Nonlinear pricing and exclusion:
I. Buyer opportunism

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Abstract

We study the exclusionary properties of nonlinear price-quantity schedules in an Aghion-Bolton style model with elastic demand and product differentiation. We distinguish three regimes depending on whether and how the price of the incumbent good is linked to the quantity purchased from the rival firm. We find that the supply of rival good is distorted downwards. Moreover, given the quantity supplied from the rival, the buyer may opportunistically purchase inefficiently many units from the incumbent to pocket quantity rebates. We show that the possibility for the buyer to dispose of unconsumed units attenuates the opportunism problem and limits the exclusionary effects of nonlinear pricing.

JEL codes: L12, L42, D82, D86

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1 Introduction

In recent years, exclusionary conduct by firms with strong market power has become a high-priority issue on the agenda of antitrust agencies. For instance, the European Commission has made it clear that the emphasis of its enforcement activities is on “ensuring that undertakings which hold a dominant position do not exclude their competitors by other means than competing on the merits of the products or services they provide.” The U.S. Department of Justice states that “whether conduct has the potential to exclude, eliminate, or weaken the competitiveness of equally efficient competitors can be a useful inquiry”, and suggests that this inquiry “may be best suited to particular pricing practices.”

In this article, we consider a wide range of pricing practices that fall under the general heading of nonlinear pricing, e.g. quantity or market-share rebates and exclusivity discounts. We organize a taxonomy of price schemes around the following main distinction: whether or not the price set by the dominant firm depends on the quantity supplied from rivals. When this is the case, we say that the dominant firm’s price schedule is “conditional” (on rival quantities). Market-share discounts enter into this category. Because enforcing a conditional price may be unfeasible or legally prohibited, we also consider the situation where the firms are restricted to use “non-conditional” price schedules. Finally we define “exclusivity-based” schedules as conditional schedules for which the price depends on whether or not the buyer supplies exclusively from the dominant firm, but does not otherwise depend on the quantities sold by competitors.

Our analysis aims to understand how these different types of price-quantity schedules affect the way large buyers split their purchase requirements between the dominant firm and rival suppliers. It relies on a game close to Aghion and Bolton (1987) with three players—an incumbent firm, a rival, and a buyer—where contract offers are sequential: first a price schedule is decided by the buyer and the incumbent; only then have the rival and the buyer a chance to interact. In the present paper, the incumbent’s market power is captured by the first-mover advantage and the corresponding commitment power. In a companion paper, Choné and Linnemer (2014), we introduce on top of incumbency the notion that the incumbent firm is, at least to some extent, an “unavoidable trading partner”—a key ingredient of dominance.
under European competition law at least since *Hoffmann-La Roche.*

As in *Aghion and Bolton* (1987), we assume that the buyer and the incumbent contract at a time when the characteristics of the rival good are not yet known, i.e., the rival’s cost and the buyer’s willingness to pay for the rival good are uncertain. To concentrate on the exclusionary effects of nonlinear pricing, we assume away any bilateral inefficiency (e.g. asymmetric information) between the buyer and each of the two suppliers. In particular, the buyer and the incumbent would have no reason to distort the traded quantity in the absence of a rival. Similarly, we assume throughout the article that the negotiation between the buyer and the rival takes place under perfect information and is efficient. Formally, the game is thus equivalent to an asymmetric information set-up where the principal would be the buyer-incumbent pair and the agent the buyer-rival pair, and we can thus use insights from the nonlinear pricing literature, see *Wilson* (1993) and *Laffont and Martimort* (2002). The fact that the buyer is part of both coalitions raises interesting theoretical questions that are discussed at the end of the paper. In particular, the buyer’s dual role might open a scope for more sophisticated screening instruments and create subtle patterns of information revelation.

In the spirit of *Martimort and Stole* (2009), we are interested in distortions of productive allocations at both the extensive and the intensive margins. In their terminology, our framework is a common-agency game: the buyer may supply exclusively from the incumbent firm, in which case the rival is driven out of the market. We carefully examine distortions at the “extensive” margin, and indeed find that complete exclusion of efficient rivals occurs with positive probability. A contribution of the present article is to consider distortions at the intensive margin as well. To this aim, we model the incumbent and the rival goods as imperfect substitutes for the buyer. The degree of substitution can vary from perfect substitutes to independent goods. We find intensive distortions of the quantity supplied by the rival, specifically that quantity is positive but distorted downwards, which is sometimes referred to as “partial foreclosure” in the antitrust literature.

We emphasize a second kind of intensive distortions, which concern the quantity of incumbent good *at given level of rival supply,* and we relate it to an opportunistic behavior of the buyer at the last stage of the game. The intuition goes as follows. The general purpose of the quantity-price schedules agreed upon by the incumbent and the buyer is to place the latter in a favorable position when bargaining with the rival. In this bargain, the buyer can argue she will lose rebates if she purchases less from the incumbent, which allows her to extract surplus

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3 Marx and Shaffer (2004) also study a model à la Aghion and Bolton with differentiated goods but they restrict themselves to a complete information setting.
from the rival. *Ex ante*, the incumbent and the buyer share expected profits and their interests are aligned. *Ex post*, however, the buyer does not take into account the production costs of the incumbent good. Due to the offered rebates, the buyer has an incentive to purchase inefficiently many units from the incumbent conditionally on the quantity supplied from the rival.

A key issue in the paper is to compare the exclusionary properties of the three considered price schedules –conditional, exclusivity-based, and non-conditional. In Aghion and Bolton (1987) the buyer’s demand is inelastic and is supplied from only one producer, making it hard to distinguish between the three types of schedules. In contrast, here, the results differ strikingly across the three pricing regimes.

When the price of the incumbent good can freely depend on the supply from the rival (conditional regime), the optimal schedule is a two-part tariff. The incumbent good is priced at marginal cost, hence the absence of buyer *ex post* opportunism. The fixed part of the tariff is increasing and concave in the quantity purchased from the rival. That fixed part can be seen as a penalty imposed for buying from the rival, in line with Aghion and Bolton (1987), but here the rival’s supply is distorted at both the extensive and the intensive margins. These distortions increase with the rival’s bargaining power *vis-à-vis* the buyer. Market-share rebates are shown to be ill-adapted to control buyer opportunism.

When the price schedule only depends on the incumbent quantity (non-conditional regime), the buyer purchases the efficient quantity of rival good given the incumbent’s quantity. This link between the two quantities creates a channel through which the buyer and the incumbent can indirectly control the rival’s activity. In equilibrium, the marginal price of the incumbent is lower than the marginal cost of production up to a certain quantity threshold. These generous rebates allow the buyer to extract a good deal from the rival but at the same time induce her to behave opportunistically *ex post*. This distortion, in turn, translates into complete or partial foreclosure of efficient rivals. The presence of complete exclusion in equilibrium implies that the price schedule is not globally concave: the price is set at a high level beyond the quantity threshold mentioned above to deter the buyer from purchasing even more units of incumbent good, hence a convex kink in the schedule.

In the exclusivity-based regime, the price schedule is the same as in the non-conditional case for low quantities of incumbent good. The exclusivity offer allows to avoid buyer opportunism when the rival is inactive. On the other hand, this offer creates locally a strong distortion at the extensive margin, excluding a bunch of efficient rivals.

Finally, we are able to extend the analysis to the case where the buyer can dispose of or resell unconsumed units of incumbent good. In practice, the magnitude of the disposal costs depends on the seller’s ability to impose or to prevent particular uses of the purchased units.
and on the buyer’s ability to avoid such monitoring by the dominant firm. Depending on
the industry, unused items can be difficult to store or dispose of making disposal costs large. On the contrary, the buyer may have access to a secondary market and resell the extra units making disposal costs negative.

Purchasing units from the incumbent with the sole purpose of pocketing rebates, and then throwing away the unneeded units, would constitute an extreme form of buyer opportunism. We show that this form is never part of an equilibrium. We find that low disposal costs prevent the incumbent from committing on too generous rebates because the buyer could purchase units and discard them. Lower disposal costs are associated to less exclusion and higher values of the expected total welfare. Antitrust authorities, therefore, should pay close attention to contracting provisions that help increase disposal costs.

It is worthwhile relating our work to recent studies on market-share discounts. In a setting with a dominant firm, a competitive fringe and two retailers, Inderst and Shaffer (2010) show that market-share discounts can be used by the dominant firm to dampen intra- and inter-brand competition. Their anticompetitive scenario, contrary to the one presented here, highlights retail competition and assume complete information. Turning to models with imperfect information, most of the literature has examined how specific forms of pricing perform in discriminating among privately informed buyers. For instance, in a discrete type model, Kolay, Shaffer, and Ordover (2004) show that all-units discounts are more effective than menus of two-part tariffs in screening out retailers with private information about the state of demand. Majumdar and Shaffer (2009) and Calzolari and Denicolo (2013) introduce market-share discounts. In the former article, a dominant firm resorts to nonlinear pricing to screen a buyer who is informed about the size of demand and who also sells a good provided by a competitive fringe. The latter article addresses the issue in a symmetric duopoly setting, considering both market-share discounts and exclusive contracts.

The article is organized as follows. Section 2 introduces the model, and Section 3 studies conditional price-quantity schedules. Assuming very large disposal costs, Sections 4 and 5 explore the non-conditional and exclusivity-based regimes. Section 6 describes the effect of moderate disposal costs. Section 7 discusses a couple of extensions regarding the timing of the game, the informational environment, and the available instruments.

2 The model and purchase decisions

A dominant firm, $I$, competes with a rival, $E$, to serve a buyer, $B$. Marginal production costs are assumed to be constant and are denoted by $c_I$ and $c_E$. The timing of events reflects the incumbency advantage of the dominant firm and the uncertainty as to the characteristics of
the rival good: 1) B and I design a price-quantity schedule to maximize (and split) their joint expected surplus. At this stage, they know \( c_I \) and the characteristics of good I, but they do not know \( c_E \) nor the willingness to pay \( v_E \) for the rival good. 2) B and E discover \( c_E \) and \( v_E \) and jointly decide on the variables under their control, namely a transfer \( p_E \) and quantities \( q_E \) and \( q_I \), knowing both the terms of the agreement between B and I and all the relevant cost and preference parameters.

At the first stage, we consider three types of price-quantity schedules that differ in how the price of the incumbent good depends on the quantity supplied from the rival: (i) under a non-conditional schedule \( T(q_I) \), the price depends only on the number of I-units purchased; (ii) under a conditional schedule \( T(q_E, q_I) \), the price of \( q_I \) units of good I freely depends on the quantity purchased from the rival; (iii) an exclusivity scheme is a pair of schedules \( (T(q_I), T^x(q_I)) \) that specifies the price of \( q_I \) units of good I if the buyer supplies exclusively from the incumbent, \( T^x(q_I) \), and if she purchases a positive number of units from the rival firm, \( T(q_I) \).

At the second stage, we assume that B and E negotiate under complete information (Nash bargaining where \( \beta \) denotes E’s bargaining power) to maximize their joint surplus. The timing of negotiations assumes that B and I cannot renegotiate once uncertainty is resolved. This assumption and a couple of variants are discussed in Section 7.

**Buyer’s demand** When the buyer consumes \( x_E \) units of good E and \( x_I \) units of good I, she earns a gross profit of \( v_E x_E + v_I x_I - h(x_E, x_I) \), where \( h \) is a convex function of \((x_E, x_I)\) with first derivatives at \((0,0)\) equal to zero and with positive cross-derivative to reflect the imperfect substitutability of the two goods.

A key feature of the model is that the buyer can dispose of unneeded units of each good at a cost \( \gamma_E \geq -c_E \) and \( \gamma_I \geq -c_I \). That is, it is always inefficient (from a welfare perspective) to produce units in order to throw them away or to resell them. Consequently, the buyer’s net utility if she purchases \( q_E \) units from the rival and \( q_I \) units from the incumbent is

\[
V(q_E, q_I) = \max_{x_E \leq q_E, x_I \leq q_I} v_E x_E + v_I x_I - h(x_E, x_I) - c_E q_E - c_I q_I.
\]  

(1)

The buyer disposes of units of good \( k, k = E, I, \) when the purchased quantity \( q_k \) is so large that the marginal utility \( v_k - \partial h(q_k, q_l)/\partial q_k \) becomes smaller than the utility loss caused by disposal, \( -\gamma_k \). In this region, the buyer net utility \( V \) decreases linearly with \( q_k \), and the marginal net utility \( \partial V/\partial q_k \) is equal to \( -\gamma_k \). When the buyer consumes all the purchased

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4We do not impose any regularity condition on the price schedules. Some authors have studied specific types of price schedules as two-part tariffs (Marx and Shaffer (1999)) and all-units discounts (Feess and Wohlschlegel (2010)) under complete information.
units of the rival and incumbent goods ("no-disposal region"), the marginal net utility is greater than $-\gamma_k$ for each good.

**Efficient quantities** We denote by $q_E^{**}$ and $q_I^{**}$ the quantities that maximize the total surplus

$$W(q_E, q_I) = V(q_E, q_I; v_E) - c_E q_E - c_I q_I.$$ 

The efficient quantities involve no disposal and hence do not depend on the magnitude of disposal costs. These quantities also maximize $\omega_E q_E + \omega_I q_I - h(q_E, q_I)$, where $\omega_E = v_E - c_E$ and $\omega_I = v_I - c_I$. Hence $q_E^{**}$ and $q_I^{**}$ are respectively nondecreasing and non-increasing in $\omega_E$. Hereafter, we denote by $\omega_E^{**}$ the maximum value of $\omega_E$ for which $q_E^{**}(\omega_E) = 0$. We now introduce the distribution of $\omega_E$, that we denote by $F$, and a set of assumptions maintained throughout the paper.

**Assumption 1.** The distribution of $\omega_E$ has a positive density $f$ on its support $[\omega_E, \varpi_E]$ with $\omega_E < \omega_E^{**} < \varpi_E$ and $q_I^{**}(\varpi_E) > 0$. The hazard rate $f/(1 - F)$ is nondecreasing in $\omega_E$.

The assumption on $\omega_E^{**}$ allows us to concentrate on the most interesting case where full exclusion is socially optimal with positive probability. Moreover, to avoid uninteresting complications, we assume that both firms are active when the surplus created by the rival is maximal, formally $q_E^{**}(\omega_E), q_I^{**}(\omega_E) > 0$.

**Definition 1.** The quantity of good $E$ that maximizes the social welfare $W$ given $q_I$ is said to be conditionally efficient and is denoted by $q_E^{*}(q_I; \omega_E)$. The conditionally efficient quantity of incumbent good, $q_I^{*}(q_E; \omega_I)$, is symmetrically defined.

If the buyer consumes all the units of good $I$ she has purchased, then $q_E^{*}(q_I; \omega_E)$ is defined by the first order condition $\partial h/\partial q_E(q_E^{*}, q_I) = \omega_E$. In this region, the function $q_E^{*}(q_I; \omega_E)$ is decreasing by substitutability. In contrast, in the region where $q_I$ is so high that the buyer disposes of some units of good $I$, $q_E^{*}$ does not vary with $q_I$ as only the consumed quantity of good $I$ is relevant to determine the conditionally efficient quantity of good $E$. In this region, total surplus $W$ decreases linearly with $q_I$, and the partial derivative $\partial W/\partial q_I$ is $-c_I - \gamma$.

**Definition 2.** The rival firm is said to be super-efficient when $q_E^{*}(q_I; \omega_E)$ is positive for any value of $q_I$.

**Example: Quadratic utility** The leading example in this paper has $h(x_E, x_I) = x_E^2/2 + x_I^2/2 + \sigma x_E x_I, 0 \leq \sigma < 1$. The efficient quantities involve no disposal cost and are given by

$$q_E^{**}(\omega_E) = \max \left( \frac{\omega_E - \sigma \omega_I}{1 - \sigma^2}, 0 \right) \quad \text{and} \quad q_I^{**}(\omega_E) = \frac{\omega_I - \sigma \omega_E}{1 - \sigma^2}. \quad (2)$$
To respect Assumption 1, we must have in the quadratic case: $\omega_E < \omega^*_E = \sigma \omega_I < \omega_I / \sigma$.

The efficient allocation is represented by the point $A$ on Figure 1. When the buyer’s utility is quadratic, the welfare isolines are ellipses centered at $A$.

If the buyer has purchased $q_I$ units of good $I$, with $q_I \geq v_I + \gamma - \sigma q_E$, she consumes $x_I = v_I + \gamma - \sigma q_E$ units of good $I$, thus an amount that is independent of $q_I$. The no-disposal region is located below the bold dashed line on the figure. Applying the envelope theorem, we find that the conditionally efficient quantity $q^*_E(q_I; \omega_E)$ is constant in this region and equal to the second argument of the following maximum:

$$q^*_E(q_I; \omega_E) = \max \left( \omega_E - \sigma q_I, \frac{\omega_E - \sigma (v_I + \gamma)}{1 - \sigma^2}, 0 \right).$$

The rival firm is super-efficient if and only if $\omega_E > \sigma (v_I + \gamma)$. This is the case represented on Figure 1, where $q^*_E(q_I; \omega_E)$ is positive for any value of $q_I$.

![Figure 1: The total welfare is maximal at A (quadratic example)](image)

Figure 1: The total welfare is maximal at $A$ (quadratic example)
**Purchase decisions**  The last stage of the game takes place under perfect information, given the price schedule $T = T(q_E, q_I)$ or $T = T(q_I)$ and the known characteristics of the rival good. The buyer and the rival choose the quantities to maximize their joint surplus

$$S_{BE} = \max_{q_E, q_I} V(q_E, q_I) - T(q_E, q_I) - c_E q_E,$$

with no consideration for the incumbent’s cost or profit. The above expression shows that under a non-conditional schedule $T(q_I)$, the quantity of rival good is efficient given that of the incumbent good, formally $q_E = q^*_E(q_I; \omega_E)$, implying that no unit of the rival good is produced and disposed of. To avoid uninteresting developments, we take the latter property as granted under conditional schedules as well.\footnote{It would be extremely counter-intuitive that the buyer and the incumbent use their pricing instrument, e.g. $T(q_E, q_I)$, to encourage production and disposal of the rival good. The following analysis finds no force pushing in that direction. A formal proof is available upon request.}

Without loss of generality, the competitor’s outside option is normalized to zero. As to the buyer, she may source exclusively from the incumbent, so her outside option is

$$V^0_B = \max_{q_I \geq 0} V(0, q_I) - T(0, q_I).$$

The reservation utility $V^0_B$, which depends on the price schedule by (4), is endogenous but independent from $\omega_E$. The surplus created by the buyer and the rival firm can thus be written as

$$\Delta S_{BE} = S_{BE} - V^0_B.$$  

Denoting by $\beta \in (0, 1)$ the competitor’s bargaining power vis-à-vis the buyer, we derive the competitor’s and buyer’s profits:

$$\Pi_E = \beta \quad \Delta S_{BE}$$

$$\Pi_B = (1 - \beta) \quad \Delta S_{BE} + V^0_B.$$  

If $\beta = 0$, the competitor has no bargaining power and may be seen as a competitive fringe from which the buyer can purchase any quantity at price $c_E$. On the contrary, the case $\beta = 1$ happens when the competitor has all the bargaining power vis-à-vis the buyer.

**Perfect information** Marx and Shaffer (2004) have shown that quantities are efficient under perfect information. When the incumbent can make an exclusivity offer to the buyer, all the surplus from the rival can be extracted by adjusting the level of that offer in such a way that the surplus created by the rival, $\Delta S_{BE}$, is zero. The situation is subtler when the sole instrument available to the buyer and the incumbent is a non-conditional schedule $T(q_I)$. In our terminology, the result can be expressed as follows: full extraction occurs if and only if the rival is not super-efficient, see Section 7.
Virtual surplus  We henceforth focus on the situation where the buyer and the incumbent commit to a price-quantity schedule before the uncertainty on the rival good is realized. In this context, the schedule is designed ex ante to maximize the expected joint surplus, equal to the total surplus minus the profit left to the competitor:

$$
\mathbb{E}_{c_E, v_E} \Pi_{BI} = \mathbb{E}_{c_E, v_E} \{W(q_E, q_I; c_E, v_E) - \Pi_E\},
$$

(7)

where $q_E$ and $q_I$ are solution to (3) and $\Pi_E$ is given by (6). The sharing of the expected joint surplus between the buyer and the incumbent, and hence the respective bargaining power of each party, play no role in the following analysis.6

As all purchased units of rival good are consumed, the surplus (3) depends on the uncertain cost and preference parameters $c_E$ and $v_E$ only through the difference $\omega_E = v_E - c_E$. The surplus $S_{BE}$ is actually a convex function of $\omega_E$ because it is the upper bound of a family of functions that are affine in $\omega_E$. By the envelope theorem, the rent left to the rival satisfies:

$$
\frac{\partial \Pi_E}{\partial \omega_E} = \beta \frac{\partial S_{BE}}{\partial \omega_E} = \beta q_E(\omega_E).
$$

(8)

Integrating by parts, we get

$$
\int_{\omega_E}^{\omega_E} \Pi_E(\omega_E)f(\omega_E)\,d\omega_E = \Pi_E(\omega_E) + \beta \int_{\omega_E}^{\omega_E} q_E(\omega_E)[1 - F(\omega_E)]\,d\omega_E.
$$

Substituting in (7), we rewrite the buyer-incumbent objective as

$$
\mathbb{E}_{\omega_E} \Pi_{BI} = \mathbb{E}_{\omega_E} S^v(q_E, q_I; \omega_E) - \Pi_E(\omega_E),
$$

(9)

where, following Jullien (2000), we have defined the “virtual surplus” $S^v$ as

$$
S^v(q_E, q_I; \omega_E) = W(q_E, q_I; \omega_E) - \beta q_E \frac{1 - F(\omega_E)}{f(\omega_E)}.
$$

(10)

The virtual surplus is the total surplus $W(q_E, q_I; \omega_E)$ adjusted for the informational rents $\beta q_E (1 - F(\omega_E)) / f(\omega_E)$ induced by the self-selection constraints.

Buyer opportunism  Expression (7) reflects a standard rent-extraction tradeoff. From the ex ante perspective, the tariff has two purposes: on the one hand, maximizing the expected welfare $W$; on the other, making $\Pi_E = \beta \Delta S_{BE}$ as small as possible. Rent extraction is obtained by placing competitive pressure on the rival firm, i.e., granting the buyer low marginal price to

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6Figueroa, Ide, and Montero (2014) restrict the ability of the buyer and the incumbent to share rents through transfers and explore the implications for inefficient exclusion in a model with inelastic demand and one-dimensional uncertainty.
force the rival to match these rebates, which may drive the rival out of the market or distort downwards the quantity it sells, \( q_E < q_E^* \).

The novelty in our analysis lies in the possible distortion on \( q_I \). We call buyer opportunism the fact that the buyer purchases too many units of good \( I \) given her supply from the rival, formally \( q_I > q_I^*(q_E; \omega_I) \). We show below that buyer opportunism is observed in equilibrium except when the buyer and the incumbent firm have the most powerful instrument \( T(q_E, q_I) \) at their disposal. Granting rebates to the buyer induces her to distort the quantity purchased from the dominant firm upwards. The buyer indeed wants to pocket the rebates and does not internalize the production cost \( c_I \) when she purchases from the dominant firm.

Purchasing and throwing away units of incumbent good would constitute an extreme form of buyer opportunism. We show that this form is never part of an equilibrium. On the contrary, we find that the possibility of disposing of units of good \( I \) alters the terms of the rent-extraction tradeoff and limits the exclusionary effects of nonlinear pricing.

3 Conditional price-quantity schedules

For each type of pricing instrument, we proceed as follows. First, we derive necessary conditions for a quantity allocation \((q_E(\omega_E), q_I(\omega_E))\) to be achieved with the considered type of price-quantity schedule. Second, we maximize the virtual surplus under those necessary conditions. Third, we check that the found allocation can indeed be implemented.

We start with conditional schedules \( T(q_E, q_I) \). As regards implementability, we simply observe that the quantity of rival good \( q_E \) is a nondecreasing function of \( \omega_E \). This follows from the convexity of the surplus function \( S_{BE}(\omega_E) \), combined to the envelope condition (8).

Maximization of the virtual surplus  The maximum of the virtual surplus (10) is achieved in the no-disposal region. At the optimum, the quantity of good \( I \) is conditionally efficient, \( q_I = q_I^*(q_E; \omega_I) \). More precisely, for each \( \omega_E \), the virtual surplus \( S^v \) is maximal at \((q_E, q_I)\) such that

\[
\omega_E - \frac{\partial h}{\partial q_E}(q_E^*, q_I^*) \leq \beta \frac{1 - F(\omega_E)}{f(\omega_E)} \quad \text{and} \quad \frac{\partial h}{\partial q_I}(q_E^*, q_I') = \omega_I.
\]

(11)

with equality in the first inequality when \( q_E^* > 0 \). In this case, the two conditions can be collapsed into

\[
\omega_E - \frac{\partial h}{\partial q_E}(q_E^*, q_I^*(q_E^*; \omega_I)) = \beta \frac{1 - F(\omega_E)}{f(\omega_E)}.
\]

(12)

By convexity of \( h \), the function \( \partial h/\partial q_E(q_E, q_I^*(q_E)) \) increases with \( q_E \), and hence the left-hand side of (12) decreases with \( q_E^* \). Under Assumption 1, the right-hand side is non-increasing.
in $\omega_E$. The function $q^*_E(\omega_E)$, therefore, is nondecreasing. Let $\omega^*_E$ be defined by

$$\omega^*_E = \beta \frac{1 - F(\omega^*_E)}{f(\omega^*_E)} - \frac{\partial h}{\partial q_E}(0, q^*_I(0; \omega_I)) = 0.$$  

The left-hand side of the above equation increases with $\omega^*_E$ and is negative for $\omega^*_E = \omega^{**}_E = \partial h/\partial q_E(0, q^*_I(0; \omega_I))$, hence $\omega^*_E > \omega^{**}_E$.

For $\omega_E > \omega^*_E$, the rival supplies a positive quantity, $q^*_E > 0$, as represented by point $C$ on Figure 2. For $\omega_E$ below $\omega^*_E$, the virtual surplus is maximum at the point $(q^*_E, q^*_I) = (0; q^*_I(0; \omega_I))$, i.e., the rival is driven out of the market—the distortion is at the extensive margin. (On Figure 2, the point $C$ would lie on the $q_I$-axis.) The dashed ellipses centered at $C$ represent the isolines of the virtual surplus.

Figure 2: The virtual surplus is maximal at $C$ (quadratic example)

**Construction of a conditional schedule** We now construct a conditional price-quantity schedule $T(q_E, q_I)$ under which for all $\omega_E$ the buyer purchases $q^*_E(\omega_E)$ and $q^*_I(\omega_E)$ from the
rival and the incumbent firm, respectively. We start by introducing a function $P(q_E)$ whose derivative is given by

$$P'(q^*_E(\omega_E)) = \beta \frac{1 - F(\omega_E)}{f(\omega_E)},$$

on the interval $[0, q^*_E(\omega_E)]$ and is zero above $q^*_E(\omega_E)$. The value of $P(0)$ determines the sharing of the surplus between the buyer and the dominant firm. It is straightforward to verify that the function $P$ is globally concave. Moreover, by definition of $q^*_E$, we have

$$\frac{\partial h}{\partial q_E}(q^*_E(\omega_E), q^*_I(q^*_E(\omega_E))) + P'(q^*_E(\omega_E)) = \omega_E,$$

(13)

for all $\omega_E$ between $\omega_E$ and $\omega_E$. We now check that the function $h(q_E, q^*_I(q_E)) + P(q_E) - \omega_I q^*_I(q_E)$ is convex in $q_E$. Indeed its derivative at some $q_E = q^*_E(\omega_E)$ in the interval $[0, q^*_E(\omega_E)]$ is the left-hand side of (13) and, therefore is increasing with $\omega_E$ and $q_E$. Hence the convexity result.

We now introduce conditional price-quantity schedules of a particular form: two-part tariffs in $q_I$ with an intercept that depends on $q_E$, $T(q_E, q_I) = c_I q_I + P(q_E)$. Under such a schedule, the buyer and the rival choose the efficient quantity of good $I$ given $q_E$, $q^*_I(q_E; \omega_I)$. Replacing $q_I$ with $q^*_I(q_E; \omega_E)$ in their common objective (3), we find that the buyer and the rival choose the quantity $q_E$ that maximizes the function $\omega_E q_E + \omega_I q^*_I(q_E) - h(q_I, q^*_I(q_E)) - P(q_E)$. This function is concave in $q_E$ from the above analysis. The quantity of good $E$, therefore, is determined by the first-order conditions, and is thus $q^*_E(\omega_E)$ for any $\omega_E$.

**Proposition 1.** The following properties hold at the second-best optimum with a conditional price-quantity schedule:

1. The buyer and the incumbent firm agree on a price schedule that is linear in $q_I$ with slope $c_I$ and nondecreasing and concave in $q_E$.

2. For any level of rival’s surplus $\omega_E$ but $\omega_E$, the quantity purchased from the rival, $q^*_E$, is distorted downwards relative to $q^{**}_E$. Exclusion is complete for $\omega_E \leq \omega^*_E$, where $\omega^*_E > \omega^{**}_E$.

3. The quantity purchased from the incumbent firm, $q^*_I = q^*_I(q^*_E; \omega_E)$, is efficient given $q^*_E$ but distorted upwards relative to $q^{**}_I$.

4. The magnitude of the disposal costs, $\gamma_I$, does not affect the buyer’s supply policy or the price-quantity schedule $T(q_E, q_I)$.

Letting the price of the incumbent good depend on the quantity purchased from the rival allows the buyer and the incumbent to neutralize buyer ex post opportunism, i.e., to make sure that the quantity of incumbent good is efficient given the quantity supplied from the rival. Conditional efficiency imposes that the partial derivative of $T$ with respect to $q_I$ is $c_I$ at the
second-best allocation. This condition is hard to meet when the price schedule \( T \) depends on the market share \( q_I/(q_E+q_I) \) rather than directly on \( q_E \), because the market share is nonlinear in \( q_I \). Market-share discounts, for this reason, appear as a less convenient way to implement the second-best allocation than two-part tariffs of the form \( c_I q_I + P(q_E) \).

Proposition 1 builds a bridge between the literatures on market foreclosure and nonlinear pricing. As in Aghion and Bolton (1987), the buyer and the incumbent jointly act like a monopoly towards the rival, setting \( P(q_E) \) to extract rent at the cost of reducing the extent of entry: \( q_E < q_E^* \), which yields inefficient market foreclosure. The efficiency-rent tradeoff leads to more inefficient exclusion as the rival’s bargaining power, \( \beta \), rises. When \( \beta \) is high, the rival has a strong bargaining power vis-à-vis the buyer, which makes rent extraction a more serious issue and pushes towards reducing \( q_E \).

Aghion and Bolton (1987) assume that the buyer’s demand was supplied entirely by a single supplier. Hence they interpret the difference \( P(1) - P(0) \) as a penalty for breach of contract. In contrast, we allow the buyer to split her purchase requirements between the two suppliers and find that inefficient foreclosure may be complete or partial: \( 0 \leq q_E < q_E^* \). We interpret the difference \( P(q_E) - P(0) \) as rebates lost when supplying from the competitor. The presence of these rebates implies a form of below-cost pricing. Specifically, when \( q_E > 0 \), the average incremental price of the “last” units of good \( I \) (units between \( q_I^*(q_E;\omega_I) \) and \( q_I^*(0;\omega_I) \)) is lower than the production cost:

\[
\frac{T(0,q_I^*(0;\omega_I)) - T(q_E,q_I^*(q_E;\omega_I))}{q_I^*(0;\omega_I) - q_I^*(q_E;\omega_I)} = c_I - \frac{P(q_E) - P(0)}{q_I^*(0;\omega_I) - q_I^*(q_E;\omega_I)} < c_I,
\]

because the penalty function is increasing, \( P(q_E) > P(0) \), and the function \( q_I^* \) is decreasing, \( q_I^*(0;\omega_I) > q_I^*(q_E;\omega_I) \). The above price-cost comparison is reminiscent of the “as-efficient competitor test”. The precise form of the test advocated by the European Commission, which involves the notion of contestable demand, is more accurately described in a model with inelastic demand (see our companion paper, Choné and Linnemer (2014)).

**Quadratic example**  With \( h(q_E,q_I; s_E) = q_E^2/2 + q_I^2/2 + \sigma q_E q_I \), the second-best quantities purchased from both suppliers under a conditional tariff are given by

\[
q_{E}^*(\omega_E) = \max \left( \omega_E - \beta \frac{1-F(\omega_E)}{f(\omega_E)} - \sigma q_I^*, 0 \right) \quad \text{and} \quad q_{I}^*(\omega_E) = q_{I}^*(q_{E}^*),
\]

The quantity purchased from the incumbent is conditionally efficient while that purchased from the rival is distorted downwards. We get the allocation that would be efficient if the rival’s efficiency index \( \omega_E \) were artificially reduced by \( \beta(1 - F(\omega_E))/f(\omega_E) \):

\[
q_{E}^*(\omega_E) = q_{E}^{**} \left( \omega_E - \beta \frac{1-F(\omega_E)}{f(\omega_E)} \right), q_{I}^*(\omega_E) = q_{I}^{**} \left( \omega_E - \beta \frac{1-F(\omega_E)}{f(\omega_E)} \right)
\]
where the efficient quantities \( q_{E}^{**} \) and \( q_{I}^{**} \) are given by (2). When \( \omega_{E} \) is uniformly distributed over the interval \([\omega_{E}, \bar{x}_{E}]\), the penalty function is quadratic, with its derivative being given by

\[
P'(q_{E}) = \frac{\beta}{1+\beta} [q_{E}^{**}(\bar{x}_{E}) - q_{E}].
\]

In the limit case where the demands for the two goods are independent \((\sigma = 0)\), the penalty is given by the formula above with \( q_{E}^{**}(\bar{x}_{E}) = \bar{x}_{E} \), and hence is increasing in \( q_{E} \). Rent-shifting appears here as pure extortion, and we now turn to more realistic price instruments.

### 4 Non-conditional price-quantity schedules

The analysis is more involved when the price schedule cannot freely depend on the quantity purchased from the rival, because buyer opportunum will materialize at the second-best allocation and the degree of buyer opportunum will depend on the magnitude of the disposal costs. To simplify the presentation, in this and the next section, we assume that the buyer must consume the all the purchased units, \( \gamma_{E} = \gamma_{I} = +\infty \), and hence \( V(q_{E}, q_{I}) = v_{E}q_{E} + v_{I}q_{I} - h(q_{E}, q_{I}). \)

The effect of disposal costs is examined Section 6.

**Implementable quantity functions** We now suppose that the buyer and the dominant firm are constrained to use a schedule of the form \( T(q_{I}) \). As the schedule does not depend on \( q_{E} \), the buyer and the rival trade the efficient quantity of good \( E \) given \( q_{I}, q_{E} = q_{E}^{*}(q_{I}; \omega_{E}). \)

Following Martimort and Stole (2009), we think of the buyer and rival joint utility as a function of the quantity purchased from the incumbent:

\[
\tilde{S}_{BE}(q_{I}; \omega_{E}) = \max_{q_{E} \geq 0} v_{I}q_{I} + \omega_{E}q_{E} - h(q_{E}, q_{I}) = v_{I}q_{I} + \omega_{E}q_{E}^{*}(q_{I}; \omega_{E}) - h(q_{E}^{*}(q_{I}; \omega_{E}), q_{I}).
\]

The function \( \tilde{S}_{BE} \) is concave in \( q_{I} \) as the marginal utility

\[
\frac{\partial \tilde{S}_{BE}}{\partial q_{I}} = v_{I} - \frac{\partial h}{\partial q_{I}}(q_{E}^{*}(q_{I}; \omega_{E}), q_{I})
\]

decreases in \( q_{I} \) by convexity of \( h \). It is nondecreasing in \( \omega_{E} \) with derivative \( q_{E}^{*}(q_{I}; \omega_{E}) \), and satisfies the single-crossing property:

\[
\frac{\partial^{2} \tilde{S}_{BE}}{\partial q_{I} \partial \omega_{E}} = \frac{\partial}{\partial q_{I}} \left( \frac{\partial \tilde{S}_{BE}}{\partial \omega_{E}} \right) = \frac{\partial q_{E}^{*}}{\partial q_{I}} \leq 0
\]

by substitutability of the two goods: the buyer and rival marginal utility for good \( I \) decreases with \( \omega_{E} \). For non super-efficient rivals, the isolines of \( \tilde{S}_{BE} \) coincide with those of \( v_{I}q_{I} - h(0, q_{I}) \) for large values of \( q_{I} \), namely in the region where \( q_{E}^{*}(q_{I}; \omega_{E}) = 0 \); in this region, the marginal
The chosen quantity of incumbent good, \( q_I(\omega_E) \), is solution to

\[
S_{BE}(\omega_E) = \max_{q_I \geq 0} \tilde{S}_{BE}(q_I; \omega_E) - T(q_I),
\]

for some price schedule \( T(q_I) \). Adapting usual arguments, we find that a quantity allocation \((q_E(\omega_E), q_I(\omega_E))\) is implementable under a non-conditional schedule if and only if the two conditions are satisfied: (i) \( q_E = q_E^*(q_I; \omega_E) \); (ii) \( q_I \) is decreasing in \( \omega_E \) where \( q_E > 0 \) and constant in \( \omega_E \) where \( q_E = 0 \).

**Constrained maximization of the virtual surplus**  We now maximize the virtual surplus under the two constraints listed above, namely \( q_E = q_E^*(q_I; \omega_E) \) and \( q_I \) non-increasing in \( \omega_E \).

To account for the former constraint, we define the constrained virtual surplus as

\[
\tilde{S}^*(q_I; \omega_E) = S^*(q_E^*(q_I; \omega_E), q_I; \omega_E).
\]

Then we maximize the constrained surplus subject to the monotonicity requirement imposed on the function \( q_I(\omega_E) \). Following Martimort and Stole (2009)’s approach in multi-principal games, we make the following regularity assumption:

**Assumption 2.** The constrained virtual surplus \( \tilde{S}^*(q_I; \omega_E) \) is strictly quasi-concave in \( q_I \) and has strict decreasing differences with respect to \( q_I \) and \( \omega_E \) where \( q_E^*(q_I; \omega_E) > 0 \).

Before proceeding to the maximization, we comment on the two conditions stated in Assumption 2. The concavity condition is equivalent to the following function being decreasing in \( q_I \):

\[
\frac{\partial \tilde{S}^*(q_I; \omega_E)}{\partial q_I} = \omega_I - \frac{\partial h}{\partial q_I}(q_E^*(q_I; \omega_E), q_I) - \beta \frac{1 - F(\omega_E)}{f(\omega_E)} \frac{\partial q_E^*(q_I; \omega_E)}{\partial q_I}.
\]

The second term of the right-hand side is indeed decreasing in \( q_I \) by convexity of \( h \), which tends to make the virtual surplus concave in \( q_I \). The last term involves the slope of the conditionally efficient quantity, \( \partial q_E^*/\partial q_I \). In the quadratic example, that slope is \(-\sigma\), and hence the virtual surplus is concave. In general, however, the slope is equal to a ratio of second-order derivatives of \( h \) whose variations with \( q_I \) depend on properties of third-order derivatives of \( h \).

The second part of Assumption 2 is equivalent to the partial derivative \( \partial \tilde{S}^*/\partial q_I \) being decreasing in \( \omega_E \). By substitutability, the second term at the right-hand side of (17) is decreasing in \( \omega_E \) when \( q_E^* > 0 \) and constant when \( q_E^* = 0 \). The last term has two factors: by Assumption 1 the hazard rate \( f/(1 - F) \) also tends to make \( \partial \tilde{S}^*/\partial q_I \) decrease with \( \omega_E \) (recall

\footnote{The derivative of \( q_E^* \) with respect to \( q_I \) is \( -\frac{\partial^2 h}{\partial q_E \partial q_I} / \partial^2 h/\partial q_E^2 \) evaluated at \( (q_E^*(q_I; \omega_E), q_I) \).}
that \( \partial q_E^u / \partial q_I \) is negative). The contribution of the last factor, however, is ambiguous as we do not know how the slope of the conditionally efficient quantity, \( \partial q_E^u / \partial q_I \), varies with \( \omega_E \). In the quadratic case, the slope is constant and the first two forces yield the desired property.

Lemma 1. Let \((\hat{q}_E^u, \hat{q}_I^u)\) denote the quantity allocation that maximizes the constrained virtual surplus for each value of \( \omega_E \). There exists \( \hat{\omega}_E^u \) in \((\omega_E, \omega_E)\) such that \((\hat{q}_E^u, \hat{q}_I^u)\) satisfies

\[
\omega_I - \frac{\partial h}{\partial q_I}(\hat{q}_E^u, \hat{q}_I^u) = \beta \frac{1 - F(\omega_E)}{f(\omega_E)} \frac{\partial q_E^u}{\partial q_I} \quad \text{and} \quad \omega_E - \frac{\partial h}{\partial q_E}(\hat{q}_E^u, \hat{q}_I^u) = 0, \tag{18}
\]

for \( \omega_E \geq \hat{\omega}_E^u \) and \( \hat{q}_E^u = q_E^u(\hat{q}_I^u; \omega_E) = 0 \) for \( \omega_E \leq \hat{\omega}_E^u \). The quantity \( \hat{q}_I^u \) decreases (increases) with \( \omega_E \) above (below) \( \hat{\omega}_E^u \).

Proof. Under Assumption 2, the virtual surplus is concave and hence its maximum is determined by the first-order conditions. These conditions are given by (18) when \( \hat{q}_E^u > 0 \). The second part of Assumption 2 guarantees that \( \hat{q}_I^u \) decreases with \( \omega_E \) as long as \( \hat{q}_E^u > 0 \). It follows that \( \hat{q}_E^u = q_E^u(\hat{q}_I^u; \omega_E) \) increases with \( \omega_E \) in this region.

The existence of the threshold \( \hat{\omega}_E^u \) follows from the assumption that \( q_E^{*u}(\omega_E^{**}) = 0 < q_E^{**}(\omega_E) \) and the observation that \( q_E^u \) is equal to (lower than or equal to) \( q_E^{*u} \) at \( \omega_E \) (at \( \omega_E^{**} \)). Below that threshold, the maximum of the constrained virtual surplus is achieved at a point where \( q_E^u = 0 \) and the value of \( \hat{q}_I^u \) is determined by conditional efficiency, i.e., by the condition \( 0 = q_E^u(\hat{q}_I^u; \omega_E) \), implying that \( \hat{q}_I^u \) is increasing in \( \omega_E \) in this region. \( \square \)

Figure 3 shows a situation in the quadratic example where the maximum of the constrained virtual surplus is achieved at a point where \( q_E > 0 \), i.e., the represented case corresponds to a value of \( \omega_E \) higher than \( \hat{\omega}_E^u \). For \( \omega_E \) lower than that threshold, the point \( U \) lies on the vertical \( q_I \)-axis. The non-monotonicity of \( \hat{q}_I^u \) is apparent on Figure 4b, see the thin dotted line.

Lemma 1 shows that the pointwise maximization of the constrained virtual surplus yields a quantity of incumbent good that is not monotonic and hence not implementable. There must therefore be bunching at the bottom of the distribution of \( \omega_E \). The next proposition characterizes the optimal quantity allocation \((q_E^u, q_I^u)\) under a non-conditional schedule.

Proposition 2. Under Assumption 2, the optimal quantities implementable with a non-conditional schedule \( T(q_I) \) satisfy the following properties:

1. There exists \( \hat{\omega}_E^u \) in \((\omega_E, \omega_E)\) such that \( \hat{q}_I^u(\omega_E) \) is constant up to \( \hat{\omega}_E^u \) and then equal to \( \hat{q}_I^u(\omega_E) \).

2. The quantity purchased from the rival is efficient given \( q_I^u \): \( q_E^u = q_E^u(\hat{q}_I^u; \omega_E) \), and distorted downwards relative to \( q_E^{*u} \) for all \( \omega_E < \omega_E \). Exclusion is complete for \( \omega_E \leq \omega_E \), with \( \omega_E^{*u} < \omega_E^{*u} < \omega_E^u \).
3. The quantity purchased from the dominant firm is distorted upwards relative to the conditionally efficient quantity, $q_u^* > q_I^*(q_E; \omega_I)$ (“buyer opportunism”) for all $\omega_E < \bar{\omega}_E$.

Proof. The condition that determines the bunching threshold $\tilde{\omega}_E^u$ is

$$\int_{\omega_E}^{\tilde{\omega}_E} \frac{\partial}{\partial q_I} \tilde{S}^v(q_I; \omega_E) \, dF(\omega_E) = 0,$$

where $\tilde{q}_I = \tilde{q}_I^*(\tilde{\omega}_E^u)$ is the constant value of $q_I$ over the interval $[\omega_E, \tilde{\omega}_E^u]$, see Figure 4b. The derivative of the constrained virtual surplus $\partial \tilde{S}^v / \partial q_I$ depends on whether $q_E$ is positive or zero. Let $\omega_E$ be defined by $q_E^*(\tilde{q}_I; \omega_E) = 0$. For $\omega_E < \omega_E$, $q_E^n = 0$, the derivative of the constrained virtual surplus is $w_I - \partial h / \partial q_I(0, \tilde{q}_I)$, which is negative because $\tilde{q}_I$ is above $\tilde{q}_I^*(\omega_E)$. For $\omega_E < \omega_E < \tilde{\omega}_E^u$, $q_E^n > 0$, the derivative features the additional (positive) term $-\beta(1 - F)/f \partial q_E^n / \partial q_I$; in this region it is positive because $\tilde{q}_I$ is below $\tilde{q}_I^*(\omega_E)$. The threshold $\tilde{\omega}_E^u$ is such that the positive and negative contributions offset each other.

The quantity purchased from the rival is undistorted for $\omega_E = \bar{\omega}_E$ and is zero below a
threshold \( \omega_E^* \) that is strictly larger than \( \omega_E^* \). The quantity purchased from the dominant firm, being distorted upwards relative to \( q^*_I(q_I; \omega_I) \), is a fortiori distorted upwards relative to \( \tilde{q}_I^* \). In particular, \( \tilde{q}_I^*(\omega_E) \) is above \( q^*_I(0; \omega_I) \) for low values of \( \omega_E \). This can be seen on Figure 4c where the “trajectories” of the quantity pairs \((q_E, q_I)\) are represented in various regimes. We see that \( q_I \) is efficient conditionally on \( q_E \) at the first-best allocation as well as under a conditional price-quantity schedule. In contrast, inefficiently many units of incumbent good given rival supply are purchased under a non-conditional schedule.

Shape of the price-quantity schedule Under Assumption 2 and assuming furthermore that \( h \) is twice continuously differentiable, \( q^*_I = \tilde{q}_I^* \) is differentiable outside the bunching region, i.e., for \( \omega_E > \tilde{\omega}_E^* \), and its derivative is positive in that region. Now we observe that the surplus function \( S_{BE}(\omega_E) = v_Iq_I + \omega_Eq_E - h(q^*_I, q_I) - T(q_I) \) is convex and hence differentiable at almost every value of \( \omega_E \). It follows that the price schedule \( T \) is almost everywhere differentiable over the range of \( q^*_I(\omega_E) \). Differentiating \( S_{BE} \) and simplifying by \( dq^*_I/\omega_E \), we get \( v_I - \frac{\partial h}{\partial q_I}(q^*_I, q_I) = T'(q_I) \), which, combined with (18), yields

\[
T'(q^*_I(\omega_E)) = c_I + \frac{1}{f(\omega_E)} \frac{\partial h}{\partial q_I} < c_I,
\]

where the slope \( \frac{\partial q^*_E}{\partial q_I} \) is evaluated at \((q^*_I(\omega_E); \omega_E)\). The monotonicity of the hazard rate tends to make the schedule concave in \( q_I \). Indeed, as \( \omega_E \) rises, the quantity \( \tilde{q}_I^* \) falls and the hazard rate pushes the the right-hand side of (20) upwards because \( \frac{\partial q^*_E}{\partial q_I} \) is negative. There is, however, the additional effect that the derivative \( \frac{\partial q^*_E}{\partial q_I} \) can itself move with \( \omega_E \); this effect is absent in the quadratic case where the derivative is constant. The following proposition presents sufficient conditions (derived from (20)) for the price schedule to be concave below the maximum quantity purchased from the incumbent, \( \tilde{q}_I \), given by (19).

**Proposition 3.** Suppose that the slope of the conditionally efficient quantity, \( \frac{\partial q^*_E}{\partial q_I} \), is non-increasing (nondecreasing) in \( q_I \) (resp. \( \omega_E \)). Then the optimal non-conditional price-quantity schedule \( T(q_I) \) is concave in \( q_I \) up to \( \tilde{q}_I \) and has a convex kink at this point.

The shape of the optimal non-conditional schedule is shown on Figure 5. The convex kink at \( \tilde{q}_I \) is due to complete exclusion and the associated bunching phenomenon at the bottom of the distribution. Indeed, the slope of the price schedule at the left of \( \tilde{q}_I \) is equal to the marginal rate of substitution for \( v_I - \frac{\partial h}{\partial q_I} \) evaluated at \((q^*_I(\tilde{q}_I; \tilde{\omega}_E^*), \tilde{q}_I)\). This rate is higher for the agents with lower type, and is the highest for \( \omega_E = \tilde{\omega}_E^* \), because these types value the rival good less, and hence the incumbent good more. To prevent these agents from purchasing more than \( \tilde{q}_I \), the price schedule must lie above the iso-utility curve of the lowest type, hence
a the convex kink. To be specific, the right derivative of the schedule at $\bar{q}_I$ must be greater than $v_I - \partial h/\partial q_I$ evaluated at $(0, \bar{q}_I)$.\footnote{The bunching at the bottom of the distribution of $\omega_E$, and the corresponding non-concavity of $T(q_I)$ at $\tilde{q}_I$, are present because $q^*_E(\omega_E) = 0$ for low values of $\omega_E$. This is due in particular to our assumption that $q^{**}_E$ is zero at the bottom of the distribution ($\omega^*_E \geq \omega_E$, see Assumption 1). We would have no bunching and a globally concave schedule if $q^*_E(q_I; \omega_E)$ were positive for all $q_I$ and $\omega_E$.}

In the limit case of two independent markets, the conditionally efficient quantity of rival good, $q^*_E$, does not depend on $q_I$, and Lemma 2 and Proposition 2 show that both quantities are fully efficient at the second-best allocation. In particular, the quantity of incumbent good does not vary with the rival’s efficiency index $\omega_E$, so the range of $q^*_I(\omega_E)$ is the singleton $\{q^{**}_I\}$, which makes the above analysis inoperative. Here we see directly that $T' = c_I$ is necessary to induce efficiency.

**Quadratic example** When the buyer’s utility is quadratic, the second-best quantities under a non-conditional schedule are given in the no-bunching region, i.e., for $\omega_E > \bar{\omega}_E$, by

$$q^n_E(\omega_E) = q^{**}_E(q^*_I; \omega_E) \quad \text{and} \quad q^n_I(\omega_E) = \omega_I + \sigma \beta \frac{1 - F(\omega_E)}{f(\omega_E)} - \sigma q^n_E(\omega_E),$$

which yields

$$q^n_E(\omega_E) = q^{**}_E(\omega_E) - \beta \frac{1 - F(\omega_E)}{f(\omega_E)} \quad \text{and} \quad q^n_I(\omega_E) = q^{**}_I(\omega_E) + \beta \frac{1 - F(\omega_E)}{f(\omega_E)}.$$

When the distribution is uniform, the bunching condition (19) can be rewritten as:

$$(\omega_I - \tilde{q}_I)(\omega^n_E - \omega_E) + \int_{\omega^n_E}^{\bar{\omega}_E} [\omega_I - (1 - \sigma^2)\tilde{q}_I - \sigma \omega_E + \beta \sigma (\bar{\omega}_E - \omega_E)] \, d\omega_E = 0, \quad (21)$$

with $\omega^n_E = \sigma \bar{q}_I$ and $q^n_I$ continuous at $\bar{\omega}_E$. The quantities are represented as a function of $\omega_E$ on Figures 4a and 4b.

As already observed, the sufficient conditions of Proposition 3, that guarantee the concavity of the price schedule, are satisfied when the buyer’s utility is quadratic. To illustrate, we compute the curvature of the schedule when the distribution of $\omega_E$ is uniform. Observing that $(1 - F)/f = \bar{\omega}_E - \omega_E$ and combining (20) with the expression of $q^*_I(\omega_E)$, we find that the first derivative of the schedule is linear in $q_I$:

$$T'(q_I) = c_I - \frac{\beta}{1 + \beta} (1 - \sigma^2) [q_I - q^{**}_I(\bar{\omega}_E)] < c_I$$

for all $q_I$ between $q^*_I(\bar{\omega}_E)$ and $\tilde{q}_I$; the schedule is quadratic (and concave) in this region.
Figure 4a: Equilibrium quantities of good $E$ under each type of price schedule

Figure 4b: Equilibrium quantities of good $I$ under each type of price schedule

Figure 4c: Buyer opportunism in regimes $u$ and $x$: $q_I$ is above $q^*_I(q_E; \omega_I)$

5 Exclusivity offer

In this section, we investigate the situation where the price of the incumbent good can be conditioned on the rival supply only through the events $q_E = 0$ and $q_E > 0$. This situation is somewhat intermediary between conditional and non-conditional schedules.

Let $(T(q_I), T^*(q_I))$ be an exclusivity price scheme constituted of a pair of non-conditional
schedules, where \( T^x \) is available to the buyer only if she supplies exclusively from the dominant firm. Under such a scheme, the buyer and the rival, should they settle on a positive quantity, choose \( q_E = q_E^*(q_I; \omega_E) \) and \( q_I \) solution to (16). If they fail to find such an agreement, the buyer earns

\[
S^x = v_I q_I - h(0, q_I) - T^x(q_I),
\]

which does not depend on \( \omega_E \). It follows that the price schedule \( T^x \) consists in fact of a single price-quantity pair. The only difference with the non-conditional situation studied in Section 4 is the availability of an independent instrument to control the outside option.

**Proposition 4.** There exists a threshold \( \omega^x_E \) such that the quantities purchased by the buyer under an exclusivity scheme satisfy \((q^x_E, q^x_I) = (0, q^*_I(0)) \) for \( \omega_E \leq \omega^x_E \) and \((q^x_E, q^x_I) = (q^u_E, q^u_I) \) for \( \omega_E \geq \omega^x_E \).

**Proof.** Maximizing the virtual surplus under the constraint \( q_E = 0 \) yields the conditionally efficient quantity of incumbent good, \( q^*_I(0; \omega_I) \). If \( q_E > 0 \), we maximize as above the constrained virtual surplus to account for the constraint \( q_E = q_E^*(q_I; \omega_E) \). The virtual surplus is \( S^v(0, q_I^*(0; \omega_I)) = W(0, q_I^*(0; \omega_I)) \) in the former situation, \( \max_{q_I} S^v(q_E^*(q_I; \omega_E), q_I; \omega_E) \) in the latter. Let \( \omega^x_E \) be the type for which the buyer and the incumbent are ex ante indifferent between these alternatives:

\[
\max_{q_I} S^v(q_E^*(q_I; \omega^x_E), q_I; \omega^x_E) = W(0, q_I^*(0; \omega_I)).
\]

Since \( \max_{q_I} S^v(q_E^*(q_I; \omega_E), q_I; \omega_E) \) increases with \( \omega_E \), the quantity allocation \((q^x_E, q^x_I)\) defined in the statement of the proposition maximizes the virtual surplus.

We now explain how to implement this allocation with a pair \((T, T^x)\). Regarding \( T \), we use the same schedule as in Section 4. We define \( T^x \) as a two-part tariff with slope \( c_I \) to ensure that \( q_I = q^*_I(0; \omega_I) \). The intercept of this tariff, and hence the price \( T^x(q^*_I(0; \omega_I)) \) associated to the exclusivity offer, is adjusted so that the buyer and the rival, for \( \omega_E = \omega^x_E \), are ex post indifferent between \((q^*_E(\omega^x_E), q^*_I(\omega^x_E)) \) and \((0, q_I^*(0; \omega_I))\). Then the types above the threshold \( \omega^x_E \), who value the incumbent good less than \( \omega^x_E \), prefer a non-exclusive arrangement and pick a point in the nonlinear schedule \( T \). In contrast, the types below \( \omega^x_E \), who value the incumbent good more than \( \omega^x_E \), are attracted by the exclusivity offer. That offer is represented by the point \( X \) on Figure 5.

Next, we compare the magnitude of exclusionary effects in the three considered pricing regimes.
Assumption 3. The nonlinear part of the buyer’s utility, $h(q_E, q_I)$ satisfies
\[
\int_{q_I}^{q_I^1} \left[ \frac{\partial^2 h}{\partial q_E^2} (q_E, q_I^1) \frac{\partial^2 h}{\partial q_I^2} (q_E, q_I) - \frac{\partial^2 h}{\partial q_E \partial q_I} (q_E, q_I^1) \frac{\partial^2 h}{\partial q_E \partial q_I} (q_E, q_I) \right] dq_I \geq 0
\]
for all $q_E$ and $q_I^1 \geq q_I^0$.

Assumption 3 holds for any convex quadratic function because the term under the integral is then constant and nonnegative. If $h$ is a convex function with positive second-order cross derivative as assumed in this paper, the assumption is true for instance when $\partial^2 h / \partial q_I^2$ and $\partial^2 h / \partial q_E \partial q_I$ are respectively non-increasing and nondecreasing in $q_I$.

Recall $\omega_E^{**}$, $\omega_E^u$, $\omega_E^c$ and $\omega_E^x$ denote the maximum values of $\omega_E$ for which the rival is inactive at the first-best optimum and at the second-best optima under respectively a non-conditional schedule $T(q_E, q_I)$, a conditional schedule $T(q_I)$, and an exclusivity price scheme. We refer to these values as the exclusion thresholds in each regime.

Proposition 5. Under Assumption 3, the quantities purchased from the rival firm in each regime are ordered as follows:

\[
0 = q_E^x(\omega_E) \leq q_E^n(\omega_E) \leq q_E^u(\omega_E) < q_E^{**}(\omega_E)
\] (23)

for $\omega_E^x < \omega_E$ and

\[
q_E^e(\omega_E) \leq q_E^n(\omega_E) = q_E^u(\omega_E) < q_E^{**}(\omega_E)
\] (24)
for $\omega_E > \omega_E^x$. The exclusion thresholds are ordered as follows:

$$\omega_E^{x^*} \leq \omega_E^u \leq \omega_E^c \leq \omega_E^x. \quad (25)$$

Proof. The first part of the proposition, relative to the ordering of $q_E$ and $q_E^x$ is proved in the appendix. The left two inequalities in (25) follow directly. We now prove the right inequality. From the analysis of Section 3, we have:

$$W(0, q_I^*(0; \omega_I)) = \max_{q_E^*, q_I} S^v(q_E, q_I; \omega_E^x).$$

Imposing the constraint that $q_E$ must be efficient conditional on $q_I$ reduces the maximum value of the virtual surplus:

$$\max_{q_I} S^v(q_E^*(q_I; \omega_E^x), q_I; \omega_E^x) < W(0, q_I^*(0; \omega_I)) = \max_{q_E^*, q_I} S^v(q_E, q_I; \omega_E^x). \quad (26)$$

The right inequality in (25) follows from the comparison of (22) and (26), combined with the observation that $\max_{q_I} S^v(q_E^*(q_I; \omega_E^x), q_I; \omega_E^x)$ increases with $\omega_E$. \qed

The above analysis implies that $q_I^*(\omega_E^x) < q_I^*(0; \omega_I)$ and hence the optimal quantity of incumbent good under an exclusivity scheme, $q_I^*$, admits a downward discontinuity at $\omega_E^x$, see the dashed line on Figure 4b for an illustration.

**Quadratic Example** For $\omega_E = \omega_E^x$, the buyer and the incumbent are ex ante indifferent between the points $(0, q_I^*(0))$ and the point $U$ on Figure 3. Geometrically, the same isoline of the virtual surplus contains the point $(0, q_I^*(0))$ and the non-conditional second-best allocation denoted by $U$ (the dashed ellipsis passing through $(0, q_I^*(0))$ is tangent to the straight line $q_E = q_E^*(q_I; \omega_E^x)$).

The quantities sold by each suppliers in each of the three regimes are represented on Figures 4a and 4b. A specificity of the quadratic case is that when $q_E > 0$ the quantity of incumbent good is the same under the conditional and non-conditional regimes. (Geometrically the points $C$ and $U$ are on the same horizontal line on Figure 3.) The ordering of $q_I$ across regimes is unclear in general.

**Welfare analysis** The welfare implications of the three pricing regimes involve two types of distortion. First, the quantity of rival good is distorted downwards, which deteriorates the social welfare. The best regime in this dimension is non-conditional pricing. Second, the quantity of incumbent good may be distorted upwards conditionally on the rival supply. The best regime in this dimension is conditional pricing because it completely eliminates buyer opportunism. In this respect, non-conditional schedules perform badly at the bottom of the
distribution because $q_I$ is larger than $q^*_I(0; \omega_I)$ in this region. Exclusivity schemes avoid the latter effect while behaving like unconditional schedules at the top of the distribution; they induce, however, the largest distortions for both goods in an intermediate range of values for the efficiency index $\omega_E$. All these effects are summarized on Figure 6.

In the quadratic case with a uniform distribution, numerical simulations suggest that the non-conditional regime is socially preferred to the conditional and exclusivity regimes and that the exclusivity regime is preferred to the conditional regime for small values of $\beta$.

6 Disposal costs

We now allow the buyer to dispose of unconsumed units at the unit cost $\gamma_I > -c_I$. We know that the total welfare and the virtual surplus linearly decrease in the region where the buyer indeed does not consume all of the purchased units of incumbent good. As noted in Section 3, the possibility of disposal is of no importance for the analysis of the conditional regime because the virtual surplus attains its maximum in the interior of the no-disposal region for all $\gamma_I > -c_I$.

In contrast, we have seen in Sections 4 and 5 that when the buyer must consume all purchased units ($\gamma_I = \infty$) the second-best allocations under a non-conditional schedule and under an exclusivity scheme are essentially determined by the maximum of the constrained
virtual surplus, see the characterization of \((\hat{q}_E^n, \tilde{q}_I^n)\) in Lemma 1. More precisely, \((q_E^n, q_I^n)\) and 
\((\hat{q}_E^n, \tilde{q}_I^n)\) coincide with \((\hat{q}_E^\gamma, \tilde{q}_I^\gamma)\) respectively for \(\omega_E \geq \hat{\omega}_E^\gamma\) and for \(\omega_E \geq \omega_E^\gamma\), with \(\omega_E^\gamma \geq \hat{\omega}_E^\gamma\). Below these thresholds, the quantities purchased from the incumbent, \(q_I^n\) and \(q_I^{\gamma}\), are constant in \(\omega_E\), equal to respectively \(\hat{q}_I^n(\hat{\omega}_E^\gamma)\) and \(q_I^{\gamma}(0; \omega_I)\).

The possibility of disposal does not change the second-best allocations if and only if the solutions found for \(\gamma_I = \infty\) remain in the no-disposal region for finite \(\gamma_I\). This is the case if and only if the buyer is strictly better off consuming all the units purchased than disposing of some of them:

\[
v_I - \frac{\partial h}{\partial q_I}(q_E^n(\omega_E), \tilde{q}_I^n(\omega_E)) = T'(\tilde{q}_I^n(\omega_E)) > -\gamma_I, \tag{27}
\]

for all \(\omega_E\) greater than \(\hat{\omega}_E^\gamma\) or \(\omega_E^\gamma\) depending on the considered regime. If this condition is violated, the maximum of the constrained virtual surplus lies on boundary of the no-disposal region. It is then determined as the intersection of that boundary, \(v_I - \partial h/\partial q_I(q_E, q_I) = -\gamma_I\), and of the conditionally efficient curve, \(q_E = q_E^\gamma(q_I; \omega_E)\). The intersection point is denoted by \(B^\gamma\) on Figure 7 for the quadratic example.\(^9\)

**Proposition 6.** Suppose the assumptions of Proposition 3 hold. Then the second-best allocations under a non-conditional schedule and under an exclusivity scheme do not vary with the magnitude of disposal costs as long as \(\gamma_I\) remains above \(-T'(\tilde{q}_I^n(\omega_E^\gamma))\) and \(-T'(q_I^n(\omega_E^\gamma))\) respectively. As \(\gamma_I\) falls below these thresholds and tends to \(-c_I\), the quantity purchased from the rival and the incumbent respectively increases and decreases, tending to \(q_E^*\) and \(q_I^*\); the slope of the price schedule tends to \(c_I\); the welfare rises to its first-best optimum.

**Proof.** Under the assumptions of Proposition 3, the optimal non-conditional schedule is concave, i.e., \(T'(\tilde{q}_I^n(\omega_E))\) increases with \(\omega_E\). The condition imposed by (27), therefore, is stronger for lower values of \(\omega_E\) or equivalently higher values of \(\hat{q}_I^n\). If (27) holds for \(\omega_E = \omega_E^\gamma\) in the non-conditional regime and for \(\omega_E = \omega_E^\gamma\) in the exclusivity regime, it holds for all \(\omega_E\) above the threshold.

Otherwise, if (27) is violated at the relevant lower bound (\(\hat{\omega}_E^\gamma\) or \(\omega_E^\gamma\)), it is violated for all values of \(\omega_E\) below some threshold. As \(\omega_E\) falls from this threshold to the lower bound, the maximum of the constrained virtual surplus is first located on the boundary of the no-disposal region (point \(B^\gamma\) on Figure 7); at some point, \(q_E\) reaches zero (point \(D^\gamma\) on Figure 7); for lower values of \(\omega_E\), the maximum of the constrained virtual surplus is determined by \(q_E = 0\) and \(q_E^\gamma(q_I; \omega_E) = 0\). Under the non-conditional regime, the latter phenomenon gives rise to bunching as in Section 4; under an exclusivity scheme, the second-best solution switches to \((0, q_I^*(0; \omega_I))\) before the point \(D^\gamma\) is reached.

\(^9\)In this case, the tangency point of the isoline of the virtual surplus (dashed ellipsis) to the straight line \(q_E = \omega_E - \sigma q_I\), lies above the boundary of the no-disposal region, \(q_I = v_I + \gamma_I - \sigma q_E\).
When \( \gamma_I \) falls to \(-c_I\), the boundary of the no-disposal region, \( v_I - \partial h/\partial q_I = -\gamma_I \), moves closer to the conditional efficiency line \( q_I = q_I^*(q_E; \omega_I) \). On Figure 7, the point \( B^\gamma \) tends to the first-best optimum \( A \). At the same time, the slope of the price schedule, \( T'(q_I) \), which lies between \(-\gamma_I\) and \( c_I \), tends to \( c_I \).

\[ W = \text{cst} \]

\[ q_I = v_I + \gamma - \sigma q_E \]

\[ q_E = v_E + \gamma - \sigma q_I \]

\[ q_I^*(q_E; \omega_I) \]

\[ q_E^*(q_I; \omega_E) \]

\[ \omega_I = \text{cst} \]

\[ \omega_E = \text{cst} \]

Figure 7: The constrained virtual surplus is maximal at \( B^\gamma \)

It is ex ante suboptimal for the buyer and the incumbent that some units of incumbent good are produced and disposed of. When the magnitude of the disposal cost is low, the purchased quantity of incumbent good cannot be too far away from the conditionally efficient quantity, \( q_I^*(q_E; \omega_I) \). In other words, the possibility of disposing of unconsumed units of good \( I \) reduces the degree of buyer opportunism present at the second-best allocation.

7 Discussion

We now consider a couple of variants in the timing of events and the instruments available to the parties.
First suppose the buyer and the incumbent can wait for the uncertainty to be resolved before deciding on the price-quantity schedule and still enjoy the same commitment power at this point. They can then implement the efficient allocation and extract all the surplus from non super-efficient rivals through a non-conditional price-quantity schedule. Indeed, if \( q_I^* \) is such that 
\[
q_I^* = \text{argmax}_{q_I} \left( \frac{V(q_I^*, q_{E}^*) - c_E q_{E}^* - T(q_I^*)}{W(q_{E}^*, q_{I}^*) - T(0)} \right) > \bar{q}_I,
\]
the following non-conditional schedule yields the first-best outcome:
\[
T(q_I) = c_I q_I + T(0) \quad \text{for} \quad q_I < \bar{q}_I,
\]
and \( T(\bar{q}_I) \) is such that
\[
V(q_{E}^*, q_{I}^*) - c_{E} q_{E}^* - T(q_I^*) = W(q_{E}^*, q_{I}^*) - T(0) \quad \text{is slightly above} \quad V(0, \bar{q}_I) - T(\bar{q}_I);
\]
and \( T(q_I) = +\infty \) beyond \( \bar{q}_I \). (The constant \( T(0) \) serves to share the surplus \( W(q_{E}^*, q_{I}^*) \).) It is easy to check that the quantities purchased in equilibrium are \( q_{E}^* \) and \( q_I^* \). If the rival and the buyer failed to agree on a price and a quantity for good \( E \), the buyer would purchase \( \bar{q}_I \) from the incumbent. It follows from the definition of \( T(\bar{q}_I) \) that the surplus \( \Delta S_{BE} \) created by from the trade with the rival is negligible, and the rival profit can be made arbitrarily close to zero. This timing, therefore, would be very favorable to the buyer and the incumbent. In contrast, this paper has assumed that the incumbent and the buyer cannot wait for the resolution of uncertainty and at the same time keep their commitment power.

Next we discuss the perhaps intriguing feature of the model that the buyer is part of two successive coalitions. One might consider an interim stage where the buyer has learnt the characteristics of the rival good but has not yet started negotiating a price and a quantity with the rival. It would then be natural to endow the buyer and the incumbent firm with a more powerful instrument consisting of a menu of price-quantity schedules, \( (T(q_I; \hat{\omega}_E)) \), and to consider the following game: (i) the buyer and the incumbent agree on such a menu; (ii) the buyer learns \( \omega_E \) and announces \( \hat{\omega}_E \); (iii) the buyer and the rival negotiate under the price-quantity schedule \( T(q_I; \hat{\omega}_E) \). At the interim stage, the buyer pursues her own interest and may therefore try to cheat on the incumbent by manipulating \( \hat{\omega}_E \). The menu should be designed to maintain truthfulness.

A fundamental observation is that at the interim stage the buyer is weakly better off colluding with the rival firm on the announcement \( \hat{\omega}_E \). In other words, it is in the buyer’s interest to agree with the rival not only on the quantities of both goods and the price of the rival good but also on the announcement. Indeed, negotiating on all variables under control weakly increases the surplus to be shared with the rival, and hence the part that goes to the buyer. We believe that in practice collusion on the announcement is unavoidable, and for this reason we have not included such an interim stage in our modeling framework.

Suppose, for the sake of the discussion, that the buyer and the rival can be prevented

\[10\] When the rival is super-efficient, dealing with the rival creates a positive surplus however large \( q_I \) becomes. Formally the decreasing function \( \beta[V(q_{E}^*(q_I; \omega_E); q_I) - c_E q_{E}^*(q_I; \omega_E) - V(0, q_I)] \) remains positive for all \( q_I \). It can be shown that the rival’s rent at the second-best optimum is equal to the lower bound of this function.
from colluding on the announcement. We now show that if the rival is never super-efficient and has all the bargaining power vis-à-vis the buyer \((\beta = 1)\), then there exists a menu of schedules \(T(q_I; \hat{\omega}_E)\) that yields the first-best outcome. Let \(\bar{q}_I\) be such that \(q_E^*(\bar{q}_I; \omega_E) = 0\) for all \(\omega_E\). We define \(T(q_I; \hat{\omega}_E)\) for each \(\hat{\omega}_E\) in the same way as in the complete information case presented above. The only difference with that case is that we choose \(T(0; \hat{\omega}_E) = W(q_E^*(\hat{\omega}_E), q_I^*(\hat{\omega}_E); \hat{\omega}_E) - \bar{V}\), where \(\bar{V}\) is a constant. The latter equality ensures that \(T(q_I; \hat{\omega}_E)\), and hence the buyer’s outside option, does not depend on \(\omega_E\) or \(\hat{\omega}_E\). Since \(\beta = 1\), the buyer, at the interim stage, gets utility \(\bar{V}\) irrespective of her announcement. If we assume that she declares the true value of \(\omega_E\) to the incumbent, then the first-best allocation obtains. Yet, as explained above, the mechanism is not collusion-proof because the rival would like \(\hat{\omega}_E\) to be as low as possible and is ready to bribe the buyer in return for such an announcement. Since an arbitrarily small bribe is sufficient to break the buyer’s indifference, truthful revelation is unrealistic.

In our companion paper, Choné and Linnemer (2014), we assume that the rival firm cannot compete for the entire buyer’s demand. Assuming that the size of the contestable demand is uncertain, we obtain concave price-quantity schedules together with full exclusion (which we do not have here), as well as many other shapes of nonlinear schedules, including retroactive rebates.

References


\footnote{The problem with \(\beta < 1\), under the assumption that the buyer decides on \(\hat{\omega}_E\) without colluding with the rival, is open. We only know that the first-best allocation cannot be achieved.}


Appendix

A Proof of Proposition 5

We first observe that the bunching procedure at the bottom of the distribution leads to increase the quantity of good $E$, i.e., $q^u_E \geq \hat{q}^u_E$, where $\hat{q}^u_E$ maximizes the constrained virtual surplus, see Lemma 1. This follows from $\hat{q}^u_E = q^u_E(\hat{q}_l; \omega_E)$ and $q^u_E = q^u_E(\tilde{q}_l; \omega_E)$, together with $\tilde{q}_l \leq \bar{q}_l$ when $\tilde{q}_l > 0$. It is therefore sufficient to prove that $\hat{q}^u_E \geq q^c_E$.

Let $\tilde{q}_l$ be defined by $q^c_E(\tilde{q}_l; \omega_E) = q^c_E$. By concavity of the modified virtual surplus, the ordering $q^c_E(\omega_E) \leq \hat{q}^u_E(\omega_E; \gamma)$ is equivalent to

$$\omega_l - \frac{\partial h}{\partial q_l}(q^c_E, \tilde{q}_l) - \beta \frac{1 - F(\omega_E)}{f(\omega_E)} \frac{\partial q^c_E}{\partial q_l}(q^c_E, \tilde{q}_l) \leq 0. \quad (28)$$

This inequality is indeed equivalent to the modified virtual surplus reaching its maximum for $q_l < \tilde{q}_l$, and hence $q_E > q^c_E$. We have, using $q^c_l = q^c_l(q^c_E)$

$$\omega_l - \frac{\partial h}{\partial q_l}(q^c_E, \tilde{q}_l) = - \int_{q^c_l}^{\tilde{q}_l} \frac{\partial^2 h}{\partial q^2_l}(q^c_E, q_l) \, dq_l \quad (29)$$

and, using $q^c_E = q^c_E(\tilde{q}_l)$

$$\beta \frac{1 - F(\omega_E)}{f(\omega_E)} = \omega_E - \frac{\partial h}{\partial q_E}(q^c_E, q^c_l) = \int_{q^c_l}^{\tilde{q}_l} \frac{\partial^2 h}{\partial q_E \partial q_l}(q^c_E, q_l) \, dq_l \quad (30)$$

Finally recall that

$$\frac{\partial q^c_E}{\partial q_l}(q^c_E, \tilde{q}_l) = - \frac{\partial^2 h}{\partial q_E \partial q_l}(q^c_E, \tilde{q}_l) \left/ \frac{\partial^2 h}{\partial q^2_E}(q^c_E, \tilde{q}_l) \right. \quad (31)$$

We get (28) by combining (29), (30), and (31) and applying the inequality of Assumption 3 with $q_E = q^c_E$, $q^0_l = q^c_l$ and $q^1_l = \tilde{q}_l$. 

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