Nonlinear pricing and exclusion:  
II. Must-stock products 

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November 13, 2015

Abstract

Dominant firms often are unavoidable trading partners. Buyers may consider switching a fraction of their requirements to rival products, but that fraction is highly uncertain in rapidly evolving industries. Nonlinear pricing serves to adjust the competitive pressure placed on rival firms, depending on the joint distribution of the buyer willingness to pay for the rival’s good and the share of contestable demand. Concave price-quantity schedules erect barriers to entry. Convex parts in schedules introduce barriers to expansion. Dominant firms use all-units discounts to create high entry barriers for rival firms with intermediate levels of contestable demand.

JEL codes: L12, L42, D82, D86

Keywords: Inefficient exclusion; disposal costs; quantity rebates; incomplete information.

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1 Introduction

This article is devoted to market situations with a dominant firm and large buyers for whom “losing [the dominant firm] as a supplier [is] not an option.”\(^1\) In such situations, a portion of any individual buyer demand is de facto uncontestable due to high brand loyalty for the dominant firm:

“Competitors may not be able to compete for an individual customer’s entire demand because the dominant undertaking is an unavoidable trading partner at least for part of the demand on the market, for instance because its brand is a “must stock item” preferred by many final consumers or because the capacity constraints on the other suppliers are such that a part of demand can only be provided for by the dominant supplier.”\(^2\)

According to U.S. Department of Justice (2008), there is a general agreement among antitrust practitioners that “when customers must carry a certain percentage of the leading firm’s products, discounts can be structured to induce purchasers to buy all or nearly all needs beyond that uncontestable percentage from the leading firm”, and may under this circumstance have anticompetitive effects. Two considerations in these statements deserve attention: the notion that a portion of demand is uncontestable, and the nature of the considered discounts.

Regarding the contestable share of demand, the above-cited Communication of the European Commission suggests to determine “how much of the customer’s purchase requirements can realistically be switched to a competitor”. This share depends on the rival capacity constraint as well as on client-specific factors that may limit how quickly a buyer can ramp-up products based on rival suppliers and therefore how much of its requirement is contestable at any given point in time. When a dominant firm faces a growing competitive threat from a competitor, it has to form expectations about its clients’ willingness to pay for rival products and about the share of their requirement the clients consider switching to the competitor. For instance, in the Intel case,\(^3\) the Commission sought to determine “what volumes were

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\(^2\)Communication on abusive exclusionary conduct by dominant undertakings (2009/C 45/02).

\(^3\)Decision of the European Commission COMP/C-3 /37.990 of 13 May 2009. Judgment of the General Court (Seventh Chamber, Extended Composition), Case T-286/09, 12 June 2014. Other cases where the dominant firm was an unavoidable trading partner include: LePage’s, Inc. et al. v. 3M Company 324 F.3d 141 (3rd Cir. 2003); Van den Bergh Foods Limited, Court of First Instance (T-65/98) 23 October 2003; British Airways, Court of First Instance (Third Chamber, Case C-95/04 P), 15 March 2007; Michelin, Judgment of the Court of First Instance (Third Chamber, T-203/01), 30 September 2003, discussed by Motta (2009); Tomra, C-549/10 P Judgment of the Court of First Instance (Third Chamber), 19 April 2012. In these cases, the dominant firm used some kind of rebate, e.g., pure quantity discounts, loyalty discounts, exclusivity rebates, market-share discounts, bundled rebates.
actually thought to be at risk during the period examined”. These volumes are inherently un-
certain, and their quantitative assessment is a difficult task for the dominant firm, particularly in industries with rapid pace of innovations where products with high technological content are frequently introduced.\footnote{See sections “V.5. Innovation in x86 CPUs”, “VI.1. The growing competitive threat from AMD”, and VII.4.2.3.1 in \textit{Intel}, op.cit.} We argue that for these reasons the contestable share should be regarded as a random variable rather than as a deterministic figure.

Turning to the nature of the considered rebates, we investigate in this article the exclusionary effects of the simplest pricing policy, namely standard nonlinear pricing, whereby the rebates granted by the dominant firm are based only on the purchased quantity. Pure quantity rebates are ubiquitous in practice, have many pro-competitive justifications, and hence are certainly not anticompetitive by object. Our purpose is to offer static scenarios of exclusion that account for the uncertainty about the contestable share of demand and can explain the various, often highly nonlinear, price schedules observed in practice. In these scenarios, the dominant firm designs a price-quantity schedule before discovering the characteristics of the rival good, namely the unit surplus brought by that good and the contestable share of the demand.

To highlight the role of each dimension of uncertainty, we present our findings first under complete information, second when only the contestable demand is known, and finally when both rival’s characteristics are unknown. Firstly, when there is no uncertainty at all, the quantity of rival good is efficient: the rival firm serves the contestable demand if and only if it brings a higher unit surplus than the incumbent firm. When the rival firm is more efficient, the buyer and the incumbent firm set the unit price of the contestable units below their production cost so as to force the rival firm to give up all of the surplus created by these units.

Secondly, consider the situation where the contestable share of the demand is observed, but the rival unit surplus is not. Under this circumstance, it is no longer possible to perfectly adjust the competitive pressure placed on the rival firm: the buyer and the incumbent firm need to tradeoff efficiency against rent extraction. To optimally solve the tradeoff, the incumbent firm and the buyer resort to a price-quantity schedule that is affine over the contestable part of the demand, with the price of the contestable units being set below cost. If the rival can match that price, it serves all the contestable demand; otherwise it is driven out of the market. Hence a barrier to entry: The quantity distortion occurs at the extensive margin only. The height of the barrier depends on the distribution of the unit rival surplus.

Thirdly, we investigate the general case where both characteristics of the rival good are unobserved. To derive the optimal price-quantity schedule, a natural starting point is the family of affine schedules that are optimal for each given level of the contestable demand (under the corresponding distribution of the rival surplus). For instance, when the two rival’s
characteristics are independent random variables, all the schedules in the family have the same slope because the distribution of the rival surplus and the height of the optimal barrier to entry are the same for any size of the contestable share; as a result, the optimal price schedule is affine over the whole range of potentially contestable units. In general, when the two characteristics are not independent, the derivation of the optimal schedule is more involved and delivers richer economic insights that we now explain.

When rival types with smaller contestable demand tend to generate a higher unit surplus, the efficiency-rent tradeoff leads the buyer and the incumbent firm to place more competitive pressure on smaller competitors. A rival type, therefore, has no incentives to mimic types with smaller contestable demands, as the latter face a higher entry barrier. Hence the same logic as above continues to prevail separately for any size of the contestable demand, with the only change that the average price of contestable units now increases with this size. Consequently, the optimal price-quantity schedule is nonlinear, and actually often concave in this situation. Competition agencies often presume that concave schedules are justified by economies of scales. Yet the average or “effective” price of contestable units, if set below cost, generates inefficient exclusion, namely in the present context total exclusion: if the rival firm cannot match the effective price, it is driven out of the market. Our findings therefore call for applying price-cost comparisons (see Section 7) even to concave schedules.

On the contrary, when the efficiency-rent tradeoff leads the buyer and the incumbent firm to place less competitive pressure on smaller competitors, it is no longer possible to solve the problem separately for each size of the contestable demand, because at the corresponding solution larger types would mimic smaller ones to enjoy less competition. The buyer and the incumbent firm have now to tradeoff efficiency against rent extraction simultaneously across different levels of contestable demand. The “averaging” of the solutions for different contestable demands leads some efficient rival types to be active and yet not to serve their entire contestable demand. These types of rival are partially foreclosed from the market; for them the quantity distortion is at the intensive margin. The optimal schedule prevents them from selling more units, acting as a barrier to expansion rather than to entry. Defendants in antitrust litigation commonly put forward that the alleged abuse did not prevent competitors from gaining significant market shares. Our analysis points out that antitrust enforcers are right to discard this line of defense as a positive market share is not incompatible with (partial) anticompetitive foreclosure.

Finally, we explain why and when “retroactive rebates” emerge in equilibrium. Such rebates, also known as “all-units discounts”, are granted for all the purchased units once a quantity threshold is reached, thus inducing downward discontinuities in price-quantity schedules.\(^5\)

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\(^5\)Retroactive rebates have received much attention from antitrust enforcers. See, among others, Waelbroeck (2005) and Faella (2008).
We show that retroactive rebates are optimal when the rent-efficiency tradeoff implies higher barriers to entry for medium-size contestable demands than for small and large ones. The incumbent firm and the buyer use retroactive rebates to erect particularly high entry barriers in intermediate quantity ranges. These rebates are a powerful instrument to foreclose efficient rival firms from the market, either totally or partially. Their exclusionary effect combines the two scenarios exposed above – barrier to entry and barrier to expansion.

Related literature. To study exclusionary pricing by a dominant firm, Marx and Shaffer (1999) solve a sequential game with variable consumption and perfect information. Our companion paper, Choné and Linnemer (2015a), generalize their framework to allow for one-dimensional uncertainty about the rival surplus, thus providing an intensive version of Aghion and Bolton (1987). As in Martimort and Stole (2009), the price-quantity schedule is used to indirectly control the quantities sold by the rival. The marginal price is set to lower the buyer’s incentives to supply from the rival without inducing excessive consumption of incumbent good – a phenomenon referred to as “buyer opportunism”. Calzolari and Denicolo (2013) investigate market-share discounts and exclusive contracts in a symmetric duopoly setting, allowing, as the articles cited above do, for one dimension of uncertainty. Calzolari and Denicolo (2015) relax the symmetry assumption by assuming that the dominant firm has a competitive (cost or quality) advantage over its rivals.6 By contrast, this and most of the studies cited below are concerned with the exclusion of more efficient rival firms.

In the present article, we introduce uncertainty on the nonlinear part of the utility provided by the rival. We assume that the buyer’s demand is inelastic and that either the rival has capacity constraint or there is satiation point for the rival good. The framework is a convenient way to express the rival’s inability to serve all the buyer demand; for this reason, it has attracted much attention in the recent literature. DeGraba (2013) models the competitor as being capacity constrained as we do.7

In Chao and Tan (2014), all-units discounts are seen as a collusive device to extract surplus from the buyer. The dominant firm uses its price leadership to soften competition, committing to relatively high (above-cost) prices for contestable units to induce less aggressive reactions from the rival. By contrast, in the present article, the dominant firm sets low prices for contestable units at the expense of the rival. Ide, Montero, and Figueroa (2015) examine the sharing of the surplus between the dominant firm and the buyer, an issue we discuss in Section 7. Closer to our study is Feess and Wohlschlegel (2010) who compares all-units rebates to exclusive dealing, showing that all-units discounts can shift rent away from the competitor to the buyer-incumbent coalition.

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6In their setting, uncertainty is about consumers’ preferences. See also Calzolari and Denicolo (2011).
7DeGraba (2013)’s approach is in the tradition of the “naked exclusion” literature pioneered by Rasmusen, Ramseyer, and Wiley (1991) and Segal and Whinston (2000).
The above-cited studies impose the form of the price scheme and assume that the contestable share of the market is known. In the present article, the contestable share as well as the rival unit surplus are both uncertain ante. The two dimensions of uncertainty are necessary to generate the variety of pricing patterns we see in practice and to understand the underlying exclusionary scenarios.

The demand inelasticity simplifies the analysis as in equilibrium the quantities of the two goods sum up to the total demand and each quantity is efficient given the other. This makes the problem formally isomorphic to that of nonlinear pricing by a single-product monopolist facing a population of consumers with two dimensions of heterogeneity. Pricing problems where the number of instruments is lower than the dimension of uncertainty are notoriously difficult. For instance, Laffont, Maskin, and Rochet (1987) consider quadratic preferences and assume that the coefficient of the linear term and the coefficient of the quadratic term in the buyer utility are independent and uniformly distributed. Our demand specification (with a satiation point rather than with a quadratic utility term) allows to accommodate flexible distributions of uncertainty, which is necessary for our purpose. Furthermore, it is well-adapted to examine the concern repeatedly expressed by antitrust enforcers that a dominant firm might leverage the non-contestable part of the demand to drive efficient rivals out of the market.

The article is organized as follows. Section 2 presents the model and the grand problem faced by the dominant firm and the buyer. Section 3 rewrite the grand problem so as to highlight the tradeoff between rent extraction and efficiency, and introduces a relaxed version of the problem. Section 4 studies the cases where the solutions of relaxed and complete problems coincide, deriving conditions under which two-part tariffs and concave schedules are optimal. Section 5 studies cases where the solutions of the two problems do not coincide, characterizes implementable second-best allocations, and explains why erecting barriers to expansion rather than barriers to expansion may be optimal. Section 6 shows that all-units discounts naturally appear under simple distributions of uncertainty. Section 7 discusses the implications of our findings for policy intervention, emphasizing the role of the “as-efficient competitor test” in assessing exclusionary effects of rebates in the presence of must-carry goods. Proofs are relegated to the appendix.

2 Model

A dominant firm $I$ and a competitor $E$ interact with a large buyer $B$. The rival good and the incumbent good are produced at constant marginal costs of $c_E$ and $c_I$, respectively. The buyer gets utility $v_E$ and $v_I$ per consumed unit of each good.
**Buyer’s demand.** The buyer needs to supply a given quantity of an intermediary product, which we normalize to one. When she purchases $q_E$ units of good $E$ and $q_I$ units of good $I$, her utility is $V(q_E, q_I) = v_E q_E + v_I q_I$. The rival firm can contest only a fraction of the market, which we denote by $s_E$. As explained in the introduction, this restriction may reflect rival production capacity or the existence of a satiation point for good $E$ (the buyer cannot absorb more than $s_E$ units of that good). Under both interpretations, the rival firm can address only the fraction $s_E$ of the buyer’s demand: $q_E \leq s_E$. Our purpose is to understand how uncertainty about the contestable share of the market, $s_E$, affects the price-quantity schedules offered by the incumbent firm.

The total surplus $W(q_E, q_I) = V(q_E, q_I) - c_E q_E - c_I q_I$ is equal to $\omega_E q_E + \omega_I q_I$, where $\omega_E = v_E - c_E$ and $\omega_I = v_I - c_I$ denote the surpluses per unit of each good. The efficient quantities maximize $\omega_E q_E + \omega_I q_I$ are given by

$$
(q^{**}_E(s_E, \omega_E), q^{**}_I(s_E, \omega_E)) = \begin{cases} 
(s_E, 1 - s_E) & \text{if } \omega_E > \omega_I \\
(0, 1) & \text{if } \omega_E < \omega_I.
\end{cases}
$$

**Timing and information.** At the first stage of the game, $B$ and $I$ face the same uncertainty as to the characteristics of the rival good. Given their prior distribution of uncertainty, they design a price-quantity schedule to maximize their joint expected surplus. At this stage, the characteristics of the incumbent good, i.e., the constant marginal cost $c_I$ and willingness to pay $v_I$, are common knowledge.\(^9\)

At the second stage of the game, the buyer and the rival observe the realization of $s_E$ and $\omega_E = v_E - c_E$. Knowing the terms of the agreement between $B$ and $I$, $B$ and $E$ agree on a transfer $p_E$ and quantities $q_E$ and $q_I$. This negotiation takes place under complete information and is modeled as a Nash bargaining.

**Purchase decisions.** At the last stage of the game, the buyer and the rival choose quantities to maximize their joint profit

$$
\Pi_{BE}(s_E, \omega_E) = \max_{q_E \leq s_E, q_E+q_I \leq 1} \omega_E q_E + v_I q_I - T(q_I),
$$

with no consideration for the incumbent’s cost or profit. For a given level of $q_I$, the buyer and the rival choose the maximal quantity of good $E$ compatible with the consumption set, namely

$$
q^{**}_E(q_I; s_E) = \min \{1 - q_I, s_E\}.
$$

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\(^{8}\)We assume here that technological constraints or contractual arrangements prevent the buyer from stocking, disposing of, or reselling, unneeded units. Hence any purchased unit must be consumed. This assumption is relaxed in our working paper Choné and Linnemer (2015b).

\(^{9}\)We do not restrict the two players’ ability to share surplus ex ante. For a discussion of the surplus-sharing issue, see Section 7.
Given $q_I$, the above quantity is efficient, i.e., maximizes $W(q_E, q_I)$.

Sharing of the surplus between $B$ and $E$. Without loss of generality, the competitor’s outside option is normalized to zero. As to the buyer, she may source exclusively from the incumbent, in which case her utility, hereafter denoted by $V_B^0$, does not depend on $s_E$ or $\omega_E$. The surplus created by the buyer and the rival firm can thus be written as $\Delta \Pi_{BE} = \Pi_{BE} - V_B^0$. Denoting by $\beta \in (0, 1)$ the competitor’s bargaining power vis-à-vis the buyer, we derive its profit

$$\Pi_E(s_E, \omega_E) = \beta \Delta \Pi_{BE}(s_E, \omega_E),$$

as well as the buyer utility, $\Pi_B = (1 - \beta) \Delta \Pi_{BE} + V_B^0$. If $\beta = 0$, the competitor has no bargaining power and may be seen as a competitive fringe from which the buyer can purchase any quantity at price $c_E$.

The grand problem. Ex ante, the buyer and the incumbent design a price-quantity schedule $T(q_I)$ to maximize their expected joint surplus, equal to the total surplus minus the profit left to the competitor:

$$\mathbb{E}\Pi_{BI} = \mathbb{E}\{W(q_E, q_I) - \Pi_E\},$$

where $q_E$ and $q_I$ are solution to (1), $\Pi_E$ is given by (3), and the expectations are to be taken against the distribution of $(s_E, \omega_E)$. We assume that the cumulative distribution function of $s_E$, denoted by $G$, admits a positive and continuous density function $g$ on $[0, \bar{s}_E]$, with $\bar{s}_E < 1$. The conditional distribution of $\omega_E$ given $s_E$ has a positive density $f(\omega_E|s_E)$ on its support $[\omega_E, \omega_E']$, with $\omega_E < \omega_I < \omega_E'$.

Two properties simplify the problem. First, we have already observed that $q_E$ is efficient given $q_I$, see equation (2). Second, as regards $q_I$, it would be inefficient not to produce and consume units of incumbent good that could bring a positive net surplus of $\omega_I$ each. It is actually both feasible and efficient to satisfy the buyer demand, i.e., to make sure that $q_E^* + q_I = 1$ for all $s_E$ and $\omega_E$. A simple way to do so is to sell all units of incumbent good below the monopoly price $v_I$. In the appendix (see Lemma A.1), we show formally that we may indeed restrict attention, without loss of generality, to price schedules that satisfy $T'(q_I) \leq v_I$ for all $q_I$, and to allocations such that $q_E + q_I = 1$ for all $s_E$ and $\omega_E$. In practice, we hereafter add the constraint $T' \leq v_I$ to the grand problem and accordingly assume that $q_E + q_I = 1$.

3 Rent-efficiency tradeoff

In this section, we compute the expected profit of the rival firm and rewrite the grand problem to highlight the ex ante tradeoff between efficiency and rent extraction. This leads us to introduce the notion of virtual surplus as well as a relaxed version of the problem.
Replacing $q_I$ with $1 - q_E$ in (1), we rewrite the joint profit of the buyer-rival pair as
\[
\Pi_{BE}(s_E, \omega_E) = \max_{q_E \leq s_E} \omega_E q_E + v_I(1 - q_E) - T(1 - q_E).
\]
By (3), the rival’s profit is \(\Pi_E = \beta(\Pi_{BE} - V_0^B)\). We observe that \(\Pi_{BE}\) is the upper bound of a family of affine functions of \(\omega_E\), and hence is convex in \(\omega_E\). It is therefore differentiable almost everywhere. Differentiating (5) with respect to \(\omega_E\) at given \(s_E\) and using the envelope theorem, we get \(\partial \Pi_E(s_E, \omega_E)/\partial \omega_E = \beta q_E(s_E, \omega_E)\), which implies that \(q_E(s_E, \omega_E)\) is nondecreasing in \(\omega_E\).

Virtual surplus. Integrating the rival profit by parts with respect to \(\omega_E\), for any given size of the contestable demand \(s_E\),
\[
\int_{\omega_E}^{\omega_I} \Pi_E(s_E, \omega_E) dF(\omega_E|s_E) = \beta \int_{\omega_E}^{\omega_I} q_E(s_E, \omega_E)[1 - F(\omega_E|s_E)] d\omega_E,
\]
because the rival with the lowest rival unit surplus \(\omega_E < \omega_I\) cannot be active. We can therefore rewrite the buyer-incumbent objective as the expectation of the virtual surplus
\[
W(q_E, 1 - q_E) - \beta q_E \frac{1 - F(\omega_E|s_E)}{f(\omega_E|s_E)} = \omega_I + \left[\omega_E - \omega_I - \beta \frac{1 - F(\omega_E|s_E)}{f(\omega_E|s_E)}\right] q_E.
\]
The virtual surplus balances the efficiency and rent extraction motives: on the one hand, efficiency requires \(q_E\) to be as large as possible when \(\omega_E\) is larger than \(\omega_I\); on the other hand, large values of \(q_E\) are associated with informational rents that are costly for the buyer-incumbent coalition. Because the surplus is linear in \(q_E\), it is natural to define the virtual surplus \(s^v(s_E, \omega_E)\) per unit of good \(E\) as the bracketed term above. We then rewrite the unit surplus as
\[
s^v(s_E, \omega_E) = \omega_E \left[1 - \frac{\beta}{\varepsilon(\omega_E|s_E)}\right] - \omega_I,
\]
with
\[
\varepsilon(\omega_E|s_E) = \frac{\omega_E f(\omega_E|s_E)}{1 - F(\omega_E|s_E)} = -\frac{\partial \ln [1 - F(\omega_E|s_E)]}{\partial \ln \omega_E}.
\]
Hereafter we refer to \(\varepsilon(\omega_E|s_E)\) as the “elasticity of entry”: when \(\omega_E\) rises by 1%, the probability that the rival unit surplus is above \(\omega_E\) decreases by \(\varepsilon(\omega_E|s_E)\)%.

Assumption 1. The elasticity of entry is nondecreasing in \(\omega_E\).

Assumption 1 is true in particular when the hazard rate \(f/(1 - F)\) is nondecreasing in \(\omega_E\), a standard condition in the literature. Lemma A.2 in the appendix explains how the elasticity of entry varies with the size of the contestable demand under various assumptions on the two-dimensional distribution of uncertainty.
Rewriting the grand problem. The buyer and the incumbent design the price-quantity schedule $T$ to maximize their expected joint profit:

$$E\Pi_{BI} = \omega_I + \int s^v(s_E, \omega_E) q_E(s_E, \omega_E) dF(\omega_E|s_E) g(s_E) ds_E,$$

where $q_E(s_E, \omega_E)$ is solution to (5), i.e., is implementable with the schedule $T$. We refer to this problem as the grand problem or the “complete” problem, and denote its solution by $q^c_E$.

Implementability is a well-known issue in one-dimensional situations: if $s_E$ were common knowledge, the quantity $q_E$ would be implementable if and only if it is nondecreasing in $\omega_E$.

In the present context where both $s_E$ and $\omega_E$ are uncertain, implementability requirements do not boil down to this simple monotonicity condition. We characterize implementability in Section 5. Yet in a number of cases, such a characterization is not needed as implementability can be omitted and checked ex post.

Introducing the relaxed problem. As a first step, we omit the implementability constraint and choose $q_E(s_E, \omega_E)$ to maximize the expected virtual surplus, namely the product $s^v(s_E, \omega_E) q_E(s_E, \omega_E)$, separately for each value of $s_E$ and $\omega_E$. We refer to this pointwise maximization problem as the “relaxed problem”, and denote its solution by $q^r_E$. If the solution to the relaxed problem is implementable with a price-quantity schedule $T(q_I)$, i.e., solves the buyer-rival program (5) for such a schedule, it is the solution of the grand problem: $q^c_E = q^r_E$.

Lemma 1. The virtual surplus achieves its maximum at the point $(q^r_E, q^r_I)$ given by

$$q^r_E(s_E, \omega_E) = \begin{cases} 0 & \text{if } \omega_E \leq \omega^r_E(s_E) \\ s_E & \text{otherwise} \end{cases} \quad \text{and} \quad q^r_I(s_E, \omega_E) = 1 - q^r_E(s_E, \omega_E),$$

where $\omega^r_E(s_E) \in (\omega_I, \omega_E)$ is the unique solution to

$$\frac{\omega^r_E(s_E) - \omega_I}{\omega^r_E(s_E)} = \frac{\beta}{\varepsilon(\omega^r_E(s_E)|s_E)}.$$  \(9\)

The threshold $\omega^r_E(s_E)$ increases with the rival’s bargaining power vis-à-vis the buyer and decreases with the elasticity of entry.

We conclude this section by considering the special case where $s_E$ is common knowledge. In this case, the solution of the relaxed problem is implementable with a two-part tariff, and the elasticity of entry is a sufficient statistic that determines the optimal price-quantity schedule.

Proposition 1. When there is no uncertainty about the contestable market share $s_E$, the buyer and the incumbent agree on a two-part tariff, selling any contestable unit at the same constant price $v_I - \omega_E(s_E)$. 

9
Under the above two-part tariff, each unit of incumbent good provides the buyer-rival coalition with a net surplus of $v_I - [v_I - \omega^E_E(s_E)] = \omega^E_E(s_E)$, which only rival types $\omega_E \geq \omega^E_E(s_E)$ are able to match. So only the types above $\omega^E_E(s_E)$ are “allowed” to enter the market. Accordingly, we interpret the threshold $\omega^E_E(s_E)$ as the height of the entry barrier that the buyer and the incumbent firm erect when the size of the contestable demand $s_E$ is known. Rival types lying between $\omega_I$ and $\omega^E_E(s_E)$ are driven out of the market while efficiency ($\omega_E > \omega_I$) would require them to serve the contestable demand: there is inefficient market foreclosure.

**Effective price.** To further illustrate the connection between elasticity of entry and barrier to entry, we introduce the important notion of effective price. By definition, the effective price $p^e(s_E)$ is the average price of the contestable units:

$$
p^e(s_E) = \frac{T(1) - T(1 - s_E)}{s_E}.
$$

(10)

When the buyer has the same willingness to pay for the two goods ($v_E = v_I$) and $s_E$ is the size of the contestable demand, the price $p^e(s_E)$ is the unit price that the rival must match to serve that demand. The effective price, therefore, is negatively related to the competitive pressure placed on the rival. Any given price-quantity schedule $T(q_I)$ can equivalently be represented by the effective price schedule $p^e(1 - q_I)$.

In the case of a two-part tariff, the effective price $p^e(1 - q_I)$ is independent of $q_I$. Under such a price schedule, we know from the above analysis that the rival firm is active if and only if the unit surplus $\omega_E$ brought by good $E$ is larger than $v_I - p^e$. Increasing $p^e$ releases the competitive pressure placed on the rival, lowers the barrier to entry $v_I - p^e$, and increases the probability that the rival serves the contestable demand. The elasticity of entry $\varepsilon$ measures the sensitivity of the probability of entry to the effective price:

$$
\frac{\partial \ln [1 - F(v_I - p^e|s_E)]}{\partial \ln p^e} = \frac{\partial \ln [1 - F(v_I - p^e|s_E)]}{\partial \ln (v_I - p^e)} \cdot \frac{\partial \ln (v_I - p^e)}{\partial \ln p^e} = \frac{\varepsilon(v_I - p^e|s_E)}{v_I - p^e}.
$$

In other words, a one percent increase in the effective price $p^e$ raises the probability of entry by $\varepsilon p^e/(v_I - p^e)$ per percent.

### 4 Barriers to entry

We now reintroduce uncertainty about the contestable share of the market. This section considers distributions of uncertainty for which the solution of the relaxed problem is implementable. Under this circumstance, the buyer and the incumbent firm design a price-quantity
schedule such that a rival type \((s_E, \omega_E)\) faces a barrier to entry of height \(\omega_E^r(s_E)\). We check implementability directly by exhibiting such a schedule. The rival firm either serves all the contestable demand or is driven out of the market.

**Two-part tariffs.** We first assume that \(s_E\) and \(\omega_E\) are independent random variables. Under this circumstance, the observation of \(s_E\) would be of no use for the buyer-incumbent coalition, as it does not affect the height of the optimal barrier to entry: the threshold \(\omega_E^r\) does not depend on \(s_E\). At the solution of the relaxed problem, the rival serves all of the contestable demand when \(\omega_E\) is higher than the constant level of \(\omega_E^r\), and is inactive otherwise. Hereafter we refer to the curve with equation \(\omega_E = \omega_E^r(s_E)\) as the “exclusion line” of the relaxed problem.

Figure 1a shows the exclusion line, which is flat in the present case, and the shape of the corresponding \(q_E^r\)-isoquants. For \(\omega_E > \omega_E^r\), \(q_E^r(s_E, \omega_E) = s_E\) is independent of \(\omega_E\), and therefore the \(q_E^r\)-isoquants are vertical. For \(\omega_E < \omega_E^r\), we have \(q_E^r(s_E, \omega_E) = 0\) irrespective of \(s_E\), see the grey area below the horizontal bold line. In this region, the rival is completely inactive, which we call “total foreclosure”. Total foreclosure is efficient when \(\omega_E < \omega_I\) and inefficient when \(\omega_E > \omega_I\).

Our first proposition shows that the above allocation is implementable with the same two-part tariff as if \(s_E\) were known, and the solutions to the relaxed and complete problems coincide: \(q_E^r = q_E^r\).

**Proposition 2.** When \(s_E\) and \(\omega_E\) are independent, the buyer and the incumbent sell contestable units of incumbent good at price \((v_I - \omega_E^r)\). Efficient rival types, lying between \(\omega_I\) and \(\omega_E^r\), are driven out of the market.

The contestable units are sold below cost because \(v_I - \omega_E^r < v_I - \omega_I = c_I\). The corresponding
price schedule is represented on Figure 1b, where we have assumed that $\omega_E^* < v_I$. It may well be the case, however, that $\omega_E^*$ is larger than $v_I$; in this case, contestable units of incumbent units would be sold at a negative price.\footnote{As explained in Section 7, the price of the non-contestable units is of little significance for outside observers. We therefore represent all the schedules over the range of potentially contestable units, i.e., for $q_I \geq 1 - \bar{s}_E$.}

Finally the constraint $T' \leq v_I$ ensuring $q_E + q_I = 1$ (recall Lemma A.1) is satisfied as the marginal price of all contestable units is $v_I - \omega_E^*$ at the optimum allocation.

**Nondecreasing effective price.** When the elasticity of entry varies with $s_E$, two-part tariffs are no longer optimal: the optimal tariff must exhibit some curvature.

We start with the case where the elasticity increases with $s_E$. According to Lemma A.2, this assumption implies that rival types with larger $s_E$ tend to generate a lower surplus $\omega_E$ and hence are more sensitive to competitive pressure. Under this circumstance, the efficiency-rent tradeoff leads the buyer and the incumbent to place less competitive pressure on larger competitors. Indeed, by Lemma 1, the height of the optimal barrier to entry $\omega_E^*$ decreases with $s_E$. At the solution of the relaxed problem, the rival serves all of the contestable demand when $\omega_E^*$ is higher than the constant level of $\omega_E^*$, and is inactive otherwise.

Figure 2a shows the exclusion line, which is decreasing, and the shape of the $q_E^*$-isoquants: for $\omega_E > \omega_E^*$, $q_E^*(s_E, \omega_E) = s_E$ is independent of $\omega_E$, and therefore the $q_E^*$-isoquants are vertical; for $\omega_E < \omega_E^*$, we have $q_E^*(s_E, \omega_E) = 0$ irrespective of $s_E$, see the grey area below the decreasing bold line. The next proposition shows that this allocation is implementable: the solutions to the relaxed and complete problems, again, coincide: $q_E^* = q_E^r$. 

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Figure 2a: When $\varepsilon(\omega_E|s_E)$ increases with $s_E$, the exclusion line is decreasing  

Figure 2b: Optimal price schedule with $v_I < \omega_E^*(0)$
Proposition 3. When the elasticity of entry $\varepsilon(\omega_E|s_E)$ increases with $s_E$, the buyer and the dominant firm exert less pressure on larger rival types. The optimal effective price, $p^e(1-q_I) = v_I - \omega^r_E(1-q_I)$, increases with $q_I$.

When the buyer and the incumbent set the effective price $p^e(s_E)$ at $v_I - \omega^r_E(s_E)$, the price-quantity schedule is given from (10) by

$$T(q_I) = T(1) - (1 - q_I) [v_I - \omega^r_E(1-q_I)].$$

(11)

Using $q_E = 1 - q_I$, the surplus created with the rival can be written as

$$\omega_E q_E + v_I (1 - q_E) - T(1 - q_E) = v_I - T(1) + [\omega_E - \omega^r_E(q_E)] q_E.$$

(12)

By assumption, $\varepsilon(\omega_E|s_E)$ increases with $s_E$, hence $\omega^r_E(q_E)$ decreases and the joint profit of the BE-coalition increases wherever it is positive. The joint profit is therefore maximum either at $q_E = 0$ or at $q_E = s_E$: the rival makes no sales if $\omega_E < \omega^r_E(s_E)$ and serves all the contestable demand if $\omega_E > \omega^r_E(s_E)$. In other words, the price-quantity schedule (11) implements the solution of the relaxed problem.

Figure 2b shows the shape of the optimal price-quantity schedule. Geometrically, the effective price is the slope of a chord drawn between the points $(1-q_I, T(1-q_I))$ and $(1, T(1))$. The chords, represented by the dotted lines, are indeed steeper as the number of concerned units rises: they are upwards-sloping for large values of $q_E = 1 - q_I$, approximately flat for intermediate values, and decreasing for low values. The latter property happens here because we have assumed $\omega^r_E(0) > v_I$, implying that the effective price $p^e(1-q_I)$ is negative for low values of $1 - q_I$ and hence that the buyer has strong incentives to supply exclusively from the dominant firm when the contestable market is small.

Corollary 1. When the elasticity of entry $\varepsilon(\omega_E|s_E)$ increases with $s_E$, the price-quantity schedule is concave in the neighborhood of $q_I = 1$. It is globally concave if $\omega^r_E$ is concave or moderately convex in $s_E$.

Concave price schedules are commonly seen in practice. They occur in particular when the seller offers so-called “incremental” rebates, i.e., rebates that apply to units purchased in excess of a threshold. Competition agencies often presume that concave schedules are justified by economies of scale. Yet any concave price schedule such that the associated effective price $p^e$ is below $c_I$ can be rationalized within our anticompetitive scenario. Indeed, for such a schedule $T$, the effective price is increasing because $(p^e)'(x) = [T'(1-x) - p^e(x)] / x \geq 0$. It follows that the function $\eta(s_E)$ defined by

$$\frac{\beta}{\eta(s_E)} = \frac{c_I - p^e(s_E)}{v_I - p^e(s_E)} \geq 0$$

13
increases with \( s_E \). Replacing \( \omega_E^r(s_E) \) with \( v_I - p^e(s_E) \) in (9), we find that the price schedule \( T \) is optimal when \( \omega_E \) conditionally on \( s_E \) follows the Pareto distribution \( F(\omega_E|s_E) = 1 - (\omega_E/\omega_E^r)^{-\eta(s_E)} \) and accordingly the elasticity of entry is \( \varepsilon(\omega_E|s_E) = \eta(s_E) \).

Two-part tariffs, concave schedules, and more generally nondecreasing effective prices, act as barriers to entry and generate complete market foreclosure. In particular, a rival can never mimic types with smaller contestable demand as those types face lower effective prices; hence the only available alternative to serving all the contestable demand is to be completely inactive. By contrast, the next sections consider distributions of uncertainty for which the quantity sold by the rival firm is distorted at the intensive margin.

## 5 Barriers to expansion

We now turn to situations where the solution of the relaxed problem is not implementable. Suppose that the exclusion line of the relaxed problem \( \omega_E = \omega_E^r(s_E) \) is non-monotonic, see the dashed line on Figure 3. In this case, the solution to the relaxed problem, namely zero below the line and \( s_E \) above, is not incentive compatible, hence not implementable. Indeed, the rival of type \( B = (\omega_B^E, s_B^E) \) would be inactive and hence would earn zero profit, while type \( A = (\omega_A^E, s_A^E) \), with \( 0 < s_A^E < s_B^E \) and \( \omega_A^E = \omega_B^E \), would serve all of the contestable demand. It follows that type \( B \) would better off mimicking type \( A \) and selling \( s_A^E \) units rather than being inactive.

![Figure 3: The relaxed solution is not implementable](image)

We therefore need to characterize the set of implementable quantity allocations. After providing such a characterization, we explain how to construct the optimal allocation. The main economic intuition is that configurations like the one represented on Figure 3 give rise to
quantity distortions at the intensive margin, i.e., some rival types face barriers to expansion. We then derive an appropriate first-order condition for these distortions.

**Implementable allocations.** As explained in Section 3, a quantity function $q_E(s_E, \omega_E)$ is implementable with a price-quantity schedule if and only if there exists a function $T(q_I)$ such that $q_E(s_E, \omega_E)$ is solution to (5) for all $(s_E, \omega_E)$.

To characterize such allocations, we observe that the buyer and the rival firm hit the constraint $q_E \leq s_E$ when the rival unit surplus $\omega_E$ is large. Indeed, as explained below (5), $q_E(s_E, \omega_E)$ is nondecreasing in $\omega_E$. Hence the existence, for any $s_E > 0$, of a threshold $\hat{\omega}_E(s_E)$ such that the buyer supplies all the contestable units from the rival, $q_E(s_E, \omega_E) = s_E$, if and only if $\omega_E \geq \hat{\omega}_E(s_E)$. Hereafter, we refer to the curve with equation $\omega_E = \hat{\omega}_E(s_E)$ in the $(s_E, \omega_E)$-space as the exclusion line associated with the quantity function $q_E(s_E, \omega_E)$.

For types above the exclusion line, the rival firm serves all the contestable demand: $q_E(s_E, \omega_E) = s_E$. For types below that line, the rival serves less than the contestable demand and hence the constraint $q_E \leq s_E$ in problem (5) is slack; it follows that the quantity $q_E(s_E, \omega_E)$ is independent of $s_E$ in this region. The next result states that any exclusion line gives rise to a unique implementable allocation.

**Lemma 2.** A quantity function $q_E(\cdot, \cdot)$ is implementable if and only if there exists an exclusion line $\omega_E = \hat{\omega}_E(s_E)$ such that

$$q_E(s_E, \omega_E) = \begin{cases} \min \{ x \leq s_E \mid \hat{\omega}_E(y) \geq \omega_E \text{ for all } y \in [x, s_E]\} & \text{if } \hat{\omega}_E(s_E) > \omega_E, \\ s_E & \text{if } \hat{\omega}_E(s_E) \leq \omega_E. \end{cases} \quad (13)$$

The intuition behind the above characterization is best understood by considering the isoquants of an implementable quantity function $q_E(s_E, \omega_E)$. Such isoquants are essentially of three types, the first two of which have been observed earlier in this study.

The first type of $q_E$-isoquants” consists of the two-dimensional set of rival types that are completely foreclosed from the market, i.e., for which $q_E(s_E, \omega_E) = 0$. This set is represented by the dark-shaded area on Figures 1a, 2a and 4.

The second type of $q_E$-isoquants are vertical lines above non-increasing portions of the exclusion line. We have encountered such isoquants in Section 4, recall Figures 1a and 2a. They also appear on the more general Figure 4, with the equations $q_E(s_E, \omega_E) = s_E^3$ and $q_E(s_E, \omega_E) = s_E^{1.5}$. They are associated with the corner solution $q_E = s_E$ to the maximization problem (5). When the constraint $q_E \leq s_E$ is binding, $B$ and $E$ would like to trade more units than $s_E$, but are prevented from doing so by the limited size of the contestable demand.

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12 We adopt the convention that $\hat{\omega}_E(s_E) = \infty$ when $q_E(s_E, \omega_E) < s_E$ for all $\omega_E$.

13 Another form of two-dimensional bunching is mentioned at the end of this section.
The third type of isoquants deserves more attention as it appears in the analysis for the first time. Isoquants of that type consist of a horizontal segment and a vertical half-line intersecting on an increasing portion of the exclusion line. For instance, rival types A and B on Figure 4 belong to the same isoquant, namely $q_E(s_E, \omega_E) = s_E^2$. Yet these two types are in different situations: A serves all of the contestable demand, while B is partially foreclosed from the market. More precisely, rival type B sells $s_E^2$ units of rival good, which is positive but lower than the size of its contestable market. Thus, horizontal parts are associated with interior solutions of the maximization problem (5): the rival firm is active ($q_E > 0$) but serves less than the contestable demand ($q_E < s_E$). We interpret such configurations as partial foreclosure:

$$0 < q_E(s_E, \omega_E) < s_E. \quad (14)$$

The partial foreclosure region is represented by the light-shaded area below the exclusion line $\omega_E = \hat{\omega}_E(s_E)$ on Figure 4. In this region, the quantity $q_E(s_E, \omega_E)$ does not depend on $s_E$. The quantity is continuous when crossing increasing parts of the exclusion line $\omega_E = \hat{\omega}_E(s_E)$, but discontinuous when crossing decreasing part of that line.\(^{14}\)

**Properties of the optimal allocation.** The buyer and the incumbent firm design the optimal price-quantity schedule $T$ while facing uncertainty about the characteristics of the rival product. According to Lemma 2, they can equivalently look for the exclusion line $\omega_E = \hat{\omega}_E(s_E)$.

\(^{14}\)For instance, the quantity function $q_E(s_E, \omega_E)$ is discontinuous at point C. For this type, the buyer and the rival firm are indifferent between the quantities $q_E = s_E^2$ and $q_E = s_E^3$. 

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below which the rival firm serves less than the contestable demand. The main primitive of the
problem are the distribution of \((s_E, \omega_E)\) and the corresponding exclusion line of the relaxed
problem \(\omega_E = \omega^*_E(s_E)\) given by (9). The following optimality properties, which exploit the
shape of implementable allocations, link the optimal exclusion line to these primitives.

**Lemma 3.** The exclusion line \(\omega_E = \omega^*_E(s_E)\) of the complete problem satisfies the following
properties:

1. For all \(s_E\), \(\omega^*_E(s_E) \geq \omega^r_E(s_E)\), and the equality holds for \(s_E = \bar s_E\);
2. If \(\omega^*_E\) is nondecreasing in a neighborhood of \(s_E\), then \(\omega^*_E\) is also nondecreasing in that
neighborhood;
3. If \(\omega^*_E\) is decreasing in a neighborhood of \(s_E\), either \(\omega^*_E\) is nondecreasing or \(\omega^*_E = \omega^r_E\) in
that neighborhood.

**Decreasing elasticity of entry.** We now address the simplest case where the solution of
the relaxed problem is not implementable. When the elasticity of entry \(\varepsilon(\omega_E|s_E)\) decreases
with \(s_E\), the efficiency-rent tradeoff requires a higher entry barrier for larger competitors:
the exclusion line of the relaxed problem \(\omega_E = \omega^*_E(s_E)\) is monotonically increasing, see the
increasing dashed line on Figure 5a. If \(q_E\) were equal to \(s_E\) above this threshold and zero
below, the quantity purchased from the rival would locally decrease with \(s_E\), which is not
incentive compatible.

We know from property 2 of Lemma 3 that the optimal allocation is characterized by a
nondecreasing exclusion line \(\omega_E = \omega^*_E(s_E)\), see the bold line on Figure 5a. For any \(q_I\) in
\((1 - \bar s_E, 1)\), consider a rival type \((s_E, \omega_E)\) such that \(s_E > 1 - q_I\) and \(\omega_E = \omega^*_E(1 - q_I)\). Such
a type belongs to the partial foreclosure region, see the light-shaded area on Figure 5a. For
this type, the solution to the buyer-rival coalition’s problem (5) is \(q_E = 1 - q_I\), meaning that
the constraints \(q_E \geq 0\) and \(q_E \leq s_E\) are slack. In other words, the solution of (5) is interior
for that type, implying that the buyer and rival’s joint objective is locally concave around \(q_E\).
It follows that \(T\) is locally convex around any \(q_I\), and hence globally convex, as represented
on Figure 5b. As a result, the schedule is almost everywhere differentiable, and the first-order
conditions of (5) yield the marginal price \(T'(q_I) = v_I - \omega^*_E(1 - q_I)\) for almost all value of \(q_I\).

**Proposition 4.** When \(\varepsilon(\omega_E|s_E)\) decreases with \(s_E\), the optimal price schedule is convex. The
equilibrium outcome exhibits inefficient exclusion, in the form of both total and partial fore-
closure.

We now explain heuristically how to construct the exclusion line for the solution of the
complete problem, \(\omega_E = \omega^*_E(s_E)\). We start with the natural candidate \(\omega^*_E = \omega^r_E\). Suppose
accompanying that the bold line and the dotted line coincide on Figure 5a; in this case, the segment $[AB]$ is reduced to point $B$. This means that rival types located on $[BC]$ all serve quantity $q^E(s_E, \omega_E) = s^B_E$. Now consider the interval $[s^B_E - \varepsilon, s^B_E]$ for a small $\varepsilon > 0$, and raise $\omega^E(s_E)$ from $\omega^r_E(s_E)$ to the constant level $\omega^r_E(s^B_E)$ on that small interval. This change affects the quantity sold by rival types with $\omega_E$ close to $\omega^r_E(s^B_E)$: (i) for types located on the segment $[BC]$, $q^E(s_E, \omega_E)$ is lowered from $s^B_E$ to $s^B_E - \varepsilon$, hence a first-order increase in the expected virtual surplus as $s^v < 0$ in this region; (ii) for types with $s_E$ in the small interval $[s^B_E - \varepsilon, s^B_E]$, the quantity is lowered by a first-order amount, hence a second-order decrease of the expected virtual surplus as $s^v > 0$ in this region. The change, therefore, increases the expected profit of the buyer-incumbent coalition. The optimum is reached when the loss from lowering the quantity sold by types with $s^v > 0$ (for instance rival types in $[AB]$) exactly compensates the gain from lowering the quantity sold by types with $s^v < 0$ (for instance types in $[BC]$).

It follows from the above analysis that at the optimum the unit virtual surplus is zero in expectation over rival types in horizontal intervals where $q^E$ is constant, see the appendix for a formal proof. This yields the unique value for $(\omega^E_E)^{-1}(\omega_E)$ given by

$$\int_{(\omega^E_E)^{-1}(\omega_E)}^{s_E} s^v(s, \omega_E) f(\omega_E | s) g(s) \, ds = 0.$$  

(15)

Along the horizontal segment $[AC]$ on Figure 5a, all rival types sell the same quantity as type $A$. The unit virtual surplus is positive for low values of $s_E$ (i.e., between points $A$ and $B$), is zero at point $B$ where $\omega_E = \omega^r_E(s_E)$, and is negative for high values of $s_E$ (i.e., between points $B$ and $C$).

The above procedure, which allows to maximize the expected virtual surplus for each $\omega_E$,
yields the solution to the complete problem provided that the solution \((\omega_E^{-1})'(\omega_E)\) of equation (15) is nondecreasing. Indeed, when the solution of (15) is non-monotonic, it does not determine a unique value of \(\omega_E(s_E)\) for each \(s_E\), see Figure 9a in the appendix. In this case, the optimal exclusion line \(\omega_E = \omega_E^c(s_E)\) admits an upward jump.\(^{15}\) The optimal allocation exhibits two-dimensional bunching: rival types in a two-dimensional set (see the light-shaded area on Figure 9b) choose the same quantity \(\hat{s}\). The quantity \(\hat{s}\) is found by writing that the expected unit surplus on that set is zero.

6 Retroactive rebates

In this section, we first revisit the two exclusionary scenarios seen so far, namely barrier to entry and barrier to expansion. Next, we combine these two scenarios to understand why and when “retroactive rebates” emerge in equilibrium. Specifically, we show that simple non-monotonic variations of the entry elasticity are sufficient to create highly nonlinear price schedules. Then we show that retroactive rebates naturally emerge as a limiting case of such non-monotonic configurations when the distribution of uncertainty has mass points.

**Barrier to entry and barrier to expansion.** In Section 4, we have considered implementable allocations such that the exclusion line \(\omega_E = \hat{\omega}_E(s_E)\) is decreasing. For such allocations, the quantity of good \(E\) chosen by the buyer-rival-coalition, i.e., the solution of the maximization problem (5), is \(s_E\) above the exclusion line and zero below. For types located exactly on the exclusion line, the buyer and the rival are indifferent between \(q_E = 0\) and \(q_E = s_E\), which yields \(v_I(1 - s_E) + \omega_E s_E - T(1 - s_E) = v_I - T(1)\) or equivalently

\[
p_E(s_E) = \frac{T(1) - T(1 - s_E)}{s_E} = v_I - \hat{\omega}_E(s_E).
\]

Hence the choice of \(q_E(s_E, \omega_E)\) is governed by the effective price: rival types serve all the contestable demand when they bring a unit surplus \(\omega_E\) larger than \(v_I - p_E(s_E)\), and are inactive otherwise. The price schedule therefore erects a barrier to entry whose height is negatively related to the effective price. If the rival can match that price, it serves the contestable demand; otherwise, it is driven out of the market. According to Proposition 3, this pattern appears at the optimum when the elasticity of entry is monotonically increasing in the size of the contestable demand. In this case, the solutions to the relaxed and complete problems coincide: \(\omega_E^c = \omega_E^r\) and \(q_E^c = q_E^r\).

\(^{15}\)Because the marginal price is almost everywhere given by \(T'(q_I) = v_I - \omega_E^c(1 - q_I)\), upward discontinuities in the exclusion line \(\omega_E = \omega_E^c(s_E)\) translate into convex kinks in the price-quantity schedule \(T\). See the appendix for details.
In Section 5, we have considered implementable allocations such that the exclusion line \( \omega_E = \hat{\omega}_E(s_E) \) is increasing. For such allocations, the quantity of good \( E \) chosen by the buyer-rival-coalition satisfies \( 0 < q_E(s_E, \omega_E) < s_E \) in a non-degenerated region below the exclusion line, see the light-shaded area on Figure 5a. In this “partial foreclosure” region, the solution of (5) is interior: the buyer and the rival compare the surplus created by an extra unit of rival good, \( \omega_E \), with the net surplus foregone by consuming one unit less of incumbent good, \( v_I - T'(1 - q_E) \). The first-order condition of (5) yields, at points where \( T \) is differentiable,

\[
T'(1 - q_E) = v_I - \hat{\omega}_E(q_E).
\]

Thus, in this region, the quantity of rival good, \( q_E(s_E, \omega_E) \), is driven by the marginal, rather than effective, price: the schedule \( T \) prevents the rival from selling more units, acting as a barrier to expansion rather than to entry. According to Proposition 4, this pattern occurs when the elasticity of entry is monotonically decreasing in the size of the contestable demand. In this case, the exclusion line of the complete problem is above that of the relaxed problem: \( \omega^c_E > \omega^r_E \) and \( q^c_E \neq q^r_E \).

**U-shaped elasticity of entry.** When the elasticity of entry \( \varepsilon(\omega_E|s_E) \) is first decreasing then increasing in the size of the contestable market \( s_E \), competitors with intermediate size are less sensitive to competitive pressure than competitors with small or large size. Under this circumstance, the efficiency-rent tradeoff leads the buyer and the incumbent to place strong competitive pressure on competitors with intermediate size and less pressure on small or large competitors: the exclusion line of the relaxed problem \( \omega_E = \omega^r_E(s_E) \) is hump-shaped as shown on Figure 6a.

**Proposition 5.** Suppose that the elasticity of entry \( \varepsilon \) is a U-shaped function of \( s_E \) and that \( \omega^r_E(s_E) \) is smaller than \( v_I \) except for intermediate values of \( s_E \).

Then the price schedule \( T(q_I) \) changes curvature, being concave for small quantities and convex for large ones. The marginal price is negative in an intermediate quantity range.

For high values of \( s_E \), the exclusion line of the relaxed problem \( \omega_E = \omega^r_E(s_E) \) is decreasing. According to property 3 of Lemma 3, the exclusion line of the complete problem is either increasing or coincides with that of the relaxed problem. In the present context, there is no objection to sticking to the solution of the relaxed problem, i.e., to setting \( \omega^c_E = \omega^r_E \). This choice indeed guarantees that the rival firm serves all the contestable demand when \( \omega_E > \omega^r_E(s_E) \) and zero otherwise, and has no impact on rival types with higher contestable demand.\(^{16}\)

\(^{16}\)Figure 8b shows a case where the strict inequality \( \omega^c_E > \omega^r_E \) holds along a decreasing part of the exclusion line of the relaxed problem. This is because the chosen quantity affects rival types with higher \( s_E \).
For low values of \( s_E \), the exclusion line of the relaxed problem \( \omega_E = \omega^r_E(s_E) \) is decreasing. In the light-shaded area below the solid curve, the quantity purchased from the competitor does not depend on the size of the contestable market. For instance, for any type on the horizontal segment \((A_1A_3)\), the rival sells \( s_1^E \) units. On this interval, the unit virtual surplus is positive for low values of \( s_E \), is zero at the point where \( \omega_E = \omega^r_E(s_E) \), and is negative for high values of \( s_E \). To find the exclusion line of the complete problem, we write that the expected unit surplus is zero on such intervals. For this purpose, we denote by \( \sigma(\omega_E) \) the highest root of the equation \( \omega^r_E(s_E) = \omega_E \), so the decreasing part of the exclusion line of the relaxed problem has equation \( s_E = \sigma(\omega_E) \). In this context, the zero expected surplus condition takes a slightly different form than (15), namely:

\[
\int_{\sigma(\omega_E)}^{(\omega_E)^{-1}(\omega_E)} s^v(s, \omega_E)f(\omega_E|s)g(s) \, ds = 0. \tag{18}
\]

This condition yields the exclusion line of the complete problem \( \omega^c_E \) provided that it defines an increasing function \((\omega^c_E)^{-1}(\omega_E)\). We refer to the end of Section 5 and to the appendix for the treatment of the general case.

Figure 6b shows the shape of the corresponding optimal price schedule. Consider first the case where the dominant firm serves a large part of the demand. Between \( A_0 \) and \( A_2 \), the exclusion line \( \omega_E = \omega^c_E(s_E) \) is increasing and the price-quantity schedule \( T \) is convex. As explained when showing equation (17), the solution of problem (5) is interior for types in the partial foreclosure region, and the first-order condition \( T'(1 - s_E) = v_I - \omega^c_E(s_E) \) holds for almost every \( s_E \). The marginal price is negative at \( A_2 \) because \( \omega^c_E > v_I \) at this point. To recover the marginal price below \( A_2 \), for instance at point \( A_3 \), we express that for such a type
the buyer and the rival pair are indifferent between trading $s_E^1$ or $s_E^2$ units of good $E$. We are then able to show that the schedule changes curvature at point $A_2$, which therefore is an inflection point: the schedule is concave below $A_2$ and convex above.

The price-quantity schedule described in Proposition 5 leads to inefficient partial exclusion for all types in the light-shaded area shown on Figure 6a. Exclusion is complete for types in the dark-shaded area, and is inefficient for types with $\omega_E > \omega_I$. The decreasing part of the schedule acts as a barrier to expansion, giving the buyer a strong incentive to supply much of her purchase requirements from the incumbent (i.e., to pick a point near $A_1$ in the schedule, see Figure 6b). We now turn to a particular form of rebates that exacerbates this exclusionary effect.

**Figure 7a: Exclusion line under a retroactive rebate**

**Figure 7b: Price schedule with retroactive rebate**

**Retroactive rebates.** Retroactive rebates, also known as “all-units discounts”, have attracted much attention from antitrust agencies. They apply to all the purchased units provided that the buyer reaches a certain quantity threshold. Hence, contrary to incremental rebates, they induce downward discontinuities in price-quantity schedules. For instance, under the schedule shown on Figure 7b, potentially contestable units below the threshold $q_I = 1 - \tilde{s}$ are sold at unit price $t_1$, while above that threshold, all potentially contestable units are sold at price $t_2 < t_1$. The price schedule, therefore, admits a downward price discontinuity at $q_I = 1 - \tilde{s}$. We denote by $\Delta > 0$ the absolute value of this discontinuity.

We first consider the buyer-rival problem assuming that the retroactive rebate of Figure 7b is offered. Figure 7a shows the corresponding allocation. When the contestable demand is small, $s_E < \tilde{s}$, the rival (e.g., type $B_1$) sells all contestable units if it can match the effective price $t_2$, i.e., if the rival unit surplus $\omega_E$ is greater than $v_I - t_2$; otherwise it is driven
out of the market. Small rivals, therefore, are not affected by the specific form of the rebate; they behave as under a two-part tariff with slope $t_2$. This part of the allocation follows the same pattern as in Figure 1a and 1b.

By contrast, when the contestable demand is large, $s_E > \tilde{s}$, the buyer-rival coalition faces more complex incentives: either the rival serves all the contestable demand, or sells $\tilde{s}$ units (light-shaded area), or is inactive. More precisely, writing that the buyer-rival surplus $\Pi_{BE} = \omega_E q_E + v_I (1 - q_E) - T (1 - q_E)$ is equal for $q_E = \tilde{s}$ and $q_E = s_E$, we get the equation of the decreasing part of the exclusion line (the upper boundary of the light-shaded area):

$$(\omega_E + t_1 - v_I) (s_E - \tilde{s}) = \Delta.$$  \hspace{1cm} (19)

For a type $(s_E, \omega_E)$ that satisfies (19), the net surplus generated by $s_E - \tilde{s}$ units of good $E$ in excess of $\tilde{s}$ exactly offset the foregone rebate $\Delta$. Rivals of types $B_2$ and $B_3$, as well as all types in the light-shaded area, are “trapped” at the threshold $\tilde{s}$ of the retroactive rebate, while rival of type $B_4$ brings a sufficiently high unit surplus to serve all of its contestable demand.\footnote{The configuration of Figures 7a and 7b assumes that for $(s_E, \omega_E) = (\bar{s}_E, v_I - t_2)$ the coalition surplus $\Pi_{BE}$ is higher for $q_E = \tilde{s}$ than for $q_E = \bar{s}_E$, which occurs when $(t_1 - t_2)(\bar{s}_E - \tilde{s}) < \Delta.$}

**Proposition 6.** For any given threshold $q_I = 1 - \tilde{s}$, there exists a distribution of uncertainty with a positive mass at $s_E = \tilde{s}$ under which a retroactive rebate at this threshold is optimal.

Proposition 6 states that the retroactive rebate of Figure 7b is indeed optimal for certain distributions of uncertainly. Suppose that the distribution $f(\omega_E | s_E)$ is such that the exclusion line of the relaxed problem $\omega_E = \omega_E^r(s_E)$ is $v_I - t_2$ for $s_E < \tilde{s}$ and satisfies (19) for $s_E > \tilde{s}$, as represented on Figure 7a. By (9), the elasticity of entry is zero at $\tilde{s}$ and is very low for $s_E$ slightly above $\tilde{s}$. Then the virtual surplus is negative below $v_I - t_2$, hence the optimal quantity is $q_E = 0$ in this region. Moreover, the virtual surplus is negative in the interior of the light-shaded area. To make sure that the expectation of the virtual surplus in this area is zero, we construct in the appendix a distribution of uncertainty that places a positive mass on the left boundary of that area (included in the vertical line $s_E = \tilde{s}$). We check that the allocation represented on Figure 7a is indeed optimal under this distribution.

7 Discussion

We have examined the exclusionary effects of nonlinear pricing by a dominant firm when (i) only a fraction of a buyer’s demand is potentially contestable; (ii) the dominant firm and the buyer have a negotiation opportunity before the rival unit surplus and the contestable demand are known. In our anticompetitive scenario, the contestable units of incumbent good
are offered at a price that is below the marginal cost, and the surplus extracted from the rival firm is shared between the dominant firm and the buyer.

We now discuss the implications of our findings for the antitrust standard applicable to nonlinear pricing by dominant firms and consider possible regulatory remedies to limit the associated exclusionary effect. As a preliminary observation, we recall from Choné and Linneker (2015a) that nonlinear schedules lose exclusionary power when buyers can dispose of unneeded units or resell them on a secondary market. Higher disposal costs are associated with a higher surplus for the dominant firm and the buyer. Thus, these two parties have an incentive to artificially increase disposal costs, for instance by agreeing on provisions that allow the dominant firm to monitor the use of purchased units by the buyer and to prevent her from reselling unused units on a secondary market. Antitrust authorities should pay close attention to any contracting provisions that help increase disposal costs.

We devote the following discussion to the legal treatment of price-quantity schedules, focusing particularly on two issues: the pricing of contestable units below cost and the sharing of the surplus between the dominant firm and the buyer. Below-cost pricing for contestable units generates a gain for the buyer—through an improved bargaining position vis-à-vis the rival—but losses for the dominant firm; at the same time, the buyer and the dominant firm share the surplus created by the non-contestable units.

**Surplus-sharing problem.** In this article, we have assumed that the dominant firm and the buyer have unrestricted ability to share surplus though ex ante transfers. This assumption is unimportant when the dominant firm has little bargaining power vis-à-vis the buyer because in this case much of the rent extracted from the rival accrues to the buyer. When on the contrary the dominant firm has all the bargaining power, it enters in the agreement with the buyer only if it can pocket the extracted rent—on top of the monopoly profit on the non-contestable units. If ex ante transfers are prohibited, as Ide, Montero, and Figueroa (2015) assume, this condition requires non-contestable units to be offered at a price that is *above* their value for the buyer. But if the contract can be terminated without any breach penalty, the buyer might ex post refuse to pay such a high price. Indeed, having supplied contestable units from the rival firm at favorable terms (thanks to the competitive pressure placed by the dominant firm), the buyer would be better off not purchasing the non-contestable units at all from the incumbent firm. Yet, the rival firm would anticipate such an opportunistic behavior on the buyer’s side, and hence disregard the dominant firm’s rebates and resist rent extraction. The anticompetitive mechanism would be undermined.

The notion that the dominant firm is an “unavoidable trading partner”, however, is at odd with the opportunistic scenario considered above. Unavoidability reflects the fact that in practice, contract termination is a risk that the buyer cannot afford to take, precisely because
the dominant firm is an unavoidable supplier. In a dynamic setting, not supplying from the dominant firm would most likely be suboptimal for the buyer as it would harm long-term business relationships with that firm. Recall that in the foreseeable future the buyer depends on the dominant firm for the largest part of its supply because by assumption only a modest fraction of the buyer’s demand is potentially contestable.\textsuperscript{18}

Furthermore, as regards policy intervention, imposing “easy terminability” is not equivalent to ruling out lump-sum transfers. Easy terminability does not prevent hidden up-front transfers from the buyer to the dominant firm. A multiplicity of on-going contracts give the parties many opportunities for such transfers, which competition agencies cannot monitor. As a general rule, regulators are ill-equipped to assess the sharing of the surplus between commercial partners because such an assessment would require knowing their valuations for any traded product, their outside options, and their bargaining power. This is why policy discussions essentially concentrate on the other side of the mechanism, namely below-cost pricing.

**As-efficient competitor test under uncertain contestable demand.** The as-efficient competitor test asks whether a hypothetical rival firm, having the same production costs and selling a product of similar quality as the dominant firm, could profitably match the price offered by that firm. The first instance of the test has been introduced by Areeda and Turner (1975) for predatory pricing. In this context, the underlying theory of harm involves short-term profit-sacrifice outweighed by long-run monopoly profits. The legal standard set in the United States by the *Brooke Group* Judgment of the Supreme Court is very high as it requires the plaintiff to prove that the prices were below an appropriate measure of defendant’s costs in the short term and that the defendant had a “dangerous probability of recouping” its investment in below-cost prices.\textsuperscript{19} The standard is not as high in Europe where proof of recoupment of losses is not required to determine predation.\textsuperscript{20}

Under the static theory of harm developed in this study, surplus sharing –instead of intertemporal recoupment in predation cases– determines the profitability of the agreement for the buyer and the dominant firm. However, as mentioned above, assessing surplus sharing is virtually impossible for antitrust enforcers due to lack of information. Our findings show that rebates have a high exclusionary potential when the customer must carry some percentage of the leading firm’s products. They suggest that under such circumstances the exclusionary analysis should be key in the legal treatment of nonlinear pricing, as also advocated by Fu-magalli and Motta (2015). For this purpose, it is natural to modify the Areeda-Turner test

\textsuperscript{18}Typically, the upper bound of the distribution of $s_E$, $\bar{s}_E$, is relatively small, around 10\% or 20\%.

\textsuperscript{19}*Brooke Group v. Brown & Williamson Tobacco Corp.*, 509 U.S. 209, 226 (1993). Since then, according to Hovenkamp (2014), few plaintiffs have won a predatory case, and the incidence of classical predatory pricing claims has declined dramatically.

\textsuperscript{20}*France Telecom*, European Court of Justice, 2 April 2009, Case C-202/07 P.
and check whether the price of contestable units covers their cost. This extended test has been implemented by the European Commission in Intel for the first time and has generally received support from economists, see e.g. Shapiro and Hayes (2006).

Yet the difficulty to define contestable units has been pointed out by many observers, and used to discard the as-efficient competitor test. For instance, Wright (2013) believes that the test would be hard to administer, emphasizing “the difficult question of how to define contestable units” in practice. He concludes that “a court should not focus on whether the defendant’s discounting has resulted in prices below cost.” This view is shared by the General Court of the European Union whose Intel judgment disregarded the as-efficient competitor analysis carried out by the Commission, sticking to the legal view that “exclusivity rebates” by a dominant firm are banned per se.\(^\text{21}\)

We have argued in this article that the contestable share of demand should not be seen as a fixed number but rather as a random variable. Accordingly, instead of looking for a precise number, it is sensible to consider a range of values for the contestable demand. One way of proceeding along these lines is to use the “minimum required share”, \(s_{\text{mr}}\), defined by the European Commission as the lowest value of \(s\) such that the effective price \([T(1) - T(1 - s)]/s\) is above \(c_I\). In words, \(s_{\text{mr}}\) is the minimum number of units that an as-efficient competitor must sell to overcome the rebates implemented by the dominant firm. Comparing the values of \(s_{\text{mr}}\) and \(\bar{s}_E\) yields a concrete assessment of exclusionary effects from an ex ante perspective. For instance, suppose that \(s_{\text{mr}}\) is found to be clearly above \(\bar{s}_E\). This means that under any reasonable expectation the buyer could not realistically switch to the rival a portion of her purchase requirements close to \(s_{\text{mr}}\). We believe that such a finding should be seen as a convincing measure of the potential foreclosure effects of the rebate scheme. If on the contrary \(\bar{s}_E\) is clearly below \(s_{\text{mr}}\), no evidence of potential foreclosure is present.

Thus, the uncertainty inherent to the contestable share of demand can be addressed in a pragmatic manner and cannot justify a formalistic approach to rebates. Formalism in this matter is inappropriate for many reasons: the huge diversity of rebate schemes observed in practice, the complexity of their effects in various economic environments, and the long list of efficiency gains that rebates can bring about. Although many considerations relevant for legal standards (the administration of the antitrust system, the costs of judicial errors, the risk of under-deterrence and over-deterrence, etc.) are left out of the scope of the present article, we believe that our findings, at the very least, support the use of price-cost tests to assess the potential exclusionary effects of nonlinear pricing by dominant firms with must-carry brands or products.

\(^{21}\)See Footnote 3.
References


Yong Chao and Guofu Tan. All-units discounts as a partial foreclosure device. *working paper*, 2014.


A Appendix

Lemma A.1. The buyer and the incumbent are better off using a schedule with slope $T'$ smaller than or equal to $v_I$. Consequently, we may assume, with no loss of generality, that the buyer does not buy less than her total requirements: $q_E + q_I = 1$ for any $(s_E, \omega_E)$.

Proof. At given $c$, the buyer and the incumbent choose $q_E^*(q_I; s_E) = \min(1 - q_I, s_E)$ units of good $E$. It follows that their joint gross surplus for $q_I$ units of good $I$ is

$$S_{BE}(q_I; s_E, \omega_E) = v_I q_I + \omega_E \min(1 - q_I, s_E).$$

When $q_I$ is below $1 - s_E$, the quantity purchased from the rival is constant, $q_E^* = s_E$, and hence $S_{BE} = v_I q_I + \omega_E s_E$. When $q_I$ is above $1 - s_E$, $S_{BE} = \omega_E + (v_I - \omega_E) q_I$. We conclude that the buyer-rival pair’s valuation of a unit of good $I$, $\partial S_{BE}/\partial q_I$, never exceeds $v_I$.

Now, starting from any price schedule $T$, we introduce the modified schedule $\tilde{T}$ given by

$$\tilde{T}(q_I) = \inf_{q \leq q_I} T(q) + v_I (q_I - q). \quad (A.1)$$

The schedule $\tilde{T}$ is derived from the schedule $T$ as follows. When the incumbent offers $q$ units at price $T(q)$, he also offers to sell more units than $q$, say $q_I > q$, at price $T(q) + v_I (q_I - q)$. The additional units are offered at the monopoly price $v_I$. By construction, the slope of $\tilde{T}$ is lower than or equal to $v_I$.

Because $\tilde{T}$ is lower than or equal to $T$, the buyer and the rival cannot be worse off after the change. On the other hand, they cannot be better off either because the modified schedule $\tilde{T}$ offers them the possibility to purchase extra units at monopoly price $v_I$ while their valuation for those units cannot exceed $v_I$. It follows from these two observations that the joint net surpluses $\max_{q_I} S_{BE} - \tilde{T}$ and $\max_{q_I} S_{BE} - T$ must be equal. For the same reason, the corresponding equality holds under exclusive supply, $\max_{q_I} V(0, q_I) - \tilde{T} = \max_{q_I} V(0, q_I) - T$. We conclude that the rival profit $\Pi_E$ is the same under the schedules $T$ and $\tilde{T}$.

Moreover, suppose that the buyer demand is not satisfied $q_E^* + q_I < 1$. When this happens, we have $q_E^* = s_E$ and $q_I < 1 - s_E$. Under the modified schedule $\tilde{T}$, the buyer has a weak incentive to increase her purchase of incumbent good from $q_I$ to $1 - s_E$, leaving $q_E^* = s_E$ and $\Pi_E$ unchanged. This change increases the total welfare $W$ because each extra unit of incumbent good increases the surplus by $\omega_I > 0$. In sum, the change from $T$ to $\tilde{T}$ does not alter the rival’s profit and does not decrease the total surplus. We conclude from (4) that the change does not decrease the expected payoff of the buyer-incumbent coalition.

Proof of Lemma 1. By linearity of the virtual surplus, the maximization problem generically yields a corner solution, either $q_E = 0$ or $q_E = s_E$. At the maximum, we have $q_E = s_E$. 

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(respectively \( q_E = 0 \)) when \( s^v > 0 \) (resp. \( s^v < 0 \)). The unit virtual surplus \( s^v \) is positive if and only if
\[
\frac{\omega_E - \omega_I}{\omega_E} > \frac{\beta}{\varepsilon(\omega_E|s_E)}.
\]
The left-hand side increases with \( \omega_E \), and the right-hand side is non-increasing in \( \omega_E \), hence the uniqueness of \( \omega_E^r(s_E) \). Moreover, the virtual surplus per unit is negative for \( \omega_E = \omega_I \) and positive for \( \omega_E = \omega_E^r \). Hence the existence of a solution to equation (9) lying between \( \omega_I \) and \( \omega_E^r \). Straightforward comparative statics shows that \( \omega_E^r \) increases with \( \beta \) and decreases with \( \varepsilon \).

Lemma A.2. The random variables \( s_E \) and \( \omega_E \) are independent if and only if the elasticity of entry, \( \varepsilon(\omega_E|s_E) \), does not depend on \( s_E \). If the elasticity of entry increases (decreases) with \( s_E \), then \( \omega_E \) first-order stochastically decreases (increases) with \( s_E \).

Proof. The elasticity of entry varies with \( s_E \) in the same way as the hazard rate \( h \) given by
\[
h(\omega_E|s_E) = \frac{f(\omega_E|s_E)}{1 - F(\omega_E|s_E)}.
\]
We have
\[
\int_{\omega_E}^{\omega_E^r} h(x|s_E) \, dx = -\ln[1 - F(\omega_E|s_E)].
\]
If the elasticity of entry does not depend on (increases with, decreases with) \( s_E \), the same is true for the hazard rate, and hence also for the cdf \( F(\omega_E|s_E) \), which yields the results.\(^{22}\)

Proof of Proposition 1. We consider a two-part tariff with slope \( v_I - \omega_E^r(s_E) \), i.e., of the form \( T(q_I) = T(1) - (1 - q_I) [v_I - \omega_E^r(s_E)] \). The surplus created with the rival can be rewritten as
\[
\omega_EQ_E + v_I(1 - Q_E) - T(1 - Q_E) = v_I - T(1) + [\omega_E - \omega_E^r(s_E)] Q_E.
\]
(A.2)
For the same reasons, we get \( V_0^B = v_I - T(1) \), and hence \( \Delta \Pi_{BE} = [\omega_E - \omega_E^r(s_E)] Q_E \), showing that the buyer supplies the contestable part of her demand from the rival if and only if \( \omega_E > \omega_E^r(s_E) \). It follows that the proposed schedule allows the buyer and the incumbent to solve the rent-efficient tradeoff.

Proof of Proposition 2. As \( \omega_E^r(s_E) \) does not depend on \( s_E \), the proof of Proposition 1 applies with no other change than dropping the argument \( s_E \).

\(^{22}\)The variable \( \omega_E \) first-order stochastically decreases (increases) with \( s_E \) if and only if \( F(\omega_E|s_E) \) increases (decreases) with \( s_E \).
Proof of Proposition 3. We have already shown that the price schedule $T$ implements the solution of the relaxed problem. It remains to verify the condition $T' \leq v_I$ guaranteeing that $q_E + q_I = 1$ is satisfied, recall Lemma A.1. From (11), the marginal price $T'(q_I)$ is given by

$$T'(q_I) = v_I - \omega_E'(1 - q_I) - (1 - q_I)(\omega_E')'(1 - q_I). \quad (A.3)$$

It follows that $T' \leq v_I$ is satisfied if $\omega_E'(s_E) + s_E(\omega_E')'(s_E)$ is nonnegative for any $s_E$, which is true if $\omega_E'$ is not too strongly decreasing. When this condition is violated, the optimal price-quantity schedule has linear parts with slope $v_I$ so as to ensure that the buyer purchases enough units of good $I$. We do not elaborate further on this issue because the competition concern that motivates this article is that the buyer purchases too many, rather than too few, units of incumbent good.

Proof of Corollary 1. To prove concavity in the neighborhood of $q_I = 1$, we differentiate equation (11), accounting for the fact that $\omega_E$ is evaluated at $s_E = 1 - q_I$. We get $T'(q_I) = v_I - \omega_E' + (q_I - 1)(\omega_E)'$ and $T''(q_I) = 2(\omega_E)' + (1 - q_I)(\omega_E)''$. The term $(\omega_E)'$, which is negative for any $q_I$, tends to make the schedule concave. Assuming that $(\omega_E)''(0)$ is not infinite, we get $T''(1) = 2(\omega_E)'(0) < 0$, hence the concavity at the top.

Proof of Lemma 2. To recover the schedule $T(q)$ from the allocation $q_E$, we replace $q$ with $q_E(s_E, \omega_E)$ in the following equation

$$T(1) - T(1 - q) = (v_I - \omega_E)q + \Delta \Pi_{BE}(s_E, \omega_E), \quad (A.4)$$

where

$$\Delta \Pi_{BE}(s_E, \omega_E) = \int_{\omega_E}^{\omega_E(s_E, x)} q_E(s_E, x) \, dx. \quad (A.5)$$

We first show that this procedure univocally defines a price-quantity schedule, $T(q)$. To this aim, we observe that $(v_I - \omega_E)q_E(s_E, \omega_E) + \Delta \Pi_{BE}(s_E, \omega_E)$ is constant on $q_E$-isolines. Indeed, both $q_E(., \omega_E)$ and $\Delta \Pi_{BE}(., \omega_E)$ are constant on horizontal isolines (located below the exclusion line). On vertical isolines (above the exclusion line), $\Delta \Pi_{BE}(s_E, .)$ is linear with slope $\omega_E$, which ensures that the above expression is constant. This shows that (A.4) univocally defines $T(1) - T(1 - q)$ on the range of the quantity function $q_E(., .)$.\(^{23}\)

Next, we prove that the buyer and the competitor with type $(s_E, \omega_E)$, facing the above defined schedule $T$, agree on the quantity $q_E(s_E, \omega_E)$. We thus have to check that

$$\Delta \Pi_{BE}(s_E, \omega_E) \geq (\omega_E - v_I)q + T(1) - T(1 - q') \quad (A.6)$$

\(^{23}\)The range of $q_E$ may be disconnected. Specifically, if $\omega_E$ is above $\omega_E$ on the interval $I = [s_E^1, s_E^2]$, then $q_E$ does not take any value between $s_E^1$ and $s_E^2$. In this case, we define $T$ as being linear with slope $v_I - \omega_E$ on the corresponding interval: $T(1 - s_E^1) - T(1 - q) = (v_I - \omega_E)(q - s_E^1)$ for $q \in I$.  

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for any \( q' \leq s_E \). When \( q' \) is the range of the quantity function, we can write \( q' = q_E(s'_E, \omega'_E) \) for some \((s'_E, \omega'_E)\), with \( q' \leq s'_E \). Observing that \( q' = q_E(q', \omega'_E) \) and using successively the monotonicity of \( \Delta \Pi_{BE} \) in \( s_E \) and its convexity in \( \omega_E \), we get:

\[
\Delta \Pi_{BE}(s_E, \omega_E) \geq \Delta \Pi_{BE}(q', \omega_E) \geq \Delta \Pi_{BE}(q', \omega'_E) + (\omega_E - \omega'_E)q',
\]

which, after replacing \( T(1) - T(1 - q') \) with its value from (A.4), yields (A.6). To check (A.6) when \( q' \) is not in the range of the quantity function \((q' \text{ belongs to a hole } (s^1_E, s^2_E)) \) as explained in Footnote 23), use (A.6) at \( s^1_E \) and the linearity of the schedule between \( s^1_E \) and \( q' \).

**Proof of Lemma 3.** Property 1. We observe that replacing \( \hat{\omega}_E \) with \( \max(\hat{\omega}_E, \omega'_E) \) diminishes \( q_E(s_E, \omega_E) \) in the region \( \omega_E \leq \omega'_E(s_E) \) where the unit virtual surplus \( s^v(s_E, \omega_E) \) is negative, and leaves \( q_E(s_E, \omega_E) \) unchanged in the region where \( s^v(s_E, \omega_E) \) is positive. This change therefore increases the expected virtual surplus. Hence \( \omega'_E \geq \omega'_E \). If \( \omega'_E(s_E) \) were greater than \( \omega'_E(s_E) \), lowering \( \omega'_E(s_E) \) to \( \omega'_E(s_E) \) would only increase the quantity sold by types with \( s_E = s_E \) and \( \omega_E > \omega'_E(s_E) \), for which \( s^v > 0 \). This change would raise the expected virtual surplus.

Property 2. Suppose that \( \omega'_E \) is nondecreasing and that \( \omega'_E \) is decreasing on an interval \((s_E, s'_E)\). We know from point 1 that \( \omega'_E \geq \omega'_E \), hence replacing \( \omega'_E \) on the interval \((s_E, s'_E)\) with the constant value \( \omega'_E(s'_E) \) would raise \( q_E \) in a region where the virtual surplus is positive.

Property 3, suppose that \( \omega'_E \) is decreasing around \( s_E \) and that \( \omega'_E(s_E) > \omega'_E(s_E) \). Then a slight decrease in \( \omega'_E \) around \( s_E \) would raise the quantity \( q_E \) for types around \((s_E, \omega'_E(s_E))\). As the virtual surplus for these types is positive and no other type is affected, the change would increase the expected virtual surplus. Hence \( \omega'_E(s_E) = \omega'_E(s_E) \), a situation we encountered in the case where the elasticity of entry \( \varepsilon(\omega_E|s_E) \) increases with \( s_E \), recall Proposition 3.

**Proof of Proposition 4.** We have already observed that the price-quantity schedule is convex because the solution of (5) is interior for all types in the partial foreclosure region. It follows that the schedule \( T(q_i) \) is almost everywhere differentiable in \( q_i \). The first-order condition yields \( T'(1 - q_E) = v_i - \hat{\omega}_E(q_E) \) almost everywhere.\(^{24}\) As a result, the condition \( T'(q_i) \leq v_i \) that guarantees that \( q_E + q_i = 1 \) (recall Lemma A.1) is automatically satisfied.

**Proof of (15) and (18)**. To construct the optimal allocation when the elasticity of entry is decreasing or U-shaped, we seek to maximize (8) separately for each value of \( \omega_E \). The method actually applies for any variation of the elasticity of entry.

For any implementable quantity function \( q_E(s_E, \omega_E) \) and any level of \( \omega_E \), we denote by \( s_i(\omega_E) \) the roots of the equation \( \omega_E = \hat{\omega}_E(s_E) \), where \( \hat{\omega}_E \) is the exclusion line of \( q_E \) defined

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\(^{24}\) As explained in Footnote 15, the price schedule has convex kink when \( \hat{\omega}_E \) admits upward discontinuities.
in Section 5. The roots \( s_i(\omega_E) \) define a partition of the interval \([0, \bar{s}_E]\) into “even” intervals \([s_{2i}(\omega_E), s_{2i+1}(\omega_E)]\) and “odd intervals” \([s_{2i-1}(\omega_E), s_{2i}(\omega_E)]\) such that the function \( q_E(\cdot, \omega_E) \) coincides with the identity map on even intervals, is constant on odd intervals, and is continuous at odd extremities. We denote by \( K \) the set of the functions thus obtained. According to Lemma 2, a function \( q_E(s, \omega_E) \) is implementable if and only if for each \( \omega_E \) the function of one variable \( q_E(\cdot, \omega_E) \) belongs to the set \( K \) defined above and the even (odd) extremities are nonincreasing (nondecreasing) in \( \omega_E \).

**Lemma A.3.** For a given level of \( \omega_E \), the one-dimensional problem

\[
\max_{q_E(\cdot, \omega_E) \in K} \int_0^{\bar{s}_E} s^v(s_E, \omega_E) f(\omega_E|s_E) g(s_E) q_E(s, \omega_E) \, ds_E
\]

admits a unique solution characterized as follows. For any interior even extremity \( s_{2i} \), the virtual surplus \( s^v \) equals zero at \( s_{2i} \) and is negative (positive) at the left (right) of \( s_{2i} \). For any interior odd extremity \( s_{2i-1} \), the virtual surplus is positive at \( s_{2i-1} \) and satisfies

\[
\int_{s_{2i-1}}^{s_{2i}} s^v(s_E, \omega_E) f(\omega_E|s_E) g(s_E) \, ds_E = 0. \tag{A.7}
\]

**Proof.** Letting \( I \) be the integral to be maximized, we have

\[
I = \sum_i \int_{s_{2i}}^{s_{2i+1}} s_E s^v(s_E, \omega_E) f(\omega_E|s_E) g(s_E) \, ds_E + \sum_i s_{2i-1} \int_{s_{2i-1}}^{s_{2i}} s^v(s_E, \omega_E) f(\omega_E|s_E) g(s_E) \, ds_E,
\]

where the index \( i \) in each of the two sums goes from either \( i = 0 \) or \( i = 1 \) to either \( i = n - 1 \) or \( i = n \) depending on whether the first and last intervals are even or odd. Differentiating with respect to an interior even extremity yields

\[
\frac{\partial I}{\partial s_{2i}} = s^v(s_{2i}, \omega_E).[s_{2i-1} - s_{2i}].
\]

The first-order condition therefore imposes \( s^v(s_{2i}, \omega_E) = 0 \). The second-order condition for a maximum shows that \( s^v \) must be negative (positive) at the left (right) of \( s_{2i} \). Differentiating with respect to an interior odd extremity and using the continuity of \( q_E \) at such extremities, we get

\[
\frac{\partial I}{\partial s_{2i-1}} = \int_{s_{2i-1}}^{s_{2i}} s^v(s_E, \omega_E) f(\omega_E|s_E) g(s_E) \, ds_E.
\]

which yields (A.7). The second-order condition for a maximum imposes that \( s^v(s_E, \omega_E) \) is positive at \( s_{2i-1} \). \( \square \)

---

\(^{25}\) Even (odd) extremities parameterize the nonincreasing (nondecreasing) parts of the exclusion line. Even (odd) intervals are located above (below) this line.
Equations (15) and (18) are particular cases of the general equation (A.7). They yield the exclusion line of the complete problem if the candidate even (odd) extremities are nonincreasing (nondecreasing) in $\omega_E$. Regarding even extremities, it follows from Lemma A.3 that $s_2(\omega_E)$ belongs to decreasing portions of the exclusion line of the relaxed problem $\omega_E = \omega^r_E(s_E)$, and therefore even extremities are automatically decreasing in $\omega_E$. Regarding odd extremities, we show in our working paper Choné and Linnemer (2015b) that $s_{2i-1}(\omega_E)$ increases with $\omega_E$ if the hazard rate $f(\omega_{E}|s_E)/(1 - F(\omega_{E}|s_E))$ is nondecreasing in $\omega_E$ (a slightly stronger condition than Assumption 1) and the range of the elasticity of entry is not too wide, specifically, the elasticity is bounded from below and from above by $\xi$ and $\bar{\xi}$ satisfying

$$\beta(\bar{\xi} - \xi)^2 \leq 4\xi \bar{\xi}. \tag{A.8}$$

For instance, if the rival has all the bargaining power vis-à-vis the buyer ($\beta = 1$), the elasticity of entry $\varepsilon(\omega_{E}|s_E)$ may vary freely between 0.6 and 3 in the set of possible rival types, hence a fairly large elasticity range. The sufficient condition is even milder when $\beta$ is smaller.

![Figure 8a: The relaxed solution locally decreases with $s_E$.](image)

![Figure 8b: Exclusion lines of the relaxed problem (dashed) and of the complete problem (solid).](image)

**Two-dimensional bunching.** We explain here how to find the exclusion line of the complete problem when a candidate odd extremity given by (A.7) is non-monotonic in $\omega_E$. Accordingly, suppose that the above construction does not yield a function of $s_E$, as in the example shown on Figure 9a. In this example, the elasticity of entry is $\xi = 1.75$ for $s_E > \tilde{s}$ and $\bar{\xi} = 9.5$ for $s_E < \tilde{s}$, with $\tilde{s} = .7$, so the exclusion line of the relaxed problem is a step function; the size of the contestable demand belongs to $(0, \tilde{s})$ with probability .95 and to $(\tilde{s}, 1)$ with probability .05, and is uniformly distributed on each of the two intervals. Finally $\omega_I = \omega_E = 1$. Under
these circumstances, the solution of (15) does not define a function of $s_E$ because the odd extremity is non-monotonic. It follows that the optimal allocation features two-dimensional bunching, see the shaded region $D$ pictured on Figure 9b. The constant value of the rival quantity on the bunching region, denoted by $\hat{s}$ on the picture, is determined by the first-order condition that the expected unit virtual surplus is zero on that region, $E(s^*|D) = 0$.

**Proof of Proposition 5.** We first explain how to derive the marginal price below point $A_2$, for instance at point $A_3$. We continue to denote by $\sigma(\omega_E)$ the highest root of the equation $\omega_E = \omega_E^c(s_E)$. We consider $s_E$ below the contestable share of type $A_2$. We know $T'(1 - s_E) = v_I - \omega_E^c(s_E)$ and want to deduce $T'(1 - \sigma(\omega_E^c(s_E)))$. To this aim, we write that for the type $(\sigma(\omega_E^c(s_E)), \omega_E^c(s_E))$, the buyer and the rival are indifferent between quantities $s_E$ and $\sigma(\omega_E^c(s_E))$:  

\[
[\omega_E^c(s_E) - v_I]s_E - T(1 - s_E) = [\omega_E^c(s_E) - v_I]\sigma(\omega_E^c(s_E)) - T(1 - \sigma(\omega_E^c(s_E))).
\]

Differentiating and using the first-order condition at $s_E$ yields

\[
(\omega_E^c)'s_E = (\omega_E^c)'\sigma(s_E) + \sigma'(\omega_E^c)\left[\omega_E^c - v_I + T'(1 - \sigma(\omega_E^c))\right].
\]

Simplifying by $(\omega_E^c)'$ we get

\[
T'(1 - \sigma(\omega_E^c(s_E))) = v_I - \omega_E^c - [\sigma(s_E) - s_E](\omega_E^c)',
\]

where $(\omega_E^c)' = 1/\sigma'$ is evaluated at $\sigma(\omega_E^c(s_E))$. We observe that $\omega_E^c(s_E)$ increases with $s_E$; $\sigma(\omega_E^c(s_E))$ decreases with $s_E$; the difference $\sigma(s_E) - s_E$ is positive and decreasing in $s_E$; $(\omega_E^c)'$

\[
\text{Figure 9a: Exclusion line of the relaxed problem (dashed). Non-monotonic solution to (15) (solid line)}.
\]

\[
\text{Figure 9b: Two-dimensional bunching area: Exclusion line of the relaxed problem (dashed) and the complete problem (bold).}
\]
evaluated at $\sigma(\omega_E^c(s_E))$ is negative and increasing in $s_E$ near point $A_2$ because this point is the maximum of $\omega_E^c$. From these observations, we conclude that $T$ is concave below point $A_2$ (in the direction of point $A_3$). This shows that the price schedule changes curvature at point $A_2$ where $\omega_E^c$ achieves its maximum.

**Proof of Proposition 6.** For simplicity, we assume $\beta = 1$. As explained in the text, we take the distribution $f(\omega_E|s_E)$ such that the entry barrier $\omega_E^r$ is $v_I - t_2$ for $s_E < \tilde{s}$ and satisfies (19) for $s_E > \tilde{s}$. The distribution of $s_E$ has to have a mass point at $\tilde{s}$, and we denote the corresponding mass by $\mu$. Outside this point, we choose a density $g(s_E)$ such that the integral over the interior of the partial foreclosure region $P$ (i.e., of the light-shaded area shown on Figure 7a)

$$J = \int_{\{P, s > s_E\}} (\omega_E - \omega_I) f(\omega_E|s_E) - [1 - F(\omega_E|s_E)] g(s_E) ds_E d\omega_E < 0$$

exists and is finite.

We now construct a distribution $F(\omega_E|\tilde{s})$ such that the bunching condition (18) holds for all horizontal intervals crossing $P$ from one end to the other. In the present context, this condition can be written as

$$\mu \{ (\omega_E - \omega_I) f(\omega_E|\tilde{s}) - [1 - F(\omega_E|\tilde{s})] \} =$$

$$- \int_{\tilde{s}}^{\sigma(\omega_E)} \{ (\omega_E - \omega_I) f(\omega_E|s_E) - [1 - F(\omega_E|s_E)] \} g(s_E) ds_E > 0,$$ (A.9)

where the function $\sigma(\omega_E)$ given by (19) parameterizes the decreasing part of the exclusion line (the upper boundary of $P$). The virtual surplus along the vertical left boundary is positive to offset the negative contribution in the interior of $P$, i.e., both sides of (A.9) are positive.

Equation (A.9), which must hold for all $\omega_E$ greater than $\omega_E^0 = v_I - t_2$, is a linear first-order ordinary differential equation with unknown function $F(\omega_E|\tilde{s})$. Integrating between $\omega_E^0$ and $\infty$ and using the limit condition $1 - F(\infty) = 0$, we get the initial value at $\omega_E^0$ through the equality $\mu(\omega_E^0 - \omega_I) [1 - F(\omega_E^0|\tilde{s})] = -J > 0$. The mass $\mu$, the density $g$, and the value of $J$ can be adjusted to make sure that the bracketed term is smaller than one as it should be. Finally, inspecting (A.9) immediately yields $f(\omega_E|\tilde{s}) > 0$, so we have constructed a well-defined distribution of $\omega_E$ conditional on $s_E = \tilde{s}$.$^{26}$

---

$^{26}$For $\omega_E < \omega_E^0$, we may set $f$ and $F$ to zero so that $(\omega_E - \omega_I) f(\omega_E|\tilde{s}) - [1 - F(\omega_E|\tilde{s})]$ is negative in this region.