-\alpha(1+a-b)-b)}$. If $\alpha \leq \overline{\alpha}$, $w_A^d \geq \overline{w}$ and no deviation of this type is profitable for A. However if $\alpha \geq \overline{\alpha}$, $w_A^d \leq \overline{w}$. If A sets w_A^d in stage 1, the resolution of the Nash condition between producer A and his retailers leads to a corner solution. The unique subgame equilibrium of the game is as follows: each pair (A, i) negotiates the corner transfer $t_{Ail} = (2-b)w_A^d - 1 + b$, each pair (B, i) negotiates the interior transfers $t_{Bi}^* = \frac{\alpha(\overline{w}+a((1-b)w_A^d-1+b))}{1-(1-\alpha)b}$, and the retail prices set in stage 3 are $p_{Ai} = w_A^d$ and $p_{Bi} = \overline{w}$. Producer $A's$ profit is then larger than $\overline{\Pi_K}$: this deviation is profitable for any $\alpha \geq \overline{\alpha}$.

Yet if $\alpha \leq \overline{\alpha}$, no deviation is profitable. First, if both producers set the unit price $\overline{w}$, and second stage negotiations lead to the transfers (10), the four retail prices will be constrained and equal to $\overline{w}$. Second, if both producers set the unit price $\overline{w}$, the only subgame equilibrium outcome defines the transfers (10). Third, it cannot be profitable for A to set a higher unit price, as retail prices would remain constrained and $A's$ profit would be lower than $\overline{\Pi_K}$. Finally, using the same method as in section $A3.1$ we show that it is not profitable to deviate by setting a lower wholesale price such that transfers are set to (10) and retail prices for A are not constrained. Here excluding B would never be possible, and the deviation without exclusion of B would give A a profit lower than $\overline{\Pi_K}$ for $\widetilde{\alpha} \leq \alpha \leq \overline{\alpha}$.

6.3 The corner transfer equilibrium: proof of lemma 5

The unit price $w_A = w_B = \widehat{w}$ sustains an equilibrium for $\overline{\alpha} \leq \alpha \leq \widehat{\alpha} = \frac{2(2-b)}{4-b}$ where each pair negotiates limit transfers $\widehat{t} = (2-b)\,\widehat{w} - 1 + b$. First, if both producers have set the unit price $\widehat{w}$ in stage 1, and all other pairs' negotiations outcomes in stage 2 are the optimal transfer $\widehat{t}$, the unique solution of the Nash condition of the negotiation for the last pair is the corner solution $\widehat{t}$ as long as $\alpha \leq \widehat{\alpha}$. Second, we show that if producer B chooses $\widehat{w}$ in stage 1, the best response of A is to set the same unit price.

7 Two-Part Tariffs

So far we have assumed that tariffs were linear, and that negotiations determined unit net transfers. We extend here our analysis with two-part tariff contracts. We assume that stages 1 and 3 of the game are unchanged, but that the stage 2 negotiations now define, for each pair of producer K and retailer i, a unit transfer t_{Ki} and a fixed fee F_{Ki}, both positive: The total transfer retailer i pays producer K for the quantity q_{Ki} is now $t_{Ki}q_{Ki} + F_{Ki}$. With two-part tariffs, profits are, with P the vector of retail prices and $D_{Ki}(P)$ the demand for product Ki :

$$\begin{aligned}
\Pi_K &= t_{Ki}D_{Ki}(P) + F_{Ki} + t_{Kj}D_{Kj}(P) + F_{Kj} \\
\Pi_i &= (p_{Ki} - t_{Ki})D_{Ki}(P) - F_{Ki} + (p_{Kj} - t_{Kj})D_{Kj}(P) - F_{Kj}
\end{aligned}$$

As with linear tariffs, we derive the equilibria first without restriction, then with RPM and finally under a price-floor.

7.1 ***No restriction***: proof of proposition 6

We solve the game in the no restriction case with a general demand before considering our linear demand case. The demand function is denoted $D_{Ki}(P)$. We assume that the demand is twice continuously differentiable, downward sloping, and symmetric with $\frac{\partial D_{Ki}}{\partial p_{Ki}} = \frac{\partial D_{Kj}}{\partial p_{Kj}} < 0$ and $\frac{\partial D_{Ki}}{\partial p_{Kj}} = \frac{\partial D_{Kj}}{\partial p_{Ki}} > 0$. Besides, we assume that direct effects dominate cross-price effects:

$$\left|\frac{\partial D_{Ki}}{\partial p_{Ki}}\right| > \left|\frac{\partial D_{Ki}}{\partial p_X}\right| \text{where } X \in \{Kj, Li, Lj\}. \tag{18}$$

Finally, we assume that, when a retailer sets his pair of final prices, the transfer he pays to one producer only affects the final price of his product, as with a linear demand: $\frac{\partial p_{Li}^{BR}}{\partial x_{Ki}} = 0$.

In the last stage, the maximisation of retailer i's profit ($i = 1, 2$) with respect to its two prices (p_{Ai}, p_{Bi}) yields a system of two first order conditions (for $\{K, L\} = \{A, B\}$ and omitting the argument when obvious).

$$(p_{Ki} - t_{Ki})\tfrac{\partial D_{Ki}(.)}{\partial p_{Ki}} + D_{Ki}(.) + (p_{Li} - t_{Li})\tfrac{\partial D_{Li}(.)}{\partial p_{Ki}} = 0 \tag{19}$$

The resolution of this system defines retailer i's pair of best response final prices $(p_{Ai}^{BR}, p_{Bi}^{BR})(\overline{p_{Kj}}, \overline{p_{Lj}}, t_{Ai}, t_{Bi})$. We denote $P = (p_{Ki}^{BR}, p_{Li}^{BR}, \overline{p_{Kj}}, \overline{p_{Lj}})$.

In the second stage, the Nash program for each of the four pairwise negotiations is as follows:

$$\underset{t_{Ki},F_{Ki}}{Max}\,(\Pi_K^a - \Pi_K^{3a})^{\alpha}(\Pi_i^a - \Pi_i^{3a})^{1-\alpha} \tag{20}$$

It yields the following two first order conditions:

$$\alpha\tfrac{\partial(\Pi_K - \Pi_K^3)}{\partial t_{Ki}}(\Pi_i - \Pi_i^3) + (1-\alpha)(\Pi_K - \Pi_K^3)\tfrac{\partial(\Pi_i - \Pi_i^3)}{\partial t_{Ki}} = 0 \tag{21}$$

$$\alpha\tfrac{\partial(\Pi_K - \Pi_K^3)}{\partial F_{Ki}}(\Pi_i - \Pi_i^3) + (1-\alpha)(\Pi_K - \Pi_K^3)\tfrac{\partial(\Pi_i - \Pi_i^3)}{\partial F_{Ki}} = 0 \tag{22}$$

The two polar cases where producers make take-it-or leave-it offers to the retailers ($\alpha = 1$) and where retailers have all bargaining power ($\alpha = 0$) provide the intuitions for the main case, with a general demand function.

- Producers offer take-it-or-leave-it contracts to retailers: $\alpha = 0$.

Assume first that producers offer take-it-or-leave-it contracts to retailers: $\alpha = 1$. In the second stage, producer K holds all the bargaining power in his negotiation with each retailer. Program (20) thus amounts to producer K maximizing his profit in (t_{Ki}, F_{Ki}) given (t_{Kj}, F_{Kj}) and anticipating the prices $(\overline{p_{Kj}}, \overline{p_{Lj}})$, subject to the retailer's participation constraint:[12]

$$\underset{t_{Ki},F_{Ki}}{Max}\, t_{Ki}D_{Ki}(P) + F_{Ki} + t_{Kj}D_{Kj}(P) + F_{Kj} \tag{23}$$

$$s.t.\Pi_i \geq \Pi_i^{SQ} \tag{24}$$

where $\Pi_i^{SQ} = (p_{Li}^3 - t_{Li})D_{Li}^3(p_{Li}^3, \overline{p_{Kj}}, \overline{p_{Lj}}) - F_{Li}$ is the outside option profit of the retailer. The binding participation constraint of the retailer determines F_{Ki} as a function of t_{Ki}:

$$F_{Ki} = (p_{Ki}^{BR} - t_{Ki})D_{Ki}(P) + (p_{Li}^{BR} - t_{Li})D_{Li}(P) + (p_{Li}^3 - t_{Li})D_{Li}^3(p_{Li}^3\overline{p_{Kj}}, \overline{p_{Lj}}) \tag{25}$$

Substituting (25) into (23) and rearranging, with $\frac{\partial p_{Ki}^{BR}}{\partial t_{Ki}}$ strictly positive, yields the following first order condition:

$$D_{Ki}(P) + p_{Ki}^{BR}\frac{\partial D_{Ki}}{\partial p_{Ki}}(P) + (p_{Li} - t_{Li})\frac{\partial D_{Li}}{\partial p_{Ki}}(P) + t_{Kj}\frac{\partial D_{Kj}}{\partial p_{Kj}}(P) = 0 \tag{26}$$

The producer chooses the optimal transfer t_{Ki} as if he were vertically integrated with downstream retailer i. Substituting (19) into (26) yields:

$$t_{Ki}\frac{\partial D_{Ki}(P)}{\partial p_{Ki}} + t_{Kj}\frac{\partial D_{Kj}(P)}{\partial p_{Ki}} = 0. \tag{27}$$

The same applies for the pair (K, j). Under (18) the only equilibrium is therefore such that: $t_{Ki} = t_{Kj} = 0$.

- Retailers offer take-it-or-leave-it contracts to producers: $\alpha = 0$.

Similarly, when $\alpha = 0$ (retailers offer take-it-or-leave-it contracts to producers), in the contracting stage, producer K's participation constraint is binding: $\Pi_K \geq \Pi_K^{SQ}$ where $\Pi_K^{SQ} = (t_{Kj})D_{Kj}^3(p_{Li}^3, \overline{p_{Ki}}, \overline{p_{Lj}})$, and determines the fixed fee F_{Ki}. The maximization of retailer $i's$ profit yields the same unit transfer $t = 0$ and resulting equilibrium prices.

- Balanced sharing of power: α in (0,1).

The system of the eight first-order conditions given by (21) and (22) yields a unique symmetric subgame equilibrium: In our linear framework, the unit transfers are set to cost ($t_{Ki} = 0$) and franchise fees to $F_{Ki}^{TP} = \frac{\alpha(1-a)(1-b)}{(1+a)(1+b)(2-b)^2}$.

[12]Note that again, the outcome of the first stage is immaterial to the subsequent decisions.

As with linear tariffs, there is no commitment in stage 1. The symmetric equilibrium retail prices are $p_{Ki}^{TP} = \frac{1-b}{2-b}$ and profits are as follows:

$$\begin{aligned} \Pi_K &= 2F_{Ki} \\ \Pi_i &= \frac{2(1-b)(1-\alpha(1-a))}{(1+a)(1+b)(2-b)^2} \end{aligned}$$

Total profit is lower with two-part tariffs than with linear tariffs, except for $\alpha \in [\frac{2b(2-b)}{2-b^2-2a(1-b)}, 1]$ (this interval exists only for low values of a and b): Two-part tariffs suppress double margin; yet when α is not too high, double margin increases joint profits by relaxing competition at both levels and increasing retail prices towards the joint-profit maximizing price $1/2$. Therefore suppressing double margin hurts joint profit, except when double margin drives prices too high, which happens when a and b are low and α is high. Producers' share of total profit is $\alpha(1-a)$: it increases in their bargaining power α and decreases in the intensity of upstream competition a.

7.2 RPM equilibria

Assume that producers impose RPM prices w_A and w_B in stage 1.

7.2.1 Proof of lemma 6

In the second stage, the four simultaneous negotiations determine the two-part wholesale tariffs. Consider the negotiation between producer K and retailer i. Final prices will be set at the RPM level and final demands are thus fixed too. Anticipating the outcome of the three other negotiations, the two first order conditions (21) and (22) can be written as follows:

$$\begin{aligned} D_{Ki}(W)[\alpha[(w_K - t_{Ki})D_{Ki}(W) - F_{Ki} + (w_L - t_{Li})(D_{Li}(W) - D_{Li}^3(.))] \\ -(1-\alpha)[t_{Ki}D_{Ki}(W) + F_{Ki} + t_{Kj}(D_{Kj}(W) - D_{Kj}^3(.))]] = 0 \\ \alpha[(w_K - t_{Ki})D_{Ki}(W) - F_{Ki} + (w_L - t_{Li})(D_{Li}(W) - D_{Li}^3(.))] \\ -(1-\alpha)[t_{Ki}D_{Ki}(W) + F_{Ki} + t_{Kj}(D_{Kj}(W) - D_{Kj}^3(.))] = 0 \end{aligned}$$

This system is degenerated and equivalent to

$$\frac{-(1-a^2)(1+b)F_{Ki} + (1-a-w_K+aw_L)(\alpha(w_K+at_{Li}^a-aw_L)-t_{Ki}+bt_{Kj}^a(1-\alpha))}{(1-a^2)(1+b)} = 0 \qquad (28)$$

Given (w_A, w_B), there exists a continuum of subgame equilibria. The degenerated system of eight Nash conditions yields a system of solutions $(F_{A1}, F_{B1}, F_{A2}, F_{B2})$ as a function of four parameters $(t_{A1}, t_{B1}, t_{A2}, t_{B2})$:

$$F_{Ki} = \frac{(1-a-w_K+aw_L)(\alpha(w_K-a(w_L-t_{Li}^a))-t_{Ki}+b(1-\alpha)t_{Kj}^a)}{(1-a^2)(1+b)} \qquad (29)$$

The net transfers vary in an interval defined by the participation constraints of all players. Note that we also assume that the net transfers are nonnegative (products cannot be subsidized) and less than the RPM price, as negative rebates are not allowed (we discuss this assumption further down).

All equilibrium pairs (t_{Ki}, F_{Ki}) yield the same profits for producer K and retailer i: For instance producer K makes the following profit with retailer i:

$$F_{Ki} + t_{Ki} D_{Ki}(W) = \frac{(1-a-w_K+aw_L)(\alpha(w_K-a(w_L-t^a_{Li}))+b(1-\alpha)t^a_{Kj})}{(1-a^2)(1+b)} \tag{30}$$

Given the outcome of the three other negotiations $(t^a_{Kj}, F^a_{Kj}, t^a_{Li}, F^a_{Li}, t^a_{Lj}, F^a_{Lj})$, producer K's total profit $\Pi_K = F_{Ki} + t_{Ki}D_{Ki}(W) + t_{Kj}D_{Kj}(W) + F_{Kj}$ does neither depend on (t_{Ki}, F_{Ki}), nor on any fixed fee, but it depends on the anticipated net tariffs negotiated by the three other pairs, which influence the status quo profits: it increases in t^a_{Kj} (because the status quo profit of producer K increases in the tariff he negotiates with retailer j) and increases in t^a_{Li} (because the status quo profit of retailer i decreases in the tariff he negotiates with producer L).

We measure the subgame equilibrium joint profit of the whole distribution channel of product K by

$$\begin{aligned} J\Pi_K &= \Pi_K + (\Pi_1 - \Pi^3_1) + (\Pi_2 - \Pi^3_2) \\ &= w_K(D_{K1} + D_{K2})(W) + (p_{K1} - t_{K1})(D_{K1} - D^3_{K1}) + (p_{K2} - t_{K2})(D_{K2} - D^3_{K2}) \\ &= \frac{2(w_K + a\frac{t_{L1}+t_{L2}}{2} - aw_L)(1-a+w_K-aw_L)}{(1-a^2)(1+b)} \end{aligned}$$

As final prices are fixed by RPM, quantities are also fixed and the joint profit of the vertical structure $(K, 1, 2)$ for the sale of product K is independent of the transfers t_{Ki} negotiated inside the structure and of all fixed fees; However it increases in the unit transfer t_{Li} negotiated by the retailers with the upstream competitor because the opportunity cost for retailer i to sell product K (i.e. the reduction in its sales of product L, compared to its status quo profit) increases with its margin on product L.

Among the subgame equilibria, there exist a continuum of equilibria satisfying symmetry across the retailers: $t_{K1} = t_{K2} = t_K$ and $F_{K1} = F_{K2} = F_K$. These equilibria, parametrized by (t_K, t_L), are characterized by:

$$F^\circ_K(w_K, w_L, t_K, t_L) = \frac{(1-a-w_K+aw_L)(\alpha(w_K-a(w_L-t_L)-bt_K)-(1-b)t_K)}{(1-a^2)(1+b)} \tag{31}$$

The participation constraints impose $t_K \leq \frac{at_L+w_K-aw_L}{b}$.[13]

Note that among the symmetric subgame equilibria, the joint profit $J\Pi_K$ is still independent of t_K and increases in t_L. However, the sharing of the profit inside the structure depends on t_K through the effect of t_{Kj} on the status quo profit of producer K in the negotiation with retailer i: we have seen that, although the fixed fees are neutral to the status quo profits, the profit derived by producer K from the sale of its product by retailer i, $t_{Ki}D_{Ki} + F_{Ki}$, is independent of t_{Ki} but increases in t_{Kj} through the effect of t_{Kj} on the producer's

[13]Assuming that $w_K \geq aw_L$ which will be satisfied in a symmetric equilibrium. Note that t_K can be large enough for the fixed fee to be negative.

status quo profit

$$\Pi_K = \frac{2(1-a-w_K+aw_L)(b(1-\alpha)t_K+\alpha(w_K-a(w_L-t_{Li})))}{(1-a^2)(1+b)} \tag{32}$$

7.2.2 Proof of Proposition 7

In stage 1, the producers anticipate one of the subgame equilibria. Assume that all producers anticipate the subgame equilibria given by (t_A, t_B), then producer A's best response to w_B is

$$w_A^{BR}(w_B, t_A, t_B) = aw_B + \frac{\alpha(1-a)-a\alpha t_B-b(1-\alpha)t_A}{2\alpha} \tag{33}$$

For each (t_A, t_B), there exists a unique (w_A, w_B) such that if the producers anticipate the subgame equilibrium outcomes $(t_A, t_B, F_A^\circ, F_B^\circ)$, then setting the RPM prices (w_A, w_B) in stage 1 is an equilibrium strategy. Finally, there is a continuum of symmetric equilibria of the game.[14] Equilibrium final prices are defined as a function of t, the symmetric unit transfer for $\alpha > 0$:

$$w^\circ(t) = \frac{1}{2}(1 - t\frac{b+\alpha(a-b)}{\alpha(1-a)}) \tag{34}$$

with t such that $0 \leq t \leq w^\circ$, *i.e.* for $0 \leq t \leq t_{RPM}$, with $t_{RPM} = \frac{\alpha(1-a)}{b+\alpha(2-a-b)}$.[15] Note that $w^\circ(t)$ decreases in the variable part of the net transfer t. As t_{RPM} decreases with α the set of equilibrium retail prices shrinks when producers' bargaining power increases.

Given α, the different equilibria are not equivalent in terms of profit sharing: producers' profits increase in t, whereas retailers' profits as well as joint profits decrease in t. Producers' profits are

$$\Pi_K = \frac{(\alpha(1-a)+t(a\alpha+b(1-\alpha))^2}{2\alpha(1-a^2)(1+b)} \tag{35}$$

Joint profit is maximum when retail prices are at the monopoly price $p^M = 1/2$, and decreases in t as $w^\circ(t)$ decreases in t. The fact that the equilibrium with two-part tariffs does not necessary maximize joint profit is original. It stems from the existence of a continuum of subgame equilibria in the negotiation stage and from the fact that profit sharing is influenced by the variable parts of the tariffs (t_K and t_L) through their effect on status quo profits. The intuition is as follows. Consider stage 1. If the producers anticipate that the stage 2 negotiations will lead to $t = 0$ and the matching fixed fees, they are going to be paid through the fixed fee only and the best they can do is to maximise

[14]With $w_A = w_B = w$!‘ and $t_A = t_B = t$.

[15]t_{RPM} is defined by the constraint $t \leq w$: rebates must be negative in the legal framework we study. However, we could technically relax this assumption. This would enlarge the set of equilibria, by creating two sets of subgame equilibria for each pair (w_A, w_B): first, enlarge the set of equilibria defined by (34) to any t such that $w \leq t \leq t^{sq}$ where t^{sq} is such that the status quo profit of the retailers becomes zero for any larger t ($t^{sq} = \frac{w(1+a-\alpha(1-a))}{b+a+\alpha(a-b)}$); Second, this also creates new equilibria of the subgame whith $t > t^{sq}$, zero status quo profit for the retailers and subsequent profit sharing inside each pair. These subgame equilibria yield equilibria with lower final prices and all our qualitative results are maintained.

the joint profit, and the equilibrium RPM price is therefore $w = 1/2$. Yet if they anticipate a positive variable part of the transfer, then each producers' status quo profit decreases with its RPM price, whereas the retailers' status quo profits increase with it: Totally differentiating producer $A's$ status quo profit given (t_{A},t_{B}) and w_B gives, in $w_A = w_B = w$ and $t_A = t_B = t$

$$\begin{aligned} \frac{d\Pi_A^{sq}}{dw_A} &= \frac{\partial\Pi_A^{sq}}{\partial w_A} + \frac{\partial\Pi_A^{sq}}{\partial F_A}\frac{\partial F_A}{\partial w_A} \leq 0 \\ &\Leftrightarrow \quad t \geq \frac{\alpha(1-a)(1-2w)}{2+\alpha(a-b)} \end{aligned}$$

yet inverting (34) yields $t = \frac{\alpha(1-a)(1-2w)}{b+\alpha(a-b)}$ *QED.*

This explains why when both producers anticipate positive t_K, then the equilibrium RPM price does not maximize total industry profit. Note that this is not only a consequence of upstream competition, as it would also happen with an upstream monopoly (or in our case with $a = 0$) : there would also exist a continuum of equilibria with different RPM prices that would not always maximise the joint profit.

As t_{RPM} decreases with α, the set of equilibrium retail prices shrinks when producers' bargaining power increases.

Note that the RPM equilibrium with linear tariffs $\widetilde{w}$ is always sustainable as an equilibrium retail price under RPM with two-part tariffs. In that equilibrium, joint profit is the same than with linear tariffs, but retailers get more and producers less.

7.3 Price-floor equilibria: proof of proposition 8

Consider now that $w^{\circ}(t)$ is only a price-floor. In stage 3, if her competitor sets prices p_{Kj} and if the issue of stage 2 negotiations with her two suppliers led to unit prices t_K, retailer i's best response prices are $p_{Ki} = \frac{1-b+t_K+bp_{Kj}}{2}$. This price is below the price-floor, which will be binding indeed, if and only if $t \leq t_{PF}$, with

$$t_{PF} = \frac{\alpha b(1-a)}{b(2-b)+\alpha(2-b(2+a-b))}$$

Therefore the minimum price that is sustainable with price-floor is

$$w^{\circ}\left(t_{PF}\right) = \frac{1}{2}\left(1 - \frac{b(b+\alpha(a-b))}{(2-b)b+\alpha(2-b(2+a-b))}\right)$$

The interval of equilibrium retail prices with two-part tariffs and a price-floor is $[w^{\circ}\left(t_{PF}\right), 1/2]$. It is smaller than under *RPM.* As t_{PF} increases in α, this interval shrinks when α increases. For $\alpha = 0$, the lowest price in equilibrium is $w^{\circ}\left(t_{PF}\right) = \frac{1-b}{2-b}$, whereas it is $w^{\circ}\left(t_{PF}\right) = \frac{1-ab}{2-ab}$ for $\alpha = 1$. Note that for all α, $w^{\circ}\left(t_{PF}\right) \geq \frac{1-b}{2-b}$: retail prices are higher than in the *no-restriction* equilibrium. Finally, retail prices of the linear tariff price-floor equilibria are sustainable as price-floor equilibria with two-part tariffs as long as they are below 1/2.

Producers' profit is given by (35). It is lower with two-part tariffs than with linear tariffs, even for $t = t_{PF}$. With two-part tariffs, producers' profit is always higher with price-floor than without. Retailers' situation is more ambiguous: their profit in the price-floor equilibrium is higher than in the *no-restriction* equilibrium with two-part tariffs with $t_K = 0$, lower with $t_K = t_{PF}$.

7.3.1 Upstream deviations

We check here that producers do not deviate from the price-floor strategies. The only deviation that has not been ruled out by the RPM analysis is one where a producer (say A) deviates by setting a lower price-floor such that retail prices are unconstrained in stage 3, and these deviations are equivalent to setting $w_A = 0$ in stage 1 (see a detailed analysis in the linear tariffs case).

In that case, prices remain constrained at w_B for B in stage 3 but not for product A; Solving the corresponding Nash conditions gives the optimal tariffs in stage 2: $t_{Ki} = 0$. The deviation profit for A is thus $\Pi_A^d = \frac{2\alpha(1-b)(1-a(1-t_B^d))^2}{(1-a^2)(1+b)(2-b)^2}$ where t_B^d is the transfer negotiated between B and a retailer in stage 2. There is a continuum of subgame equilibria where the negotiation outcomes for B are given by $F_B^d = g(w_B, t_B^d)$. Among these, the subgame equilibrium with $t_B^d = 0$ leads to $\Pi_A^d = \frac{2\alpha(1-b)(1-a)}{(1+a)(1+b)(2-b)^2}$ less than $\underline{\Pi_K^{PF}}(t = 0)$, the minimum profit A gets in the price-floor equilibrium: the deviation is not profitable. Consequently, the strategies defined implement an equilibrium with price-floors.

8 Robustness and extensions

8.1 Individual price-floor

Consider that producers and retailers now negotiate in stage 2 over a total transfer $t_{Ki} - f_{Ki}$, where the price-floor is t_{Ki}.

Assume that retail prices are constrained. In stage 2, each pair negotiates over two variables t_{Ki} and f_{Ki} so that eight first-order conditions determine the equilibrium outcomes: $t_{Ki}^\diamond = 1 - \frac{1}{2-\alpha a-(1-\alpha)b}$ and $f_{Ki}^\diamond = \frac{(1-\alpha)(1-b)}{2-\alpha a-(1-\alpha)b}$. In stage 3, retailer $i's$ best response prices are p_{Ki}^{BR}. This price is always larger than the individual price-floor $t_{Ki}^\diamond$ which is therefore not binding.

Producers thus have to raise the price-floor in order to constrain the retailers. The minimum binding price-floor is $\overline{t_{Ki}^\diamond} = \frac{1-a\alpha-b(1-\alpha)}{2-\alpha(1+a-b)-b}$, such that $p_{Ki}^{BR} = \overline{t_{Ki}^\diamond}$ for both retailers. The corresponding $\overline{f_{Ki}^\diamond}$ is inferred from the first-order conditions of stage 2 negotiations: $\overline{f_{Ki}^\diamond} = \frac{(1-a)\alpha}{2-\alpha(1+a-b)-b}$. This does not define an equilibrium as there is always a unilateral incentive for a pair (K, i) to deviate towards a higher price-floor. Furthermore, any price-floor above this level will also fail to sustain an equilibrium.

As there is renegotiation in stage 2, no binding price-floor can sustain an equilibrium. Therefore there is no constrained equilibrium.

8.2 Incomplete breakdown

No restriction case

Consider the negotiations between producer A and retailer 1. In case of a breach, 1 may choose to resell product A. Its optimal final prices are then: $p_{A1}^{BR} = \frac{(1-b(1-p_{A2})+w_A)}{2}$ and $p_{B1}^{BR} = \frac{(1-b(1-p_{B2})+t_{B1})}{2}$. Let us define

$$\widehat{w_A}(p_{A2}^a, p_{B2a}, t_{B1}^a) = 1 - b(1-p_{A2}) + ab(1-p_{B2}) - a(1-t_{B1}) \tag{36}$$

- if $w_A \leq \widehat{w_A}(p_{A2}, p_{B2}, t_{B1})$, the anticipated demand for product $A1$ at the price p_{A1}^{BR} is positive and selling the two products is profitable for retailer 1. The status-quo profits of the two firms are then modified:
  $\Pi_1^{SQ} = (p_{A1}^{BR} - w_A) D_{A1}(p_{A1}^{BR}, p_{A2}, p_{B1}^{BR}, p_{B2}) + (p_{B1}^{BR} - t_{B1}) D_{B1}(p_{A1}^{BR}, p_{A2}, p_{B1}^{BR}, p_{B2}) - F_{B1}$
  $\Pi_A^{SQ} = w_A D_{A1}(p_{A1}^{BR}, p_{A2}, p_{B1}^{BR}, p_{B2}) + t_{A2} D_{A2}(p_{A1}^{BR}, p_{A2}, p_{B1}^{BR}, p_{B2}) + F_{A2}$
- If $w_A > \widehat{w_A}(p_{A2}, p_{B2}, t_{B1})$, it is not profitable for retailer 1 to buy product A at price w_A so that the status quo profit are those of the *complete breakdown* case: Π_1^{SQ} and Π_A^{SQ} (see section 7.1).

Assuming that $w_A = w_B \leq \widehat{w_A}$, the resolution of the Nash bargaining yields the following equilibrium contract, denoted by the exponent $*s$:
$t_{Ki} = t^{*s} = 0$ for $i = 1, 2$ and $K = A, B$
$F_{Ki} = F^{*s} = \frac{w_K(4(1-a(1-b)-w_K)+2\alpha w_K - b(4-(2-\alpha)w_K)}{4(1-a^2)(2-b-2b^2+b^3)}$ for $i = 1, 2$ and $K = A, B$.

Solving the game as in section A.5 yields the following symmetric equilibrium outcomes: $w_A = w_B = w^{*s} = \frac{2(1-a)(1-b)}{(2-\alpha)(2-b)}$ where $w^{*s} \leq \widehat{w_A}(p_{A2}^s, p_{B2}^s, t_{B1}^s)$. Note that final price is unchanged ($p^{*s} = p^{TP} = \frac{1-b}{2-b}$), since it depends only on the equilibrium transfers which are equal to 0 as in the *complete breakdown* case. Producers' equilibrium profit is: $\Pi_K^{*s} = \frac{2(1-a)(1-b)}{(1+a)(2-\alpha)(2-b)^2(1+b)}$. Note that, under incomplete breakdown assumption, when α tends towards 0, producers now obtain a strictly positive profit. The insight is as follows: even if producers have no power in the bargaining, they can break the negotiation and then act as leaders in the classical Stackelberg game, setting their wholesale price first and letting retailers set their final prices afterwards. In the latter case, double marginalization tends to reduce bilateral joint profits, but the producer and the retailer both obtain a strictly positive profit. Thus, even if producers have no power in the bargaining stage, retailers have to give them a strictly positive rent. Note also that w^{*s} is strictly increasing in α. Indeed, as producers have more power, they have more incentives to succeed in the bargaining which provides higher joint profits (no double-marginalization)[16] and thus, by setting a

[16] Given the equilibrium wholesale price w^{*s}, the final price in case of a breakdown in bilateral negotiations would be $p^b = \frac{3-\alpha-a}{2-\alpha}\frac{(1-b)}{(2-b)} > \frac{(1-b)}{(2-b)} = p^{*s}$. Thus, the final price in case of a breakdown would be strictly higher than the equilibrium price p^{*s} that maximizes bilateral joint profits. This is the double-marginalization inefficiency.

higher wholesale price, producers deteriorate the retailers' outside option. In the extreme case where $\alpha = 1$, the wholesale price is so high that there would be no demand left for the product at the retailer 1 in case of a breakdown, and thus the profit producers obtain under incomplete breakdown assumption tends towards the equilibrium profit under the complete breakdown assumption in the no restriction case. However, it is immediate to prove that for $\alpha < 1$, the producers' profit is always improved under the incomplete breakdown assumption. Indeed, as producers set the wholesale price, they thus have control on the level of both their status-quo and the status-quo of the retailer they are bargaining with. This tool enables producer to raise their profit to the detriment of retailers.

Note that among the equilibria obtained under the complete breakdown assumption, those satisfying $w_A^{TP} > \widehat{w_A}(p_{A2}^{TP}, p_{B2}^{TP}, t_{B1}^{TP})$ still exist (the others equilibria are suppressed).[17] Producers' profit is larger with the new "*s" equilibrium.

Price-Floor

Assume that final prices are constrained: $p_{A1} = p_{A2} = w_A$ and $p_{B1} = p_{B2} = w_B$. Imagine that the bargaining between A and 1 breaks. Rewriting the condition (36), we obtain a threshold $\underline{t}(w_A, w_B) = 1 - b + bw_B - \frac{(1-b)(1-w_A)}{a}$ such that:

- If $t_{B1} \geq \underline{t}(w_A, w_B)$ the status-quo profit of firm 1 is: $\Pi_1^{SQ} = (p_{A1}^{BR} - w_A)D_{A1}(p_{A1}^{BR}, w_A, w_B, w_B) + (w_B - t_{B1})D_{B1}(p_{A1}^{BR}, w_A, w_B, w_B) - F_{B1}$ and the status-quo profit for producer A is: $\Pi_A^{SQ} = w_A D_{A1}(p_{A1}^{BR}, w_A, w_B, w_B) + t_{A2}D_{A2}(p_{A1}^{BR}, w_A, w_B, w_B) + F_{A2}$.
- If $t_{B1} < \underline{t}(w_A, w_B)$ status-quo profits are those obtained under the complete breakdown assumption i.e. Π_1^3 and Π_A^3.

First note that $t_{PF} < \underline{t}(w_A, w_B)$ for any $a > \frac{1}{2}$, α and b: Therefore all price-floor equilibria such that $0 < t_{Ki} < t_{PF}$ defined under the complete breakdown assumption still exist in our new framework when $a > \frac{1}{2}$.

Let us now look for another type of equilibrium such that $t_{B1} \geq \underline{t}(w_A, w_B)$. The simultaneous Nash bargaining leads to the following symmetric equilibrium, for $K = A, B$ and $i = 1, 2$:

$w_K = w^s(t) = \frac{(1-b)(1+\alpha(2-b(1-t)-bt)-a(1-b+\alpha(2-b)(1-b+t)-t)}{2-(2-a)b+\alpha(3-a(2-b)^2-4b+b^2)}$ and $t_{Ki} = t^s = t$.

$F_{Ki} = F^s = \frac{(1-w)(2(w(1-2a)-(2(1-a)-b)t)-\alpha(1-b-2a(1+t-b-(1-w)+2w))+3w-b(2t+w)}{4(1-a^2)(1+b)}$.

The price-floor is binding iff $t \leq \overline{t^s} = \frac{\alpha+b-2a}{2(1-a)+b(2-b)+\alpha((3(1-b)+b^2))}$. Yet it is straightforward that $\overline{t^s} < \underline{t^s} = \underline{t}(w_A, w_B)$ when $a > \frac{1}{2}$ which contradicts our initial existence condition. There are no new price-floor equilibria when $a > \frac{1}{2}$. However, when $a < Min[\frac{\alpha+b}{2}, \frac{1}{2}]$ there exist new equilibria as defined above for $\underline{t^s} < t < \overline{t^s}$. Among the equilibria, producers select one equilibrium which

[17] Note that $\widehat{w_A}(p_{A2}^{TP}, p_{B2}^{TP}, t_{B1}^{TP}) = \frac{2(1-a-b+ab)}{2-b}$

maximizes its profit and thus chooses the optimal wholesale price:

$$\underline{w^s} = \frac{1-2a+b(1-b)+\alpha(2-2b+b^2)}{2(1-a)+b(2-b)+\alpha((3(1-b)+b^2))}$$

Producers obtain the corresponding profit $\Pi_A(\underline{w^s})$. We obtain three cases:

- If $a > \frac{1}{2}$, equilibria are identical to those existing under the complete breakdown assumption and producers may get $\Pi_A(\underline{w^{PF}})$.
- If $a < \frac{1}{2}$, new types of equilibria may give a profit $\Pi_A(\underline{w^s})$ to producers and old type equilibria which may give a profit $\Pi_A(\underline{w})$ and:
  - If $\alpha > 1-b$ there a new types of equilibria whatever $a \in [0, \frac{1}{2}]$ also as old type equilibria if $a \in [\frac{1-b}{2-b}, \frac{1}{2}]$.
  - For $\frac{(2-b)^2-2}{2-b} < \alpha \leq 1-b$, there are new type equilibria for $a \in [0, \frac{1-b}{2-b}]$, both old and new types equilibria when $a \in [\frac{1-b}{2-b}, \frac{\alpha+b}{2}]$ and only old type equilibria when $a \in [\frac{\alpha+b}{2}, \frac{1}{2}]$.
  - If $\alpha < \frac{(2-b)^2-2}{2-b}$ there are new type equilibria for $a \in [0, \frac{\alpha+b}{2}]$, no equilibrium for $a \in [\frac{\alpha+b}{2}, \frac{1-b}{2-b}]$ and old type equilibria for $a \in [\frac{1-b}{2-b}, \frac{1}{2}]$.

Note that the threshold $\frac{(2-b)^2-2}{2-b}$ is strictly positive only for $b < \sqrt{2}(\sqrt{2}-1)$.

When both equilibria co-exist, the new status-quo equilibrium improves producers' profit only if a is not too high $\Pi_A(\underline{w^s}) > \Pi_A(\underline{w})$ if $a < \hat{a}(\alpha, b)$.

8.3 Secret vs. public contracts

Assume that $a = 0$ (upstream monopoly) and the outcome of each negotiation is published at the end of stage 2. Without legal restriction, in stage 3, each retailer knows the issue of all negotiations, so that the retail prices are given by $p_i = \frac{2-b(1+b)+2t_i+bt_j}{4-b^2}$. In stage 2, if there is a failure in negotiation between the producer and retailer 1, retailer 2 sets the monopoly price $\frac{1+t_2}{2}$ which is larger than the price he sets without EPO. Quantity sold thus decreases, as does the producer's status-quo profit which is now $\frac{(1-t_2)t_2}{2}$; retailer 1's status-quo profit is still 0. As a consequence, the transfers negotiated are lower than with secret contracts and retail equilibrium prices as well.

Under the law and without *EPO*, the producer would choose $\widetilde{w} = \frac{1}{2}$ for $\alpha \leq \widetilde{\alpha} = \frac{b}{(1+b)}$. EPO leads to different status-quo profits in stage 2: If the unit price set in stage 1 is w, when negotiation breaks with retailer i, the other retailer may set his price above w. There exists $\{\underline{w^o}, \overline{w^o}\}$ such that

(*i*) if $w \leq \underline{w^o}$, retailers' pricing strategies are not constrained in stage 3 ($p_i \geq w$);

(*ii*) if $\underline{w^o} \leq w \leq \overline{w^o}$, retailers' pricing strategies are constrained in stage 3 but not if one negotiation breaks: If negotiation breaks between A and 1, retailer

2 will set the optimal monopoly price $p_2^1 = \frac{1+t_2}{2} \geq w$, and the producer's status-quo profits is $\Pi_P^1 = t_2(\frac{1-t_2}{2})$. The negotiations determine optimal net transfers such that the producer's profit is concave in w.

(iii) if $w \geq \overline{w^o}$ retailers' pricing strategies are constrained in stage 3 and in the status-quo; the optimal net transfers are thus $t_i = \frac{\alpha w}{1-b(1-\alpha)}$ and the producer's profit decreases in w for $w \geq \overline{w^o}$.

We show that $0 \leq \underline{w^o} \leq \overline{w^o} \leq 1$ and $\overline{w^o} \geq 1/2$. Furthermore $\underline{w^o} \leq 1/2$ for all $\alpha \leq \alpha_{\text{lim}} = \frac{b(2-b-b^2)}{2-b^2-b^3}$, with $\alpha_{\text{lim}} \geq \widetilde{\alpha}$.

For all $\alpha \leq \alpha_{\text{lim}}$, if the producer sets the wholesale price $w = 1/2$, retailers are constrained in equilibrium but not in the status-quo case. The producer's profit is thus larger than his unconstrained profit. Furthermore, it increases in w for $w \leq \frac{1}{2}$, so that the optimal binding wholesale price is in $]\frac{1}{2}, \overline{w^o}]$. The producer is better off with a binding wholesale price larger than $\widetilde{w} = \frac{1}{2}$ and in a wider zone than without EPO (at least for $\alpha \leq \alpha_{\text{lim}}$ which is larger than $\widetilde{\alpha}$).