

Optimal taxation in the extensive model

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Abstract

We study optimal taxation under extensive preferences: the agents' utilities are constant for positive actions up to a maximal productivity level. Utilities may be discontinuous at the origin, reflecting fixed costs of participation. Allowing for general distributions of work opportunity costs and productivity and for income effects, we characterize optimal, incentive-compatible tax schedules. We then give sufficient conditions for society to desire redistribution. When these conditions hold, upwards distortions of the financial incentives to work only can occur for low-skilled workers. Such upwards distortions are indeed always present when the fixed participation costs are pecuniary.

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1 Introduction

The notions of extensive and intensive margins are familiar to economists. The former relates to entry and exit, for instance of workers into the labor force or of firms into markets. The latter refers to a continuous action, the number of hours worked or the quantity sold on the market. Most of the optimal taxation literature is set in a framework where effort is continuous, in an intensive model, and does not deal with the extensive setup. One aim of this paper is to fill in the gap, to give a precise definition and to provide a general treatment of the extensive model, while clarifying its relationship with the existing literature.

The traditional framework with asymmetric information à la Spence-Mirrlees allows to represent the ideas of the extensive model, provided that preferences are specified adequately. The action of the agents is made of two parts: a binary decision, whether to participate or not, and a continuous decision when they participate, the effort level (in the labor supply terminology, hours worked). Two features characterize the extensive model: first, participating, i.e. doing any work however small, involves a possible discontinuous change in utility; second, once participating utility does not depend on the number of hours, up to a maximum quantity. If the action entails a private cost for the agent, this cost does not vary with the intensity of the action.

The extensive model thus defined has the same basic structure as the intensive model which the optimal taxation literature has mostly focused on. The main goal of this article is to investigate how unobserved heterogeneity affects optimal allocations. To this end, we allow for the largest possible scope of unobserved heterogeneity, on top of the agent's productivity. Individual characteristics that the government cannot observe affect both utility functions (in case of participation and nonparticipation) in the most general way. The utilitarian government thus faces a multidimensional screening problem with type-dependent reservation utility. Using the taxation principle, it is possible to show that any optimal, feasible and incentive compatible mechanism can be implemented by a nondecreasing after-tax income schedule. We then present sufficient conditions for the absence of pooling at the second best optimum.

The social preference for redistribution follows from the properties of the cardinal utilities. In the intensive model, when the utility function is constant across agents and does not depend on productivity, the mere concavity of that function gives society a taste for equality. Completing the understanding of the intensive case, Hellwig (2007) provides minimum criteria for society to favor redistribution when utility depends on productivity. We undertake the same task in the extensive model with multidimensional heterogeneity. We define a redistributive society as one where cardinal utilities induce a marginal utility of income of the workers that decreases with both income and productivity. We give a set of conditions on the economic fundamentals (shape of utility functions and distribution of heterogeneity) which warrant these properties.

We investigate the shape of optimal tax schedules when both productivities and the fixed participation costs are unknown. We look in particular at the sign of *average* tax rates. A negative sign at some productivity level means that the financial incentive to work exceeds productivity at this point: for the concerned agents, after tax income is larger than productivity when they work. Absent income effects, such work subsidies distort labor supply upwards compared to both *efficiency* and *laissez-faire*. We provide sufficient conditions on the distribution of unobserved heterogeneity for the average tax rate to be first nonpositive, then nonnegative, as productivity rises. When participation costs are pecuniary, i.e. translate into a reduction of consumption, it is optimal to subsidize low-skilled work. In contrast work subsidies are never optimal when only one parameter (productivity) is unknown to the government.

These results parallel those of Choné and Laroque (2010) and of Beaudry, Blackorby, and Szalay (2009) in the intensive framework. Although very different from a technical perspective, the latter papers establish a similar rationale for negative *marginal* tax rates, based on rent extraction. These papers highlight the impact of multidimensional heterogeneity on the direction of distortions, and emphasize the role of the correlation between productivity and the agents' preference for leisure.

The paper is organized as follows. Section 2 reviews the relevant literature. Section 3 describes the labor supply model, while Section 4 sets the government problem in the mechanism design language, and presents the tax instruments. The first order conditions satisfied by the tax schedule are derived in Section 5. Section 6 presents in turn the properties of general second best tax schedules and the restrictions that they satisfy under a redistributive government. Section 7 presents the full set of equations satisfied by an optimum, and Section 8 discusses the role of heterogeneity. Proofs are gathered in the appendix.

2 Related literature

The standard Mirrlees setup has a continuous unidimensional action and a continuous unidimensional information. The agent utility function is concave increasing, and the principal is utilitarian. Our setting has one binary and one continuous action, and a multidimensional information setup. The participation utility is constant up to the unobserved productivity, and typically differs from the reservation utility as effort tends to zero (fixed cost of participation). The extensive model has been first examined by Diamond (1980), who puts the emphasis on non convexity issues in a simple example.

Saez (2002) works with a discrete model: the action space, the types and the information all are discrete, and for most of the analysis, income effects are ruled out. Unlike this author, we allow an agent to work in any occupation which requires a skill below her type. The incentives to mimic agents with lower

productivities give rise to monotonicity constraints and, possibly, pooling which we study closely. We consider income effects and a general information structure, with possible correlation between productivities and work opportunity costs.

A related work, more in the spirit of the present article, is that of Beaudry, Blackorby, and Szalay (2009). In their model, the economic agents differ along two dimensions, productivity on the market and productivity at home which both take a finite number of values. The fruits of non market activity are not observable by the government. The agents can continuously allocate their time between work on the market and work at home, and the government transfers income contingent on the time allocated to market employment. Although the latter feature relates to the intensive margin, the authors' assumption that the technology is linear makes the model close to an extensive setup. One of their main results, Proposition 4, that low market productivity types are subsidized while high productivity types are taxed, is reminiscent of the properties of the pecuniary cost model here (our Proposition 5).

In other contexts, the principal-agent literature has studied similar issues. Jullien (2000) provides the first general approach to type-dependent participation. Agents have one binary and one continuous action, while type is continuous unidimensional. Fixed costs create a type-dependent reservation utility and a discontinuity in the utility at zero effort. We differ from this setup most notably by the multiple dimension of heterogeneity, which explains the sharp contrast in the conclusions. A robust result in Jullien (2000) is that asymmetric information reduces the benefits from trade and always induces underparticipation at the second best. This does not hold here.

Armstrong (1996) and Rochet and Choné (1998) examine a multidimensional adverse selection problem, where action and information have equal dimensions. They do not have random participation. Our framework is closer to Rochet and Stole (2002), who have both random participation and more than one dimension of heterogeneity. Like us, they examine how these two features may generate pooling.¹ We differ, however, from Rochet and Stole (2002) by the form of preferences and the principal's objective.

3 Heterogeneity and the description of the private economy

We consider an economy with a continuum of participants of measure 1. The agents' only decision is whether to work or not. The agents differ along several dimensions. First, they differ by their productivity levels ω , the before tax income that they generate when they work. Second, there is a possibly multidimensional

¹In the optimal taxation Mirrlees setup, Ebert (1992) presents a thorough treatment of bunching, but does not model the participation decision.

heterogeneity parameter α that describes other idiosyncratic characteristics, such as the costs of going to work. Allowing for a lot of heterogeneity accords with empirical studies where marital status, family composition, human capital and culture often appear to be determinants of the labor supply decision.

Let c_E be the income of someone who decides to work, c_U her income when unemployed. Utility is described by

$$\begin{cases} \tilde{u}(c_E; \alpha) & \text{if she participates,} \\ \tilde{v}(c_U; \alpha) & \text{if she does not work,} \end{cases} \quad (1)$$

and the decision is the one that yields the highest utility. The utility functions \tilde{u} and \tilde{v} are assumed to be twice continuously differentiable. They are increasing and concave in (nonnegative) consumption.

The *work opportunity cost* is the (possibly negative) sum of money which, given to an agent if she works on top of the subsistence income she has while unemployed, makes her indifferent between working or not. Given the utility level of the agent, it is *efficient* to put her to work whenever her work opportunity cost is smaller than her productivity. Formally, from the monotonicity of \tilde{u} , for each agent α and for each level of consumption when not working c_U , there exists a number $\delta = \Delta(\alpha, c_U)$ such that the agent wants to work ($\tilde{u} > \tilde{v}$) if $c_E > c_U + \delta$, does not want to work if $c_E < c_U + \delta$ and is indifferent if $c_E = c_U + \delta$, i.e. δ is solution to the equation²

$$\tilde{u}(c_U + \delta, \alpha) = \tilde{v}(c_U, \alpha). \quad (2)$$

The threshold δ is the work opportunity cost of the agent. The financial incentive to work is the difference $c_E - c_U$, and an agent works whenever her net gain exceeds her opportunity cost of work. The subsistence income or other income unrelated to work is c_U . Labor supply is subject to an income effect when the work opportunity cost $\Delta(\alpha, c_U)$ depends on other income c_U . If leisure is a normal good, labor supply weakly decreases with other income, i.e. $\Delta(\alpha, c_U)$ is nondecreasing in c_U .

An allocation is efficient if the agents' utilities are obtained at the lowest possible cost. Here participation of an agent is efficient at utility \bar{U} when her productivity ω is larger than her work opportunity cost $\Delta(\alpha, c_U)$, where c_U is the consumption level that gives the agent utility \bar{U} when not working, $\bar{U} = \tilde{v}(c_U, \alpha)$. Conversely, if ω is smaller than $\Delta(\alpha, c_U)$, it is efficient to leave the agent unemployed. When leisure is a normal good, there is typically a cutoff utility level below which it is efficient for the agent to work, while above she should stay unemployed.

²We let δ equal to $-c_U$ for persons who always want to work, and δ equal $+\infty$ for persons that never work. We do not rule out $\delta < 0$: some agents may be ready to pay to go to work.

An economy is defined by a pair of utility functions (\tilde{u}, \tilde{v}) and a distribution of characteristics (α, ω) . It is convenient to derive a reduced form by substituting the work opportunity cost δ for the multidimensional parameter α . We define the average utilities of the agents who have the same work opportunity cost δ as

$$\begin{cases} u(c_E; \delta, c_U) &= \mathbb{E}_\alpha [\tilde{u}(c_E; \alpha) \mid \delta = \Delta(\alpha, c_U), c_U], \\ v(c_U; \delta) &= \mathbb{E}_\alpha [\tilde{v}(c_U; \alpha) \mid \delta = \Delta(\alpha, c_U), c_U]. \end{cases} \quad (3)$$

By construction, the reduced-form model satisfies:

$$u(c_E; \delta, c_U) \geq v(c_U; \delta) \iff c_E \geq c_U + \delta. \quad (4)$$

Note that the function u generally depends on the subsistence level c_U , except when there are no income effects (Δ independent of c_U). The function u inherits from \tilde{u} its monotonicity and concavity in c_E . Moreover, when the idiosyncratic heterogeneity is unidimensional (the dimension of α is one), the marginal rate of substitution u_δ/u_{c_U} of the utility of the employee in the reduced form is independent of the level of income when employed c_E . The latter result, however, does not extend to higher dimensions of heterogeneity. We conjecture that the structural model does not restrict the shape of the reduced form utilities. Proposition A.1 in Appendix states these properties in a formal way.

While the structural model puts little restrictions on the shape of the utility functions, it has important consequences on the distribution of work opportunity costs. For each value of c_U , the distribution of the underlying parameters induces a distribution of (δ, ω) . We denote $F(\cdot|\omega, c_U)$ the cumulative distribution of work opportunity costs of persons of productivity ω when out of work income is equal to c_U : $F(c_E - c_U|\omega, c_U)$ is the proportion of agents of productivity ω that want to work when income at work is equal to c_E and income out of work is c_U . Note that, from the monotonicity of \tilde{v} , the function $F(c_E - c_U|\omega, c_U)$, seen as a function of c_U , is nonincreasing in c_U . Also, when leisure is a normal good, an increase in c_U , keeping constant the incentive to work $c_E - c_U$, decreases labor supply. To summarize, when F is differentiable with respect to its arguments, we have

$$\frac{d}{dc_U} F(c_E - c_U|\omega, c_U) = -\frac{\partial F(\delta|\omega, c_U)}{\partial \delta} + \frac{\partial F(\delta|\omega, c_U)}{\partial c_U} \leq 0, \quad (5)$$

and, when leisure is a normal good,

$$\frac{\partial F(\delta|\omega, c_U)}{\partial c_U} \leq 0.$$

The structural and the reduced forms of the model are linked through two channels. The shape of the function $\Delta(\alpha, c_U)$ is one of them. The implicit function theorem, when valid, gives

$$\tilde{u}'_1 \frac{\partial \Delta}{\partial \alpha} = \tilde{v}'_\alpha - \tilde{u}'_\alpha.$$

The heterogeneity of the work opportunity cost δ , given the person's productivity ω which is observed by the government for the workers, is an important feature of the economy (see Section 8 for a discussion): for the model to be interesting, the utility functions must have a (generically) non zero $\tilde{v}'_\alpha - \tilde{u}'_\alpha$. The other component that determines the reduced form is the distribution of α conditional on ω .

Most of the results carry over to a situation where the distributions have mass points, provided that there is genuine heterogeneity in the work opportunity costs. To simplify the presentation, we focus on the case where distributions are continuous. Throughout the paper, except in Section 8, we maintain the following assumption.

Assumption 1. *The marginal distribution of productivities ω has support $\Omega = [\underline{\omega}, \bar{\omega}]$, an interval of the positive line. Its cumulative distribution function G has a continuous positive derivative g everywhere on the support.*

The distribution of work opportunity costs δ , conditional on ω and c_U , is continuous with support $[\underline{\delta}, \bar{\delta}]$, $\underline{\delta} < \bar{\delta}$, and cumulative distribution function $F(\cdot|\omega, c_U)$. Its probability distribution function $f(\cdot|\omega, c_U)$ is positive and continuous on $(\underline{\delta}, \bar{\delta})$ and $F/f(\delta|\omega, c_U)$ tends to 0 as δ goes to $\underline{\delta}$.³

Assumption 1 plays a critical role in the analysis. Section 8 shows that our results are driven by the presence of genuine heterogeneity in work opportunity costs: for our results to hold, the inequality $\underline{\delta} < \bar{\delta}$ must be strict (as stated in the assumption).

Pecuniary cost of work: A simple specification of particular interest is the following. The parameter α is unidimensional and the utility functions are defined through a concave twice differentiable increasing function U , such that

$$\tilde{u}(c_E; \alpha) = U(c_E - \alpha), \quad \tilde{v}(c_U; \alpha) = U(c_U).$$

Then direct substitutions yield $\Delta(\alpha, c_U) = \alpha$ and

$$u(c_E; \delta, c_U) = U(c_E - \delta), \quad v(c_U; \delta) = U(c_U).$$

The work opportunity cost is simply equal to α . It is independent of the value of the subsistence income c_U : there are no income effects. Also the value attached by the government to the welfare of the unemployed agents only depend on their income, and does not vary with their characteristics (α, ω) .

4 From general mechanisms to tax schedules

We write the government problem in the mechanism design language and show that the problem can be expressed in terms of two instruments, an income tax

³The last part of the assumption is satisfied in particular when $f(\underline{\delta}|\omega, c_U) > 0$ and when $f(\underline{\delta}|\omega, c_U) = 0$ and f is differentiable at $\underline{\delta}$ with $f'(\underline{\delta}|\omega, c_U) > 0$.

schedule and a subsistence income for nonworkers.⁴

4.1 Mechanism and information

The government does not a priori observe the agents' private characteristics (α, ω) . He sees the agents' incomes y and provides consumptions c . Under the revelation principle, a mechanism is a couple $(c(\alpha, \omega), y(\alpha, \omega))$ which satisfies incentive and feasibility constraints. We are looking for properties of the optimal mechanism for a utilitarian government, which maximizes the sum of utilities of the agents in the economy.

When agent α produces an output y , $y \leq \omega$, her utility is given by:

$$\tilde{U}(c, y; \alpha) = \begin{cases} \tilde{u}(c; \alpha) & \text{if the agent participates } (\omega \geq y > 0) \\ \tilde{v}(c; \alpha) & \text{if she does not work } (y = 0). \end{cases}$$

In contrast with the intensive setting, there is no variable disutility of work: $\tilde{u}(c; \alpha)$ does not depend on y , for $y \leq \omega$. Beyond that maximum productivity level, the utility is $-\infty$ so that

$$y(\alpha, \omega) \leq \omega$$

for all (α, ω) . As (α, ω) is private information, an agent can mimic any agent who produces less than her own productivity. Incentive compatibility constraints are given by

$$\tilde{U}(c(\alpha, \omega), y(\alpha, \omega); \alpha) \geq \tilde{U}(c(\alpha', \omega'), y(\alpha', \omega'); \alpha) \quad (6)$$

for all (α', ω') such that $y(\alpha', \omega') \leq \omega$. An allocation $(c(\alpha, \omega), y(\alpha, \omega))$ is feasible if

$$\mathbb{E}_{\alpha, \omega} [c(\alpha, \omega) - y(\alpha, \omega)] \leq 0. \quad (7)$$

The utilitarian government maximizes $\mathbb{E}_{\alpha, \omega} \tilde{U}$ subject to the incentive constraints (6) and the feasibility constraint (7).

4.2 The taxation principle

Any optimal mechanism can be implemented with a tax scheme, made of a subsistence income s given to the non-workers and of a nondecreasing after tax income $R(y)$. The agents behavior is to work full time or stay unemployed depending on whether $\tilde{u}(R(\omega), \alpha) \geq \tilde{v}(s, \alpha)$ or not.

To show this property, first observe that a nonworker can mimic any nonworker. From (6), we get:

$$\tilde{v}(c(\alpha, \omega); \alpha) \geq \tilde{v}(c(\alpha', \omega'); \alpha)$$

⁴We thank a referee for urging us to give a mechanism design foundation to our approach.

for all (α, ω) and (α', ω') with $y(\alpha, \omega) = y(\alpha', \omega') = 0$; thus $c(\alpha, \omega) \geq c(\alpha', \omega')$, and, by symmetry: $c(\alpha, \omega) = c(\alpha', \omega')$, implying that the consumption of non-workers does not depend on their type. Hereafter, this level is noted s .

Second, note that a worker can mimic a worker with the same productivity. Incentive compatibility then implies:

$$\tilde{u}(c(\alpha, \omega); \alpha) \geq \tilde{u}(c(\alpha', \omega); \alpha)$$

for all α and α' with $y(\alpha, \omega) > 0$ and $y(\alpha', \omega) > 0$. Thus $c(\alpha, \omega) \geq c(\alpha', \omega)$, and, by symmetry: $c(\alpha, \omega) = c(\alpha', \omega)$. The consumption level of workers only depends on their productivity.

Third, without loss of generality, workers can be assumed to work at full productivity. Indeed, given any feasible, incentive compatible mechanism (c, y) , define

$$\bar{y} = \begin{cases} \omega & \text{if } y > 0 \\ 0 & \text{if } y = 0. \end{cases}$$

Suppose that agent (α, ω) can mimic agent (α', ω') under the mechanism (c, \bar{y}) , meaning that $\bar{y}(\alpha', \omega') \leq \omega$. Since $\bar{y}(\alpha', \omega') = \omega'$, we know that $y(\alpha', \omega') \leq \omega' \leq \omega$: agent (α, ω) can mimic (α', ω') under the original mechanism (c, y) as well. Applying (6) and noting that utility functions do not depend on production levels, we conclude that the mechanism (c, \bar{y}) is incentive compatible. Feasibility is obvious, as workers produce more and consumption is unchanged. If a positive mass of workers work below productivity under the original mechanism (c, y) , the new mechanism (c, \bar{y}) strictly increases available resources, possibly allowing to enhance the utilitarian objective. It follows that we may assume $y(\alpha, \omega) = \omega$ as soon as $y(\alpha, \omega) > 0$.

Combining the above results shows the existence of a map $c = R(y)$, linking production $y = \omega$ to after-tax income c . Finally, a worker can mimic any worker with lower productivity, and incentive compatibility can be written

$$\tilde{u}(R(\omega); \alpha) \geq \tilde{u}(R(\omega'); \alpha)$$

for all $\omega \geq \omega'$. Incentive constraints (6) are therefore equivalent to R being nondecreasing.

It follows that any optimal, feasible and incentive compatible mechanism (c, y) can be implemented by a nondecreasing after-tax income schedule $c = R(y)$, together with a subsistence income for nonworkers, s .

4.3 Tax schemes and labor supply

From now on, we work with a tax scheme, made of a subsistence income s , $s \geq 0$, and of a nondecreasing after tax income schedule $R(\cdot)$, which associates to any

before tax income y , $y > 0$, a nonnegative disposable income $R(y)$. Given the income schedule, the *participation tax rate* $\tau(y)$ faced by the worker is

$$\tau(y) = \frac{y - (R(y) - s)}{y} = 1 - \frac{R(y) - s}{y}.$$

The tax rate faced by a person of productivity ω is *positive* if and only if $\omega > R(\omega) - s$, i.e. when the financial incentive to work is smaller at the second best allocation than at *laissez-faire*.

Taxes typically distort labor supply. Under the maintained assumption that leisure is a normal good, we measure *distortion* with respect to two alternative references : *laissez-faire* and efficiency. Labor supply is distorted upwards (resp. downwards) when its level at the second best allocation is larger (resp. smaller) than at the chosen reference.

4.3.1 The *laissez-faire* reference

At *laissez-faire*, ($R(y) = y, s = 0$), the tax rate $\tau(y)$ is equal to zero. A person of productivity ω decides to work when her financial incentive ω is larger than her work opportunity cost $\Delta(\alpha, 0)$, where the second argument of Δ , the subsistence income, is zero in the absence of a government safety net. In a second best allocation, the person works when

$$R(\omega) - s > \Delta(\alpha, s).$$

Then, for *positive* tax rates, $R(\omega) - s < \omega$, if leisure is a normal good, that is Δ is nonincreasing in its second argument, labor supply is unambiguously distorted downwards compared to *laissez-faire*: any unemployed person at *laissez-faire*, with $\omega < \Delta(\alpha, 0)$, is still unemployed at second best, since $R(\omega) - s < \omega < \Delta(\alpha, 0) \leq \Delta(\alpha, s)$.

When the tax rate is *negative* and $R(\omega) > s + \underline{\delta}$, the financial incentive to work is larger at the second best than at *laissez-faire*, $\omega < R(\omega) - s$. Absent income effects, every unemployed agent at the second best, with $\Delta > R(\omega) - s$, is idle at the *laissez faire*, $\Delta > \omega$: labor supply is distorted upwards.⁵

⁵Income effects blur the picture. Formally, the labor supply of agents with productivity ω is (weakly) distorted upwards when they work more under *laissez-faire* than at the second best, i.e. if and only if any agent with productivity ω who works under *laissez faire* also works at the second best:

$$\omega \geq \Delta(\alpha, 0) \implies R(\omega) \geq s + \Delta(\alpha, s).$$

Applying the nondecreasing function R to the left hand side inequality, a sufficient condition for this to be satisfied is

$$R(\Delta(\alpha, 0)) \geq s + \Delta(\alpha, s). \quad (8)$$

The negativity of the average tax rate only tells us that $R(\Delta(\alpha, 0)) \geq s + \Delta(\alpha, 0)$, a weaker condition than (8) when labor is a normal good.

4.3.2 The efficiency reference

Agents are efficiently unemployed if and only if their work opportunity cost $\Delta(\alpha, s)$ is larger than their productivity ω . At the second best equilibrium all the agents with work opportunity costs larger than $R(\omega) - s$ are out of work, and efficiency therefore requires $\omega \leq R(\omega) - s$, that is $\tau(\omega) \leq 0$.

Agents are efficiently employed if and only if their work opportunity cost $\Delta(\alpha, c_U)$ is larger than their productivity ω , where c_U is the consumption level that would give them the same utility if they were not participating: $\tilde{v}(c_U, \alpha) = \tilde{u}(R(\omega); \alpha)$. At the second best, as the agents work, c_U is larger than s . If leisure is a normal good, therefore $\Delta(\alpha, c_U) > \Delta(\alpha, s)$. If $\tau(\omega) < 0$, there are agents with $\omega < \Delta(\alpha, s) < R(\omega) - s$ who work, which is inefficient as $\omega < \Delta(\alpha, s) < \Delta(\alpha, c_U)$. Efficiency thus requires $\tau(\omega) \geq 0$.

To summarize, when leisure is a normal good, labor supply distortions with respect to efficiency are downwards when the participation tax rate is positive, upwards when it is negative. Recall that in the absence of income effects the same is true for the laissez-faire reference.

4.4 Writing the government problem

The government objective, as well as the feasibility constraint, can be written in terms of the reduced-form model. The utilitarian government maximizes the sum of the utilities of the participants in the economy. Its objective takes the form

$$\mathbb{E}_{\omega, \alpha} \max [\tilde{u}(R(\omega); \alpha), \tilde{v}(s; \alpha)] = \mathbb{E}_{\omega, \alpha} \{ \tilde{u} \mathbf{1}_{R(\omega) - s \geq \delta} + \tilde{v} \mathbf{1}_{R(\omega) - s < \delta} \}.$$

Noticing that the participation decision only depends on δ and ω and applying the law of iterated expectations, we can replace the expectation over (ω, α) with an expectation over (ω, δ) :

$$\mathbb{E}_{\omega, \alpha} \max [\tilde{u}(R(\omega); \alpha), \tilde{v}(s; \alpha)] = \mathbb{E}_{\omega, \delta} \{ u \mathbf{1}_{R(\omega) - s \geq \delta} + v \mathbf{1}_{R(\omega) - s < \delta} \}.$$

The government objective thus is the expectation with respect to ω of

$$\int_{\underline{\delta}}^{R(\omega) - s} u(R(\omega); \delta, s) dF(\delta|\omega, s) + \int_{R(\omega) - s}^{\bar{\delta}} v(s; \delta) dF(\delta|\omega, s). \quad (9)$$

The feasibility constraint is

$$\int_{\Omega} [\omega - R(\omega) + s] F(R(\omega) - s|\omega, s) dG(\omega) = s. \quad (10)$$

Let λ be the multiplier associated with the feasibility constraint (10). The Lagrangian of the problem is $\mathbb{E}_{\omega} L(R(\omega), s; \omega)$, with

$$\begin{aligned} L(R, s; \omega) &= \int_{\underline{\delta}}^{R - s} u(R; \delta, s) dF(\delta|\omega, s) + \int_{R - s}^{\bar{\delta}} v(s; \delta) dF(\delta|\omega, s) \\ &\quad + \lambda[\omega - R + s] F(R - s|\omega, s) - \lambda s. \end{aligned}$$

The optimization takes place over (R, s, λ) , where R is a nonnegative nondecreasing function and s and λ are nonnegative. The Lagrangian L is not concave in its arguments, since the cdf of the work opportunity cost F is typically not concave. Any utilitarian optimum however must satisfy first order necessary conditions, which turn out to be especially fruitful here. In the next two sections, we concentrate on the study of $R(\cdot)$, for given values of s and λ , before commenting briefly on the full program in Section 7.

Without loss of generality, we impose the additional constraint: $R(\omega) - s \geq \underline{\delta}$ for all $\omega \in \Omega$. Indeed, the objective does not depend on the precise value taken by $R(\omega)$ in the region where the set of workers is negligible, i.e. whenever $R(\omega) - s$ is smaller than or equal to the minimal work opportunity cost $\underline{\delta}$. That is, if $R(\omega) - s < \underline{\delta}$ is optimal, then $R(\omega) - s = \underline{\delta}$ is optimal as well. Given this extra constraint, the financial incentives to work for agents of productivity ω are distorted downwards if and only if $R(\omega) < \omega + s$; they are distorted upwards if and only if $R(\omega) > \max(\omega + s, \underline{\delta} + s)$; they are undistorted if and only if $R(\omega) = \max(\omega + s, \underline{\delta} + s)$.

5 First order conditions for $R(\omega)$

Consider a value R of after-tax income, larger than $s + \underline{\delta}$. Suppose that the agents of productivity ω earn R when working and s when unemployed. Then a positive fraction of these agents work, and we define their average social weight as

$$\begin{aligned} p_E(R, s|\omega) &= \mathbb{E}_\alpha [\tilde{u}'_1(R; \alpha) \mid \delta \leq R - s, \omega, s] \\ &= \frac{1}{F(R - s|\omega, s)} \int_{\underline{\delta}}^{R-s} u'_1(R; \delta, s) dF(\delta|\omega, s). \end{aligned} \quad (11)$$

By continuity, we can set: $p_E(s + \underline{\delta}, s|\omega) = u'_1(s + \underline{\delta}; \underline{\delta}, s)$. The derivative of the Lagrangian L with respect to R can be expressed in terms of the social weight p_E :

$$\frac{\partial L}{\partial R}(R, s; \omega) = \lambda[\omega - R + s]f(R - s|\omega, s) - F(R - s|\omega, s) [\lambda - p_E(R, s|\omega)]. \quad (12)$$

The expression of $\partial L/\partial R$ has a direct economic interpretation. The first term $\lambda[\omega - R + s]f(R - s|\omega, s)$ is the gain in government income obtained from the new $f(R - s|\omega, s)$ workers who participate following an increase in R : they produce ω , they are paid $R(\omega)$ but do not receive the subsistence income s anymore. The second term $F(R - s|\omega, s)[\lambda - p_E(R, s)]$ is the loss on the existing workers: the marginal cost is λ per worker, while the social value of this distribution of income is equal to the average social weight of the employees of productivity ω .

At any point ω where the income schedule $R(\cdot)$ is strictly increasing ($R > s + \underline{\delta}$ and no pooling), it satisfies the first order condition for a pointwise maximum:

$$\frac{\partial L}{\partial R}(R, s; \omega) = 0. \quad (13)$$

The average tax rate supported by the workers of productivity ω is $\tau(\omega) = [\omega - R(\omega) + s]/\omega$, so that the first order condition can be rewritten as

$$\omega - R + s = \omega\tau(\omega) = \frac{F(R - s|\omega, s)}{f(R - s|\omega, s)} \left[1 - \frac{p_E(R, s|\omega)}{\lambda} \right]. \quad (14)$$

It may also help to rewrite this expression using labor supply elasticities, as in Proposition 1 of Saez (2002). The elasticity of the labor supply $F(R - s|\omega, s)$ of the agents of productivity ω with respect to their after tax income R , when subsistence income is s , is

$$\varepsilon_R(\omega, s) = R \frac{f(R - s|\omega, s)}{F(R - s|\omega, s)},$$

so that (14) can be rewritten as

$$\omega\tau(\omega) = \frac{R}{\varepsilon_R(\omega, s)} \left[1 - \frac{p_E(R, s|\omega)}{\lambda} \right].$$

At a point ω where the tax schedule $R(\omega)$ is strictly increasing and $R - s$ is in the interval $(\underline{\delta}, \bar{\delta})$, i.e. some, but not all, agents of productivity ω want to work, we have

- either $p_E(R(\omega), s|\omega) < \lambda$, the financial incentive to work $R(\omega) - s$ is smaller than before tax income ω , labor supply is inefficiently distorted downwards;
- or $p_E(R(\omega), s|\omega) = \lambda$, the financial incentive to work $R(\omega) - s$ equals before tax income ω , labor supply is efficient;
- or $p_E(R(\omega), s|\omega) > \lambda$, the financial incentive to work $R(\omega) - s$ is larger than before tax income ω : it is distorted upwards with respect to both laissez-faire and efficiency.

Social weights larger than λ , corresponding to a group of employees whose average social weight is larger than the marginal cost of public funds, receive a financial incentive to work $R(\omega) - s$ larger than their productivity ω , distorted upwards compared with laissez-faire.⁶

⁶In the intensive model also, the sign of the *marginal* tax rate follows from the comparison of the cost of public funds with the average social weights of a specific group of workers. But the relevant groups differ in the intensive and extensive models. In the former case, what matters is the population of workers whose productivity is greater than or equal to a given level; in the latter, only workers with a given productivity are considered. This difference follows from the informational structures: the workers' productivities are observed in the extensive model, whereas this information has to be extracted in the intensive case.

The optimum may involve pooling, with regions where R stays constant because of the monotonicity condition. In a pooling interval $[\omega_1, \omega_2]$, whenever $R - s$ is in the interior of the support of work opportunity costs, the first order conditions become

$$\int_{\omega_1}^{\omega_2} \frac{\partial L}{\partial R}(R, s; \omega) dG(\omega) = 0 \quad (15)$$

and

$$\int_{\omega}^{\omega_2} \frac{\partial L}{\partial R}(R, s; \omega) dG(\omega) \leq 0 \quad (16)$$

for all $\omega_1 \leq \omega \leq \omega_2$. Applied in a neighborhood of the extremities of the pooling interval, the last condition implies

$$\frac{\partial L}{\partial R}(R, s; \omega_1) \geq 0 \quad \text{and} \quad \frac{\partial L}{\partial R}(R, s; \omega_2) \leq 0. \quad (17)$$

Note that in a region of no-pooling, under regularity conditions, the derivative of after tax income $R(\omega)$ can be computed with the implicit function theorem

$$\frac{\partial^2 L}{\partial R^2} \frac{\partial R}{\partial \omega} + \frac{\partial^2 L}{\partial R \partial \omega} = 0.$$

At a local maximum, $\partial^2 L / \partial R^2$ is negative, so that for R to be increasing, one must have $\partial^2 L / \partial R \partial \omega \geq 0$. Conversely, pooling only prevails if there is a region where $\partial^2 L / \partial R \partial \omega$ is negative (see equation (17)).

6 The shape of optimal tax schemes

The derivation of the optimal tax schedule is closely linked to the drawing describing the sign of $\partial L / \partial R$ in the plane (ω, δ) . We study this link, first by giving conditions for the absence of pooling in the general second best model, second by restricting the attention to utilities associated with a redistributive government.

6.1 Unrestricted second best tax schedules

How the distribution of work opportunity costs varies with productivity unsurprisingly is an important determinant of the shape of the optimal tax schedules. The two following assumptions help to describe the structure of the optimal tax problem, and to single out a class of economies where pooling does not occur at the optimum.

Assumption 2. *The economy satisfies one of the three conditions stated below:*

1. *The distribution of work opportunity costs $F(\cdot | \omega, c_U)$ does not depend on the productivity level ω .*

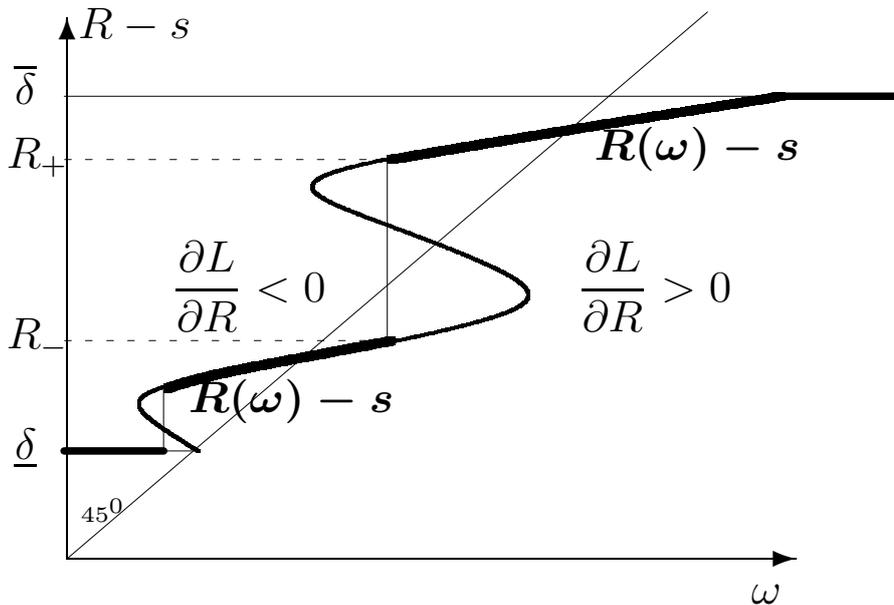


Figure 1: Financial incentives to work

2. The distribution of work opportunity costs $F(\cdot|\omega, c_U)$ first order stochastically increases in ω . The marginal utility of income $u'_1(c_E; \delta, c_U)$ is increasing with δ .
3. The distribution of work opportunity costs $F(\cdot|\omega, c_U)$ first order stochastically decreases in ω . The marginal utility of income $u'_1(c_E; \delta, c_U)$ is decreasing with δ .

Diamond (1980) worked under independence (case 1). To the best of our knowledge, there is no clear evidence as to whether work opportunity costs increase or decrease with productivity (the latter seems in line with the idea of specialization). As far as the marginal utility of income is concerned, case 2 seems most likely in circumstances where, as in the pecuniary cost model, the work opportunity cost acts as a reduction of income, thus increasing its marginal utility.

The next assumption is best stated in terms of the joint distribution of productivities and work opportunity costs. Given a level of income out of work c_U , if the government gives financial incentives r to the agents of productivity ω , a proportion $F(r|\omega, c_U)$ of those agents will work, so that tax receipts equal $(\omega - r)F(r|\omega, c_U)$. A marginal increase in r changes the receipts by $(\omega - r)f(r|\omega, c_U) - F(r|\omega, c_U)$. We shall assume that this change is increasing with productivity ω , a property that always holds when the distribution of work opportunity costs is independent of productivity:

Assumption 3. *The tax receipts of the government on the persons of productivity ω when their incentives to work is r , $(\omega - r)F(r|\omega, c_U)$, have a positive second order cross partial derivative with respect to r and ω . For $\underline{\delta} < r < \bar{\delta}$:*

$$\frac{\partial^2}{\partial r \partial \omega} [(\omega - r)F(r|\omega, c_U)] > 0.$$

Assumptions 1 to 3 ensure that the cross-derivative of the Lagrangian is positive: $\partial^2 L / \partial R \partial \omega > 0$. The analysis of Section 5 shows that there is no pooling: Pointwise maximization of the Lagrangian for each ω yields the optimal tax scheme. Given a couple (s, λ) , for every R such that $\underline{\delta} < R - s < \bar{\delta}$, there is a unique (possibly infinite) nonnegative number $M(R; s, \lambda)$, such that

$$\frac{\partial L}{\partial R} \begin{matrix} > \\ < \end{matrix} 0 \text{ if and only if } \omega \begin{matrix} > \\ < \end{matrix} M(R; s, \lambda). \quad (18)$$

The geometrical construction of a schedule is shown on Figure 1, where productivity is on the horizontal axis while work opportunity costs or financial incentives to work are on the vertical axis. By construction, for ω smaller than M , on the left of the graph of M , the derivative of the Lagrangian with respect to R is negative, while on the right of the graph it is positive.

To look for the pointwise maximum of the Lagrangian, draw the vertical line through ω . When it does not intersect the graph of M (i.e. ω is outside the range of M), then the optimum is to have everybody unemployed ($R - s \leq \underline{\delta}$) when the line is on the left of the graph, or to have everyone employed ($R - s \geq \bar{\delta}$) when it is on the right of the graph.

Consider now an intersection point: if the slope of M is negative at the point, this is a local minimum ($\partial L / \partial R$ is negative for smaller R 's, positive for larger R 's); the intersection points where the slope of M is positive are local maxima. Furthermore, the optimal tax scheme is continuous whenever M is nondecreasing everywhere. If the support of productivities is included in the range of M , the tax scheme exhibits some discontinuity whenever M is decreasing on part of its domain. The pointwise maximum is the point that yields the higher L , comparing all the local maxima and the two corners (full employment, or no employment). All this procedure is summarized in the following proposition.

Proposition 1. *Under Assumptions 1 to 3, there is no pooling at the optimum. Any optimal income scheme is increasing in the range of productivity for which the probability of being employed lies in $(0, 1)$. Furthermore, such a scheme is continuous if and only if the function M defined in (18) is globally increasing.*

The no-pooling property holds in the interval of productivities for which there are non negligible sets of both employees and unemployed, say $[\omega_\ell, \omega_h]$. Then R is increasing and no employee has interest in faking a smaller productivity than her own: she then would receive a smaller after tax income. People with

lower productivities than ω_ℓ are unemployed and receive the subsistence income s : they are treated as all the unemployed whose productivities are not observed by the government. Persons with a higher productivity than ω_h are all working. Typically, the optimal function R is constant for ω larger than ω_h , and the high skilled workers under consideration are indifferent between working at their full productivities ω or at any productivity in the interval $[\omega_h, \omega]$. We suppose that they do not shirk.

To clarify the logic of the argument, we spell out what happens at a discontinuity point of the tax schedule, for instance when one jumps from $R_- < \omega + s$ to $R_+ > \omega + s$ at some productivity ω . At R_- , there is a (relatively) small number of employees, $F(R_- - s | s)$ and their average social weights $p_E(R_-, s)$ is smaller than λ , since we supposed that the point lies below the 45 degree line on the graph. At R_+ , there are many more employees: all the persons with a work opportunity cost in the interval $[R_- - s, R_+ - s]$ who are unemployed when their productivities are slightly smaller than ω take a job when their productivity is larger than ω . Since the M curve then is above the 45 degree line, the average social weight of the employees $p_E(R_+, s)$ is larger than λ . For a discontinuity to occur across the 45 degree line, when the distributions are smooth (Assumption 1), *the average social weight of the persons of work opportunity costs in $[R_- - s, R_+ - s]$ has to be larger than that of persons with work opportunity costs lower than $(R_- - s)$.*

6.2 Redistribution

The previous paragraph stresses that, as in Stiglitz (1982), the social weights p_E play a crucial role in shaping the income tax schedule. Intuitively, the type of discontinuities in the schedule that we just described is only likely to occur empirically when the parameter α is multidimensional, and some component kicks in when the work opportunity cost becomes larger than $R_- - s$, making social weight locally an increasing function of income. We now lay down conditions under which the government objective involves standard redistribution, that is social weights are decreasing with incomes and/or productivities.

6.2.1 The social weights of the employees

Typically in the intensive model the more productive the agents, the richer they are at the optimum: with identical preferences social weights decrease with income. This behavior of the social weights, p_E decreases with R , is not straightforward in the extensive model.⁷

⁷We provide here structural foundations to Saez (2002)'s statement, page 1049:

If the government values redistribution, then the lower the earnings level of the individual, the higher the social marginal value of an extra dollar for that individual.

Indeed brute differentiation yields

$$\begin{aligned} \frac{\partial p_E}{\partial R} &= \frac{1}{F(R-s|\omega, s)} \int_{\delta}^{R-s} u''_{11}(R; \delta, s) dF(\delta|\omega, s) \\ &\quad - \frac{f(R-s|\omega, s)}{F(R-s|\omega, s)} p_E(R, s|\omega) \\ &\quad + u'_1(R; R-s, s) \frac{f(R-s|\omega, s)}{F(R-s|\omega, s)}. \end{aligned}$$

The first two terms are negative, following the intuition, but the last term is positive: an increase in R brings newcomers into employment, whose weights may be high. Nevertheless one can derive the desired property under two restrictive assumptions that bear on the two factors in the last term. The first one states that the new entrants in the labor force do not have too high a marginal utility of income:

Assumption 4. *The marginal utility of income of the new entrants into the labor force is nonincreasing in R for all s and ω : $u'_1(R; R-s, s)$ is nonincreasing in R .*

This assumption does not seem overly restrictive. For instance, in the pecuniary cost model, the marginal social utility of newcomers equals $U'(s)$ and does not depend on R . The second assumption, the log concavity of the cumulative distribution of work opportunity costs, is a stronger requirement, but is satisfied by a number of distributions.

Assumption 5. *The distribution of work opportunity costs is log-concave.*

The latter two assumptions guarantee that the first aspect of redistribution mentioned earlier prevails, namely that the social weights of workers are nonincreasing in income.

Proposition 2. *Under Assumptions 4 and 5, the average social weight of the employees of productivity ω , $p_E(R, s|\omega)$, is a nonincreasing function of R .*

The second redistributive property concerns the monotonicity of social weights with productivity. The following assumption is relevant in that respect.

Assumption 6. *1. The distribution of work opportunity costs stochastically decreases with productivity (i.e. the function $F(\delta|\omega, s)$ increases in ω).*

2. The second derivative $u''_{R\delta}$ is nonnegative.

The first part of the assumption is satisfied whenever higher productivity on the market goes with smaller opportunity cost of going to work, or less productivity in tasks at home. The second part of the assumption, which holds in the pecuniary cost model ($u(R; \delta, s) = U(R - \delta)$), is associated with social weights

which increase with work opportunity cost, all other things equal: individuals suffer from having a large opportunity cost, akin to a handicap and society wants to compensate them for this. The assumption fails when work opportunity costs reflect laziness and carry a social stigma. Note that there is a tension between Assumption 4 and Assumption 6.2. The former supposes that $u''_{RR} + u''_{R\delta} \leq 0$, so that for both to be satisfied, one needs $-u''_{RR} \geq u''_{R\delta} \geq 0$.

Proposition 3. *Under Assumption 6, the average social weight of the employees $p_E(R, s|\omega)$ is a nonincreasing function of ω .*

Proof: It is a straightforward consequence of the assumptions. From (11),

$$\begin{aligned} \frac{\partial p_E(R, s|\omega)}{\partial \omega} &= -\frac{p_E(R, s|\omega)}{F(R - s|\omega, s)} \frac{\partial F(R - s|\omega, s)}{\partial \omega} \\ &\quad + \frac{1}{F(R - s|\omega, s)} \frac{\partial}{\partial \omega} \int_{\underline{\delta}}^{R-s} u'_1(R; \delta, s) dF(\delta|\omega, s). \end{aligned}$$

The first term is negative from the fact that F increases with ω , while the negativity of the second follows from Assumption 6 and the properties of stochastic dominance. \blacksquare

6.2.2 Utilitarian tax schedules

We proceed to describing the shapes of the optimal tax schedules under Assumptions 4 to 6, which ensure that the social weights of the employees decrease both with their incomes and their productivities. Note that these assumptions do not preclude pooling (Proposition 1 does not apply). The next proposition states that, when the government has a redistributive objective, either taxes distort downwards the incentives to work everywhere, or they distort it *upwards* up to some threshold productivity level, and downwards for larger productivities. Furthermore taxes are a continuous function of income in the region where labor supply is distorted downwards. Discontinuities can occur for low incomes, when financial incentives to work are distorted upwards (see Section 7.2).

Proposition 4. *Suppose that Assumptions 1 (continuity of the distributions) and 4 to 6 (redistribution) hold.*

Then there exists a productivity threshold ω_m , $\omega_m < \bar{\delta}$, such that the financial incentive to work is undistorted or distorted downwards for $\omega > \omega_m$, while it is undistorted or distorted upwards for $\omega_m \geq \omega \geq \underline{\delta}$. In the region where incentives are distorted downwards, after tax income is a continuous function of before tax income.

According to Proposition 4, if the financial incentive to work is distorted downwards (upwards) at a given productivity level, it must be distorted downwards

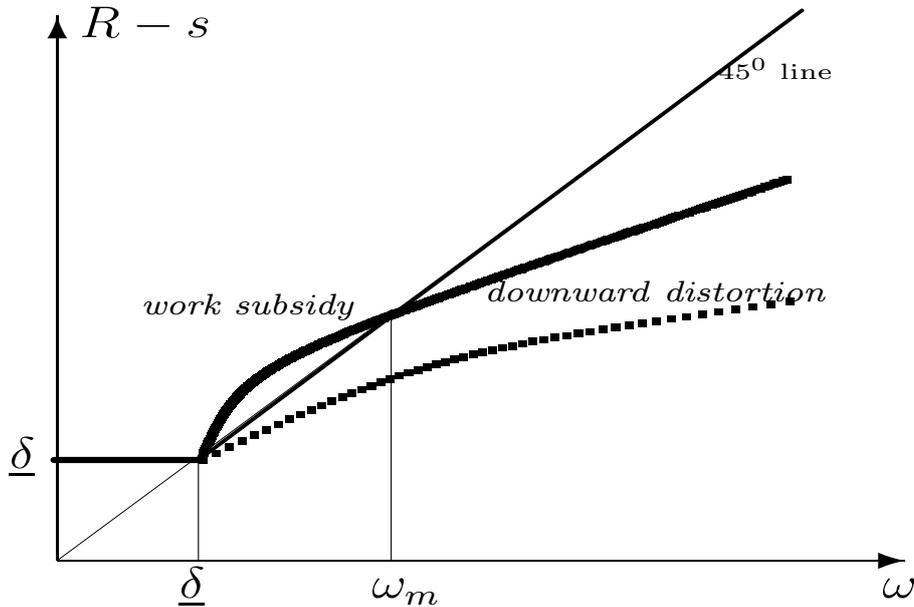


Figure 2: Two ‘well behaved’ optimal tax schemes

(upwards) or undistorted for higher (lower) productivities.⁸ Figure 2 illustrates the Proposition. Along the curve $R - s = \max(\underline{\delta}, \omega)$, labor supply is efficient and is not distorted with respect to laissez-faire in the absence of income effects. The dotted curve corresponds to the case $\omega_m = \underline{\delta}$. The graph of after tax income lies below the 45 degree line;⁹ all the workers are taxed; their financial incentive to work is distorted downwards compared to laissez-faire, where they would be better off than in the second best. This situation is typical of a highly redistributive government and holds in particular under the Rawlsian objective, where p_E is zero (see e.g. Choné and Laroque (2005)). The bold solid curve illustrates the case $\omega_m > \underline{\delta}$.¹⁰ The figure shows a case where there is no pooling and where incentives to work $R(\omega) - s$ are a continuous increasing function of productivity. There is a low skilled region, $\underline{\delta} \leq \omega \leq \omega_m$, where incentives are distorted upwards. Absent income effects, labor supply is distorted upwards in this region, both compared to its efficient and laissez-faire levels.¹¹

⁸The laissez-faire (no distortion at all) is a possible configuration in Proposition 4.

⁹More precisely, we have: $R(\omega) \leq \max(\omega + s, \underline{\delta} + s)$ for all ω .

¹⁰It is similar to Figure IIa in Saez (2002), who discusses from a more applied perspective the occurrence of negative marginal tax rates.

¹¹Other properties of the optimal tax schedules, in relationship with the Rawlsian criterion may be worth recalling. Theorem 6 of Choné and Laroque (2005) applies here: all the utilitarian optimal incentive schemes are located above the Rawlsian curve. Theorem 3 of Laroque (2005) also applies: any incentive scheme above the Laffer curve which does not overtax and such that $R(\omega) - s \leq \omega$ is associated with a second best optimal allocation.

The following remark, proved in the Appendix, gives a condition under which after tax income has a slope smaller than 1 below the 45 degree line, as on Figure 2.

Remark 6.1. *Suppose that in addition to the assumptions of Proposition 4, the distribution of work opportunity costs is independent of productivity. Then the marginal tax rate is nonnegative in the region where incentives to work are distorted downwards.*

7 The full program

7.1 The marginal cost of public funds

The preceding section has studied the shape of the optimal tax schemes given levels of subsistence income s and of the marginal cost of public funds λ . Two equations are needed to determine these two quantities. One is the feasibility condition (10). The other is a first order necessary condition, associated with a small translation of all incomes, i.e. an equal marginal change in both s and $R(\omega)$ for all ω . It is derived in the Appendix in general circumstances, allowing for pooling. The social weights of the employees, defined in (11), and of the unemployed

$$p_U(R, s|\omega) = \mathbb{E}_\alpha [\tilde{v}'_1(s; \alpha) \mid \delta > R - s, \omega, s], \quad (19)$$

are the ingredients of the Lemma.

Lemma 1. *Under Assumption 1, there exists a function $\rho(\omega)$, $\rho(\omega) < 1$, such that at an optimum*

$$\int_{\Omega} \{(1-\rho) F p_E(R(\omega), s|\omega) + (1-F) p_U(R(\omega), s|\omega)\} dG(\omega) = \int_{\Omega} \lambda(1-\rho F) dG(\omega), \quad (20)$$

where F stands for $F(R(\omega) - s|\omega, s)$, the proportion of employed among agents with productivity ω .

Absent income effects, ρ is identically equal to 0. When leisure is a normal good, ρ is a non positive function of ω .

In the case where work opportunity costs do not depend on the level of the subsistence income (no income effects), the lemma indicates that the familiar equality of the marginal cost of public funds to the average of the agents marginal utilities of income holds here: ρ is equal to zero, and (20) simplifies into

$$\int_{\Omega} \left[\int_{\underline{\delta}}^{R-s} u'_1(R; \delta, s) dF(\delta|\omega, s) + \int_{R-s}^{\bar{\delta}} v'_1(s; \delta) dF(\delta|\omega, s) \right] dG(\omega) = \lambda.$$

In the presence of income effects, the coefficients of p_E and of p_U are weighted. The weights are nonnegative and sum up to the coefficient of λ : $F(1 - \rho) + (1 - F) = 1 - \rho F$. When leisure is a normal good, ρ is negative and the employees are given more importance than in the absence of income effects.

7.2 When are negative tax rates optimal?

When does a redistributive government implement negative taxes? Theory helps to answer the question. From the first order condition (14), the workers of productivity ω face negative taxes when their average social weight is larger than the marginal cost of public funds. Social weights are easy to track in the pecuniary cost model, where the workers have utility $U(R(\omega) - \delta)$ and the unemployed $U(s)$. The intuition is the following. Marginal employees are indifferent between working and not working, while the utilities of the unemployed do not depend on their work opportunity costs: a utilitarian government assign the unemployed the same social weight as that of the marginal employees. From (20), it follows that the social weight of the lower income employees is larger than the marginal cost of public funds if there is any redistribution at all.¹² Then, by continuity, there is a range of productivities at the bottom of the skill distribution where the social weights of the workers is larger than the marginal cost of public funds. It follows that there are *upward labor supply distortions at low productivities* at the optimum.

The following proposition shows that this argument holds generally, accounting for pooling and other complications, for any redistributive objective that satisfies the assumptions of Section 6.2:

Proposition 5. *Consider a pecuniary cost economy satisfying Assumption 1, with $\underline{\omega} \leq \underline{\delta}$. Then, at the optimum, labor supply is distorted upwards for the lowest skilled workers.*

The proof of Proposition 5 (see Appendix F) shows that there exists a productivity ω_0 , $\omega_0 > \underline{\delta}$, such that $R(\omega) > \omega + s$ for all ω in $(\underline{\delta}, \omega_0)$ and that one of the two following properties holds:

1. None of the agents of productivity smaller than $\underline{\delta}$ work at the optimum, $R(\underline{\delta}) - s = \underline{\delta}$, and the slope of the income schedule at $\underline{\delta}$ is larger than 1. Such a configuration is shown on Figure 2.
2. There are (a non negligible set of) agents with productivity $\underline{\delta}$ who work: $R(\underline{\delta}) > s + \underline{\delta}$. Furthermore, if $\underline{\omega} < \underline{\delta}$, a nonnegligible set of agents of productivity smaller than $\underline{\delta}$ work at the optimum. Such a configuration is shown on Figure 3.

¹²Otherwise all the social weights on the left hand side of (20) are at most equal to λ , so that for the equality to hold, all weights must be equal to λ , which corresponds to the laissez-faire equilibrium.

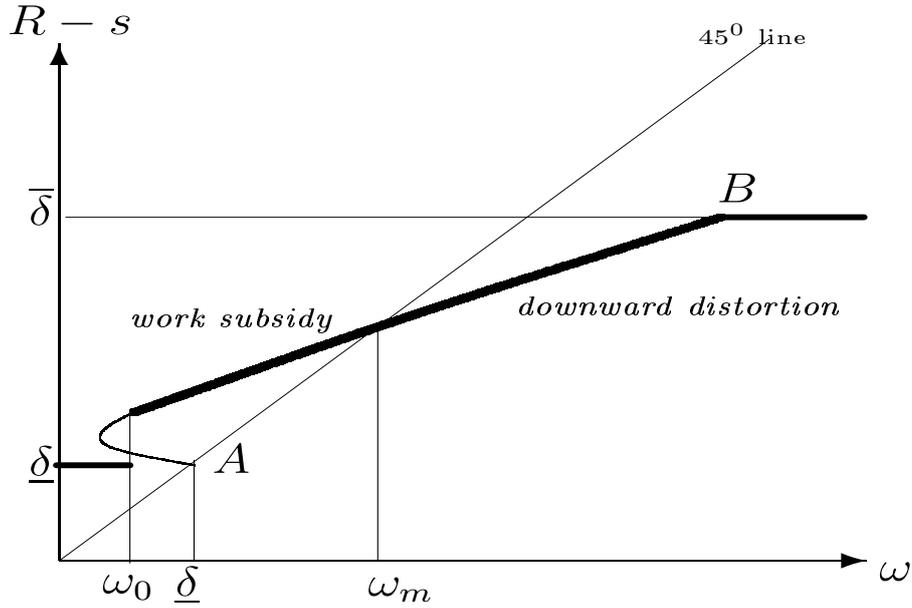


Figure 3: Strong upward distortion (pecuniary cost economy)

The configurations of Figures 2 and 3 respectively occur depending on whether the social weight of the unemployed is smaller or greater than twice the marginal cost of public funds, i.e. twice the average social weight of all the agents in the economy. When global transfers s are unable to reduce the utility differences, so that the marginal utilities of income of the unemployed and marginal workers stay very different from those of the richest, it is second best efficient to increase the welfare of (some of) the low skilled through negative taxes.

When the distribution of work opportunity costs is uniform and independent of productivity, analytic computations can be carried out in some detail (see Appendix F.2).¹³ The bold line represents the optimal tax scheme. None of the agents with very low productivities, $\omega < \omega_0$, work. But for all ω larger than or equal to ω_0 , a fraction of the agents does. In fact the upward distortion to labor supply here is particularly strong: some agents with productivity smaller than the minimal cost of going to work participate in the labor force. Note that here, with a uniform distribution of work opportunity costs, after tax income is a concave function of income, implying a progressive tax system for productivities above ω_0 .

Moving away from the standard labor supply model, there are situations where the social weight attached to the unemployed agents is larger than that attached to the employees: for instance this may be the case in the presence of ‘involuntary’ unemployment, or when a large opportunity cost to work is associated with a

¹³Figure 3 is drawn in this particular situation.

handicap (the marginal social weight $v'(s; \delta, \omega)$ is increasing with δ). It is then easy to think of economies where at the optimum the average social weight of the unemployed is larger than the marginal cost of public funds and the social weight of the lowest paid workers is smaller. In these economies, after tax income is everywhere smaller than productivity.

Similarly, the analysis has proceeded under the assumption that the social welfare function is smooth, so that the distribution of the agents' weights has no mass point. The case of a Rawlsian planner who puts all the weight on the least favored agent in the economy corresponds here to a situation where p_E is equal to zero everywhere. Then the tax rate is always positive. This is in line with the results of Choné and Laroque (2005). On the other hand, other social choice criteria, such as the one advocated in Fleurbaey and Maniquet (2006), may put the weight on the deserving 'working poors'. As a consequence, in an intensive model with multiple dimensions of heterogeneity, Fleurbaey and Maniquet (2006) show that optimal taxation implies subsidizing as much as possible the poorest workers.

8 An example without heterogeneity

The previous analysis crucially hinges on the existence of heterogeneity in work opportunity costs. This section shows how the results break down where everyone has the same work opportunity cost, as in Homburg (2002), a situation where Assumption 1 does not hold. Let δ_0 be the common value of the work opportunity cost. In a pecuniary cost economy (see end of Section 3), the objective takes the form

$$L(R, s; \omega) = \begin{cases} U(s) - \lambda s & \text{if } R - s < \delta_0, \\ U(R - \delta_0) + \lambda[\omega - R] & \text{if } R - s > \delta_0, \end{cases}$$

The Lagrangian is equal to any of the two above quantities when $R - s = \delta_0$: the agent is indifferent between working or not, and the planner can choose the preferred outcome. The problem is to maximize the integral of the Lagrangian subject to the feasibility constraint over R , R nondecreasing.

Absent pooling, the first order condition (13) with respect to $R(\omega)$ for an unconstrained worker of productivity ω would be:

$$U'(R(\omega) - \delta_0) = \lambda.$$

Since the solution $R(\omega)$ does not depend on productivity, there is full pooling. Furthermore the feasibility constraint (10) here becomes

$$\int_{\Omega} [\omega - R + s] \mathbf{1}_{\text{workers}} dG(\omega) = s.$$

An optimum is characterized by two quantities (R, s) linked by the feasibility constraint. Furthermore it has to specify the set of workers when $R - s$ is equal to δ_0 . Consequently, the first order conditions lead to three possibilities:

1. If $R - \delta_0 < s$, nobody works. By feasibility s is equal to zero. The value of social welfare, $U(0)$, is at a global minimum when the economy is productive enough.
2. If $R - \delta_0 > s$, everybody works. The constant income R is equal to the average productivity in the economy $\int_{\Omega} \omega dG(\omega)$. The social welfare is equal to

$$U \left(\int_{\Omega} \omega dG(\omega) - \delta_0 \right),$$

which is only defined when $\int_{\Omega} \omega dG(\omega) \geq \delta_0$.

3. Finally, when there is indifference between working or not, $R - \delta_0 = s$, and looking at the expression of $L(R, s; \omega)$, the planner decides to put to work the agents with productivity ω at least as large as $R - s$. The feasibility condition gives the value of s

$$\int_{\Omega} \max[\omega - \delta_0, 0] dG(\omega) = s \geq \int_{\Omega} \omega dG(\omega) - \delta_0.$$

This last case is the optimum whenever there is a non negligible set of agents with $\omega > \delta_0$. Then there is no upward distortion of labor supply. The utilitarian optimum does not leave any surplus to the workers and everyone is treated equally. Heterogeneity in the form of some dispersion of work opportunity costs gives more scope for redistribution, associated with the unknown value of δ .

9 Conclusion

Since Mirrlees (1971), the theory of optimal taxation has largely been developed in the intensive model setup. Until recently, much less attention had been devoted to the extensive model, even though its empirical relevance is well established (e.g. Heckman (1993)). Following Saez (2002), a number of authors (Laroque (2005), Beaudry, Blackorby, and Szalay (2009) among others) have undertaken to fill the gap.

In the present article, we have set the problem in the framework of mechanism design theory. Income effects are allowed and heterogeneity enters the participation and reservation utilities with little restrictions. From this general framework, we have derived a two-dimensional reduced form involving productivity and work opportunity cost. A transposition of the taxation principle allows to go from mechanism design to a public finance environment: we characterize the set of incentive-compatible tax schedules available to the government and provide a full treatment of the principal agent problem under general informational asymmetry and extensive preferences. We give sufficient conditions for the social weights of the employed workers to decrease with both income and productivity. When

these conditions hold, work subsidies can occur for low-skilled workers. Under a simple specification of utility, we show that work subsidies indeed arise for a positive mass of agents.

This study complements those of Beaudry, Blackorby, and Szalay (2009) and Choné and Laroque (2010). Taken together, these papers suggest that optimal negative (average or marginal) tax rates are to be expected when the government cannot observe at least one agent characteristics on top of her productivity.

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Appendix

A Relationship between the structural and the reduced forms

The following proposition provides a characterization of reduced-form specification which holds only in the one-dimensional setting.

Proposition A.1. *When there is one dimension of heterogeneity (α is scalar), the reduced-form utility satisfies*

$$u_\delta(c_E; \delta, c_U)/u_{c_U}(c_E; \delta, c_U) \text{ does not depend on } c_E. \quad (21)$$

Conversely, any reduced-form model (u, v) satisfying conditions (4) and (21) derives from a structural model (\tilde{u}, \tilde{v}) with unidimensional heterogeneity.

When there is more than one dimension of heterogeneity, the restriction (21) is not necessary.

To prove Proposition A.1, we consider successively the case where the dimension of α is one and the case where it is higher than 1.

A.1 The unidimensional case

Consider the structural model (1) and assume that α is scalar and that, for any given c_U , the map $\alpha \rightarrow \delta(\alpha; c_U)$ is one-to-one. We note $\delta^{-1}(\cdot; c_U)$ the inverse function: $\delta \rightarrow \delta^{-1}(\delta; c_U)$. We have

$$u(c_E; \delta, c_U) = \tilde{u}(c_E; \delta^{-1}(\delta; c_U)) \text{ and } v(c_U; \delta) = \tilde{v}(c_U; \delta^{-1}(\delta; c_U)). \quad (22)$$

Condition (2) is equivalent to condition (4). It follows from (22) that

$$u_\delta(c_E; \delta, c_U) = \frac{\tilde{u}_\alpha(c_E; \delta^{-1}(\delta; c_U))}{\delta_\alpha(\delta^{-1}(\delta, c_U); c_U)} \text{ and } u_{c_U}(c_E; \delta, c_U) = \frac{\tilde{u}_\alpha(c_E; \delta^{-1}(\delta; c_U))}{\delta_{c_U}(\delta^{-1}(\delta, c_U); c_U)}.$$

which implies (21). Now, note that

$$u(c_E; \delta(\alpha, c_U), c_U) = \tilde{u}(c_E, \alpha).$$

It follows that, for any given (c_E, α) , the function $s \rightarrow \delta(\alpha; c_U)$ satisfies the following ordinary differential equation:

$$u_\delta(c_E; \delta(\alpha; c_U), c_U)\delta_{c_U}(\alpha; c_U) + u_{c_U}(c_E; \delta(\alpha; c_U), c_U) = 0. \quad (23)$$

The income effect, that is the dependence of $\delta(\alpha; c_U)$ on c_U , is given by (23).

Conversely, consider two functions $u(c_E; \delta, c_U)$ and $v(c_U; \delta)$ satisfying conditions (4) and (21). Thanks to (21), the ordinary differential equation (23) *does not* depend on c_E . For a given initial condition at some point x_0 , the solution of this equation does not depend on c_E . Using α to parameterize the initial condition, say $\delta(\alpha; x_0) = \alpha$, we get a family of functions $c_U \rightarrow \delta(\alpha; c_U)$ that characterize the behavior of the agent and the income effect. We define

$$\begin{cases} \tilde{u}(c_E; \alpha) = u(c_E; \delta(\alpha; c_U), c_U) \\ \tilde{v}(c_U; \alpha) = v(c_U; \delta(\alpha; c_U)). \end{cases}$$

Thanks to (23), the definition of \tilde{u} is consistent, i.e. \tilde{u} does not depend on c_U . ■

A.2 Counter-example with multiple heterogeneity

When the dimension of α is greater than 1, the condition (21) is not necessary any more, as the following example shows. We fix $M > 1$ and suppose that (α_1, α_2) is uniformly distributed on the square $[0, 1]^2$. Let \tilde{u} and \tilde{v} be defined by

$$\begin{cases} \tilde{u}(c_E; \alpha) = -\frac{1}{M + c_E - \alpha_1} \\ \tilde{v}(c_U; \alpha) = -\frac{1}{M + (1 + \alpha_2)c_U}. \end{cases} \quad (24)$$

The functions \tilde{u} and \tilde{v} are strictly increasing and strictly concave in income for all $\alpha = (\alpha_1, \alpha_2)$. It is easy to check that (2) holds with $\delta(\alpha_1, \alpha_2; c_U) = \alpha_1 + c_U \alpha_2$ and that α_1 , conditionally on δ , is uniformly distributed.¹⁴ If we note $[\underline{\alpha}_1(\delta, c_U), \bar{\alpha}_1(\delta, c_U)]$ the support of the conditional distribution, we have

$$u(c_E; \delta, c_U) = \frac{-1}{\bar{\alpha}_1(\delta, c_U) - \underline{\alpha}_1(\delta, c_U)} \int_{\underline{\alpha}_1(\delta, c_U)}^{\bar{\alpha}_1(\delta, c_U)} \frac{1}{M + c_E - \alpha_1} d\alpha_1.$$

In the region where $c_U < \delta < 1$, we have $\underline{\alpha}_1(\delta, c_U) = \delta - c_U$, $\bar{\alpha}_1(\delta, c_U) = \delta$ and

$$u(c_E; \delta, c_U) = \frac{1}{c_U} \ln \frac{M + c_E - \delta}{M + c_E - \delta + c_U}$$

which does not satisfy (21). Note also that the distribution of the opportunity cost is given, from small δ , by $F(\delta|\omega, c_U) = \delta^2/c_U$. This distribution satisfies Assumption 1 as F/f tends to 0 as δ goes to $\underline{\delta} = 0$.

B Proof of Proposition 1

The existence and properties of $M(R; s, \lambda)$ follow from the fact that, under Assumptions 2 and 3, the cross derivative $\partial^2 L / \partial R \partial \omega$ is positive. Indeed:

$$\frac{1}{\lambda} \frac{\partial L}{\partial R} = \frac{\partial(\omega - \delta)F(\delta|\omega)}{\partial R} + \frac{1}{\lambda} \int_{\underline{\delta}}^{R-s} u'_1(R; \delta, s) dF(\delta|\omega, s).$$

¹⁴Leisure is a normal good as δ increases with c_U .

The first term is increasing in ω from Assumption 3, the second is nondecreasing from Assumption 2.

Pointwise maximization leads to the global maximum ignoring the monotonicity constraint, so that the only thing to prove is the monotonicity of the pointwise maximizer. It is a straightforward consequence of single crossing:

$$L(R(\omega'), \omega') - L(R(\omega'), \omega) - L(R(\omega), \omega') + L(R(\omega), \omega) = \int_{R(\omega)}^{R(\omega')} \int_{\omega}^{\omega'} \frac{\partial^2 L}{\partial R \partial \omega} dR d\omega.$$

The left hand side is non negative from pointwise maximization. The right hand side is of the same sign as $[R(\omega') - R(\omega)](\omega' - \omega)$. ■

C Proof of Proposition 2

In this section, we show that

$$p_E(R, s|\omega) = \frac{1}{F(R-s|\omega, s)} \int_{\underline{\delta}(\omega)}^{R-s} \frac{\partial u}{\partial R}(R; \delta) dF(\delta|\omega, s) = E_{\delta} \left[\frac{\partial u}{\partial R} | \delta \leq R-s, \omega, s \right] \quad (25)$$

decreases with R . To simplify notations, we drop the s and ω arguments which are kept constant in the proof. An integration by parts yields

$$p_E(R) = \frac{\partial u}{\partial R}(R; R-s) - \frac{1}{F(R-s)} \int_{\underline{\delta}(\omega)}^{R-s} \frac{\partial^2 u}{\partial R \partial \delta}(R; \delta) F(\delta) d\delta,$$

or

$$p_E(R) = \frac{\partial u}{\partial R}(R; R-s) - E_{\delta} \left[\frac{\partial^2 u}{\partial R \partial \delta}(R; \delta) \frac{\tilde{F}(\delta)}{f(\delta)} | \delta \leq R-s \right].$$

Now differentiating (25) gives

$$\frac{\partial p_E}{\partial R}(R) = E_{\delta} \left[\frac{\partial^2 u}{\partial R^2}(R; \delta) | \delta \leq R-s \right] - \frac{f(R-s)}{F(R-s)} p_E(R) + \frac{f(R-s)}{F(R-s)} \frac{\partial u}{\partial R}(R; R-s).$$

Substituting the expression obtained for $p_E(R)$:

$$\frac{\partial p_E}{\partial R}(R) = E_{\delta} \left[\frac{\partial^2 u}{\partial R^2}(R; \delta) + \frac{\partial^2 u}{\partial R \partial \delta}(R; \delta) \frac{f(R-s)}{F(R-s)} \frac{F(\delta)}{f(\delta)} | \delta \leq R-s \right].$$

By the log-concavity of F , the factor

$$\frac{f(R-s)}{F(R-s)} \frac{F(\delta)}{f(\delta)}$$

is smaller than 1. By concavity of the utility function, the first term is negative. By the assumption on the marginal utility of income of the newcomer into the labor force

$$\frac{\partial^2 u}{\partial R^2}(R; \delta) + \frac{\partial^2 u}{\partial R \partial \delta}(R; \delta)$$

is non positive. The result follows. ■

D Proof of Proposition 4

Under the assumptions of the proposition, pooling cannot be ruled out. Accordingly, the following proof accounts for the possibility of pooling.

D.1 Position of the after tax income curve with respect to the 45 degree line

By Propositions 2 and 3

$$\frac{\partial p_E(R, s|\omega)}{\partial R} + \frac{\partial p_E(R, s|\omega)}{\partial \omega} \leq 0.$$

It follows that $p_E(R, s|\omega)$ is nondecreasing as one goes up the 45 degree line. Consider in turn the three possibilities: $p_E(\omega + s, s|\omega)$ is everywhere smaller than λ , $p_E(\omega + s, s|\omega)$ is everywhere equal to or larger than λ , or finally there is some ω in $(\underline{\delta} + s, \bar{\delta} + s)$ where $p_E(\omega + s, s|\omega)$ is equal to λ (and some where it is smaller).

1. For all ω , $p_E(\omega + s, s|\omega) \leq \lambda$. It follows from the monotonicity properties of p_E that $p_E \leq \lambda$ above the 45 degree line $\omega = R - s$. Now, from (12), $\partial L/\partial R$ is negative for all ω such that $R > \max(\omega + s, \underline{\delta} + s)$. Then any nondecreasing function $R(\omega)$ above the 45 degree line is dominated by the nondecreasing function \tilde{R} , defined by $\tilde{R}(\omega) = \min(\max(\omega + s, \underline{\delta} + s), R(\omega))$. It follows that R must coincide with \tilde{R} , that is to say: $R \leq \max(\omega + s, \underline{\delta} + s)$ for all ω , which expresses that the financial incentives to work are anywhere distorted downwards or undistorted. Proposition 4 therefore holds with $\omega = \underline{\omega}$ (as well as with $\omega = \underline{\delta}$).
2. For all ω , $p_E(\omega + s, s|\omega) \geq \lambda$. It follows from the monotonicity properties of p_E that $p_E \geq \lambda$ below the 45 degree line $\omega = R - s$. From (12), $\partial L/\partial R$ is positive for all ω such that $s + \underline{\delta} < R < s + \omega$. Then any nondecreasing function $R(\omega)$ below the 45 degree line is dominated by the nondecreasing function \tilde{R} , defined by $\tilde{R}(\omega) = \max(\omega + s, R(\omega))$. It follows that $R \geq \omega + s$ for all ω . But then, by feasibility (10), the subsistence income s is equal to zero and $\omega = R(\omega)$. This is the laissez-faire allocation.
3. There is an ω , say ω_0 , such that $p_E(\omega_0 + s, s|\omega_0) = \lambda$. It follows from the monotonicity properties of p_E that $p_E \leq \lambda$ for $R \geq \omega + s$ and $\omega \geq \omega_0$ and that $p_E \geq \lambda$ for $R \leq \omega + s$ and $\omega \leq \omega_0$. From (12), $\partial L/\partial R$ is positive for all (R, ω) such that $\omega_0 > \omega > R - s$, and negative for all (R, ω) such that $\omega_0 < \omega < R - s$. Take any function $R(\omega)$. Define \tilde{R} , associated with R , through

$$\tilde{R}(\omega) = \begin{cases} \min[R(\omega_0), \max(\omega + s, R(\omega))] & \text{if } \omega \leq \omega_0, \\ \max[R(\omega_0), \min(\omega + s, R(\omega))] & \text{if } \omega \geq \omega_0. \end{cases}$$

Then, whenever R is nondecreasing, \tilde{R} is also nondecreasing and yields a social utility level at least as high as R . As a consequence, any optimal R must be such that the associated \tilde{R} coincides with R . Letting $\omega_m = R(\omega_0) - s$, the optimal tax scheme is above or on the 45 degree line for $\omega \leq \omega_m$ and is below or on the 45 degree line for $\omega \geq \omega_m$.

D.2 Continuity of taxes when incentives are distorted downwards

From (12), $\partial L/\partial R$ has the same sign as the function H defined by

$$H(R, \omega) = (\omega - R + s) - \frac{F(R - s|\omega, s)}{f(R - s|\omega, s)} \left[1 - \frac{p_E(R, s|\omega)}{\lambda} \right].$$

A straightforward computation gives:

$$\frac{\partial H}{\partial R} = -1 - \left(1 - \frac{p_E(R, s|\omega)}{\lambda} \right) \frac{\partial(F/f)}{\partial R} + \frac{F(R - s|\omega, s)}{f(R - s|\omega, s)} \frac{\partial p_E}{\partial R},$$

In the half plane $\omega \geq R - s$, any point such that $H(R, \omega)$ is equal to zero satisfies $p_E(R, s|\omega) \leq \lambda$. From the log-concavity of F and the monotonicity of p_E (which follows from the assumptions of the Proposition), it follows that at any such point

$$\frac{\partial H}{\partial R} \leq -1.$$

Therefore, from the implicit function theorem, the equality $H(R, \omega) = 0$ can be solved in the half plane into a continuously differentiable function $R = \rho(\omega)$.

We now are in a position to proceed with the proof. Consider an optimal tax scheme that enters the region $\omega > R - s$ where labor supply is distorted downwards, at the point of abscissa ω_m defined in the first part of the Proposition. From the first order conditions, there are two possibilities, depending on whether there is pooling or not, at ω_m :

1. There is no pooling at ω_m and $p_E(\omega_m + s, s|\omega_m)$ is equal to λ . Then the solution for $\omega \geq \omega_m$ is obtained by applying an ironing technique to the graph of $H(R, \omega)$. From the properties of H and the function ρ , the solution is continuous.
2. There is pooling at ω_m . From (17), at the end of the pooling interval, say at $\omega_1 > \omega_m$, $\partial L/\partial R$ and H are less than or equal to zero. We claim that these quantities are zero: indeed by the first order conditions, we must have $\partial L/\partial R \geq 0$ and $R \geq R(\omega_1)$ in the right neighborhood of ω_1 , while $\partial H/\partial R$ is negative. It follows that as in 1. the remainder of the solution is obtained by ironing the graph of $H(R, \omega)$, for $\omega \geq \omega_1$.

D.3 Proof of Remark 6.1

When the distribution of work opportunity costs is independent of ω , the equation $\partial L/\partial R = 0$ is equivalent to:

$$\omega = R - s + \frac{F(R - s|s)}{f(R - s|s)} \left[1 - \frac{p_E(R, s)}{\lambda} \right].$$

From the logconcavity of F and the fact that p_E decreases with R , the right hand side is an increasing function of R with a slope at least as large as 1 in the region where p_E is smaller than λ . The result follows directly at points where $\partial L/\partial R = 0$; it holds trivially in a pooling region, where the marginal tax rate is 1.

E Proof of Lemma 1

Writing the effect of a translation of the overall income (increasing $R(\omega)$ for all ω and s by the same small amount) shows that the expectation in w of

$$F(R(\omega) - s|\omega, s)p_E(R(\omega), s|\omega) + [1 - F(R(\omega) - s|\omega, s)]p_U(R(\omega), s|\omega) + \lambda \left\{ [\omega - R + s] \frac{\partial F}{\partial s}(R(\omega) - s|\omega, s) - 1 \right\} \quad (26)$$

is zero.

Consider first points ω where $R(\cdot)$ is strictly increasing and the first order condition (13) holds. For these points, we set $\rho = \frac{1}{f} \frac{\partial F}{\partial s}$. Thanks to (5), we get: $\rho \leq 1$. It follows from the first order condition (13) that

$$\lambda[\omega - R + s] \frac{\partial F}{\partial s}(R(\omega) - s|\omega, s) = \lambda[\omega - R + s]\rho f = \rho F(\lambda - p_E).$$

Now consider a pooling interval $[\omega_1, \omega_2]$ where R is constant and equal to \bar{R} . Suppose first that $\bar{R} - s \leq \omega_1$. Then using (5) and the first pooling condition (15), we get

$$\lambda \int_{\omega_1}^{\omega_2} (\omega - \bar{R} + s) \frac{\partial F}{\partial s} dG(\omega) \leq \lambda \int_{\omega_1}^{\omega_2} (\omega - \bar{R} + s) f dG(\omega) = \int_{\omega_1}^{\omega_2} (\lambda - p_E) F dG(\omega).$$

For $\omega \in [\omega_1, \omega_2]$, we set

$$\rho = \frac{\lambda \int_{\omega_1}^{\omega_2} (\omega - \bar{R} + s) \frac{\partial F}{\partial s} dG(\omega)}{\int_{\omega_1}^{\omega_2} (\lambda - p_E) F dG(\omega)}.$$

Since $\int_{\omega_1}^{\omega_2} (\lambda - p_E) F dG(\omega) > 0$, we get $\rho \leq 1$.

Suppose now that $\bar{R} - s \geq \omega_2$. The same computation yields

$$\int_{\omega_1}^{\omega_2} (\omega - \bar{R} + s) \frac{\partial F}{\partial s} dG(\omega) \geq \lambda \int_{\omega_1}^{\omega_2} (\omega - \bar{R} + s) f dG(\omega) = \int_{\omega_1}^{\omega_2} (\lambda - p_E) F dG(\omega).$$

We define ρ as above, and again $\rho \leq 1$. (Notice that, in this case, $\int_{\omega_1}^{\omega_2} (\lambda - p_E) F dG(\omega) < 0$.)

Finally suppose that $\omega_1 < \bar{R} - s < \omega_2$. Then, using (5) and the second pooling condition (16), we get

$$\int_{\bar{R}-s}^{\omega_2} \lambda (\omega - \bar{R} + s) \frac{\partial F}{\partial s} dG \leq \int_{\bar{R}-s}^{\omega_2} \lambda (\omega - \bar{R} + s) f dG \leq \int_{\bar{R}-s}^{\omega_2} (\lambda - p_E) F dG.$$

For $\omega \in [\bar{R} - s, \omega_2]$, we define ρ as

$$\rho = \frac{\int_{\bar{R}-s}^{\omega_2} (\omega - \bar{R} + s) \frac{\partial F}{\partial s} dG(\omega)}{\int_{\bar{R}-s}^{\omega_2} (\lambda - p_E) F dG(\omega)}$$

and get $\rho \leq 1$. Similarly we have

$$\lambda \int_{\omega_1}^{\bar{R}-s} (\omega - \bar{R} + s) \frac{\partial F}{\partial s} dG \geq \lambda \int_{\omega_1}^{\bar{R}-s} (\omega - \bar{R} + s) f dG \geq \int_{\omega_1}^{\bar{R}-s} (\lambda - p_E) F dG$$

For $\omega \in [\omega_1, \bar{R} - s]$, we define ρ as

$$\rho = \frac{\int_{\omega_1}^{\bar{R}-s} (\omega - \bar{R} + s) \frac{\partial F}{\partial s} dG(\omega)}{\int_{\omega_1}^{\bar{R}-s} (\lambda - p_E) F dG(\omega)}$$

and get, again, $\rho \leq 1$. (Notice that $\int_{\omega_1}^{\bar{R}-s} (\lambda - p_E) F dG(\omega) < 0$.)

Replacing in (26) the term $\lambda[\omega - R + s]\partial F/\partial s$ (or its integral on pooling intervals) by the expressions derived above yields the desired result.

Finally recall that absent income effects: $\partial F/\partial s = \rho = 0$, while when leisure is a normal good, $\partial F/\partial s$ is everywhere less than or equal to 0. ■

F The pecuniary cost economy

F.1 Proof of Proposition 5

Recall that in a pecuniary cost economy (see the end of Section 3) workers get utility $U(R - \delta)$ and unemployed agents get utility $U(s)$. We note the (unconditional) social weight of the whole population of unemployed agents

$$p_U(s) = \mathbb{E}_{\alpha, w} [\tilde{v}'_1(s; \alpha) \mathbf{1}_{R(\omega) - s < \delta}].$$

Here $p_U(s) = U'(s)$. There are no income effects ($\partial F/\partial s = 0$) and equation (20) simplifies into

$$\lambda = \iint U'[\max(s, R(\omega) - \delta)] dF(\delta|\omega) dG(\omega).$$

It follows that $\lambda \leq U'(s)$. The inequality has to be strict: otherwise $R(\omega) - s \leq \delta$ almost everywhere, and essentially nobody works. Then feasibility would imply $s = 0$, and everybody is unemployed with zero income. But this allocation is dominated by laissez-faire, a contradiction. Consequently $p_U(s) = U'(s) > \lambda$.

Let R_0 be such that

$$U'(R_0 - \underline{\delta}) = \lambda.$$

From the concavity of the utility function, $R_0 - \underline{\delta} > s$, and the weight of the employees p_E is larger than λ whenever the income $R(\omega)$ is smaller than R_0 . Define ω_0 as $R_0 - s$: ω_0 is larger than $\underline{\delta}$. From the expression (12) of $\partial L/\partial R$, we have

$$\frac{\partial L}{\partial R}(R, s; \omega) > 0 \quad \text{for all } \underline{\delta} + s < R \leq R_0 \text{ and } R - s \leq \omega. \quad (27)$$

For ω in $(\underline{\delta}, \omega_0)$, the above inequality directly yields $R(\omega) > \omega + s$ when there is no pooling at ω . Suppose now there is pooling at ω and consider the upper end of the pooling interval. The first order condition (17) imposes: $\partial L/\partial R \leq 0$ at this point. The inequality then shows that this point is above the 45 degree line $R = \omega + s$.¹⁵ It follows that the pooling interval entirely lies above the 45 degree line. As a consequence

$$R(\omega) > s + \omega \quad \text{for all } \omega \text{ such that } \underline{\delta} < \omega \leq \omega_0.$$

All the low skilled workers (if $\omega = \underline{\delta}$, there are no workers with productivity $\underline{\delta}$) of productivity smaller than ω_0 have financial incentives to work distorted upwards, which completes the proof. ■

We now investigate what happens at productivity $\omega = \underline{\delta}$, in order to substantiate points 1. and 2. of the comment of the Proposition: what is the optimal $R(\underline{\delta})$? The derivative $\partial L/\partial R$ is zero for $R = s + \underline{\delta}$, so that $s + \underline{\delta}$ is an extremum of the function $L(\cdot, s; \underline{\delta})$. A straightforward calculation gives

$$\frac{\partial^2 L}{\partial R^2}(s + \underline{\delta}, s; \underline{\delta}) = (p_U - 2\lambda)f(\underline{\delta}|\underline{\delta}),$$

where we use the fact that the marginal worker is indifferent between working or being unemployed, so that p_E is equal to p_U at this point.¹⁶ For $p_U > 2\lambda$,

¹⁵This point contrasts with the proof of Proposition 4. The condition (27) holds for all $R - s < \omega$, while in point 3 of Section D.1, the additional restriction $\omega \leq \omega_0$ was necessary to ensure $\partial L/\partial R(R, s; \omega) > 0$.

¹⁶For simplicity, the proof is presented under the assumption that $f(\underline{\delta}|\underline{\delta}) > 0$. It is easy to extend it to other cases, e.g. $f(\underline{\delta}|\underline{\delta}) = 0$ and $f'(\underline{\delta}|\underline{\delta}) > 0$.

the second derivative is positive, so that $s + \underline{\delta}$ is a local *minimum* in R of L . This corresponds to point 2. The value of $L(R(\underline{\delta}), s; \underline{\delta})$ then is larger than $L(s + \underline{\delta}, s; \underline{\delta}) = U(s) - \lambda s$, the value of the Lagrangian when everybody is unemployed. By continuity, the optimum tax scheme puts to work some agents of productivity ω smaller than $\underline{\delta}$, whenever $\underline{\omega} < \underline{\delta}$.

F.2 Example with a uniform distribution of work opportunity costs

When the distribution of work opportunity costs is uniform, the social weight of the employees is

$$p_E(R) = \frac{1}{F(R-s)} \int_{\underline{\delta}}^{R-s} U'[R-\delta] \frac{d\delta}{\bar{\delta}-\underline{\delta}} = \frac{U(R-\underline{\delta}) - U(s)}{R-s-\underline{\delta}}.$$

Integrating and substituting yields

$$M(R; s, \lambda) = 2(R-s) - \underline{\delta} - \frac{U(R-\underline{\delta}) - U(s)}{\lambda}.$$

The function $M(R; s, \lambda)$ is strictly convex in R , $M(s + \underline{\delta}; s, \lambda) = \underline{\delta}$ and $M'_R(s + \underline{\delta}; s, \lambda) = 2 - p_U(s)/\lambda < 1$.

1) Case $\lambda < p_U(s) \leq 2\lambda$. $M(R; s, \lambda)$ is strictly increasing in R and Proposition 4 applies. The convexity of $M(\cdot; s, \lambda)$ implies the concavity of $R(\omega)$. The slope of R at $\omega = \underline{\delta}$ is the inverse of that of M , so it is strictly greater than 1, as represented by the bold curve on Figure 2.

2) Case $p_U(s) > 2\lambda$. Here the slope of M at $s + \underline{\delta}$ is negative (see Figure 3). We study the pointwise maximum of $L(R, s; \omega)$ for $R - s \geq \underline{\delta}$. As it turns out to be increasing in ω , it satisfies the monotonicity condition and is the optimum.

Recall that $L(s + \underline{\delta}, s; \omega) = U(s) - \lambda s$, which we shall note L_{\min} . Now,

$$\frac{\partial L}{\partial R}(R, s; \omega) = \lambda(\omega - M(R; s, \lambda))f(R-s) = \frac{\lambda}{\bar{\delta} - \underline{\delta}} (\omega - M(R; s, \lambda))$$

for $\underline{\delta} \leq R - s \leq \bar{\delta}$ is a concave function of R which becomes negative for large enough R . We consider three cases:

- a. For $\omega > \underline{\delta}$, $\partial L / \partial R(s + \underline{\delta}, s; \omega)$ is positive. There is a single income $R(\omega)$ in $[s + \underline{\delta}, s + \bar{\delta}]$, solution to $\omega = M(R; s, \lambda)$, which maximizes $L(R, s; \omega)$.
- b. For $\omega = \underline{\delta}$, $\partial L / \partial R(s + \underline{\delta}, s; \omega)$ is equal to zero. $\partial^2 L / \partial R^2(s + \underline{\delta}, s; \omega) = (p_U(s) - 1 - \lambda) / (\bar{\delta} - \underline{\delta})$ is positive, so that there is another root $R(\underline{\delta})$, larger than $s + \underline{\delta}$ ($R = s + \underline{\delta}$ is a local minimum of L).

c. Finally consider $\omega < \underline{\delta}$. The function $\partial L/\partial R(\cdot, s; \omega)$ is linear increasing in ω : when ω decreases from $\underline{\delta}$, its smallest root increases, its largest root (a local maximum of L), say $\Delta(\omega)$, decreases, until eventually they both disappear, say at ω_1 , $\omega_1 < \underline{\delta}$. Note that $L(\Delta(\omega), s; \omega)$ is an increasing function of ω . Since $L(s + \underline{\delta}, s; \underline{\delta}) = L_{\min}$, $L(\Delta(\omega_1), s; \omega_1)$ is smaller than L_{\min} . Let ω_2 , $\omega_2 > \omega_1$, be such that $L(\Delta(\omega_2), s; \omega_2)$ is equal to L_{\min} . Define $\omega_0 = \max(\underline{\omega}, \omega_2)$, $R(\omega) - s = \Delta(\omega)$ for $\omega_0 \leq \omega \leq \underline{\delta}$, and $R(\omega) - s = \underline{\delta}$ for ω smaller than ω_0 .

It is easy to check that the $R(\omega)$ function thus defined indeed is the solution of the problem. ■