Introduction

- Importance of distinguishing cognitive and non-cognitive skills: both explain large part of the variance of many outputs.
- Many works on test scores but few works on non-cognitive skills and its evolution.
- The idea of this paper is to jointly model the formation and evolution of cognitive and non-cognitive skills. It allows for complementarity between skills.
Modeling a skill production function

Taking skills accumulation as the result of a production function:

- Inputs are the past state of the outputs: previous level of cognitive and non-cognitive skills.
- Investments at each period: parental/school, etc.

Literature on skill production functions is large Todd and Wolpin (2003, 2007), Hanushek, etc.
Modeling a skill production function

Three main issues arise in this framework:

▶ What is the right measurement for skills and investment?
  ▶ In data we observe many test scores, how to use all this information to account for skills.
  ▶ We also observe many parental behaviors (buying books, going to the theater, telling stories, time spend with the children, etc.)

▶ Investments are likely to be endogenous:
  ▶ When parents choose the level of investment, they take into account unobserved heterogeneity of children.

▶ Skills have no natural unit:
  ▶ The existing literature use the test score standard deviation as a unit.
  ▶ Need to find the impact of skills on an output that have a well defined scale.
What they do:

Cunha and Heckman answer these issues:

- Relying on factor models previously derived by Joreskog and Goldberger (1975), they model skills and investments as factors and scores/parental inputs as noisy signals of these factors.
- The technology of skill formation is a dynamic factor model.
- Endogeneity of inputs is solved by using instrumental variable framework. Instrument are justified by the budget constraint of households.
- Scaling problem is solved by anchoring test scores in adult outcomes, in particular earnings.
Define two skills:

- $\theta^C$: Cognitive skill
- $\theta^N$: Non-cognitive skill

Investment in skill $k \in \{C, N\}$ at period $t$ is denoted $\theta^l_{k,t}$. The technology production of skill $k$ is given by:

$$\theta^k_{t+1} = f^k_t(\theta_t, \theta^l_k)$$

with $\theta_t = (\theta^C_t, \theta^N_t)$.

Adult human capital is defined as period $T$:

$$h = g(\theta^C_{T+1}, \theta^N_{T+1})$$
Model:

The model allows for:
- measuring crossed impacts of skills.
- isolating key periods for investments.
Model: linear technology

Identification and estimations are derived for the linear model:

\[
\begin{pmatrix}
\theta^N_{t+1} \\
\theta^C_{t+1}
\end{pmatrix}
= 
\begin{pmatrix}
\gamma_1^N & \gamma_2^N \\
\gamma_1^C & \gamma_2^C
\end{pmatrix}
\begin{pmatrix}
\theta^N_t \\
\theta^C_t
\end{pmatrix}
+ 
\begin{pmatrix}
\gamma_3^N \\
\gamma_3^C
\end{pmatrix}
\theta^I_t 
+ 
\begin{pmatrix}
\eta^N_t \\
\eta^C_t
\end{pmatrix}
\]

with:

\[
\text{cov}(\eta^k_t, \eta^{k'}_{t'}) = 0
\]
Skill are not directly measured. We only observe imperfect measurements for it; To take it into account, CH use a dynamic factor structure:

\[ Y_{j,t}^k = \mu_{j,t}^k + \alpha_{j,t}^k \theta_t^k + \varepsilon_{j,t}^k, \quad \forall j \in \{1, \ldots, m_t^k\}, \quad k \in \{C, N, I\} \]

with:

- \( Y_{j,t}^k \) measurements: test scores, observed behaviors, observed investments (books, storytelling, etc.)
- \( \alpha_{j,t}^k \) are factor loadings. For all latent variable, one factor loading is set to 1 for all periods: \( \alpha_{1,t}^k = 1 \)
Simple case:

2 measurements by skill.
Additional assumptions:

(A1) : \( \varepsilon_{j,t} \perp \varepsilon_{j,t'} \ \forall t' \neq t \)

(A2) : \( \varepsilon_{j,t} \perp \theta_{t'}^{k'} \ \forall j, k, k', t, t' \)

(A3) : \( \varepsilon_{j,t} \perp \varepsilon_{i,t} \) for \( i \neq j; k = l \) or \( i = j; k \neq l \)
Identification: factor loadings

Remember that we have:

\[ Y_{j,t}^k = \mu_{j,t}^k + \alpha_{j,t}^k \theta_t^k + \varepsilon_{j,t}^k \]

Under the normalization of one factor loading by skill \((\alpha_{1,t}^C = \alpha_{1,t}^N = 1 \ \forall t)\), we have:

\[
\begin{align*}
\text{cov}(Y_{1,t}^C, Y_{1,t+1}^C) &= \text{cov}(\theta_t^C, \theta_{t+1}^C) \\
\text{cov}(Y_{2,t}^C, Y_{1,t+1}^C) &= \alpha_{2,t}^C \text{cov}(\theta_t^C, \theta_{t+1}^C) \\
\text{cov}(Y_{2,t}^C, Y_{1,t+1}^C) &= \alpha_{2,t+1}^C \text{cov}(\theta_t^C, \theta_{t+1}^C)
\end{align*}
\]

From well chosen ratios, factor loadings are identified from the data. Same idea for \(k = N, I\)
Identification: distribution of skills

It is then possible to identify distribution of skills. First rewrite the measurement model:

\[
\frac{Y^k_{j,t}}{\alpha^k_{j,t}} = \frac{\mu^k_{j,t}}{\alpha^k_{j,t}} + \theta^k_t + \frac{\varepsilon^k_{j,t}}{\alpha^k_{j,t}}
\]

then we can find (first) moments of the joint distribution of \(\{(\theta^C_t, \theta^N_t, \theta^I_t)\}_t\):

\[
\text{cov} \left( \frac{Y^k_{i,t}}{\alpha^k_{i,t}}, \frac{Y^k_{j,t}}{\alpha^k_{j,t}} \right) = \text{var}(\theta^k_t)
\]

\[
\text{cov}(Y^k_{1,t}, Y^k_{1,t'}) = \text{cov}(\theta^k_t, \theta^k_{t'})
\]
Identification: technology

Focussing on the evolution of one skill:

$$\theta_{t+1}^N = \gamma_0^N + \gamma_1^N \theta_t^N + \gamma_2^N \theta_t^C + \gamma_3^N \theta_t^I + \eta_t^N$$

From zero-mean measurements $\tilde{Y}_t^k$, if we estimate the corresponding relation, we have biased estimates:

$$\tilde{Y}_{k,t+1}^N = \gamma_0^N + \gamma_1^N \tilde{Y}_{k,t}^N + \gamma_2^N \tilde{Y}_{k,t}^C + \gamma_3^N \tilde{Y}_{k,t}^I + \omega_{t+1}^N$$

since:

$$\omega_{t+1}^N = \varepsilon_{k,t+1}^N - \gamma_1^N \varepsilon_{k,t}^N - \gamma_2^N \varepsilon_{k,t}^C - \gamma_3^N \varepsilon_{k,t}^I$$

and $\tilde{Y}_{k,t}^N, \tilde{Y}_{k,t}^C$ and $\tilde{Y}_{k,t}^I$ are endogenous.

Using any other measure $k'$ solves the problem from the independence of measurement error and the independence between $\eta_t$ and measurement error.
Different case

Replacing:

\[(A3) : \varepsilon_{j,t}^k \perp \varepsilon_{i,t}^l \text{ for } (i \neq j; k = l) \text{ or } (i = j; k \neq l)\]

by

\[(A4) : \varepsilon_{1,t}^k \perp \varepsilon_{i,t'}^l \forall i \neq 1, t, t', k, l\]

\[\varepsilon_{1,t}^k \perp \varepsilon_{i,t'}^k \forall t \neq t'\]

- Instead of assuming independence of all contemporaneous measurement error, we only assume that one measurement error is independent.
- This allows for a minimum of correlation between measurements at a given period.
- Identification proved in the paper with the same proof structure.
Correlated Omitted inputs

Suppose $\eta_t^k$ is correlated with $\theta_t$ and assume the following specification for $\eta_t^k$:

\[
\begin{align*}
\eta_t^N &= \gamma_4^N \lambda + \nu_t^N \\
\eta_t^C &= \gamma_4^C \lambda + \nu_t^C
\end{align*}
\]

with $\lambda$ potentially correlated with $\theta_t$ but $\nu_t^k$ independent from $\theta_t$, $\lambda$ and $\nu_{t'}$.

Identification is obtained by:

- Taking the first difference of the production function to eliminate the individual “fixed effect”.
- As before, substituting skills by the corresponding measurements in the specification.
- Instrumenting by other measurements.
- Identification can also be obtained for time varying technology ($\gamma_{j,t}^k$).
Anchoring factors

To find a relevant scale to compare skills. They anchor skills in adult earning:

$$\ln Y = \mu_T + \delta_N \theta_T^N + \delta_C \theta_T^C + \varepsilon$$

then defining $D$ as:

$$D = \begin{pmatrix} \delta_N & 0 \\ 0 & \delta_C \end{pmatrix}$$

Skills can be transform to an earning metric easily:

$$D\theta_{t+1} = (DAD^{-1})(D\theta_t) + DB\theta_t^l + D\eta_t$$
Estimation: Likelihood

The likelihood is derived under normality assumption. The strategy is based on the Kalman filter:

- Observed data is composed of measurements
  \((Y^N_{1,t}, \ldots, Y^N_{m^N_t, t}, Y^C_{1,t}, \ldots, Y^C_{m^C_t, t}, Y^I_{1,t}, \ldots, Y^I_{m^I_t, t})\).

- Denote by \(Y_t\) observation at time \(t\) and by \(Y^t\) observations up to period \(t\).

- The Kalman Filter method allows to derive the distribution of \(Y_{t+1}|X, Y^t\) (\(X\) are control added in the specifications)

- Then the likelihood can easily be written:

\[
L(\beta) = f(Y_{i,1}, \ldots, Y_{i,T}|X, \beta) = f(Y_{i,1}|X, \beta) \prod_{t=2}^{T} f(Y_{i,1}|X, Y_{i,t-1})
\]

and estimated...

- From this specification, missing at random measurements can be integrated out...
Estimation

Data:

- 1053 white males from CNLSY/1979
- Measurements for cognitive skills:
  - PIAT Mathematics
  - PIAT Reading recognition
- Measurements for non cognitive skills:
  - behavior problems: antisocial, anxious/depressed, headstrong, hyperactive, peer problem
- Measurements for investments
  - Number of books available to the child
  - child has musical instrument
  - family receives a daily newspaper
  - child receive special lessons
  - how often he goes to the museum
  - goes to the theater
  - (sometimes) parental income
Results

Table 2
Unanchored Technology Equations: Measurement Error is Classical, Absence of Omitted Inputs Correlated with $\theta_t$ White Males, CNLSY/79

<table>
<thead>
<tr>
<th>Independent Variable</th>
<th>Noncognitive Skill ($\theta_{t+1}^N$)</th>
<th>Cognitive Skill ($\theta_{t+1}^C$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>Lagged noncognitive skill, ($\theta_t^N$)</td>
<td>0.884</td>
<td>0.884</td>
</tr>
<tr>
<td></td>
<td>(0.021)</td>
<td>(0.021)</td>
</tr>
<tr>
<td>Lagged cognitive skill, ($\theta_t^C$)</td>
<td>0.003</td>
<td>0.003</td>
</tr>
<tr>
<td></td>
<td>(0.013)</td>
<td>(0.012)</td>
</tr>
<tr>
<td>Parental investment, ($\theta_t^I$)</td>
<td>0.072</td>
<td>0.078</td>
</tr>
<tr>
<td></td>
<td>(0.020)</td>
<td>(0.021)</td>
</tr>
<tr>
<td>Mother’s education, $S$</td>
<td>0.004</td>
<td>0.004</td>
</tr>
<tr>
<td></td>
<td>(0.008)</td>
<td>(0.008)</td>
</tr>
<tr>
<td>Mother’s cognitive skill, $A$</td>
<td>-0.006</td>
<td>-0.006</td>
</tr>
<tr>
<td></td>
<td>(0.006)</td>
<td>(0.006)</td>
</tr>
</tbody>
</table>

a. Let $\theta_t = (\theta_t^N, \theta_t^C, \theta_t^I)$ denote the noncognitive, cognitive and investment dynamic factors, respectively. Let $S$ denote mother’s education and $A$ denote mother’s cognitive ability. The technology equations are:

$$
\theta_{t+1}^k = \gamma_1^k \theta_t^N + \gamma_2^k \theta_t^C + \gamma_3^k \theta_t^I + \psi_1^k S + \psi_2^k A + \eta_t^k.
$$

In this table we show the estimated parameter values and standard errors (in parentheses) of $\gamma_1^k, \gamma_2^k, \gamma_3^k, \psi_1^k, \psi_2^k$ in Columns 1–6. In Columns 1 and 4, the parental investment factor is normalized on the log-family income equation. In Columns 2 and 5, the parental investment factor is normalized on trips to the museum. In Columns 3 and 6, we normalize the parental investment factor on trips to the theater.
Table 3
Contemporaneous Correlation Matrices: Measurement Error is Classical, Absence of Omitted Inputs Correlated with $\theta_t$, White Males, CNLSY/79

<table>
<thead>
<tr>
<th></th>
<th>Noncognitive</th>
<th>Cognitive</th>
<th>Investments</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Period 1 — Children ages 6 and 7</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Noncognitive</td>
<td>1.0000</td>
<td>0.1892</td>
<td>0.3426</td>
</tr>
<tr>
<td>Cognitive</td>
<td>0.1892</td>
<td>1.0000</td>
<td>0.2921</td>
</tr>
<tr>
<td>Investments</td>
<td>0.3426</td>
<td>0.2921</td>
<td>1.0000</td>
</tr>
<tr>
<td><strong>Period 2 — Children ages 8 and 9</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Noncognitive</td>
<td>1.0000</td>
<td>0.2334</td>
<td>0.4065</td>
</tr>
<tr>
<td>Cognitive</td>
<td>0.2334</td>
<td>1.0000</td>
<td>0.3835</td>
</tr>
<tr>
<td>Investments</td>
<td>0.4065</td>
<td>0.3835</td>
<td>1.0000</td>
</tr>
<tr>
<td><strong>Period 3 — Children ages 10 and 11</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Noncognitive</td>
<td>1.0000</td>
<td>0.2643</td>
<td>0.4785</td>
</tr>
<tr>
<td>Cognitive</td>
<td>0.2643</td>
<td>1.0000</td>
<td>0.4892</td>
</tr>
<tr>
<td>Investments</td>
<td>0.4785</td>
<td>0.4892</td>
<td>1.0000</td>
</tr>
<tr>
<td><strong>Period 4 — Children ages 12 and 13</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Noncognitive</td>
<td>1.0000</td>
<td>0.2845</td>
<td>0.5511</td>
</tr>
<tr>
<td>Cognitive</td>
<td>0.2845</td>
<td>1.0000</td>
<td>0.6111</td>
</tr>
<tr>
<td>Investments</td>
<td>0.5511</td>
<td>0.6111</td>
<td>1.0000</td>
</tr>
</tbody>
</table>
Results: Non classical measurement error

Table 4
Unanchored Technology Equations: Measurement Error is Nonclassical, Absence of Omitted Inputs Correlated with $\theta_t$, White Males, CNLSY/79

<table>
<thead>
<tr>
<th>Independent Variable</th>
<th>Noncognitive Skill ($\theta_{t+1}^N$)</th>
<th>Cognitive Skill ($\theta_{t+1}^C$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lagged noncognitive skill, ($\theta_t^N$)</td>
<td>0.8672 (0.024)</td>
<td>0.0264 (0.011)</td>
</tr>
<tr>
<td>Lagged cognitive skill, ($\theta_t^C$)</td>
<td>0.0045 (0.014)</td>
<td>0.9739 (0.038)</td>
</tr>
<tr>
<td>Parental investment, ($\theta_t^I$)</td>
<td>0.0801 (0.018)</td>
<td>0.0647 (0.012)</td>
</tr>
<tr>
<td>Maternal education, $S$</td>
<td>0.0041 (0.008)</td>
<td>0.0026 (0.010)</td>
</tr>
<tr>
<td>Maternal cognitive skill, $A$</td>
<td>$-0.0092$ (0.006)</td>
<td>0.0252 (0.009)</td>
</tr>
</tbody>
</table>

a. Let $\theta_t = (\theta_t^N, \theta_t^C, \theta_t^I)$ denote the noncognitive, cognitive and investment dynamic factors, respectively. Let $S$ denote mother’s education and $A$ denote mother’s cognitive ability. The technology equations are:

$$
\theta_{t+1}^k = \gamma_1^k \theta_t^N + \gamma_2^k \theta_t^C + \gamma_3^k \theta_t^I + \psi_1^k S + \psi_2^k A + \eta_t^k.
$$

In this table we show the estimated parameter values and standard errors (in parenthesis) of $\gamma_1^k, \gamma_2^k, \gamma_3^k, \psi_1^k,$ and $\psi_2^k$ for noncognitive ($k = N$) and cognitive ($k = C$) skills. Investment is normalized in family income.
## Results: Non classical measurement error

### Table 7

*Unanchored Technology Equations:* Measurement Error is Classical, Allows for Omitted Input \( \lambda \) Correlated with \( \theta_i \), White Males, CNLSY/79

<table>
<thead>
<tr>
<th>Independent Variable</th>
<th>Noncognitive Skill ( (\theta_{t+1}^N) )</th>
<th>Cognitive Skill ( (\theta_{t+1}^C) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lagged noncognitive skill, ( (\theta_t^N) )</td>
<td>0.8848 ( (0.021) )</td>
<td>0.0276 ( (0.013) )</td>
</tr>
<tr>
<td>Lagged cognitive skill, ( (\theta_t^C) )</td>
<td>0.0022 ( (0.013) )</td>
<td>0.9891 ( (0.039) )</td>
</tr>
<tr>
<td>Parental investment, ( (\theta_t^I) )</td>
<td>0.0797 ( (0.020) )</td>
<td>0.0844 ( (0.017) )</td>
</tr>
<tr>
<td>Omitted correlated inputs, ( \lambda )</td>
<td>0.2835 ( (0.134) )</td>
<td>1.0000 ( )</td>
</tr>
</tbody>
</table>

\( \lambda \) is correlated with \( \theta_t \).

- Let \( \theta_t' = (\theta_t^N, \theta_t^C, \theta_t^I) \) denote the noncognitive, cognitive and investment dynamic factors, respectively. Let \( \lambda \) denote omitted inputs that are potentially correlated with \( \theta_t \). The technology equations are:

\[
\theta_{t+1}^k = \gamma_1^k \theta_t^N + \gamma_2^k \theta_t^C + \gamma_3^k \theta_t^I + \gamma_4^k \lambda + \nu_t^k.
\]

In this table we show the estimated parameter values and standard errors (in parentheses) of \( \gamma_1^k, \gamma_2^k, \gamma_3^k \) and \( \gamma_4^k \). Note that for identification purposes we normalize \( \gamma_4^C = 1 \). Investment is normalized on family income.
### Table 9

**Unanchored Stage Specific Technology Equations:** Measurement Error is Classical, No Omitted Inputs Correlated with θ_t, White Males, CNLSY/79

<table>
<thead>
<tr>
<th>Independent Variable</th>
<th>Noncognitive Skill ( (θ_{t+1}^N) )</th>
<th>Cognitive Skill ( (θ_{t+1}^C) )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Stage 1</td>
<td>Stage 2</td>
</tr>
<tr>
<td>Lagged noncognitive skill, ( (θ_t^N) )</td>
<td>0.9849 (0.014)</td>
<td>0.9383 (0.015)</td>
</tr>
<tr>
<td>Lagged cognitive skill, ( (θ_t^C) )</td>
<td>0.0508 (0.043)</td>
<td>-0.0415 (0.041)</td>
</tr>
<tr>
<td>Parental investment, ( (θ_t^I) )</td>
<td>0.0533 (0.013)</td>
<td>0.1067 (0.022)</td>
</tr>
<tr>
<td>Maternal education, ( S )</td>
<td>0.0034 (0.007)</td>
<td>-0.0028 (0.007)</td>
</tr>
<tr>
<td>Maternal cognitive skill, ( A )</td>
<td>0.0007 (0.001)</td>
<td>-0.0077 (0.001)</td>
</tr>
</tbody>
</table>

a. Let \( θ_t = (θ_t^N, θ_t^C, θ_t^I) \) denote the noncognitive, cognitive and investment dynamic factors, respectively. Let \( S \) denote mother’s education and \( A \) denote mother’s cognitive ability. The technology equations are:

\[
θ_{t+1}^k = γ_{1,t}^k θ_{t}^N + γ_{2,t}^k θ_{t}^C + γ_{3,t}^k θ_{t}^I + ψ_{1,t}^k S + ψ_{2,t}^k A + η_t^k.
\]

In this table we show the estimated parameter values and standard errors (in parentheses) of \( γ_{1,t}^k, γ_{2,t}^k, γ_{3,t}^k, ψ_{1,t}^k, \) and \( ψ_{2,t}^k \). Stage 1 is the transition from ages 6–7 to ages 8–9. Stage 2 refers to the transition from ages 8–9 to 10–11. Stage 3 is the transition from ages 10–11 to 12–13.
Counterfactual Experiment

From the estimates, one is able to derive weights for measurements in order to estimate latent variables (skills or investments): from:

\[
\frac{Y_{i,j,t}^l}{\alpha_{j,t}^l} - \frac{\mu_{j,t}^k}{\alpha_{j,t}^k} = \theta_{i,t}^l + \frac{\varepsilon_{i,j,t}^l}{\alpha_{j,t}^l}
\]

\[
\tilde{Y}_{i,j,t}^l = \theta_{i,t}^l + \varepsilon_{i,j,t}^l
\]

Notation:

\[
\varepsilon_t^l = \left(\frac{\varepsilon_{1,j,t}^l}{\alpha_{1,t}^l}, \ldots, \frac{\varepsilon_{m_t,j,t}^l}{\alpha_{m_t,t}^l}\right)
\]

We then have the vector of weights:

\[
\omega_t = \frac{1}{e' V_t^{-1} e} V_t^{-1} e
\]
<table>
<thead>
<tr>
<th></th>
<th>Ages 6 and 7</th>
<th>Ages 8 and 9</th>
<th>Ages 10 and 11</th>
<th>Ages 12 and 13</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Log family income</td>
<td>Log family income</td>
<td>Log family income</td>
<td>Log family income</td>
</tr>
<tr>
<td></td>
<td>0.0787 – 0.1188</td>
<td>0.0646 – 0.0686</td>
<td>0.0721 – 0.0731</td>
<td>0.0862 – 0.0349</td>
</tr>
<tr>
<td></td>
<td>Number of books</td>
<td>Number of books</td>
<td>Number of books</td>
<td>Number of books</td>
</tr>
<tr>
<td></td>
<td>0.0919 – 0.1667</td>
<td>0.0987 – 0.1667</td>
<td>0.1310 – 0.1667</td>
<td>0.1314 – 0.1667</td>
</tr>
<tr>
<td></td>
<td>Musical instrument</td>
<td>Musical instrument</td>
<td>Musical instrument</td>
<td>Musical instrument</td>
</tr>
<tr>
<td></td>
<td>0.0917 – 0.1667</td>
<td>0.1338 – 0.1667</td>
<td>0.1566 – 0.1667</td>
<td>0.1109 – 0.1667</td>
</tr>
<tr>
<td></td>
<td>Newspaper</td>
<td>Newspaper</td>
<td>Newspaper</td>
<td>Newspaper</td>
</tr>
<tr>
<td></td>
<td>0.1083 – 0.1667</td>
<td>0.0828 – 0.1667</td>
<td>0.0973 – 0.1667</td>
<td>0.0968 – 0.1667</td>
</tr>
<tr>
<td></td>
<td>Child has special lessons</td>
<td>Child has special lessons</td>
<td>Child has special lessons</td>
<td>Child has special lessons</td>
</tr>
<tr>
<td></td>
<td>0.2251 – 0.1667</td>
<td>0.1990 – 0.1667</td>
<td>0.1386 – 0.1667</td>
<td>0.1036 – 0.1667</td>
</tr>
<tr>
<td></td>
<td>Child goes to museums</td>
<td>Child goes to museums</td>
<td>Child goes to museums</td>
<td>Child goes to museums</td>
</tr>
<tr>
<td></td>
<td>0.2019 – 0.1667</td>
<td>0.1912 – 0.1667</td>
<td>0.1785 – 0.1667</td>
<td>0.1890 – 0.1667</td>
</tr>
<tr>
<td></td>
<td>Child goes to theater</td>
<td>Child goes to theater</td>
<td>Child goes to theater</td>
<td>Child goes to theater</td>
</tr>
<tr>
<td></td>
<td>0.2025 – 0.1667</td>
<td>0.2299 – 0.1667</td>
<td>0.2260 – 0.1667</td>
<td>0.2821 – 0.1667</td>
</tr>
</tbody>
</table>

Table 12
The Weights in the Construction of the Investment Factor

Estimated
Weights\(^a\)
| Ad Hoc
Weights\(^b\)
| Share of Total
Residual Variance
due to Factors\(^c\)
| Share of Total
Residual Variance
due to Uniqueness\(^d\)

a. See text for derivation. We assume mutually uncorrelated measurement errors.
b. Ad hoc weighting is uniform weighting. If there are \( m \) measures, each measure has weight \( \frac{1}{m} \).
c. \( \text{Var} \left( \hat{Y}_{k,i} \right) = \left( \alpha_{k,i}^2 \right) \text{Var} \left( \theta^f_i \right) + \text{Var} \left( \epsilon_{k,i} \right) \). The share of the variance due to the factor is \( \left( \alpha_{k,i}^2 \right) \text{Var} \left( \theta^f_i \right) / \text{Var} \left( \hat{Y}_{k,i} \right) \).
d. \( \text{Var} \left( \epsilon_{k,i} \right) / \text{Var} \left( \hat{Y}_{k,i} \right) \).
Counterfactual Experiment

Once weights are computed, one can derive counterfactual experiments:

- Increase investments at a given period by 10%
Table 17a
The Percentage Impact on Log Earnings at Age 23 of an Exogenous Increase by 10 Percent in Investments at Different Periods, White Males, CNLSY/79

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Exclusively through Cognitive Skills</td>
<td>Exclusively through Noncognitive Skills</td>
</tr>
<tr>
<td>Period 1</td>
<td>0.2487</td>
<td>0.1247</td>
<td>0.1240</td>
</tr>
<tr>
<td></td>
<td>(0.0302)</td>
<td>(0.0151)</td>
<td>(0.0150)</td>
</tr>
<tr>
<td>Period 2</td>
<td>0.3065</td>
<td>0.0445</td>
<td>0.2620</td>
</tr>
<tr>
<td></td>
<td>(0.0358)</td>
<td>(0.0052)</td>
<td>(0.0306)</td>
</tr>
<tr>
<td>Period 3</td>
<td>0.2090</td>
<td>0.0540</td>
<td>0.1550</td>
</tr>
<tr>
<td></td>
<td>(0.0230)</td>
<td>(0.0059)</td>
<td>(0.0170)</td>
</tr>
</tbody>
</table>

Note: Let $\tilde{Y}_{j,t}$ denote the $j^{th}$ measurement on the parental investment dynamic factor $\theta_{i}$ with the mean removed. We obtain the predicted parental investment $\hat{\theta}_{i}$ by applying the weights reported in Table 12 and measurements in the following way:

$$\hat{\theta}_{i} = \sum_{j=1}^{m_{i}} \omega_{j,t} \tilde{Y}_{j,t}.$$ 

We then simulate the model and obtain the adult level of cognitive and noncognitive skills. Using the anchoring equation, we then predict baseline log earnings, log $E$. We then perform a counterfactual simulation. We investigate the level of adult skills if investments at different periods were increased by 10 percent and we check the impact on log earnings, log $E_{\tau}$, where $E_{\tau}$ is the counterfactual earnings if investment in period $\tau$ were 10 percent higher, $\tau = 1, 2, 3$. In this table, we report the percentage change in earnings, that is $\log E_{\tau} – \log E$. 


<table>
<thead>
<tr>
<th>Total Percentage Impact</th>
<th>Percentage Impact through Cognitive Skills</th>
<th>Percentage Impact Exclusively through Noncognitive Skills</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Period 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.6441</td>
<td>0.5480</td>
<td>0.0961</td>
</tr>
<tr>
<td>(0.0789)</td>
<td>(0.0672)</td>
<td>(0.0118)</td>
</tr>
<tr>
<td>Period 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.3980</td>
<td>0.1951</td>
<td>0.2029</td>
</tr>
<tr>
<td>(0.0466)</td>
<td>(0.0229)</td>
<td>(0.0238)</td>
</tr>
<tr>
<td>Period 3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.3565</td>
<td>0.2366</td>
<td>0.1198</td>
</tr>
<tr>
<td>(0.0389)</td>
<td>(0.0258)</td>
<td>(0.0131)</td>
</tr>
</tbody>
</table>

Note: Let \( \tilde{Y}_{j,t} \) denote the \( j^{th} \) measurement on the parental investment dynamic factor \( \theta_t^l \) with the mean removed. We obtain the predicted parental investment \( \hat{\theta}_t^l \) by applying the weights reported in Table 12 and measurements in the following way:

\[
\hat{\theta}_t^l = \sum_{j=1}^{m_t} \omega_{j,t} \tilde{Y}_{j,t}.
\]

We then simulate the model and obtain the adult level of cognitive and noncognitive skills. Using the anchoring equation, we then predict the probability of graduating from high school, \( p \). We then perform a counterfactual simulation. We investigate the level of adult skills if investments at different periods were increased by 10 percent and we check the impact on the probability of graduating from high school, \( p_t \), where \( p_t \) is the counterfactual graduation probability if investment in period \( \tau \) were 10 percent higher. In this table, we report the percentage change in probability of graduating, that is \( \log p_t - \log p \). Standard errors in parentheses.