Nonlinear pricing and exclusion:
II. Must-stock products

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Abstract

Dominant firms often are unavoidable trading partners. Buyers may consider switching a fraction of their requirements to rival products, but that fraction is highly uncertain in rapidly evolving industries. Nonlinear pricing can thus serve to adjust the competitive pressure placed on rivals. Concave schedules act as entry barriers, whose height is governed by the average price of contestable units. Convex schedules act as barriers to expansion, distorting the rival supply at the intensive margin. All-units discounts combine the two mechanisms, erecting especially high barriers for intermediate levels of contestable demand. Such highly nonlinear schedules, however, lose exclusionary power when buyers can dispose of unneeded units or resell them on a secondary market.

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1 Introduction

This article is devoted to market situations with a dominant firm and large buyers for whom “losing [the dominant firm] as a supplier [is] not an option.”\textsuperscript{1} In such situations, a portion of any individual buyer demand is de facto uncontestable due to high brand loyalty for the dominant firm:

“Competitors may not be able to compete for an individual customer’s entire demand because the dominant undertaking is an unavoidable trading partner at least for part of the demand on the market, for instance because its brand is a “must stock item” preferred by many final consumers or because the capacity constraints on the other suppliers are such that a part of demand can only be provided for by the dominant supplier.”\textsuperscript{2}

According to U.S. Department of Justice (2008), there is a general agreement among antitrust practitioners that “when customers must carry a certain percentage of the leading firm’s products, discounts can be structured to induce purchasers to buy all or nearly all needs beyond that uncontestable percentage from the leading firm”, and may under this circumstance have anticompetitive effects. Two considerations in these statements deserve attention. First, the notion that a portion of demand is uncontestable is difficult to handle, as amply demonstrated by the recent Intel case.\textsuperscript{3} Second, given the variety of discounts observed in practice, the nature of the considered discounts need to be carefully specified.\textsuperscript{4}

Regarding the contestable share of demand, the above-cited Communication of the European Commission suggests to determine “how much of the customer’s purchase requirements can realistically be switched to a competitor”. This share depends on the rival capacity constraint as well as on client-specific factors that may limit how quickly a buyer can ramp-up products based on rival suppliers and therefore how much of its requirement is contestable at any given point in time. When a dominant firm faces a growing competitive threat from a competitor, it has to form expectations about its clients’ willingness to pay for rival products.

\textsuperscript{2}Communication on abusive exclusionary conduct by dominant undertakings (2009/C 45/02).
\textsuperscript{4}Other cases where the dominant firm was an unavoidable trading partner include: LePage’s, Inc. et al. v. 3M Company 324 F.3d 141 (3rd Cir. 2003); Van den Bergh Foods Limited, Court of First Instance (T-65/98) 23 October 2003; British Airways, Court of First Instance (Third Chamber, Case C-95/04 P), 15 March 2007; Michelin, Judgment of the Court of First Instance (Third Chamber, T-203/01), 30 September 2003, discussed by Motta (2009); Tomra, C-549/10 P Judgment of the Court of First Instance (Third Chamber), 19 April 2012. In these cases, the dominant firm used some kind of rebate, e.g., pure quantity discounts, loyalty discounts, exclusivity rebates, market-share discounts, bundled rebates.
and the share of their requirement they consider switching to the competitor. For instance, in *Intel*, the Commission sought to determine “what volumes were actually thought to be at risk during the period examined”. These volumes are inherently uncertain, and their quantitative assessment is a difficult task for the dominant firm, particularly in industries with rapid pace of innovations where products with high technological content are frequently introduced.\(^5\) For these reasons, the contestable share should be regarded as a random variable rather than as a deterministic figure.

Turning to the nature of the considered rebates, we investigate in this article the exclusionary effects of the simplest pricing policy, namely standard nonlinear pricing, whereby the rebates granted by the dominant firm are based only on the purchased quantity. Pure quantity rebates are ubiquitous in practice, have many pro-competitive justifications, and hence are certainly not anticompetitive by object. Our purpose is to offer a static scenario of exclusion that accounts for the uncertainty about the contestable share of demand and can explain the various, often highly nonlinear, price schedules observed in practice. In this scenario, the dominant firm designs a price-quantity schedule before discovering the characteristics of the rival good, hence ignoring both the unit surplus brought by that good and the contestable share of the demand. The general intuition is that by lowering the price of potentially contestable units, the dominant firm forces the rival to match lower prices, which, depending on its efficiency, he may not be able to do profitably. Hence an entry barrier whose height is negatively related to the average or “effective” price of contestable units. For a given contestable share, this price is set to extract rent from the rival without inducing too much exclusion.

The uncertainty about the contestable share in general causes optimal price-quantity schedules to be nonlinear. We are able to relate the shape of these schedules to the joint distribution of the rival unit surplus and the size of the contestable demand. We find that the optimal schedule is linear over the range of contestable units only when these two parameters are independent. The price schedule tends to be concave when a larger contestable demand is associated with a higher unit surplus. Under this circumstance, the average or “effective” price of contestable units governs the shape of the optimal schedule and its exclusionary effects: either the rival can match the effective price and then serves the contestable demand; or it cannot and is driven out of the market. The strong asymmetry between the dominant firm and the rival thus translates into a quantity distortion at the extensive margin, i.e., into the strongest form of market foreclosure. Competition agencies tend to see concave schedules as relatively innocuous, often presuming they are justified by economies of scales. Our findings call for caution in this respect.

\(^5\)See sections “V.5. Innovation in x86 CPUs”, “VI.1. The growing competitive threat from AMD”, and VII.4.2.3.1 in *Intel*, op.cit.
Our main finding is to show that retroactive rebates are optimal under simple patterns of correlation between contestable demand and rival efficiency. Retroactive rebates, also known as “all-units discounts”, are granted for all the purchased units once a quantity threshold is reached, thus inducing downward discontinuities in price-quantity schedules—a pattern that has received much attention from antitrust enforcers. Specifically, “retroactive rebates” prevail in equilibrium when the highest values of the rival surplus are obtained for intermediate sizes of the contestable demand. Under this circumstance, the height of the entry barrier that solves the rent-efficiency tradeoff is non-monotonic in contestable demand: the highest level of the optimal barrier is achieved for rivals of intermediate size. Retroactive rebates, or more generally strongly non linear price schedules with decreasing parts, cause sizeable masses of efficient rival types to be partially excluded from the market, hence quantity distortions at the intensive margin. Defendants in antitrust litigation commonly put forward that the alleged abuse did not prevent competitors from gaining significant market shares. Our analysis points out that antitrust enforcers are right to discard this line of defense as a positive market share is not incompatible with (partial) anticompetitive foreclosure.

The analysis must be modified when the buyer is allowed to dispose of or to resell unconsumed units on a secondary market because strong quantity rebates, and a fortiori decreasing parts in a price-quantity schedule, might then induce her to purchase unneeded units of incumbent good with the sole purpose of pocketing the rebates. We show that disposal does not occur in equilibrium. Yet the mere possibility of disposal constrains the shape of optimal schedules and limits the extent of inefficient exclusion when disposal costs are weak.

Related literature To study exclusionary pricing by a dominant firm, Marx and Shaffer (1999) solve a sequential game with variable consumption and perfect information. Our companion paper, Choné and Linnemer (2015), generalize their framework to allow for one-dimensional uncertainty about the rival surplus, thus providing an intensive version of Aghion and Bolton (1987). As in Martimort and Stole (2009), the price-quantity schedule is used to indirectly control the quantities sold by the rival. Contrary to the present article, the exclusionary effects are determined by the marginal price only. This price is set to lower the buyer’s incentives to supply from the rival without inducing excessive consumption of incumbent good—a phenomenon we called “buyer opportunism”.

In the present article, we introduce uncertainty on the nonlinear part of the utility provided by the rival. We assume that the buyer’s demand is inelastic and that either the rival has capacity constraint or there is satiation point for the rival good. The framework is a convenient way to express the rival’s inability to serve all the buyer demand; for this reason, it has attracted much attention in the recent literature. DeGraba (2013) models the competitor as

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being capacity constrained as we do. His approach is in the tradition of the “naked exclusion” literature pioneered by Rasmusen, Ramseyer, and Wiley (1991) and Segal and Whinston (2000). In Chao and Tan (2014), all-units discounts are seen as a collusive device to extract surplus from the buyer. The dominant firm uses its price leadership to soften competition, committing to relatively high (above-cost) prices for contestable units to induce less aggressive reactions from the rival. By contrast, in the present article, the dominant firm sets low prices for contestable units at the expense of the rival. Ide, Montero, and Figueroa (2015) examine the sharing of the surplus between the dominant firm and the buyer, an issue we discuss in Section 8. Closer to our study is Feess and Wohlschlegel (2010) who compare all-units rebates to exclusive dealing, show that all-units discounts can shift the rent from the entrant to the coalition between the incumbent and the buyer.

The above-cited studies impose the form of the price scheme and assume that the contestable share of the market is known. Yet that share as well as the rival surplus depends on the characteristics of the rival good. The rival surplus is related to vertical differentiation, while the contestable share reflects the extent of horizontal substitutability between the incumbent and rival goods. The present study adopts a symmetric informational assumption for these two parameters.

The demand inelasticity discussed above simplifies the analysis as in equilibrium the quantities of the two goods sum up to the total demand and each quantity is efficient given the other—hence no buyer opportunism. This makes the problem formally isomorphic to that of nonlinear pricing by a single-product monopolist facing a population of consumers with two dimensions of heterogeneity. The two dimensions of uncertainty, namely the rival surplus and the contestable share of the market, are on the buyer-rival side, while the buyer-dominant firm coalition (the “principal”) has only one instrument, namely the price-quantity schedule for the incumbent good. Pricing problems where the number of instruments is lower than the dimension of uncertainty are notoriously difficult. For instance, Laffont, Maskin, and Rochet (1987) consider quadratic preferences and assume that the coefficient of the linear term and the coefficient of the quadratic term in the buyer utility are independent and uniformly distributed. Modeling utility with a satiation point rather than with a quadratic term, we are able to offer a constructive method of optimal prices that works under fairly general distributional assumptions. Accommodating flexible distributions of uncertainty is critical for our purpose because it allows to generate the variety of pricing patterns we see in practice.

The article is organized as follows. Section 2 presents the model. Section 3 computes the height of the entry barrier that solves the tradeoff between rent extraction and efficiency. Section 4 studies two-part tariffs and concave schedules. Section 5 tackles the general problem, characterizing the implementable second-best allocations and presenting a general method to construct optimal price schedules. Section 6 shows that all-units discounts naturally appear
under simple distributions of uncertainty. Section 7 explains that the buyer’s ability to dispose of unneeded units limits the extent of inefficient exclusion. Section 8 discusses the implications of our findings for policy intervention, emphasizing the role of the “as-efficient competitor test” in assessing exclusionary effects of rebates in the presence of must-carry goods.

2 Model

A dominant firm $I$ and a competitor $E$ interact with a large buyer $B$. The rival good and the incumbent good are produced at constant marginal costs of $c_E$ and $c_I$, respectively. The buyer gets utility $v_E$ and $v_I$ per consumed unit of each good.

**Buyer’s demand** Up to Section 7, we assume that technological constraints or contractual arrangements prevent the buyer from stocking, disposing of, or reselling, unneeded units. As a result, any purchased unit must be consumed. Thus, when the buyer purchases $q_E$ units of good $E$ and $q_I$ units of good $I$, her utility is $V(q_E, q_I) = v_E q_E + v_I q_I$. The consumption set, however, is constrained in two ways. First, her total consumption in both goods cannot exceed a fixed amount, which we normalize to one. Second, there is a satiation point for good $E$: no more than $s_E$ units of that good can be consumed. It follows that the purchased quantities necessarily satisfy: $q_E \leq s_E$ and $q_E + q_I \leq 1$.

The total surplus $W(q_E, q_I) = V(q_E, q_I) - c_E q_E - c_I q_I$ is equal to $\omega_E q_E + \omega_I q_I$, where $\omega_E = v_E - c_E$ and $\omega_I = v_I - c_I$ denote the surpluses per unit of each good. The efficient quantities maximize $\omega_E q_E + \omega_I q_I$ are given by

$$\left(q^*_E(s_E, \omega_E), q^*_I(s_E, \omega_E)\right) = \begin{cases} (s_E, 1 - s_E) & \text{if } \omega_E > \omega_I \\ (0, 1) & \text{if } \omega_E < \omega_I. \end{cases}$$

(1)

The quantity $q_E$ that maximizes the social welfare $W$ given $q_I$ plays a critical role in the analysis. It is given by

$$q^*_E(q_I; s_E, \omega_E) = \min \left\{ 1 - q_I, s_E \right\}.$$  

(2)

**Timing and information** At the first stage of the game, $B$ and $I$ design a price-quantity schedule to maximize their joint expected surplus, knowing the characteristics of the incumbent good, i.e., the constant marginal cost $c_I$ and willingness to pay $v_I$. We do not restrict the two players’ ability to share surplus ex ante. At this stage, $B$ and $I$ do not know the characteristics of the rival good, i.e., the constant marginal cost $c_E$, the size of the contestable demand $s_E$, and the willingness to pay $v_E$.

At the second stage of the game, the buyer and the rival discover $c_E$, $s_E$ and $v_E$, the terms of the agreement between $B$ and $I$ being common knowledge. $B$ and $E$ jointly decide on a
transfer $p_E$ and quantities $q_E$ and $q_I$. The negotiation takes place under complete information and is modeled as Nash bargaining where $\beta$ denotes the rival’s bargaining power.

**Purchase decisions** At the last stage of the game, the buyer and the rival choose quantities to maximize their joint surplus

$$S_{BE} = \max_{q_E,q_I} V(q_E,q_I) - T(q_I) - c_E q_E,$$

(3)

with no consideration for the incumbent’s cost or profit. As $T$ depends only on $q_I$, the quantity $q_E$ is efficient given that of the incumbent good, $q_E = q_E^*(q_I; s_E, \omega_E)$. Hence an alternative interpretation of the model where the non-contestable share of the market reflects rival production capacity rather than buyer satiation. Under both interpretations, the rival firm can address only the fraction $s_E$ of the buyer’s demand.

**Sharing of the surplus between $B$ and $E$** Without loss of generality, the competitor’s outside option is normalized to zero. As to the buyer, she may source exclusively from the incumbent, in which case her utility, hereafter denoted by $V^0_B$, does not depend on $s_E$ or $\omega_E$. The surplus created by the the buyer and the rival firm can thus be written as $\Delta S_{BE} = S_{BE} - V^0_B$. Denoting by $\beta \in (0,1)$ the competitor’s bargaining power vis-à-vis the buyer, we derive its profit

$$\Pi_E = \beta \Delta S_{BE},$$

(4)

as well as the buyer utility, $\Pi_B = (1 - \beta)\Delta S_{BE} + V^0_B$. If $\beta = 0$, the competitor has no bargaining power and may be seen as a competitive fringe from which the buyer can purchase any quantity at price $c_E$. On the contrary, the case $\beta = 1$ happens when the competitor has all the bargaining power vis-à-vis the buyer.

**The grand problem** Ex ante, the buyer and the incumbent design a non-conditional price-quantity schedule $T(q_I)$ to maximize their expected joint surplus, equal to the total surplus minus the profit left to the competitor:

$$\mathbb{E}\Pi_{BI} = \mathbb{E}\{W(q_E,q_I) - \Pi_E\},$$

(5)

where $q_E$ and $q_I$ are solution to (3) and $\Pi_E$ is given by (4).

The problem can be simplified in two ways. First, we have already observed that $q_E$ is efficient given $q_I$, formally $q_E = q_E^*(q_I; s_E, \omega_E)$. Second, as regards $q_I$, it would be inefficient not to produce and consume units of incumbent good that could bring a positive net surplus of $\omega_I$ each. It is actually both feasible and efficient to satisfy the buyer demand, i.e., to make sure that $q_E^* + q_I \geq 1$ for all $s_E$ and $\omega_E$. A simple way to do so is to sell all units of incumbent
good below the monopoly price $v_I$. In the appendix, we show formally that we may indeed restrict attention, without loss of generality, to price schedules that satisfy $T'(q_I) \leq v_I$ for all $q_I$, and to allocations such that $q_E + q_I \geq 1$ for all $s_E$ and $\omega_E$. Knowing that the buyer cannot purchase and consume more units than her total demand, the latter result yields the equality $q_E + q_I = 1$ for any buyer type.

The expectations in (5) are to be taken against the distribution of the rival good’s characteristics $(c_E, s_E, v_E)$. As any produced unit of rival good is consumed, the surplus depends on the uncertain cost and preference parameters $c_E$ and $v_E$ through the difference $\omega_E = v_E - c_E$. It follows that only the joint distribution of $s_E$ and $\omega_E$ matters. We assume that the cumulative distribution function of $s_E$, denoted by $G$, admits a positive and continuous density function $g$ on $[s_E, \bar{s}_E]$. The conditional distribution of $\omega_E$ given $s_E$ has a positive density $f(\omega_E|s_E)$ on its support $[\underline{\omega}_E, \bar{\omega}_E]$, with $\underline{\omega}_E < \omega_I < \bar{\omega}_E$.

## 3 Rent-efficiency tradeoff

Replacing $q_I$ with $1 - q_E$ in (3), we rewrite the joint surplus of the buyer-rival pair as

$$S_{BE} = \max_{q_E \leq s_E} \omega_E q_E + v_I(1 - q_E) - T(1 - q_E).$$

(6)

By (4), the rival’s profit is $\pi_E = \beta(S_{BE} - V^0_B)$. We observe that $S_{BE}$ is the upper bound of a family of affine functions of $\omega_E$, and hence is convex in $\omega_E$. Differentiating with respect to $\omega_E$ at given $s_E$ and using the envelope theorem yields $\partial \Pi_E/\partial \omega_E = \beta q_E$.

### Virtual surplus

Integrating the rival profit by parts with respect to $\omega_E$, for any given size of the contestable demand $s_E$,

$$\int_{\underline{\omega}_E}^{\bar{\omega}_E} \Pi_E(s_E, \omega_E) \, dF(\omega_E|s_E) = \beta \int_{\underline{\omega}_E}^{\bar{\omega}_E} q_E(s_E, \omega_E) \left[ 1 - F(\omega_E|s_E) \right] \, d\omega_E,$n

because the rival with the lowest rival unit surplus $\underline{\omega}_E < \omega_I$ cannot be active. We rewrite the buyer-incumbent objective as the expectation of the virtual surplus defined as

$$S^v(q_E, q_I) = W(q_E, q_I) - \beta q_E \frac{1 - F(\omega_E|s_E)}{f(\omega_E|s_E)}.$$

As usual in the contract theory literature, the virtual surplus balances the efficiency and rent extraction motives. Given that $q_E + q_I = 1$ and $W(q_E, q_I) = \omega_E q_E + \omega_I q_I$, the virtual surplus can be written as a linear function of $q_E$, namely $S^v = \omega_I + s^v(s_E, \omega_E) q_E$, where $s^v(s_E, \omega_E) = \omega_E - \omega_I - \beta[1 - F(\omega_E)]/f(\omega_E)$ is the virtual surplus per unit of rival good. The unit virtual surplus can itself be rewritten as

$$s^v(s_E, \omega_E) = \omega_E [1 - \beta/\varepsilon(\omega_E|s_E)] - \omega_I,$n

(7)
where we define the elasticity of entry $\varepsilon(\omega_E|s_E)$ as the percentage decrease in the probability that the rival unit surplus is above $\omega_E$ when $\omega_E$ rises by 1%:

$$\varepsilon(\omega_E|s_E) = \frac{\omega_E f(\omega_E|s_E)}{1 - F(\omega_E|s_E)} = -\frac{\partial \ln [1 - F(\omega_E|s_E)]}{\partial \ln \omega_E}. \tag{8}$$

The elasticity is constant in $\omega_E$ in the particular case where $\omega_E$ conditionally on $s_E$ follows a Pareto distribution, i.e. the conditional cdf is given by $1 - F(\omega_E|s_E) = (\omega_E/\omega_E) - \varepsilon(s_E)$, with $\varepsilon(s_E) > 0$.

**The relaxed problem** Our ultimate goal is to maximize the expected virtual surplus over all allocations that are implementable with a non-conditional price schedule satisfying $T' \leq v_I$. We start by maximizing the product $s^v(s_E, \omega_E)q_E$ pointwise, i.e., separately for each value of $s_E$ and $\omega_E$, which we call the “relaxed problem”.

**Assumption 1.** The elasticity of entry is nondecreasing in $\omega_E$.

**Lemma 1.** Assume that any purchased unit must be consumed and that Assumption 1 holds. Then the virtual surplus achieves its maximum subject to $q_E = q_E^c(q_I)$ at the point $(q_E, q_I = 1 - q_E)$ given by

$$q_E(s_E, \omega_E) = \begin{cases} 0 & \text{if } \omega_E \leq \hat{\omega}_E(s_E) \\ s_E & \text{otherwise}, \end{cases}$$

where $\hat{\omega}_E(s_E) \in (\omega_I, \omega_E)$ is the unique solution to

$$\frac{\hat{\omega}_E(s_E) - \omega_I}{\hat{\omega}_E(s_E)} = \frac{\beta}{\varepsilon(\hat{\omega}_E(s_E)|s_E)}. \tag{9}$$

The fraction of efficient types that are inactive increases with the rival’s bargaining power vis-à-vis the buyer and decreases with the elasticity of entry.

**Proof.** By linearity of the virtual surplus, the maximization problem generically yields a corner solution, either $q_E = 0$ or $q_E = s_E$. At the maximum, we have $q_E = s_E$ (respectively $q_E = 0$) when $s^v > 0$ (resp. $s^v < 0$). The unit virtual surplus $s^v$ is positive if and only if

$$\frac{\omega_E - \omega_I}{\omega_E} > \frac{\beta}{\varepsilon(\omega_E|s_E)}.$$

The left-hand side increases in $\omega_E$, and the right-hand side is non-increasing in $\omega_E$, hence the uniqueness of $\hat{\omega}_E(s_E)$. Moreover, the virtual surplus per unit is negative for $\omega_E = \omega_I$ and positive for $\omega_E = \omega_E$. Hence the existence of a solution to equation (9) lying between $\omega_I$ and $\omega_E$. Straightforward comparative statics shows that $\hat{\omega}_E$ increases with $\beta$ and decreases with $\varepsilon$. \qed
We interpret the threshold \( \hat{\omega}_E(s_E) \) as the height of the entry barrier that the buyer and the incumbent would want to erect if the size of the contestable demand \( s_E \) were known. When \( \omega_E \) is Pareto-distributed and the elasticity of entry \( \varepsilon(s_E) \) is constant in the rival unit surplus \( \omega_E \), the barrier to entry is explicitly given by

\[
\hat{\omega}_E(s_E) = \frac{\omega_I}{1 - 1/\varepsilon(s_E)}.
\]

Next, we consider the variations of the elasticity of entry \( \varepsilon \) with respect to the size of the contestable demand, \( s_E \). When the elasticity increases (respectively decreases) with \( s_E \), the barrier is lower (resp. higher) as the size of the contestable demand rises. The next lemma relates the variations of \( \varepsilon(\omega_E|s_E) \) with \( s_E \) to the primitives of the model.

**Lemma 2.** The random variables \( s_E \) and \( \omega_E \) are independent if and only if the elasticity of entry, \( \varepsilon(\omega_E|s_E) \), does not depend on \( s_E \). If the elasticity of entry increases (decreases) with \( s_E \), then \( \omega_E \) first-order stochastically decreases (increases) with \( s_E \).

**Proof.** The elasticity of entry varies with \( s_E \) in the same way as the hazard rate \( h \) given by

\[
h(\omega_E|s_E) = \frac{f(\omega_E|s_E)}{1 - F(\omega_E|s_E)}.
\]

We have

\[
\int_{\omega_E}^{\hat{\omega}_E} h(x|s_E) \, dx = -\ln[1 - F(\omega_E|s_E)].
\]

If the elasticity of entry does not depend on (increases with, decreases with) \( s_E \), the same is true for the hazard rate, and hence also for the cdf \( F(\omega_E|s_E) \), which yields the results.\(^8\)

### 4 Concave price schedules

**Two-part tariffs** When \( s_E \) and \( \omega_E \) are independent, the threshold \( \hat{\omega}_E(s_E) \) is flat, as represented on Figure 1a: the rival serves all of the contestable demand when \( \omega_E \) is higher than the constant level of \( \hat{\omega}_E \), and is inactive otherwise. We now show that this allocation is implementable by a two-part tariff with slope \((v_I - \hat{\omega}_E)\).

**Proposition 1.** Assume that any purchased unit must be consumed and that Assumption 1 holds. When \( s_E \) and \( \omega_E \) are independent, the buyer and the incumbent sell contestable units of incumbent good at price \((v_I - \hat{\omega}_E)\). Efficient rival types, lying between \( \omega_I \) and \( \hat{\omega}_E \), are driven out of the market. Partial foreclosure is not present.

\(^8\)The variable \( \omega_E \) first-order stochastically decreases (increases) with \( s_E \) if and only if \( F(\omega_E|s_E) \) increases (decreases) with \( s_E \).
Proof. We consider a two-part tariff with slope \((v_I - \hat{\omega}_E)\), i.e., of the form \(T(q_I) = T(1) + (v_I - \hat{\omega}_E)(q_I - 1)\). The surplus created with the rival can be rewritten as

\[
S_{BE} = v_E q_E + v_I q_I - c_I q_E - T(1) - (v_I - \hat{\omega}_E)(q_I - 1) = v_I - T(1) + (\omega_E - \hat{\omega}_E)q_E. \tag{10}
\]

For the same reasons, we get \(V_0^B = v_I - T(1)\), and hence \(\Delta S_{BE} = (\omega_E - \hat{\omega}_E)q_E\), showing that the buyer supplies the contestable part of her demand from the rival if and only if \(\omega_E > \hat{\omega}_E\). It follows that the proposed tariff allows the buyer and the incumbent to solve the rent-efficient tradeoff when the elasticity of entry \(\varepsilon(\omega_E|s_E)\) and the corresponding entry threshold \(\hat{\omega}_E(s_E)\) do not depend on the size of the contestable demand \(s_E\).

The contestable units are sold below cost as \(v_I - \hat{\omega}_E < v_I - \omega_I = c_I\). The corresponding price schedule is represented on Figure 1b assuming that \(\hat{\omega}_E < v_I\). It may well be the case, however, that \(\hat{\omega}_E\) is larger than \(v_I\); in this case, contestable units of incumbent units would be sold at a negative price.\(^9\)

**Effective price**  We define the “effective price” of the incumbent good as the average price of the last units:

\[
p^e(x) = \frac{T(1) - T(1 - x)}{x}. \tag{11}
\]

This is the price the rival must match to supply the contestable demand when the buyer has the same willingness to pay for the two goods, \(v_E = v_I\). The effective price, therefore, is negatively related to the competitive pressure placed on the rival: the lower the effective price, the higher the entry barrier and the more pressure placed on the rival.

\(^9\)As explained in Section 8, the price of the non-contestable units is of little significance for outside observers. We therefore represent all the schedules over the range of potentially contestable units, i.e., for \(q_I \geq 1 - \hat{s}_E\).
For the two-part tariff studied above, the effective price is constant and equal to $p^e = v_I - \hat{\omega}_E$. Increasing $p^e$ releases the competitive pressure placed on the rival and reduces the height of the entry barrier $\hat{\omega}_E$ and thus the probability that the rival is driven out of the market. In other words, the effective price serves as an instrument for the dominant firm to indirectly control the height of the entry barrier. The elasticity of entry measures the sensitivity of the rival to competitive pressure. Using the chain rule to indirectly control the height of the entry barrier.

The elasticity of entry measures the market. In other words, the effective price serves as an instrument for the dominant firm height of the entry barrier.

we find that a 1% increase in effective price increases entry by $(\varepsilon p^e / \hat{\omega}_E)\%$.

**Nondecreasing effective price** From now on, we consider cases where the elasticity of entry varies with $s_E$ and hence two-part tariffs are no longer optimal: the optimal tariff must exhibit some curvature. We start with the case where the elasticity increases with $s_E$: larger competitors, i.e., competitors with a larger contestable demand, are more sensitive to competitive pressure. Under this circumstance, the efficiency-rent tradeoff leads the buyer and the incumbent to place less competitive pressure on larger competitors.

**Proposition 2.** Assume that any purchased unit must be consumed and that Assumption 1 holds. When the elasticity of entry $\varepsilon(\omega_E|s_E)$ increases with $s_E$, the buyer and the incumbent set the effective price $p^e(s_E)$ at $v_I - \hat{\omega}_E(s_E)$. The price schedule is concave in the neighborhood of $q_I = 1$. It is globally concave if $\hat{\omega}_E$ is concave or moderately convex in $s_E$.

**Proof.** When $\varepsilon(\omega_E|s_E)$ increases with $s_E$, the threshold $\hat{\omega}_E$ given by (9) decreases with $s_E$, see Figure 2a. Suppose the buyer and the incumbent set the effective price $p^e(s_E)$ at $v_I - \hat{\omega}_E(s_E)$, which increases in $s_E$. From the definition of the effective price, (11), we recover the price schedule as

$$ T(q_I) = T(1) + (v_I - \hat{\omega}_E)(q_I - 1), \quad (12) $$

where $\hat{\omega}_E$ is evaluated at $s_E = 1 - q_I$. The same observations as in the proof of Proposition 1 yield the expression (10) for the surplus $S_{BE}$. Since $\hat{\omega}_E$ decreases in $q_E$, the surplus is maximum either at $q_E = 0$ or at $q_E = s_E$. The rival makes no sales if $\omega_E < \hat{\omega}_E(s_E)$ and serves all the contestable demand if $\omega_E > \hat{\omega}_E(s_E)$.

To prove concavity in the neighborhood of $q_I = 1$, we differentiate equation (12), accounting for the fact that $\hat{\omega}_E$ is evaluated at $s_E = 1 - q_I$. We get $T'(q_I) = v_I - \hat{\omega}_E + (q_I - 1)\hat{\omega}_E'$ and $T''(q_I) = 2\hat{\omega}_E' + (1 - q_I)\hat{\omega}_E''$. The term $\hat{\omega}_E'$, which is negative for any $q_I$, tends to make the tariff concave. Assuming that $\hat{\omega}_E''(0)$ is not infinite, we get $T''(1) = 2\hat{\omega}_E''(0) < 0$, hence the concavity at the top.

Proposition 2 assumes that the elasticity of entry is nondecreasing in the size of the contestable demand. According to Lemma 2, this assumption implies that rival types with
larger $s_E$ tend to generate a lower surplus $\omega_E$ and hence are more sensitive to competitive pressure. The buyer and the dominant firm therefore exert less pressure on larger rival types, and the optimal effective price $p^e(q_E) = [T(1) - T(1 - q_E)]/q_E$ increases with $q_E$. Geometrically, the effective price is the slope of a chord drawn from the point $(1, T(1))$. The chords, represented by the dotted lines on Figure 2b, are indeed steeper as the number of concerned units rises: they are upwards-sloping for large values of $q_E$, approximately flat for intermediate values, and decreasing for low values. The latter property happens here because we have assumed $\hat{\omega}_E(0) > v_I$, implying that the effective price $p^e(q_E)$ is negative for low values of $q_E$ and hence that the buyer has strong incentives to supply exclusively from the dominant firm when the contestable market is small.

Concave price schedules are commonly seen in practice. They occur in particular when the seller offers “incremental rebates”. Such rebates indeed apply to the units purchased in excess of given thresholds, and therefore cause the marginal price to fall.\footnote{Increasing the number of thresholds allows to approximate any concave schedule with a series of incremental rebates.} We now check that any concave price schedule $T$ such that the associated effective price $p^e$ is below $c_I$ is optimal for certain distributions of uncertainty. We first observe that the effective price is increasing because $(p^e)'(x) = [T'(1 - x) - p^e(x)]/x \geq 0$. It follows that the function $\varepsilon(s_E)$ defined by

$$\frac{\beta}{\varepsilon(s_E)} = \frac{c_I - p^e(s_E)}{v_I - p^e(s_E)} \geq 0$$

increases with $s_E$. Replacing $\hat{\omega}_E$ with $v_I - p^e$ in (9), we find that the price schedule $T$ is optimal when $\omega_E$ conditionally on $s_E$ follows the Pareto distribution $F'(\omega_E|s_E) = 1 - (\omega_E/\omega_E)^{-\varepsilon(s_E)}$.\footnote{Increasing the number of thresholds allows to approximate any concave schedule with a series of incremental rebates.}
Two-part tariffs, concave schedules, and more generally nondecreasing effective prices, act as barriers to entry and generate complete market foreclosure. In particular, a rival can never mimic types with smaller contestable demand as those types face lower effective prices; hence the only available alternative to serving all the contestable demand is to be completely inactive. This mechanism is in sharp contrast with Choné and Linnemer (2015) where the price schedules are essentially concave as in this section, but the exclusionary effects are driven by the marginal rather than effective price and concavity can be associated with partial rather than complete exclusion.

5 Derivation of the optimal allocation

We now turn to situations where optimal price schedules generate more complex incentives. When the elasticity of entry is not constant or increasing in the size of the contestable demand, solving the problem for each $s_E$ separately does not yield an incentive compatible allocation.

To illustrate, suppose that the entry barrier $\hat{\omega}_E$ depends on $s_E$ as shown on Figure 3. In this case, the solution to the relaxed problem, which is zero below the dotted line and $s_E$ above, is not incentive compatible. Indeed, the rival of type $B = (\omega^B_E, s^B_E)$ would be inactive and hence would earn zero profit, while type $A = (\omega^A_E, s^A_E)$, with $s^A_E < s^B_E$ and $\omega^A_E = \omega^B_E$, would serve all of the contestable demand. It follows that type $B$ would have an incentive to mimic type $A$ and to sell $s'_E$ (rather than $s_E$) units.

![Figure 3: The relaxed solution is not implementable](image_url)

We therefore need to characterize the set of implementable quantity allocations. After providing such a characterization, we explain how to construct the optimal allocation. The main economic intuition is that configurations like the one represented on Figure 3 give rise to quantity distortions at the intensive margin, which we interpret as partial foreclosure. We
derive an appropriate first-order condition for these distortions. The reader who is primarily interested in the qualitative properties of optimal price schedules should proceed directly to Section 6.

**Implementable allocations**  A quantity function $q_E(s_E, \omega_E)$ is implementable with a non-conditional price schedule if and only if there exists a function $T(q_I)$ such that $q_E(s_E, \omega_E)$ is solution to (6) for all $(s_E, \omega_E)$.

The buyer and the rival, when solving (6), hit the constraint $q_E \leq s_E$ when the rival unit surplus $\omega_E$ is large. Because $q_E$ increases in $\omega_E$, there exists, for any $s_E > 0$, a threshold $\Psi(s_E)$ such that the buyer supplies all the contestable units from the competitor, $q_E(s_E, \omega_E) = s_E$, if and only if $\omega_E \geq \Psi(s_E)$. We define the boundary line $\omega_E = \Psi(s_E)$ associated to the quantity function $q_E(s_E, \omega_E)$ by

$$\Psi(s_E) = \inf \{ x \in [\omega_E, \omega_E] \mid q_E(x, s_E) = s_E \},$$

with the convention $\Psi(s_E) = \omega_E$ when the above set is empty. Above the boundary line, $q_E(s_E, \omega_E)$ equals $s_E$; below that line, $q_E(s_E, \omega_E)$ is independent from $s_E$.

As shown on Figure 4, an implementable quantity function is entirely described by the associated boundary line. In particular, the bunching sets, i.e., the sets on which the quantity $q_E(s_E, \omega_E)$ is constant, are determined by the boundary line. They can be of three types: (i) vertical segments above decreasing portions of the boundary line (e.g. $q_E = s^3_E$ and $q_E = s^4_E$).
on the Figure); (ii) two-dimensional areas whose left border is vertical, being included either in the $\omega_E$-axis (then $q_E = 0$, see the shaded area on Figure 4) or in a vertical part of the boundary line (see the light shaded area on Figure 10b); (iii) “L”-shaped unions of a vertical segment and a horizontal segment intersecting on an increasing portion of the boundary line (e.g. $q_E = s_E^1$, $q_E = s_E^2$, and $q_E = s_E^5$). For instance, the types represented by points $A$ and $B$ on Figure 4 belong to such a L-shaped bunch: both types have $q_E = s_E^2$. The formal derivation of the quantity function $q_E(s_E, \omega_E)$ from the bunching pattern is done in Lemma A.2.

Properties of boundary lines and price schedules We check in the appendix that replacing $q$ with $q_E(s_E, \omega_E)$ in the following equation

$$T(1) - T(1 - q) = (v_I - \omega_E)q + \Delta S_{BE}(s_E, \omega_E),$$

where

$$\Delta S_{BE}(s_E, \omega_E) = \int_{\omega_E}^{\omega_E} q_E(s_E, x) \, dx,$$

unambiguously defines a non-conditional price-quantity schedule, $T(q)$. We also check that the buyer and the rival facing that schedule indeed agree on the considered quantity function $q_E(s_E, \omega_E)$.

The exclusionary properties of a price schedule differ according to the variations of its boundary line. Consider first the regions where $\Psi$ increases with $s_E$. Such increasing parts translate into horizontal bunching segments (or into two-dimensional bunching areas), and hence into partial foreclosure: $0 < q_E(s_E, \omega_E) < s_E$ for some types located below the boundary. To illustrate, type $B$ on Figure 4 sells $s_E^2$ units of rival good, which is lower than the size of its contestable market. More generally, the constraint $q_E \leq s_E$ is slack in the light-shaded area below the boundary line: a higher $s_E$ does not allow the competitor to enter at a larger scale. In other words, the solution of the buyer-rival problem (6) is interior. The first-order condition thus yields $T'(1 - s_E) = v_I - \Psi(s_E)$. It follows that the price schedule is convex when the boundary line is upward-slopping.\(^{11}\) The above first-order condition also ensures that $T' \leq v_I$ and $q_E + q_I = 1$ as explained in Section 2 and Lemma A.1. In the limiting case where the boundary line is flat, we find that the price schedule is linear (see Figures 1a and 1b).

By contrast, in the regions where $\Psi$ is decreasing in $s_E$, the quantity $q_E$ is discontinuous when crossing the boundary line for $\omega_E = \Psi(s_E)$.\(^{12}\) This implies that $q_E = s_E$ is a corner solution of (6) when the rival type is $(s_E, \omega_E)$. It follows that the above first-order condition does not hold and that the condition $T' \leq v_I$ must be checked ex post. In Lemma A.3, we show

\(^{11}\)An upward discontinuity in the boundary line, that induces two-dimensional bunching (see Figure 10b), corresponds to a convex kink in the schedule.

\(^{12}\)For instance, type $C$ on Figure 4 does not belong to the same bunching set as types $A$ and $B$: $q_A = q_B = s_E^2 < q_C = s_E^3$.?
that the curvature of the price schedule may change along decreasing parts of the boundary: the schedule is concave near local maxima of the boundary line and convex near local minima.

**Construction of the optimal allocation** We now explain intuitively how to correct the height of the entry barrier \( \hat{\omega}_E(s_E) \) when \( s_E \) is unknown. A formal construction of the optimal boundary line \( \omega_E = \Psi(s_E) \) is presented in the appendix.\(^{13}\)

Consider a type \((s_E, \omega_E)\) with \( \omega_E > \hat{\omega}_E(s_E) \). If the virtual surplus is always positive at the right of this point, there is no objection to setting \( q_E = s_E \). In contrast, if the virtual surplus is negative at the right of this point, setting \( q_E = s_E \) implies that \( q_E \) will have to be positive in an area where the virtual surplus is negative. We show in the appendix that the expected virtual surplus on horizontal bunching segments is zero, as under the standard ironing procedure. Denoting by \((AB)\) such a segment (see Figure 5b) and using the unit virtual surplus defined in (7), we get

\[
E(\ s^v(s_E, \omega_E) \ | \ [AB]\ ) = 0,
\]

with the boundary conditions that the virtual surplus is positive at \( A \) and zero at \( B \). This leads to the following construction of the optimal boundary line \( \omega_E = \Psi(s_E) \). We first draw the line \( \omega_E = \hat{\omega}_E(s_E) \). For \( s_E = \bar{s}_E \), we set \( \Psi(\bar{s}_E) = \hat{\omega}_E(\bar{s}_E) \). Then we consider lower values of \( s_E \). If \( \hat{\omega}_E \) decreases at \( \bar{s}_E \), we stick to the original entry barrier \( \hat{\omega}_E \), as long as it remains decreasing. When \( \hat{\omega}_E \) starts increasing (possibly at \( \bar{s}_E \)), we know that there is horizontal bunching. Equation (16) provides a unique value for \( \Psi(s_E) \). If the candidate boundary hits the line \( \omega_E = \hat{\omega}_E(s_E) \) at some lower value of \( s_E \), it must be on a decreasing part of that line and, from that value on, the optimal boundary again coincides with \( \hat{\omega}_E \) (as long as \( \hat{\omega}_E \) remains decreasing). Proposition 3, proved in the appendix, presents sufficient conditions for the above construction to yield the optimal allocation.

To avoid uninteresting complications, we now make assumptions on the conditional distributions \( F \) that are stronger than Assumption 1. We consider in particular the monotonicity of the hazard rate—a standard assumption in contract theory.

**Proposition 3.** Assume that any purchased unit must be consumed. Assume furthermore that one of the following conditions holds: (i) The conditional density \( f(\omega_E|s_E) \) is nondecreasing in \( \omega_E \); or (ii) The hazard rate \( f/(1-F) \) is nondecreasing in \( \omega_E \) and the elasticity of entry is bounded from below and from above by \( \bar{\varepsilon} \) and \( \varepsilon \) satisfying

\[
\beta(\bar{\varepsilon} - \varepsilon)^2 \leq 4\varepsilon \bar{\varepsilon}.
\]

Then the optimal boundary line \( \Psi \) can be constructed from the following properties:

\(^{13}\)Deneckere and Severinov (2009) propose a similar method for solving a general class of screening problems, which relies on a characterization of “isoquants”.
Figure 5a: The relaxed solution locally decreases with $s_E$.

Figure 5b: Entry barrier $\hat{\omega}_E$ for known $s_E$ (dashed). Optimal boundary $\Psi$ (solid)

1. $\Psi(\bar{s}_E) = \hat{\omega}_E(\bar{s}_E)$;

2. $\Psi = \hat{\omega}_E$ where $\Psi$ is non-increasing;

3. $\Psi$ is given by (16) where it is increasing.

Under hazard rate monotonicity, the construction yields the optimal allocation provided that the sufficient condition (17) holds, i.e., the range of the entry elasticity is not too wide. For instance, if the rival has all the bargaining power vis-à-vis the buyer ($\beta = 1$), the elasticity of entry $\varepsilon(\omega_E|s_E)$ may vary freely between 0.6 and 3 in the set of possible rival types, hence a fairly large elasticity range. The sufficient condition is even milder when $\beta$ is smaller.\footnote{When none of the two sufficient conditions of Proposition 3 holds, two-dimensional bunching may arise, and the first-order condition (16) must be adapted as explained in the appendix.}

Finally, we have seen that the condition $T'(q_I) \leq v_I$ is automatically satisfied when $\Psi$ is increasing at $1 - q_I$. When $\Psi$ is decreasing, the marginal price $T'$ is given by (A.5) with $\Psi(s_E) = \hat{\omega}_E(s_E)$ according to the above construction. It follows that $T' \leq v_I$ is satisfied if $\hat{\omega}_E$ is not too strongly decreasing, in particular it is true if $\hat{\omega}_E(s_E) + s_E\hat{\omega}'_E(s_E)$ is positive for any $s_E$. When this condition is violated, the optimal price schedule may have linear parts with slope $v_I$ so as to ensure that the buyer purchases enough units of good $I$ (recall Lemma A.1 in the appendix). We do not elaborate further on this technical issue because the competition concern that motivates this article is that the buyer purchases too many, rather than too few, units of incumbent good.

**Monotonic elasticity of entry** To illustrate the above construction, suppose that the elasticity of entry is monotonic in the contestable demand. The case where $\varepsilon$ is nondecreasing
in $s_E$ has been studied in Section 4. Here the boundary line solves the rent-efficiency tradeoff pointwise: $\Psi(s_E) = \hat{\omega}_E(s_E)$ for each $s_E$. Foreclosure is complete and is driven by the effective price, i.e., by the average price of contestable units.

By contrast, when the elasticity of entry $\varepsilon(\omega_E|s_E)$ decreases with $s_E$, the efficiency-rent tradeoff requires a higher entry barrier for larger competitors: the threshold $\hat{\omega}_E(s_E)$ is monotonically increasing in $s_E$. If $q_E$ were equal to $s_E$ above this threshold and zero below, the quantity purchased from the rival would locally decrease with $s_E$, which is not incentive compatible. Hence the presence of bunching along the $s_E$-dimension, such as the horizontal interval $(AC)$ represented on Figure 6a. A rival whose type belongs to $(AC)$ sells $s^1_E$ units, where $s^1_E$ denotes the left extremity of the bunching interval. The unit virtual surplus $s^v(s_E, \omega_E)$ is positive on $(AB)$ and negative on $(BC)$ as $\omega_E$ is respectively above and below $\hat{\omega}_E(s_E)$ on these intervals. The condition (16) that the expected virtual surplus is zero along bunching segments can be rewritten as

$$\int_{s_E}^{\hat{s}_E} s^v(s, \omega_E) f(\omega_E|s) g(s) \, ds = 0.$$ 

Under the sufficient conditions of Proposition 3, it defines an increasing relationship between $\omega_E$ and $s_E$, denoted by $\omega_E = \Psi(s_E)$ on Figure 6a, which yields the following result.

**Proposition 4.** Assume that any purchased unit must be consumed and that one of the sufficient conditions of Proposition 3 holds. Assume furthermore that $\varepsilon(\omega_E|s_E)$ decreases with $s_E$. Then the optimal price schedule is convex. The equilibrium outcome exhibits inefficient exclusion, in the form of both full and partial foreclosure.

When $T(q_I)$ is convex in $q_I$, the objective of the buyer-rival pair, $(\omega_E - v_I)q_E - T(1 - q_E)$, is concave in $q_E$. The buyer and the rival compare the surplus created by an extra unit of
rival good, \( \omega_E \), with the surplus foregone by consuming one unit less of incumbent good, \( v_I - T'(1 - q_E) \). The light-shaded area on Figure 6a represents the set of types for which the solution is interior, \( 0 < q_E(s_E, \omega_E) < s_E \), and hence the rival is partially foreclosed from the market, i.e., the quantity distortion is at the intensive margin. In this region, the price schedule prevents the rival from selling more units, acting as a barrier to expansion rather than to entry.

6 Retroactive rebates

So far, we have identified two different exclusionary scenarios. On the one hand, a concave price schedule imposes a barrier to entry whose height is negatively related to the effective price, i.e., to the average price of contestable units. If the rival can match that price, it serves the contestable demand; otherwise, it is driven out of the market. This scenario occurs when the elasticity of entry is monotonically increasing in the size of the contestable demand, recall Section 4. On the other hand, a convex schedule generates partial exclusion and plays the role of a barrier to expansion. The rival quantity is then driven by the marginal, rather than effective, price. This scenario occurs when the elasticity of entry is monotonically decreasing in the size of the contestable demand as explained at the end of Section 5.

We now combine the two scenarios to understand why and when “retroactive rebates” emerge in equilibrium. First we show that simple variations of the entry elasticity are sufficient to create highly nonlinear price schedules such as the one represented on Figure 7b.

Proposition 5. Assume that any purchased unit must be consumed and that one of the sufficient conditions of Proposition 3 holds. Assume furthermore that the elasticity of entry \( \varepsilon \) is a U-shaped function of \( s_E \) and that \( \hat{\omega}_E \) is smaller than \( v_I \) except for intermediate values of \( s_E \).

Then the price schedule changes curvature: increasingly generous rebates translate into negative marginal prices in an intermediate quantity range; for high quantities the schedule becomes convex and the marginal price moves up again.

Proof. The entry barrier \( \hat{\omega}_E \) that results from the rent-efficiency tradeoff is increasing in \( s_E \) for low values of \( s_E \), see the dashed line on Figure 7a. Starting from point \( A_2 \), \( \hat{\omega}_E \) decreases and hence by Proposition 3 coincides with the boundary line \( \Psi \), see the solid line up to point \( A_4 \) and beyond. The increasing part of \( \Psi \), represented by the solid line \((A_0A_2)\), is obtained from the condition that the expected virtual surplus is zero on the horizontal bunching intervals, equation (16).

Above the solid curve on Figure 7a, the competitor serves all of the contestable demand. In the light-shaded area below the solid curve, the quantity purchased from the competitor does not depend on the size of the contestable market. For instance, a rival whose type lies
on the horizontal segment \((A_1A_3)\) sells \(s^1_E\) units. On such an interval, the unit virtual surplus is negative at the right of the dashed line \(\omega_E = \hat{\omega}_E(s_E)\) and positive at its left.

Figure 7b shows the shape of the corresponding optimal price schedule. Between \(A_0\) and \(A_2\), the quantity chosen by the buyer and the competitor is an interior solution of their surplus maximization problem and is therefore given by the first-order condition: \(T'(1-s_E) = v_I - \Psi(s_E)\); the price-quantity schedule \(T\) is convex in this region. The marginal price is positive at \(q_I = 1\) if and only \(A_0\) is below \(v_I\) or equivalently \(\int_0^{s^2_E} s^\nu(s;v_I)f(v_I|s)g(s)\, ds > 0\).

Between \(A_2\) and \(A_4\), we recover the tariff by expressing that the quantity purchased from the rival is constant on the bunching segments. For example, if the rival is at \(A_3\), the buyer-rival pair is indifferent between buying \(s^1_E\) or \(s^3_E\): 

\[
(\omega_E - v_I)s^1_E - T(1-s^1_E) = (\omega_E - v_I)s^3_E - T(1-s^3_E).
\]

As \(T(1-s^1_E)\) is known, one can infer \(T(1-s^3_E)\). At points \(A_1\) and \(A_3\), we have \(\omega_E = v_I\), and hence \(T(1-s^1_E) = T(1-s^3_E)\).

The schedule changes curvature at point \(A_2\), which therefore is an inflection point, see property 4 of Lemma A.3. On Figure 7b, we see that the schedule is concave below \(A_2\) and convex beyond.

When the elasticity of entry is first decreasing then increasing as the size of the contestable demand rises, competitors with intermediate size are less sensitive to competitive pressure than competitors with small or large size. Under this circumstance, the efficiency-rent tradeoff leads the buyer and the incumbent to place strong competitive pressure on competitors with intermediate size and less pressure on small or large competitors: the entry barrier \(\hat{\omega}_E(s_E)\) is hump-shaped as shown on Figure 7a. This pattern generates inefficient partial exclusion for all types in the light-shaded area shown on Figure 7a. Exclusion is complete for types in
the dark-shaded area, and is inefficient for types with $\omega_E > \omega_I$. The decreasing part of the schedule gives the buyer a strong incentive to supply much of her purchase requirements from the incumbent, picking a point near $A_1$ in the schedule, see Figure 7b.

![Figure 8a: Quantity allocation under a retroactive rebate](image1)

![Figure 8b: Price schedule with retroactive rebate](image2)

**Retroactive rebates** We now show that retroactive rebates naturally emerge as a limiting case of the non-monotonic configuration presented in Proposition 5 when the distribution of uncertainty has mass points.

Retroactive rebates, also known as “all-units discounts”, have attracted much attention from antitrust agencies. They apply to all the purchased units provided that the buyer reaches a certain quantity threshold. Hence, contrary to incremental rebates, they induce downward discontinuities in price-quantity schedules. For instance, under the schedule shown on Figure 8b, potentially contestable units below the threshold $1 - \tilde{s}$ are sold at unit price $t_1$, while above that threshold, all potentially contestable units are sold at price $t_2 < t_1$. We denote by $\Delta > 0$ the absolute value of the resulting downward price discontinuity at $1 - \tilde{s}$.

We first consider the buyer-rival problem assuming that the retroactive rebate of Figure 8b is offered. Figure 8a shows the induced rival quantity. When the contestable demand is small, $s_E < \tilde{s}$, the rival (e.g., type $B_1$) sells all contestable units if it can match the effective price $t_2$, i.e., if the rival unit surplus $\omega_E$ is greater than $v_I - t_2$; otherwise it is driven out of the market. Small rivals, therefore, are not affected by the specific form of the rebate; they behave as under a two-part tariff with slope $t_2$. This part of the allocation follows the same pattern as in Figure 1a and 1b.

By contrast, when the contestable demand is large, $s_E > \tilde{s}$, the buyer-rival coalition faces more complex incentives: either the rival serves all the contestable demand, or sells $\tilde{s}$ units.
(light-shaded area), or is inactive. More precisely, writing that the buyer-rival surplus $S_{BE} = \omega_E q_E + v_I (1 - q_E) - T (1 - q_E)$ is equal for $q_E = \bar{s}$ and $q_E = s_E$, we get the equation of the upper boundary of the light-shaded area:

$$(\omega_E + t_1 - v_I)(s_E - \bar{s}) = \Delta.$$  

(18)

For a type $(s_E, \omega_E)$ that satisfies (18), the net surplus generated by $s_E = \bar{s}$ rival units in excess of $\bar{s}$ exactly offset the foregone rebate $\Delta$. Rivals of types $B_2$ and $B_3$, as well as all types in the light-shaded area, are “trapped” at the threshold $\bar{s}$ of the retroactive rebate, while rival of type $B_4$ brings a sufficiently high unit surplus to serve all of its contestable demand.\textsuperscript{15}

Second, we check the converse of the above property, namely the fact that the retroactive rebate of Figure 8b is indeed optimal for certain distributions of uncertainty. Suppose that the distribution $f(\omega_E | s_E)$ is such that the entry barrier $\hat{\omega}_E$ that results from the rent-efficiency tradeoff is $v_I - t_2$ for $s_E < \bar{s}$ and satisfies (18) for $s_E > \bar{s}$. Then the virtual surplus is negative below $v_I - t_2$, hence the optimal quantity is $q_E = 0$ in this region. The virtual surplus is negative in the interior of the light-shaded area, therefore to respect bunching conditions it must be positive along the vertical left boundary $s_E = \bar{s}$, and the distribution of the contestable share must have a positive mass at this point. We check in the appendix that we can choose the distribution of uncertainty such that the expectation of the virtual surplus is zero on any horizontal interval crossing the light-shaded area from its left end to its right end, which guarantees that the expected virtual surplus is maximized separately for each value of $\omega_E$ as in the constructive method of Proposition 3, and therefore that the allocation represented on Figure 8a is optimal.

7 Disposal costs

The analysis so far has assumed that the buyer must consume any purchased unit. We now relax this assumption, allowing the buyer to dispose of unused incumbent units at cost $\gamma_I$ (per discarded unit). The buyer might even be able to resell units on a secondary market, in which case $\gamma_I$ would be negative, but reselling is assumed to be inefficient, i.e., $\gamma_I$ cannot be lower than $-c_I$. The above analysis implicitly assumed that $\gamma_I$ was infinite. A formal definition of the utility function $V$ is given in the appendix.

To illustrate, suppose that the rival of type $B_3$ represented on Figure 8a brings a unit surplus $\omega_E^3$ that is higher than $v_I + \gamma_I$. This means that the buyer and the rival are jointly better off trading all contestable units ($q_E = s_E^3$) while the buyer still purchases $q_I = 1 - \bar{s}$ from the dominant firm and pockets the retroactive rebate, but now discards $s_E^3 - \bar{s}$ units

\textsuperscript{15}The configuration of Figures 8a and 8b assumes that for $(s_E, \omega_E) = (\bar{s}_E, v_I - t_2)$ the coalition surplus $S_{BE}$ is higher for $q_E = \bar{s}$ than for $q_E = \bar{s}_E$, which occurs when $(t_1 - t_2)(\bar{s}_E - \bar{s}) < \Delta$. 

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of incumbent good. Doing so, the buyer foregoes utility $v_I$ and incurs disposal cost $\gamma_I$ per discarded unit, but together with the rival earns $\omega_E^3$ per unit of rival good, hence a net gain for the coalition. It follows that the allocation represented on Figure 8a is no longer implementable.

More generally, we say that rivals such that $\omega_E > v_I + \gamma_I$ are super-efficient. Efficiency requires that the buyer supplies all of the contestable part of her demand from the rival, and, if need be, disposes of the purchased units of incumbent good in excess of $1 - s_E$. In other words, the conditionally efficient quantity of rival good, $q_E(q_I; s_E, \omega_E)$, is equal to $s_E$ however large $q_I$ becomes. Hence, as $q_E$ is efficient given $q_I$, we necessarily have $q_E = s_E$ for super-efficient rivals, which implies that the boundary line must satisfy $\Psi \leq v_I + \gamma_I$. The converse property is shown in Lemma A.5: if the inequality $\Psi \leq v_I + \gamma_I$ holds, then the associated price schedule satisfies $T' \geq -\gamma_I$, and therefore it is never ex post profitable for the buyer to purchase and discard a unit of incumbent good.

In fact, we know from Choné and Linnemer (2015) that the possibility of buyer opportunism makes it suboptimal to offer marginal prices below $-\gamma_I$. Starting from any price schedule $T$, it is optimal for the dominant firm to offer the buyer the possibility of purchasing less units and of paying the corresponding disposal costs directly to the dominant firm, i.e., to replace $T$ with the modified schedule

$$\hat{T}(q_I) = \inf_{q \geq q_I} T(q) + \gamma_I(q - q_I).$$  \hspace{1cm} (19)

Figure 9a: Optimal allocation under finite disposal costs with increasing elasticity of entry

Figure 9b: Optimal price schedule under finite disposal costs

**Proposition 6.** Under the sufficient conditions of Proposition 3, the magnitude of the disposal costs, $\gamma_I$, affects the optimal allocation as follows:

1. When $\gamma_I$ is larger than $\max \hat{\omega}_E - v_I$, the second-best allocation is the same as if $\gamma_I = \infty$;
2. When \( \gamma_I \) is smaller than \( \max \omega_E - v_I \), the optimal non-conditional schedule \( T \) corresponding to \( \gamma_I = \infty \) should be trimmed according to (19).

The expected profit of the buyer-incumbent pair, \( E\Pi_{BI} \), is nondecreasing and total welfare is non-increasing in \( \gamma_I \).

Proof. According to the above discussion, the buyer’s ex post temptation to purchase units of incumbent good with the sole purpose of pocketing the rebates translates into the constraint \( \Psi \leq v_I + \gamma_I \). Under the sufficient conditions of Proposition 3, the expected surplus is maximized separately for each \( \omega_E \), which makes the above constraint easy to accommodate. If the entry barrier \( \hat{\omega}_E \) that results from the rent-efficiency tradeoff does not lead to exclude super-efficient rivals, i.e., if \( \hat{\omega}_E \leq v_I + \gamma_I \), then the boundary line \( \Psi \) is located below \( v_I + \gamma_I \) as well, and the finiteness of disposal costs does not change the allocation.

If on the other hand \( \hat{\omega}_E \) sometimes exceeds \( v_I + \gamma_I \), the unconstrained boundary line \( \Psi \) exceeds \( v_I + \gamma_I \) on some intervals. On those intervals, it must be replaced with \( v_I + \gamma_I \), and the resulting schedule is linear with \( T' = -\gamma_I \) (see Lemma A.3 item 1). The constructive method applies for \( \omega_E \) below \( v_I + \gamma_I \) in an unchanged manner, so \( \Psi(s_E) \) must simply be replaced with \( \min(\Psi(s_E), v_I + \gamma_I) \). As shown on Figures 9a and 9b, this change trims portions of the schedule that decrease more rapidly than \( -\gamma_I \), in such a way that the marginal price of contestable units is everywhere above \( -\gamma_I \).

In sum, the buyer’s ability to dispose of unneeded units of incumbent good limits the competitive pressure that can be placed on the rival, protecting super-efficient rivals from exclusion. Lower disposal costs, therefore, reduce the extent of inefficient market foreclosure. In the limiting case where \( \gamma_I = -c_I \), the constraint \( T'(q_I) \geq -\gamma_I \) leaves no scope for anticompetitive exclusion. \( \square \)

8 Discussion

We have examined the exclusionary effects of nonlinear pricing by a dominant firm when (i) only a fraction of a buyer’s demand is potentially contestable; (ii) the dominant firm and the buyer have a negotiation opportunity before the rival unit surplus and the contestable share of demand are known. Under these circumstances, we have shown that the dominant firms sells potentially contestable units below costs, which causes efficient competitors to be foreclosed from the market.

We now discuss the implications of our findings for the antitrust standard applicable to nonlinear pricing by dominant firms and consider possible regulatory remedies to limit the associated exclusionary effect. As a preliminary observation, we recall from Section 7 that higher disposal costs are associated with a higher surplus for the dominant firm and the buyer.
These two parties thus have an incentive to artificially increase disposal costs, for instance by agreeing on provisions that allow the dominant firm to monitor the use of purchased units by the buyer and to prevent her from reselling unused units on a secondary market. Also, we know from Chöne and Linnemer (2015) that letting the price of incumbent good depend on the supply from the rival is a way to render disposal costs infinite. As higher disposal costs are associated with more inefficient exclusion, antitrust authorities should pay close attention to contracting provisions that help increase them.

We devote the following discussion to the legal treatment of the “non-conditional schedules” studied in the present article, focusing particularly on two issues: the pricing of contestable units below cost and the sharing of the surplus between the dominant firm and the buyer. Below-cost pricing for contestable units generates a gain for the buyer –through an improved bargaining position vis-à-vis the rival– but losses for the dominant firm; at the same time, the buyer and the dominant firm share the surplus created by the non-contestable units.

**Surplus-sharing problem** In practical cases, consistently with the notion of “unavoidable trading partner”, only a modest fraction of the buyer’s demand may be potentially contestable, i.e., \( \bar{s}_E \) may be relatively small (around 10% or 20%). Furthermore, although we have assumed in Section 2 the same marginal utility \( v_I \) for any unit of incumbent good, it might well be the case that the first units of incumbent good have a much higher value for the buyer, which, again, would be consistent with the dominant firm’s unavoidability.\(^{16}\) It follows that the surplus created by non-contestable units might well be much higher than that created by contestable ones. Hence, if the relative bargaining positions of the buyer and the dominant firm are not too unbalanced, the transfer caused by below-cost pricing for contestable units is easily absorbed by adjusting the sharing of the surplus for non-contestable ones.

Only in extreme cases does the sharing of the expected surplus between the dominant firm and the buyer involve selling the non-contestable units at a price \( T(1 - \bar{s}_E) \) that is above the buyer valuation for those units. To prevent such supra-monopolistic prices, Ide, Montero, and Figueroa (2015) suggest imposing an “easy terminability constraint” whereby the buyer could terminate the contract without any breach penalty. Yet contract termination often is a risk that the buyer cannot afford in practice, precisely because the dominant firm is an unavoidable supplier. Moreover, easy terminability does not prevent hidden up-front transfers from the buyer to the dominant firm. Long-term business relationships and a multiplicity of on-going contracts give the parties many opportunities for such transfers, which competition agencies cannot monitor. More generally, regulators are ill-equipped to assess the sharing of the surplus between commercial partners because such an assessment would require knowing

\(^{16}\)The model is easily adapted to accommodate any utility function that is strictly concave up to \( 1 - \bar{s}_E \), and linear with slope \( v_I \) beyond that point.
their valuations for any traded product, their outside options, and their bargaining power.

**As-efficient competitor test and legal standard** More relevant to policy intervention is the issue of below-cost pricing for contestable units. Indeed, the proper use of the so-called “as-efficient competitor test” in competition cases has been subject of intense discussions among practitioners for a couple of years. Generally speaking, the test asks whether a hypothetical rival firm, having the same production costs and selling a product of similar quality as the dominant firm, could profitably match the price offered by that firm.

The first instance of the test has been introduced by Areeda and Turner (1975) for predatory pricing. In this context, the underlying theory of harm involves short-term profit-sacrifice outweighed by long-run monopoly profits. The legal standard set in the United States by the Brooke Group Judgment of the Supreme Court is very high as it requires the plaintiff to prove that the prices were below an appropriate measure of defendant’s costs in the short term and that the defendant had a “dangerous probability of recouping” its investment in below-cost prices.\(^{17}\) The standard is not as high in Europe where proof of recoupment of losses is not required to determine predation.\(^{18}\)

The profit-sacrifice test has subsequently been extended to address strong asymmetries between the dominant firm and its rivals, such as the existence of non-contestable units. When the customer must carry some percentage of the leading firm’s products, we have provided a static theory of harm where the contestable units are sold below costs and the surplus extracted from the rival is shared between the dominant firm and the buyer. Our results support Fumagalli and Motta (2015)’s point that rebates combined with strong asymmetries have a high exclusionary potential and call for a different legal treatment than predation. Surplus sharing in our anticompetitive scenario –instead of intertemporal recoupment in predation cases– makes below-cost pricing profitable. Yet we have shown that assessing surplus sharing is virtually impossible for antitrust enforcers due to lack of information.

Our findings therefore suggest that the exclusionary analysis should be key in the legal treatment of nonlinear pricing when part of the demand is non-contestable. For this purpose, it is natural to modify the Areeda-Turner test and check whether the price of contestable units covers their cost. This extended test has been implemented by the European Commission in *Intel* for the first time and has generally received support from economists, see e.g. Shapiro and Hayes (2006).

\(^{17}\) *Brooke Group v. Brown & Williamson Tobacco Corp.*, 509 U.S. 209, 226 (1993). Since then, according to Hovenkamp (2014), few plaintiffs have won a predatory case, and the incidence of classical predatory pricing claims has declined dramatically.

\(^{18}\) *France Telecom*, European Court of Justice, 2 April 2009, Case C-202/07 P.
Test implementation under uncertain contestable share  On the other hand, the difficulty to define contestable units has been pointed out by many observers, and used to discard the as-efficient competitor test. For instance, Wright (2013) believes that the test would be hard to administer, emphasizing “the difficult question of how to define contestable units” in practice. He concludes that “a court should not focus on whether the defendant’s discounting has resulted in prices below cost.” This view is shared by the General Court of the European Union whose Intel judgment disregarded the as-efficient competitor analysis carried out by the Commission, sticking to the legal view that “exclusivity rebates” by a dominant firm are banned per se.19

We have argued in this article that the contestable share of demand should not be seen as a fixed number but rather as a random variable. Accordingly, instead of looking for a precise number, it is sensible to consider a range of values. One way of proceeding along these lines is to use the “minimum required share”, $s_{mr}$, defined by the European Commission as the lowest value of $s$ such that the effective price $[T(1) - T(1 - s)]/s$ is above $c_I$. In words, $s_{mr}$ is the minimum number of units that an as-efficient competitor must sell to overcome the rebates implemented by the dominant firm. Comparing the values of $s_{mr}$ and $\bar{s}_E$ yields a concrete assessment of exclusionary effects from an ex ante perspective. For instance, suppose that $s_{mr}$ is found to be clearly above $\bar{s}_E$. This means that under any reasonable expectation the buyer could not realistically switch to the rival a portion of her purchase requirements close to $s_{mr}$. We believe that such a finding should be seen as a convincing measure of the potential foreclosure effects of the rebate scheme. If on the contrary $\bar{s}_E$ is clearly below $s_{mr}$, no evidence of potential foreclosure is present.

The uncertainty inherent to the contestable share of demand can be addressed in a pragmatic manner and cannot justify a formalistic approach to rebates. Formalism in this matter is inappropriate for many reasons: the huge diversity of rebate schemes observed in practice, the complexity of their effects in various economic environments, and the long list of efficiency gains that rebates can bring about. Although many considerations relevant for legal standards (the administration of the antitrust system, the costs of judicial errors, the risk of under-deterrence and over-deterrence, etc.) are left out of the scope of the present article, we believe that our findings, at the very least, support the use of price-cost tests to assess the potential exclusionary effects of nonlinear pricing by dominant firms with must-carry brands or products.

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19See Footnote 3.
References


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A Appendix

Buyer utility function under finite disposal costs The buyer utility function is formally written as

$$V(q_E, q_I; s_E, v_E) = \max_{(x_E, x_I) \in X(q_E, q_I)} v_E x_E + v_I x_I - \gamma_E (q_E - x_E) - \gamma_I (q_I - x_I)$$ (A.1)

where $X(q_E, q_I)$ includes the constraints $x_E \leq q_E$, $x_I \leq q_I$, $x_E + x_I \leq 1$, and $x_E \leq s_E$, i.e., the buyer cannot consume more of each good than the purchased quantity, more of both goods together than her total requirement, and more of good $E$ than the contestable demand.\footnote{For the sake of completeness, we allow the buyer to dispose of units of rival good at a unit cost $\gamma_E$, see (A.1). Yet as the negotiation between the buyer and the rival is efficient, they never trade more than $s_E$ units, and hence no unit of rival good is disposed of. It follows that the disposal cost $\gamma_E$ plays no role in the analysis.}

We prove the concavity of $V$ in the vector $(q_E, q_I)$. Consider two vectors $q^0 = (q^0_E, q^0_I)$, $q^1 = (q^1_E, q^1_I)$ and a weight $\alpha$ with $0 \leq \alpha \leq 1$. Let $x^0 = (x^0_E, x^0_I)$ and $x^1 = (x^1_E, x^1_I)$ denote the consumption levels chosen by the buyer after having purchased $q_0$ and $q_1$. By linearity, $\alpha x^0 + (1 - \alpha) x^1$ belongs to the set $X(\alpha q^0 + (1 - \alpha) q^1)$. The result follows immediately.

Demand is satisfied for any buyer type

Lemma A.1. The buyer and the incumbent are better off using a tariff with slope $T'$ smaller than or equal to $v_I$. Consequently, we may assume, with no loss of generality, that the buyer does not buy less than her total requirements: $q_E + q_I \geq 1$ for any $(s_E, \omega_E)$. The result holds irrespective of whether or not the buyer has the ability to dispose of unconsumed units.

Proof. Suppose first that any purchased unit must be consumed. Given that the buyer and the rival firm choose quantities $q_I$ and $q^*_E(q_I; s_E) = \min(1 - q_I, s_E)$, their joint gross surplus is

$$\hat{S}_{BE}(q_I; s_E, \omega_E) = v_I q_I + \omega_E \min(1 - q_I, s_E).$$

When $q_I$ is below $1 - s_E$, the quantity purchased from the rival is constant, $q^*_E = s_E$, and hence $\hat{S}_{BE} = v_I q_I + \omega_E s_E$. When $q_I$ is above $1 - s_E$, $\hat{S}_{BE} = \omega_E + (v_I - \omega_E) q_I$. We conclude that the buyer-rival pair’s valuation of a unit of good $I$, $\partial \hat{S}_{BE}/\partial q_I$, never exceeds $v_I$.

Now, starting from any price schedule $T$, we introduce the modified schedule $\tilde{T}$ given by

$$\tilde{T}(q_I) = \inf_{q \leq q_I} T(q) + v_I (q_I - q).$$ (A.2)

The tariff $\tilde{T}$ is derived from the tariff $T$ as follows. When the incumbent offer $q$ units at price $T(q)$, he also offers to sell more units than $q$, say $q_I > q$, at price $T(q) + v_I (q_I - q)$. The additional units are offered at the monopoly price $v_I$. By construction, the slope of $\tilde{T}$ is lower than or equal to $v_I$. 

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Lemma A.2. Characterization of implementable allocations

Because $\tilde{T}$ is lower than or equal to $T$, the buyer and the rival cannot be worse off after the change. On the other hand, they cannot be better off either because the modified schedule $\tilde{T}$ offers them the possibility to purchase extra units at monopoly price $v_I$ while their valuation for those units cannot exceed $v_I$. It follows from these two observations that the joint net surpluses $\max_q, \tilde{S}_{BE} - \tilde{T}$ and $\max_q, \tilde{S}_{BE} - T$ must be equal. For the same reason, the corresponding equality holds under exclusive supply, $\max_q, V(0, q_I) - \tilde{T} = \max_q, V(0, q_I) - T$. We conclude that the rival profit $\Pi_E$ is the same under the schedules $T$ and $\tilde{T}$.

Moreover, suppose that the buyer demand is not satisfied $q_E^* + q_I < 1$. When this happens, we have $q_E^* = s_E$ and $q_I < 1 - s_E$. Under the modified schedule $\tilde{T}$, the buyer has a weak incentive to increase her purchase of incumbent good from $q_I$ to $1 - s_E$, leaving $q_E^* = s_E$ and $\Pi_E$ unchanged. This change increases the total welfare $W$ because each extra unit of incumbent good increases the surplus by $\omega_I > 0$. In sum, the change from $T$ to $\tilde{T}$ does not alter the entrant’s profit and does not decrease the total surplus. We conclude from (5) that the change does not decrease the expected payoff of the buyer-incumbent coalition.

The proof carries over with little change when the buyer can dispose of unconsumed units of good $I$ at unit cost $\gamma_I$. In this situation, the joint gross surplus of the buyer and the rival is

$$\tilde{S}_{BE}(q_I; s_E, \omega_E) = v_I q_I + \omega_E q_E^*(q_I; s_E, \omega_E) - \gamma_I \max(q_E^*(q_I; s_E, \omega_E) + q_I - 1, 0).$$

When $q_I$ is above $1 - s_E$, two cases may happen: (i) if the rival is not super-efficient, $q_E^* = 1 - q_I$; (ii) if the rival is super-efficient, $q_E^* = s_E$. In both cases, $\tilde{S}_{BE}$ is equal to the sum of $v_I q_I$ and of a function that decreases with $q_I$, hence $\partial \tilde{S}_{BE} / \partial q_I < v_I$ in this region. We conclude that the buyer-rival pair’s valuation of a unit of good $I$, $\partial \tilde{S}_{BE} / \partial q_I$, never exceeds $v_I$. The rest of the proof is unchanged. \hfill \Box

Characterization of implementable allocations

Lemma A.2. A quantity function $q_E(., .)$ is implementable if and only if there exists a boundary function $\Psi(.)$ defined on $[0, 1]$ such that

$$q_E(s_E, \omega_E) = \begin{cases} 
\min \{ x \leq s_E \mid \Psi(y) \geq \omega_E \text{ for all } y \in [x, s_E] \} & \text{if } \Psi(s_E) > \omega_E, \\
\min s_E & \text{if } \Psi(s_E) \leq \omega_E.
\end{cases} \quad (A.3)$$

When (6) has multiple solutions, the above definition selects the highest. For instance, type $C$ on Figure 4 is indifferent between $s_E^2$ and $s_E^4$ and, by convention, is assumed to choose $s_E^3$. Notice that the quantity function jumps from $s_E^2$ to $s_E^3$ at $C$; more generally, implementable quantity functions are discontinuous along decreasing parts of their boundary line.

Proof. We first observe that $(v_I - \omega_E)q_E(s_E, \omega_E) + \Delta S_{BE}(s_E, \omega_E)$ is constant on $q_E$-isolines. Indeed, both $q_E(., .)$ and $\Delta S_{BE}(., .)$ are constant on horizontal isolines (located below the
boundary line $\Psi$). On vertical isolines (above the boundary), $\Delta S_{BE}(s_E,.)$ is linear with slope $\omega_E$, which ensures that the above expression is constant. This shows that (14) unambiguously defines $T(1) - T(1 - q)$ on the range of the quantity function $q_E(.,.)$.\footnote{Notice that the range of $q_E$ may be disconnected when $\Psi$ is above $\varpi_E$ on some intervals. Specifically, if $\Psi$ is above $\varpi_E$ on the interval $I = [s^1_E, s^2_E]$, then $q_E$ does not take any value between $s^1_E$ and $s^2_E$. In this case, we define $T$ as being linear with slope $v_I - \varpi_E$ on the corresponding interval: $T(1) - T(1 - q) = (v_I - \varpi_E)(q - s^1_E)$ for $q \in I$.}

We now prove that the buyer and the competitor, facing the above defined tariff $T$, agree on the quantity $q_E(s_E, \omega_E)$. We thus have to check that

$$\Delta S_{BE}(s_E, \omega_E) \geq (\omega_E - v_I)q' + T(1) - T(1 - q') \quad (A.4)$$

for any $q' \leq s_E$. When $q'$ is the range of the quantity function, we can write $q' = q_E(s'_E, \omega'_E)$ for some $(s'_E, \omega'_E)$, with $q' \leq s'_E$. Observing that $q' = q_E(q', \omega'_E)$ and using successively the monotonicity of $\Delta S_{BE}$ in $s_E$ and its convexity in $\omega_E$, we get:

$$\Delta S_{BE}(s_E, \omega_E) \geq \Delta S_{BE}(q', \omega_E) \geq \Delta S_{BE}(q', \omega'_E) + (\omega_E - \omega'_E)q',$$

which, after replacing $T(1) - T(1 - q')$ with its value from (14), yields (A.4). To check (A.4) when $q'$ is not in the range of the quantity function ($q'$ belongs to a hole $(s^1_E, s^2_E)$ as explained in Footnote 21), use (A.4) at $s^1_E$ and the linearity of the tariff between $s^1_E$ and $q'$.

\[\square\]

**From the boundary line to the price schedule**

**Lemma A.3.** The shape of the boundary line $\Psi$ and the curvature of the price schedule $T$ are linked in the following way:

1. If $\Psi$ is increasing (resp. constant) around $s_E$, then the tariff is strictly convex (resp. linear) around $1 - s_E$.

2. If $\Psi$ decreases and is concave around $s_E$, then the tariff is concave around $1 - s_E$.

3. If $\Psi$ decreases and is convex around $s_E$ and $s_E$ is close to a local minimum of $\Psi$, then the tariff is convex around $1 - s_E$.

4. If $\Psi$ has a local maximum at $s_E$, then the tariff has an inflection point at $1 - s_E$.

**Proof.** First, suppose that $\Psi$ is nondecreasing on a neighborhood of $s_E$. Let $s'_E$ slightly above $s_E$. Then $q_E = s_E$ is an interior solution of the buyer-rival pair’s problem (6) for $s'_E$ and $\omega_E = \Psi(s_E)$. It follows that the first order condition $\Psi(s_E) - v_I + T'(1 - s_E) = 0$ holds, implying property 1 of the lemma.
Next, suppose that the boundary line decreases around $s_E$. Here we assume that $\Psi$ is twice differentiable. We denote by $[\sigma(s_E), s_E]$ the set of value $s_E$ such that $q_E(s_E, \omega_E) = \sigma(s_E)$, where $\omega_E = \Psi(s_E)$. The buyer-rival surplus $\Delta S_{BE}(s_E, \omega_E)$ is convex and hence continuous in $\omega_E$. It can be computed slightly below or above $\Psi(s_E)$. At $(s_E, \Psi(s_E))$, the buyer and the rival are indifferent between quantities $s_E$ and $\sigma(s_E)$:

$$\Delta S_{BE}(s_E, \Psi(s_E)) = [\Psi(s_E) - v_I]s_E - T(1 - \sigma(s_E)) = [\Psi(s_E) - v_I]s_E - T(1 - s_E).$$

Differentiating and using the first-order condition at $\sigma(s_E)$ yields

$$T'(1 - s_E) = -\Psi'(s_E)[s_E - \sigma(s_E)] - \Psi(s_E) + v_I. \quad \text{(A.5)}$$

Differentiating again yields

$$T''(1 - s_E) = \Psi''(s_E)[s_E - \sigma(s_E)] + \Psi'(s_E)[2 - \sigma'(s_E)].$$

In the above equation, the two bracketed terms are nonnegative (use $\sigma' \leq 0$), and the slope $\Psi'$ is negative by assumption, which yields item 2 of the lemma. Around a local minimum of $\Psi$, $\Psi'$ is small, and the first term is positive, hence property 3. Property 4 follows from items 1 and 2. \hfill \Box \\

**Proof of Proposition 3**  
First, we offer a convenient parametrization of horizontal bunching intervals. Second, we state and prove a one-dimensional optimization result, which serves to maximize the expected virtual surplus for a given level of $\omega_E$. Third, we rewrite the complete problem as the maximization of the expected virtual surplus under monotonicity constraints. Finally, we show that these constraints are not binding under fairly mild conditions.

1) **Parameterizing horizontal bunching intervals**

Consider an implementable quantity function $q_E$. For any $\omega_E$, the function of one variable $q_E(\cdot, \omega_E)$ is nondecreasing on $[0, 1]$, being either constant or equal to the identity map: $q_E = s_E$. By convention, we call regions where it is constant “odd intervals”, and regions where $q_E = s_E$ “even intervals”.

We are thus led to consider any partition of the interval $[0, 1]$ into “even intervals” $[s_{2i}, s_{2i+1})$ and “odd intervals” $(s_{2i+1}, s_{2i+2})$, where $(s_i)$ is a finite, increasing sequence with first term zero and last term one.\footnote{For notational consistency, we denote the first term of the sequence by $s_0 = 0$ if the first interval is even and by $s_1 = 0$ if the first interval is odd. Similarly, we denote the last term by $s_{2n} = 1$ if the last interval is odd and by $s_{2n+1} = 1$ if the last interval is even.} We associate to any such partition the function of one variable that coincides with the identity map on even intervals, is constant on odd intervals, and is continuous at odd extremities. We denote by $K$ the set of the functions thus obtained.

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For any implementable quantity function \( q_{E} \), the functions of one variable, \( q_{E}(., \omega_{E}) \), belong to \( K \) for all \( \omega_{E} \). Conversely, any quantity function such that \( q_{E}(., \omega_{E}) \) belong to \( K \) for all \( \omega_{E} \) is implementable if and only if even (odd) extremities do not increase (decrease) as \( \omega_{E} \) rises. Hereafter, we call the conditions on the extremities the “monotonicity constraints”.

Even (odd) extremities constitute decreasing (increasing) parts of the boundary line. Odd intervals, \( [s_{2i+1}, s_{2i+2}] \), constitute horizontal bunching segments, or, more precisely, the horizontal portions of the L-shaped bunching regions.

ii) A one-dimensional optimization result

Lemma A.4. Let \( a(.) \) be a continuous function on \([0, 1]\). Then the problem

\[
\max_{r \in K} \int_{0}^{1} a(s)r(s) \, ds
\]

admits a unique solution \( r^{*} \) characterized as follows. For any interior even extremity \( s_{2i}^{E} \), the function \( a \) equals zero at \( s_{2i}^{E} \) and is negative (positive) at the left (right) of \( s_{2i}^{E} \). For any interior odd extremity \( s_{2i+1}^{E} \), the function \( a \) is positive at \( s_{2i+1}^{E} \) and satisfies

\[
\int_{s_{2i+1}^{E}}^{s_{2i+2}^{E}} a(s) \, ds = 0. \tag{A.6}
\]

If \( a(1) > 0 \), then \( r^{*}(s) = s \) at the top of the interval \([0, 1]\). If \( a(1) < 0 \), then \( r^{*} \) is constant at the top of the interval.

Proof. Letting \( I(r) = \int_{0}^{1} a(x)r(x) \, dx \), we have

\[
I(r) = \sum_{i} \int_{x_{2i}}^{x_{2i+1}} xa(x) \, dx + \sum_{i} x_{2i+1} \int_{x_{2i+1}}^{x_{2i+2}} a(x) \, dx,
\]

where the index \( i \) in the two sums goes from either \( i = 0 \) or \( i = 1 \) to either \( i = n - 1 \) or \( i = n \), in accordance with the conventions exposed in Footnote 22. Differentiating with respect to an interior even extremity yields

\[
\frac{\partial I}{\partial x_{2i}} = a(x_{2i})[x_{2i-1} - x_{2i}].
\]

The first-order condition therefore imposes \( a(x_{2i}^{*}) = 0 \). The second-order condition for a maximum shows that \( a \) must be negative (positive) at the left (right) of \( x_{2i}^{*} \).

Differentiating with respect to an interior odd extremity yields

\[
\frac{\partial I}{\partial x_{2i+1}} = \int_{x_{2i+1}}^{x_{2i+2}} a(x) \, dx.
\]

The first-order condition therefore imposes \( \int_{x_{2i+1}}^{x_{2i+2}} a(x) \, dx \). The second-order condition for a maximum imposes that \( a \) is nonnegative at \( x_{2i+1}^{*} \).
If $a(1) > 0$, then it is easy to check that $r^*(x) = x$ at the top, namely on the interval $[x_{2n}^*, x_{2n+1}^*]$ with $x_{2n}^*$ being the highest zero of the function $a$ and $x_{2n+1}^* = 1$. If the function $a$ admits no zero, it is everywhere positive and hence $r^*(x) = x$ on the whole interval $[0, 1]$.

If $a(1) < 0$, then $r^*$ is constant at the top, namely on the interval $[x_{2n}^* - 1, x_{2n}^*]$, with $x_{2n}^* = 1$ and $\int_{x_{2n}^* - 1}^{1} a(x) \, dx = 0$. If the integral $\int_{y}^{1} a(x) \, dx$ remains negative for all $y$, then $r^*$ is constant and equal to zero on the whole interval $[0, 1]$.

### iii) Solving the complete problem

The complete problem consists in maximizing the expected virtual surplus subject to the even (odd) extremities being nonincreasing (nondecreasing). The latter conditions are called hereafter the “monotonicity constraints”.

Applying Lemma A.4 with $a(s_E) = s(v(s_E, \omega_E))$ for any given $\omega_E$, we find that the virtual surplus is zero at candidate even extremities: $s(v(x_{2i}(\omega_E), \omega_E)) = 0$ and is negative (positive) at the left (right) of these extremities. In other words, $\Psi = \hat{\omega}_E$ at candidate even extremities. Thus, as regards even extremities, the monotonicity constraints are never binding.

Lemma A.4 also implies that the virtual surplus is positive at odd extremities. At these extremities, we must therefore have $\Psi > \hat{\omega}_E$. By the first-order condition (A.6), the expected virtual surplus is zero on horizontal bunching intervals:

$$E(s^v|H) = 0, \quad (A.7)$$

where $H$ is a horizontal bunching interval with extremities $s_{E}^{2i+1}$ and $s_{E}^{2i+2}$. The virtual surplus on a bunching interval is first positive, then negative as $s_E$ rises, and its mean on the interval is zero. The segment $(AB)$ on Figure 5b is an example of horizontal bunching interval (in fact the horizontal part of an “L”-shaped bunching set). Unfortunately, the first-order condition (A.7) does not imply that candidate odd extremities $x_{2i+1}(\omega_E)$ are nondecreasing in $\omega_E$: odd extremities might decrease with $\omega_E$ in some regions, generating two-dimensional bunching.

### iv) Sufficient conditions

We now check that each of the three conditions mentioned in Proposition 3 is sufficient for the odd extremities $s_{E}^{2i+1}(\omega_E)$ to be nondecreasing in $\omega_E$. We can restrict attention to efficient rival types, $\omega_E \geq \omega_I$.\(^{23}\) We rewrite equation (A.7) as $A(s_{E}^{2i+1}, \omega_E) = 0$ with

$$A(s_{E}^{2i+1}, \omega_E) = \int_{s_{E}^{2i+1}}^{s_{E}^{2i+2}} s^v(s, \omega_E) f(\omega_E|s) g(s) \, ds$$

$$= \int_{s_{E}^{2i+1}}^{s_{E}^{2i+2}} [(\omega_E - \omega_I) f(\omega_E|s) - \beta(1 - F(\omega_E|s))] g(s) \, ds.$$\(^{23}\)For $\omega_E < \omega_I$, the virtual surplus is negative for all $s_E$ and the solution is $q_E = 0$ for all $s_E$. 35
The function $A$ is nonincreasing in $s_E^{2i+1}$, as the virtual surplus is nonnegative at this point:
\[
\frac{\partial A}{\partial s_E^{2i+1}}(s_E^{2i+1}, \omega_E) = -s^v(s_E^{2i+1}, \omega_E)f(\omega_E|s_E^{2i+1})g(s_E^{2i+1}) \leq 0.
\]

Differentiating with respect to $\omega_E$, we get
\[
\frac{\partial A}{\partial \omega_E}(s_E^{2i+1}, \omega_E) = \int_{s_E^{2i+1}}^{s_E^{2i+2}} [(\omega_E - \omega_I)f'(\omega_E|s) + f(\omega_E|s) + \beta f(\omega_E|s)]g(s) \, ds,
\]
where we denote by $f'$ the derivative of $f$ in $\omega_E$.

When $f$ is nondecreasing in $\omega_E$, or $f' \geq 0$, we have $\partial A/\partial \omega_E \geq 0$, and hence the odd extremities are nondecreasing in $\omega_E$. We now examine successively the cases where the hazard rate is nondecreasing in $\omega_E$ (a weaker condition than $f' \geq 0$) and the elasticity of entry is nondecreasing in $\omega_E$ (an even weaker condition).

We now assume that the hazard rate, $f/(1-F)$, is nondecreasing in $\omega_E$, which can be expressed as $f' \geq -\varepsilon f/\omega_E$. Using $\omega_E \geq \omega_I$, we find that
\[
\frac{\partial A}{\partial \omega_E} \geq \int_{s_E^{2i+1}}^{s_E^{2i+2}} \left[ -\frac{(\omega_E - \omega_I) \varepsilon}{\omega_E} + 1 + \beta \right] f(\omega_E|s)g(s) \, ds
\]
\[
= \int_{s_E^{2i+1}}^{s_E^{2i+2}} \left\{ \varepsilon \left[ \frac{\omega_I}{\omega_E} - 1 + \frac{\beta}{\varepsilon} \right] + 1 \right\} f(\omega_E|s)g(s) \, ds.
\]

On a horizontal interval $H$, the variable $\omega_E$ is constant, and only the elasticity $\varepsilon$ may vary. Hence, the first order condition (A.7) yields: $\mathbb{E}(1 - \beta/\varepsilon \mid H) = \omega_I/\omega_E$. The right-hand side of the above inequality is equal, up to a positive multiplicative constant, to $1 - \text{cov}(\varepsilon, 1 - \beta/\varepsilon \mid H)$.

We now look for a sufficient condition for this expression to be nonnegative for any distribution of $\varepsilon$. Noting $m = \mathbb{E}(\varepsilon \mid H)$ the expectation of $\varepsilon$ on $H$, the condition can be rewritten as
\[
\mathbb{E} \left[ (\varepsilon - m) \left( 1 - \frac{\beta}{\varepsilon} \right) \mid H \right] \leq 1.
\]

The function $(\varepsilon - m)(1 - \beta/\varepsilon)$ is convex in $\varepsilon$. We denote by $[\underline{\varepsilon}, \overline{\varepsilon}]$ the support of the distribution of $\varepsilon$. For given values of $\underline{\varepsilon}$, $\overline{\varepsilon}$ and $m = \mathbb{E}(\varepsilon \mid H)$, the expectation of this convex function is maximal when the distribution of $\varepsilon$ has two mass points at $\underline{\varepsilon}$ and $\overline{\varepsilon}$, associated with the respective weights $(\overline{\varepsilon} - m)/(\overline{\varepsilon} - \underline{\varepsilon})$ and $(m - \varepsilon)/(\overline{\varepsilon} - \underline{\varepsilon})$. We thus need to make sure that
\[
(\overline{\varepsilon} - m)(\underline{\varepsilon} - m) \left( 1 - \frac{\beta}{\underline{\varepsilon}} \right) + (m - \underline{\varepsilon})(\overline{\varepsilon} - m) \left( 1 - \frac{\beta}{\overline{\varepsilon}} \right) \leq \overline{\varepsilon} - \underline{\varepsilon},
\]
for any $m \in [\underline{\varepsilon}, \overline{\varepsilon}]$. The left-hand side of the above inequality is maximal for $m = (\underline{\varepsilon} + \overline{\varepsilon})/2$. It follows that the inequality holds for all $m \in [\underline{\varepsilon}, \overline{\varepsilon}]$ if and only if the condition (17) is satisfied.

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Two-dimensional bunching  When none of the two sufficient conditions of Proposition 3 holds, the above construction may not yield a function of $s_E$, as in the example shown on Figure 10a. In this example, the elasticity of entry is $\varepsilon = 1.75$ for $s_E > \bar{s}$ and $\bar{\varepsilon} = 9.5$ for $s_E < \bar{s}$, with $\bar{s} = .7$; the size of the contestable demand belongs to $(0, \bar{s})$ with probability .95 and to $(\bar{s}; 1)$ with probability .05, and is uniformly distributed on each of the two intervals. Finally $\omega_I = \omega_E = 1$. Under these circumstances, the optimal allocation features two-dimensional bunching, see the shaded region $D$ pictured on Figure 10b. The constant value of the rival quantity on the bunching region, denoted by $\hat{s}$ on the picture, is determined by the first-order condition that the expected unit virtual surplus is zero on that region, $\mathbb{E}(s|^D) = 0$.

Retroactive rebates  For simplicity, we assume $\beta = 1$. As explained in the text, we take the distribution $f(\omega_E|s_E)$ such that the entry barrier $\hat{\omega}_E$ is $\nu_1 - t_2$ for $s_E < \bar{s}$ and satisfies (18) for $s_E > \bar{s}$. The distribution of $s_E$ has to have a mass point at $\bar{s}$, and we denote the corresponding mass by $\mu$. Outside this point, we choose a density $g(s_E)$ such that the integral over the partial foreclosure region $P$ (the light-shaded area shown on Figure 8a)

$$J = \int_P (\omega_E - \omega_I) f(\omega_E|\bar{s}) - [1 - F(\omega_E|\bar{s})] g(s) \, ds \, d\omega_E < 0$$

exists and is finite.

We now construct a distribution $F(\omega_E|\bar{s})$ such the bunching condition (16) holds for all horizontal intervals crossing $P$ from one end to the other. In the present context, this condition
can be written as
\[
\mu \{(\omega_E - \omega_I)f(\omega_E|\bar{s}) - [1 - F(\omega_E|\bar{s})]\} = \\
- \int_{\bar{s}}^{s_E(\omega_E)} \{(\omega_E - \omega_I)f(\omega_E|s) - [1 - F(\omega_E|s)]\} g(s) \, ds,
\]  \tag{A.8}
where the function \( s_E(\omega_E) \) given by (18) and parameterizes the upper boundary of \( \mathcal{P} \). The virtual surplus along the vertical left boundary is positive to offset the negative contribution in the interior of \( \mathcal{P} \), i.e., both sides of (A.8) are positive.

Equation (A.8), which must hold for all \( \omega_E \) greater than \( \omega_E^0 = v_I - t_2 \), is a linear first-order ordinary differential equation with unknown function \( F(\omega_E|\bar{s}) \). Integrating between \( \omega_E^0 \) and \( +\infty \) and using the limit condition \( 1 - F(\infty) = 0 \), we get the initial value at \( \omega_E^0 \) through the equality \( \mu(\omega_E^0 - \omega_I) [1 - F(\omega_E^0|\bar{s})] = -J > 0 \). The mass \( \mu \), the density \( g \), and the value of \( J \) can be adjusted to make sure that the bracketed term is smaller than one as it should be.

Finally, inspecting (A.8) immediately yields \( f(\omega_E|\bar{s}) > 0 \), so we have constructed a well-defined distribution of \( \omega_E \) conditional on \( s_E = \bar{s} \).\footnote{For \( \omega_E < \omega_E^0 \), we may set \( f \) and \( F \) to zero so that \( (\omega_E - \omega_I)f(\omega_E|\bar{s}) - [1 - F(\omega_E|\bar{s})] \) is negative in this region.}

**Implementable allocation under finite disposal costs**

**Lemma A.5.** When the buyer can discard units of incumbent good at unit cost \( \gamma_I \), an allocation is implementable if and only if the associated boundary line \( \Psi \) satisfies \( \Psi \leq v_I + \gamma_I \).

**Proof.** We have already proved the necessary part. We show here that if the boundary line satisfies \( \Psi \leq v_I + \gamma_I \), then the associated price schedule satisfies \( T' \geq \gamma_I \) and hence there is no incentive to discard any unit of incumbent good. We recover the schedule using (6)
\[
T(1) - T(1 - s_E) = [v_I - \omega_E] s_E + \Delta S_{BE}(s_E, \omega_E),
\]  \tag{A.9}
with \( \omega_E = \Psi(s_E) \). Differentiating with respect to \( s_E \) and observing that the terms coming from \( \omega_E \) cancel out by the envelope theorem, we get
\[
T'(1 - s_E) = v_I - \Psi(s_E) + \frac{\partial \Delta S_{BE}}{\partial s_E}.
\]  \tag{A.10}
Since \( \Delta S_{BE} \) is nondecreasing in \( s_E \) and that \( \Psi \) is below \( \max \hat{\omega}_E \) (see Section 5), we get \( T'(1 - s_E) \geq v_I - \max \hat{\omega}_E \). It follows that when \( \gamma_I \) is larger than \( \max \hat{\omega}_E - v_I \), all units are sold at a price above \( -\gamma_I \), and the buyer has no incentive to purchase unneeded units. \( \square \)