Nonlinear pricing and exclusion:
I. Buyer opportunism

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Abstract

We study the exclusionary properties of nonlinear price-quantity schedules in an Aghion-Bolton style model with elastic demand and product differentiation. We distinguish three regimes depending on whether and how the price charged by the dominant firm depends on the quantity purchased from the rival firm. We find that the supply of rival good is distorted downwards. Moreover, given the quantity supplied from the rival, the buyer may opportunistically purchase inefficiently many units from the dominant firm to pocket quantity rebates. We show that the possibility for the buyer to dispose of unconsumed units attenuates the opportunism problem and limits the exclusionary effects of nonlinear pricing.

JEL codes: L12, L42, D82, D86

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1 Introduction

In recent years, exclusionary conduct by firms with strong market power has become a high-priority issue on the agenda of antitrust agencies. For instance, the European Commission has made it clear that the emphasis of its enforcement activities is on "ensuring that undertakings which hold a dominant position do not exclude their competitors by other means than competing on the merits of the products or services they provide." The U.S. Department of Justice states that "whether conduct has the potential to exclude, eliminate, or weaken the competitiveness of equally efficient competitors can be a useful inquiry", and suggests that this inquiry "may be best suited to particular pricing practices."[1]

In this article, we consider a wide range of pricing practices that fall under the general heading of nonlinear pricing, e.g. quantity or market-share rebates and exclusivity discounts. We organize a taxonomy of price schemes around the following main distinction: whether or not the price set by the dominant firm depends on the quantity supplied from rivals. When this is the case, we say that the dominant firm's price schedule is "conditional" (on rival quantities). Market-share discounts enter into this category. Because enforcing a conditional price may be infeasible or legally prohibited, we also consider the situation where the firms are restricted to use "non-conditional" price schedules. Finally we define "exclusivity-based" schedules as conditional schedules for which the price depends on whether or not the buyer supplies exclusively from the dominant firm, but does not otherwise depend on the quantities sold by competitors.

Our analysis aims to understand how these different types of price-quantity schedules affect the way large buyers split their purchase requirements between the dominant firm and rival suppliers. As in Aghion and Bolton (1987), we assume that a buyer and a dominant firm contract at a time when the characteristics of a rival good are not yet known, i.e., the rival's cost and the buyer's willingness to pay for the rival good are uncertain.[2] To concentrate on the exclusionary effects of nonlinear pricing, we assume away any bilateral inefficiency (e.g. asymmetric information) between the buyer and each of the two suppliers. In particular,

[1]See, respectively, European Commission (2009) and U.S. Department of Justice (2008). High-profile exclusionary cases involving pricing practices include *Virgin v British Airways* (S.D.N.Y. 1999 and 2nd Cir. 2001), *Concord Boat* (8th Cir. 2000), *Lepage's v 3M* (3d Cir. 2003) in the United States; *Virgin/British Airways* (2000/74/EC of 14 July 1999), *Michelin* (COMP/E-2/36.041/PO of 20 June 2001), *Tomra* (COMP/E-1/38.113, 2006), and *Intel* (COMP/C-3 /37.990, 2009) in Europe. Figueroa et al. (2014) present a Chilean case: *Chile v. Unilever.*

[2]For instance, Intel's rival, AMD, launched new products on a quarterly basis, and these products were considered a "growing competitive threat" by Intel, see pages 45-53 of the Commission's decision cited in Footnote 1.

the buyer and the dominant firm would have no reason to distort the traded quantity in the absence of a rival. Similarly, we assume throughout the article that the negotiation between the buyer and the rival takes place under perfect information and is efficient. Formally, the game is equivalent to an asymmetric information set-up where the principal would be the buyer-dominant firm pair and the agent the buyer-rival pair, and we can thus use insights from the nonlinear pricing literature, see Wilson (1993) and Laffont and Martimort (2002). The fact that the buyer is part of both coalitions raises interesting theoretical questions that are discussed at the end of the article. In particular, the buyer's dual role might open a scope for more sophisticated screening instruments and create subtle patterns of information revelation.

In the spirit of Martimort and Stole (2009), we are interested in distortions of productive allocations at both the extensive and the intensive margins. In their terminology, our framework is a common-agency game: the buyer may supply exclusively from the dominant firm, in which case the rival is driven out of the market. We carefully examine distortions at the "extensive" margin, and indeed find that complete exclusion of an efficient rival occurs with positive probability. A contribution of the present article is to consider distortions at the intensive margin as well. To this aim, we model the incumbent and the rival goods as imperfect substitutes for the buyer. The degree of substitution can vary from perfect substitutes to independent goods.[3] We find intensive distortions of the quantity supplied by the rival, specifically that quantity is positive but distorted downwards, which is sometimes referred to as "partial foreclosure" in the antitrust literature.

We emphasize a second kind of intensive distortions, which pertains to the quantity of incumbent good *at given level of rival supply*. This distortion is linked to an opportunistic behavior of the buyer at the last stage of the game, hereafter referred to as "buyer opportunism". The intuition goes as follows. The general purpose of the quantity-price schedules agreed upon by the dominant firm and the buyer is to place the latter in a favorable position when bargaining with the rival. In this bargain, the buyer can argue she will lose rebates if she purchases less from the dominant firm, which allows her to extract surplus from the rival. *Ex ante*, the dominant firm and the buyer share expected profits and their interests are aligned. *Ex post*, however, the buyer does not take into account the production costs of the incumbent good. Due to the offered rebates, the buyer has an incentive to purchase inefficiently many units from the dominant firm conditional on the quantity supplied from the rival.

A key issue in the article is to compare the exclusionary properties of the three considered price schedules – conditional, exclusivity-based, and non-conditional. In Aghion and Bolton

[3]Marx and Shaffer (2004) also study a model à la Aghion and Bolton with differentiated goods but they restrict themselves to a complete information setting.

(1987) the buyer's demand is inelastic and is supplied from only one producer, making it hard to distinguish between the three types of schedules. In contrast, here, the results differ strikingly across the three pricing regimes.

When the price of the incumbent good can freely depend on the supply from the rival (conditional regime), the optimal schedule is a two-part tariff. The incumbent good is priced at marginal cost, hence the absence of buyer ex post opportunism. The fixed part of the tariff is increasing and concave in the quantity purchased from the rival. That fixed part can be seen as a penalty imposed for buying from the rival, in line with Aghion and Bolton (1987), but here the rival's supply is distorted at both the extensive and the intensive margins. These distortions increase with the rival's bargaining power vis-à-vis the buyer. Market-share rebates are shown to be ill-adapted to control buyer opportunism.

When the price schedule only depends on the incumbent quantity (non-conditional regime), the buyer purchases the efficient quantity of rival good given the incumbent's quantity. This link between the two quantities creates a channel through which the buyer and the dominant firm can indirectly control the rival's activity. In equilibrium, the marginal price of the dominant firm is lower than the marginal cost of production up to a certain quantity threshold. These generous rebates allow the buyer to extract a good deal from the rival but at the same time induce her to behave opportunistically *ex post*. This distortion, in turn, translates into complete or partial foreclosure of efficient rival types. The presence of complete exclusion in equilibrium implies that the price schedule is not globally concave: the price is set at a high level beyond the quantity threshold mentioned above to deter the buyer from purchasing even more units of incumbent good, hence a convex kink in the schedule.

In the exclusivity-based regime, the price schedule is the same as in the non-conditional case for low quantities of incumbent good. The exclusivity offer prevents buyer opportunism when the rival is inactive. On the other hand, this offer creates locally a strong distortion at the extensive margin, excluding a bunch of efficient rival types.

Finally, we are able to extend the analysis to the case where the buyer can dispose of or resell unconsumed units of incumbent good. In practice, the magnitude of the disposal costs depends on the seller's ability to impose or to prevent particular uses of the purchased units and on the buyer's ability to avoid such monitoring by the dominant firm. Depending on the industry, unused items can be difficult to store or dispose of making disposal costs large. On the contrary, the buyer may have access to a secondary market and resell the extra units making disposal costs negative.

Purchasing units from the dominant firm with the sole purpose of pocketing rebates, and then throwing away the unneeded units, would constitute an extreme form of buyer opportunism. We show that this form is never part of an equilibrium. We find that low disposal costs prevent the dominant firm from committing on too generous rebates because the buyer could purchase units and discard them. Lower disposal costs are associated to less exclusion and higher values of the expected total welfare. Antitrust authorities, therefore, should pay close attention to contracting provisions that help increase disposal costs.

It is worthwhile relating our work to recent studies on market-share discounts. In a setting with a dominant firm, a competitive fringe and two retailers, Inderst and Shaffer (2010) show that market-share discounts can be used by the dominant firm to dampen intra- and inter-brand competition. Their anticompetitive scenario, contrary to the one presented here, highlights retail competition and assumes complete information. Turning to models with imperfect information, most of the literature has examined how specific forms of pricing perform in discriminating among privately informed buyers. For instance, in a discrete type model, Kolay et al. (2004) show that all-units discounts are more effective than menus of two-part tariffs in screening out retailers with private information about the state of demand. Majumdar and Shaffer (2009) and Calzolari and Denicolo (2013) introduce market-share discounts. In the former article, a dominant firm resorts to nonlinear pricing to screen a buyer who is informed about the size of demand and who also sells a good provided by a competitive fringe. The latter article addresses the issue in a symmetric duopoly setting, considering both market-share discounts and exclusive contracts. Calzolari and Denicolo (2014) relaxes the symmetry assumption by assuming that the dominant firm has a competitive advantage over its rival. Finally, in a companion article, Choné and Linnemer (2014), we introduce on top of incumbency the notion that the dominant firm is, at least to some extent, an "unavoidable trading partner" – a key ingredient of dominance under European competition law at least since *Hoffmann-La Roche*.[4]

The article is organized as follows. Section 2 introduces the model, and Section 3 studies conditional price-quantity schedules. Assuming very large disposal costs, Sections 4 and 5 explore the non-conditional and exclusivity-based regimes. Section 6 describes the effect of moderate disposal costs. Section 7 discusses a couple of extensions regarding the timing of the game, the informational environment, and the available instruments.

2 The model and purchase decisions

A dominant firm, I, competes with a rival, E, to serve a buyer, B. Marginal production costs are assumed to be constant and are denoted by c_I and c_E. The timing of events reflects the incumbency advantage of the dominant firm and the uncertainty as to the characteristics of

[4]Judgment of the European Court of Justice of 13 February 1979. Hoffmann-La Roche & Co. AG v Commission of the European Communities. Case 85\76.

the rival good: 1) B and I design a price-quantity schedule to maximize (and split) their joint expected surplus. At this stage, they know c_I and the characteristics of good I, but they do not know c_E nor the willingness to pay v_E for the rival good. 2) B and E discover c_E and v_E and jointly decide on the variables under their control, namely a transfer p_E and quantities q_E and q_I, knowing both the terms of the agreement between B and I and all the relevant cost and preference parameters.

At the first stage, we consider three types of price-quantity schedules that differ in how the price of the incumbent good depends on the quantity supplied from the rival: (i) under a non-conditional schedules $T(q_I)$, the price depends only on the number of I-units purchased; (ii) under a conditional schedule $T(q_E, q_I)$, the price of q_I units of good I freely depends on the quantity purchased from the rival; (iii) an exclusivity scheme is a pair of schedules $(T(q_I), T^{\mathrm{x}}(q_I))$ that specifies the price of q_I units of good I if the buyer supplies exclusively from the dominant firm, $T^{\mathrm{x}}(q_I)$, and if she purchases a positive number of units from the rival firm, $T(q_I)$.[5]

At the second stage, we assume that B and E negotiate under complete information (Nash bargaining where β denotes E's bargaining power) to maximize their joint surplus. The timing of negotiations assumes that B and I cannot renegotiate once uncertainty is resolved. This assumption and a couple of variants are discussed in Section 7.

Buyer's demand When the buyer consumes x_E units of good E and x_I units of good I, she earns a gross profit of $v_E x_E + v_I x_I - h(x_E, x_I)$, where h is a convex function of (x_E, x_I) with first derivatives at $(0, 0)$ equal to zero and with positive cross-derivative to reflect the imperfect substitutability of the two goods.

A key feature of the model is that the buyer can dispose of unneeded units of each good at a cost $\gamma_E \geq -c_E$ and $\gamma_I \geq -c_I$. That is, it is always inefficient (from a welfare perspective) to produce units in order to throw them away or to resell them. Consequently, the buyer's net utility if she purchases q_E units from the rival and q_I units from the dominant firm is

$$V(q_E, q_I) = \max_{x_E \leq q_E, x_I \leq q_I} v_E x_E + v_I x_I - h(x_E, x_I) - \gamma_E(q_E - x_E) - \gamma_I(q_I - x_I). \tag{1}$$

The buyer disposes of units of good k, $k = E, I$, when the purchased quantity q_k is so large that the marginal utility $v_k - \partial h(q_k, q_l)/\partial q_k$ becomes smaller than the utility loss caused by disposal, $-\gamma_k$. In this region, the buyer net utility V decreases linearly with q_k, and the marginal net utility $\partial V/\partial q_k$ is equal to $-\gamma_k$. When the buyer consumes all the purchased

[5] We do not impose a priori restrictions on the shape of the price schedules. Some authors have studied specific types of schedules such as two-part tariffs (Marx and Shaffer (1999)) and all-units discounts (Feess and Wohlschlegel (2010)) under complete information.

units of the rival and incumbent goods ("no-disposal region"), the marginal net utility is greater than $-\gamma_k$ for each good.

Efficient quantities We denote by q_E^{**} and q_I^{**} the quantities that maximize the total surplus

$$W\left(q_E, q_I\right) = V\left(q_E, q_I; v_E\right) - c_E q_E - c_I q_I.$$

The efficient quantities involve no disposal and hence do not depend on the magnitude of disposal costs. These quantities also maximize $\omega_E q_E + \omega_I q_I - h(q_E, q_I)$, where $\omega_E = v_E - c_E$ and $\omega_I = v_I - c_I$. Hence q_E^{**} and q_I^{**} are respectively nondecreasing and non-increasing in ω_E. Hereafter, we denote by ω_E^{**} the maximum value of ω_E for which $q_E^{**}(\omega_E) = 0$. We now introduce the distribution of ω_E, that we denote by F, and a set of assumptions maintained throughout the article.

Assumption 1. *The distribution of ω_E has a positive density f on its support $[\underline{\omega}_E, \overline{\omega}_E]$, with $\underline{\omega}_E < \omega_E^{**} < \overline{\omega}_E$ and $q_I^{**}(\overline{\omega}_E) > 0$. The hazard rate $f/(1-F)$ is nondecreasing in ω_E.*

The assumption on ω_E^{**} allows us to concentrate on the most interesting case where full exclusion is socially optimal with positive probability. Moreover, to avoid uninteresting complications, we assume that both firms are active when the surplus created by the rival is maximal, formally $q_E^{**}(\overline{\omega}_E), q_I^{**}(\overline{\omega}_E) > 0$.

Definition 1. *The quantity of good E that maximizes the social welfare W given q_I is said to be conditionally efficient and is denoted by $q_E^*(q_I; \omega_E)$. The conditionally efficient quantity of incumbent good, $q_I^*(q_E; \omega_I)$, is symmetrically defined.*

If the buyer consumes all the units of good I she has purchased, then $q_E^*(q_I; \omega_E)$ is defined by the first order condition $\partial h/\partial q_E(q_E^*, q_I) = \omega_E$. In this region, the function $q_E^*(q_I; \omega_E)$ is decreasing by substitutability. In contrast, in the region where q_I is so high that the buyer disposes of some units of good I, q_E^* does not vary with q_I as only the *consumed* quantity of good I is relevant to determine the conditionally efficient quantity of good E. In this region, total surplus W decreases linearly with q_I, and the partial derivative $\partial W/\partial q_I$ is $-c_I - \gamma$.

Definition 2. *The rival firm is said to be* super-efficient *when $q_E^*(q_I; \omega_E)$ is positive for any value of q_I.*

Example: Quadratic utility The leading example in this article has $h(x_E, x_I) = x_E^2/2 + x_I^2/2 + \sigma x_E x_I$, $0 \le \sigma < 1$. The efficient quantities involve no disposal cost and are given by

$$q_E^{**}(\omega_E) = \max\left(\frac{\omega_E - \sigma\omega_I}{1-\sigma^2}, 0\right) \text{ and } q_I^{**}(\omega_E) = \frac{\omega_I - \sigma\omega_E}{1-\sigma^2}. \tag{2}$$

To respect Assumption 1, we must have in the quadratic case: $\underline{\omega}_E < \omega_E^{**} = \sigma\omega_I < \overline{\omega}_E < \omega_I/\sigma$. The efficient allocation is represented by the point A on Figure 1. When the buyer's utility is quadratic, the welfare isolines are ellipses centered at A.

If the buyer has purchased q_I units of good I, with $q_I \geq v_I + \gamma - \sigma q_E$, she consumes $x_I = v_I + \gamma - \sigma q_E$ units of good I, thus an amount that is independent of q_I. The no-disposal region is located below the bold dashed line on the figure. Applying the envelope theorem, we find that the conditionally efficient quantity $q_E^*(q_I; \omega_E)$ is constant in this region and equal to the second argument of the following maximum:

$$q_E^*(q_I; \omega_E) = \max\left(\omega_E - \sigma q_I\,,\ \frac{\omega_E - \sigma(v_I + \gamma)}{1-\sigma^2}, 0\right).$$

The rival firm is super-efficient if and only if $\omega_E > \sigma(v_I + \gamma)$. This is the case represented on Figure 1, where $q_E^*(q_I; \omega_E)$ is positive for any value of q_I.

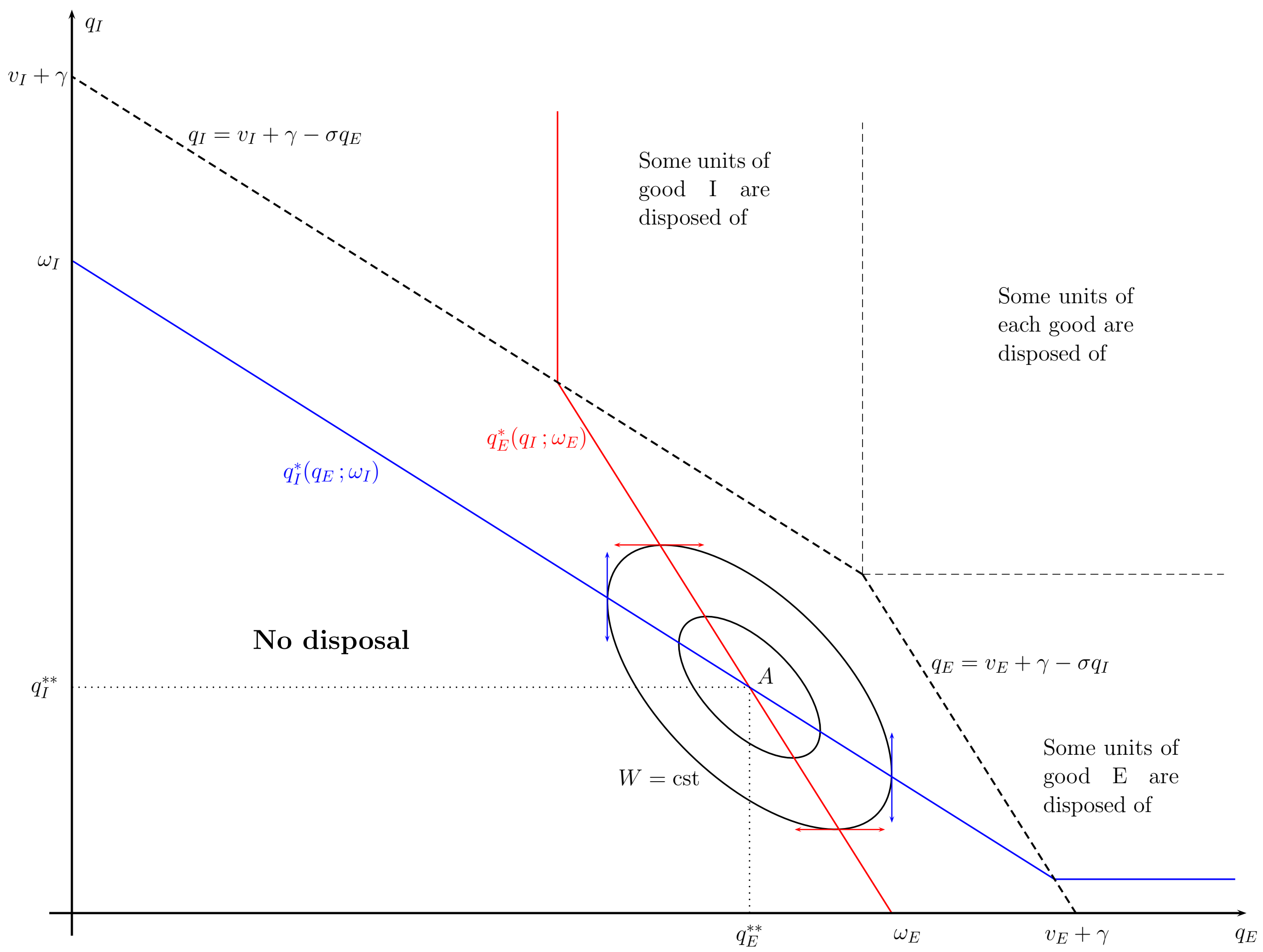

Figure 1: The total welfare is maximal at A (quadratic example)

Purchase decisions The last stage of the game takes place under perfect information, given the price schedule $T = T(q_E, q_I)$ or $T = T(q_I)$ and the known characteristics of the rival good. The buyer and the rival choose the quantities to maximize their joint surplus

$$S_{BE} = \max_{q_E, q_I} V(q_E, q_I) - T(q_E, q_I) - c_E q_E, \tag{3}$$

with no consideration for the dominant firm's cost or profit. The above expression shows that under a non-conditional schedule $T(q_I)$, the quantity of rival good is efficient given that of the incumbent good, formally $q_E = q_E^*(q_I; \omega_E)$, implying that no unit of the rival good is produced and disposed of. To avoid uninteresting developments, we take the latter property as granted under conditional schedules as well.[6]

Without loss of generality, the competitor's outside option is normalized to zero. As to the buyer, she may source exclusively from the dominant firm, so her outside option is

$$V_B^0 = \max_{q_I \geq 0} V(0, q_I) - T(0, q_I). \tag{4}$$

The reservation utility V_B^0, which depends on the price schedule by (4), is endogenous but independent from ω_E. The surplus created by the buyer and the rival firm can thus be written as

$$\Delta S_{BE} = S_{BE} - V_B^0. \tag{5}$$

Denoting by $\beta \in (0, 1)$ the competitor's bargaining power vis-à-vis the buyer, we derive the competitor's and buyer's profits:

$$\begin{array}{rcccl} \Pi_E = & \beta & \Delta S_{BE} & & \\ \Pi_B = & (1-\beta) & \Delta S_{BE} & + & V_B^0. \end{array} \tag{6}$$

If $\beta = 0$, the competitor has no bargaining power and may be seen as a competitive fringe from which the buyer can purchase any quantity at price c_E. On the contrary, the case $\beta = 1$ happens when the competitor has all the bargaining power vis-à-vis the buyer.

Virtual surplus We henceforth focus on the situation where the buyer and the dominant firm commit to a price-quantity schedule before the uncertainty on the rival good is resolved. In this context, the schedule is designed ex ante to maximize the expected joint surplus, equal to the total surplus minus the profit left to the competitor:

$$\mathbb{E}_{c_E, v_E} \Pi_{BI} = \mathbb{E}_{c_E, v_E} \left\{ W(q_E, q_I; c_E, v_E) - \Pi_E \right\}, \tag{7}$$

[6] It would be extremely counter-intuitive that the buyer and the dominant firm use their pricing instrument, e.g. $T(q_E, q_I)$, to encourage production and disposal of the rival good. The following analysis finds no force pushing in that direction. A formal proof is available upon request.

where q_E and q_I are solution to (3) and Π_E is given by (6). The sharing of the expected joint surplus between the buyer and the dominant firm, and hence the respective bargaining power of each party, play no role in the following analysis.[7]

As all purchased units of the rival good are consumed, the surplus (3) depends on the uncertain cost and preference parameters c_E and v_E only through the difference $\omega_E = v_E - c_E$. The surplus S_{BE} is a convex function of ω_E because it is the upper bound of a family of functions that are affine in ω_E; hence S_{BE} is almost everywhere differentiable in ω_E. By the envelope theorem, the rent left to the rival satisfies:

$$\frac{\partial \Pi_E}{\partial \omega_E} = \beta \frac{\partial \Delta S_{BE}}{\partial \omega_E} = \beta q_E(\omega_E). \tag{8}$$

Integrating by parts, we get

$$\int_{\underline{\omega}_E}^{\overline{\omega}_E} \Pi_E(\omega_E) f(\omega_E)\, \mathrm{d}\omega_E = \Pi_E(\underline{\omega}_E) + \beta \int_{\underline{\omega}_E}^{\overline{\omega}_E} q_E(\omega_E)[1 - F(\omega_E)]\, \mathrm{d}\omega_E.$$

Substituting in (7), we rewrite the buyer-dominant firm objective as

$$\mathbb{E}_{\omega_E} \Pi_{BI} = \mathbb{E}_{\omega_E} S^{\mathrm{v}}(q_E, q_I; \omega_E) - \Pi_E(\underline{\omega}_E), \tag{9}$$

where, following Jullien (2000), we have defined the "virtual surplus" S^{v} as

$$S^{\mathrm{v}}(q_E, q_I; \omega_E) = W(q_E, q_I; \omega_E) - \beta q_E \frac{1 - F(\omega_E)}{f(\omega_E)}. \tag{10}$$

The virtual surplus is the total surplus $W(q_E, q_I; \omega_E)$ adjusted for the informational rents $\beta q_E \left(1 - F(\omega_E)\right) / f(\omega_E)$ induced by the self-selection constraints.

Buyer opportunism Expression (7) reflects a standard rent-extraction tradeoff. From the ex ante perspective, the tariff has two purposes: on the one hand, maximizing the expected welfare W; on the other, making $\Pi_E = \beta \Delta S_{BE}$ as small as possible. Rent extraction is obtained by placing competitive pressure on the rival firm, i.e., granting the buyer low marginal price to force the rival to match these rebates, which may drive the rival out of the market or distort downwards the quantity it sells, $q_E < q_E^{**}$.

The novelty in our analysis lies in the possible distortion of q_I. We call buyer opportunism the fact that the buyer purchases too many units of incumbent good given her supply from the rival, formally $q_I > q_I^*(q_E; \omega_I)$. We show below that buyer opportunism is observed in equilibrium except when the buyer and the dominant firm have the most powerful instrument

[7] Figueroa et al. (2014) restrict the ability of the buyer and the dominant firm to share rents through transfers and explore the implications for inefficient exclusion in a model with inelastic demand and one-dimensional uncertainty.

$T(q_E, q_I)$ at their disposal. Granting rebates to the buyer induces her to distort the quantity purchased from the dominant firm upwards. The buyer indeed wants to pocket the rebates and does not internalize the production cost c_I when she purchases from the dominant firm.

Purchasing and throwing away units of incumbent good would constitute an extreme form of buyer opportunism. We show that this form is never part of an equilibrium. On the contrary, we find that the possibility of disposing of units of good I alters the terms of the rent-extraction tradeoff and limits the exclusionary effects of nonlinear pricing.

3 Conditional price-quantity schedules

For each type of pricing instrument, we proceed as follows. First, we derive necessary conditions for a quantity allocation $(q_E(\omega_E), q_I(\omega_E))$ to be achieved with the considered type of price-quantity schedule. Second, we maximize the virtual surplus under those necessary conditions. Third, we check that the identified allocation can indeed be implemented.

We start with conditional schedules $T(q_E, q_I)$. As regards implementability, we simply observe that the quantity of rival good q_E is a nondecreasing function of ω_E. This follows from the convexity of the surplus function $S_{BE}(\omega_E)$, combined with the envelope condition (8).

Maximization of the virtual surplus The maximum of the virtual surplus (10) is achieved in the no-disposal region. At the optimum, the quantity of good I is conditionally efficient, $q_I = q_I^*(q_E; \omega_I)$. More precisely, for each ω_E, the virtual surplus S^{v} is maximal at (q_E^c, q_I^c) such that

$$\omega_E - \frac{\partial h}{\partial q_E}(q_E^c, q_I^c) \leq \beta \frac{1 - F(\omega_E)}{f(\omega_E)} \quad \text{and} \quad \frac{\partial h}{\partial q_I}(q_E^c, q_I^c) = \omega_I. \tag{11}$$

with equality in the first inequality when $q_E^c > 0$. In this case, the two conditions can be collapsed into

$$\omega_E - \frac{\partial h}{\partial q_E}(q_E^c, q_I^*(q_E^c; \omega_I)) = \beta \frac{1 - F(\omega_E)}{f(\omega_E)}. \tag{12}$$

By convexity of h, the function $\partial h / \partial q_E(q_E, q_I^*(q_E))$ increases with q_E, and hence the left-hand side of (12) decreases with q_E^c. Under Assumption 1, the right-hand side is non-increasing in ω_E. The function $q_E^c(\omega_E)$, therefore, is nondecreasing. Let ω_E^c be defined by

$$\omega_E^c - \beta \frac{1 - F(\omega_E^c)}{f(\omega_E^c)} - \frac{\partial h}{\partial q_E}(0, q_I^*(0; \omega_I)) = 0.$$

The left-hand side of the above equation increases with ω_E^c and is negative for $\omega_E = \omega_E^{**} = \partial h / \partial q_E(0, q_I^*(0; \omega_I))$, hence $\omega_E^c > \omega_E^{**}$.

For $\omega_E > \omega_E^c$, the rival supplies a positive quantity, $q_E^c > 0$, as represented by point C on Figure 2. For ω_E below ω_E^c, the virtual surplus is maximized at the point $(q_E^c, q_I^c) =$

$(0; q_I^*(0;\omega_I))$, i.e., the rival is driven out of the market – the distortion is at the extensive margin. (On Figure 2, the point C would lie on the q_I-axis.) The dashed ellipses centered at C represent the isolines of the virtual surplus.

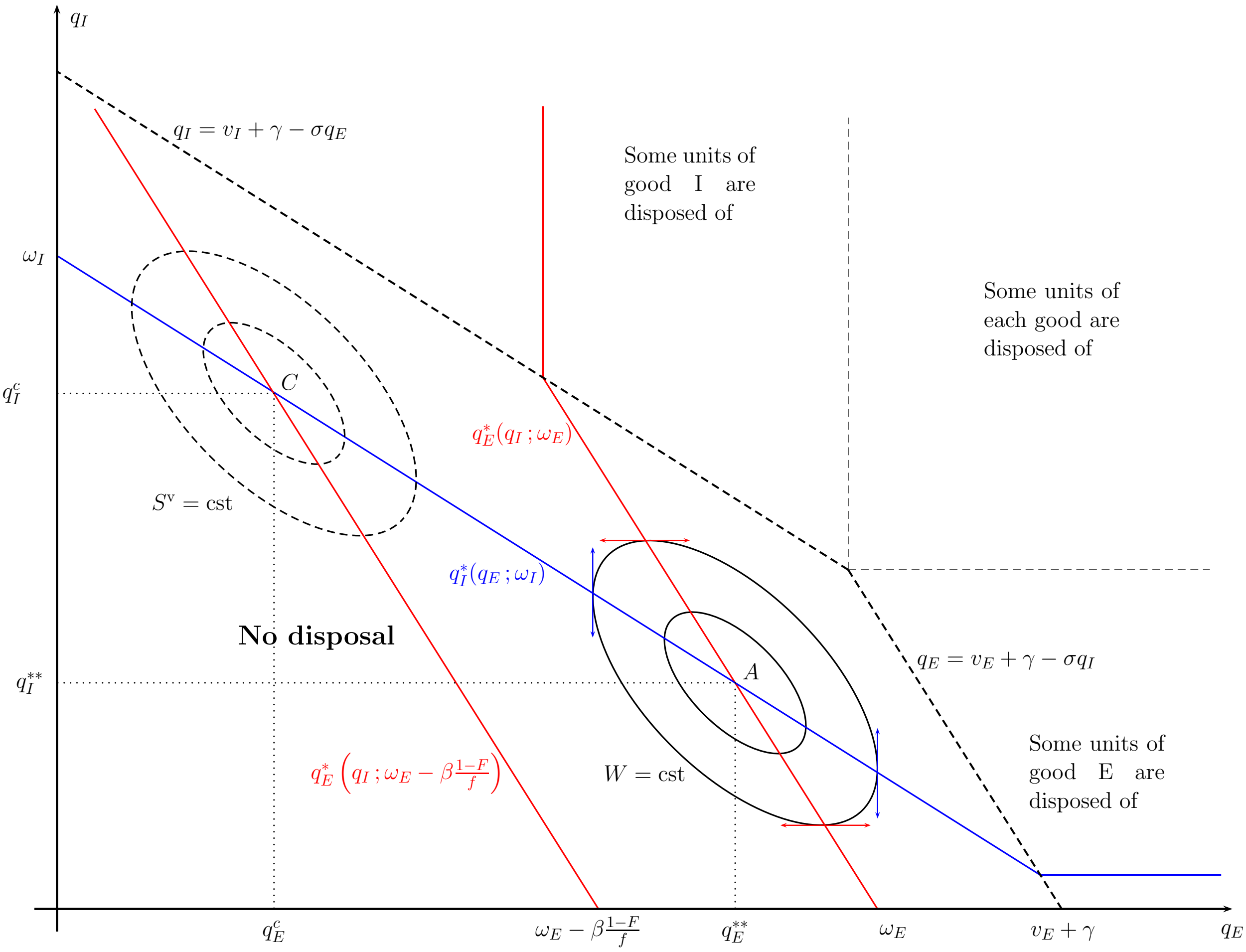

Figure 2: The virtual surplus is maximal at C (quadratic example)

Implementation The quantity allocation $(q_E^c(\omega_E), q_I^c(\omega_E))$ can be represented by a curve in the (q_E, q_I)-space, see Figure 4c for an example. Conditional schedules $T(q_E, q_I)$ are defined on the whole space $q_E \geq 0, q_I \geq 0$. Considering perturbations of T far away from the quantity allocation shows that many schedules $T(q_E, q_I)$ yield the same allocation. Yet there are necessary conditions for implementation. We have seen that the surplus

$$S_{BE}(\omega_E) = \omega_E q_E^c(\omega_E) + v_I q_I^c(\omega_E) - h(q_E^c(\omega_E), q_I^c(\omega_E)) - T(q_E^c(\omega_E), q_I^c(\omega_E))$$

is almost everywhere differentiable in ω_E, with its derivative being $q_E^c(\omega_E)$. It follows that T is differentiable along the quantity allocation, but in general it need not be differentiable with respect to (q_E, q_I). The simplest way to implement the allocation $(q_E^c(\omega_E), q_I^c(\omega_E))$ is a two-part tariff of the form $T(q_E, q_I) = c_I q_I + P(q_E)$. In this case, the right-hand side depends on ω_E through $q_E^c(\omega_E)$ but not through $q_I^c(\omega_E) = q_I^*(q_E^c(\omega_E))$ by definition of q_I^*. It follows that the function P is differentiable. Differentiating with respect to ω_E and simplifying by $\mathrm{d}q_E^c/\,\mathrm{d}\omega_E$ in the region where q_E^c is increasing, we get $\omega_E - \partial h/\partial q_E = P'(q_E^c(\omega_E))$, where the partial derivative is evaluated at $(q_E^c(\omega_E), q_I^c(\omega_E))$. Combining with (12), we find

$$P'(q_E^c(\omega_E)) = \beta \frac{1 - F(\omega_E)}{f(\omega_E)}. \tag{13}$$

Proposition 1. *The following properties hold at the second-best optimum with a conditional price-quantity schedule:*

1. *For any level of rival's surplus ω_E but $\overline{\omega}_E$, the quantity purchased from the rival, q_E^c, is distorted downwards relative to q_E^{**}. Exclusion is complete for $\omega_E \leq \omega_E^c$, where $\omega_E^c > \omega_E^{**}$.*

2. *The quantity purchased from the dominant firm, $q_I^c = q_I^*(q_E^c; \omega_E)$, is efficient given q_E^c but distorted upwards relative to q_I^{**}.*

3. *The magnitude of the disposal costs, γ_I, does not affect the buyer's supply policy or the price quantity schedule $T(q_E, q_I)$.*

4. *The buyer and the dominant firm may agree on a price schedule that is linear in q_I with slope c_I and nondecreasing and concave in q_E.*

Letting the price of the incumbent good depend on the quantity purchased from the rival allows the buyer and the dominant firm to neutralize buyer ex post opportunism, i.e., to make sure that the quantity of incumbent good is efficient given the quantity supplied from the rival. Conditional efficiency imposes that the partial derivative of T with respect to q_I is c_I at the second-best allocation. This condition is hard to meet when the price schedule T depends on the market share $q_I/(q_E+q_I)$ rather than directly on q_E, because the market share is nonlinear in q_I. Market-share discounts, for this reason, appear as a less convenient way to implement the second-best allocation than two-part tariffs of the form $c_I q_I + P(q_E)$.

Proposition 1 builds a bridge between the literatures on market foreclosure and nonlinear pricing. As in Aghion and Bolton (1987), the buyer and the dominant firm jointly act like a monopoly towards the rival, setting $P(q_E)$ to extract rent at the cost of reducing the extent of entry: $q_E < q_E^{**}$, which yields inefficient market foreclosure. The efficiency-rent tradeoff leads to more inefficient exclusion as the rival's bargaining power, β, rises. When β is high, the rival has a strong bargaining power vis-à-vis the buyer, which makes rent extraction a more serious issue and pushes towards reducing q_E.

Aghion and Bolton (1987) assume that the buyer's demand was supplied entirely by a single supplier. Hence they interpret the difference $P(1) - P(0)$ as a penalty for breach of contract. In contrast, we allow the buyer to split her purchase requirements between the two suppliers and find that inefficient foreclosure may be complete or partial: $0 \leq q_E < q_E^{**}$. We interpret the difference $P(q_E) - P(0)$ as rebates lost when supplying from the competitor. The presence of these rebates implies a form of below-cost pricing. Specifically, when $q_E > 0$, the average incremental price of the "last" units of good I (units between $q_I^*(q_E;\omega_I)$ and $q_I^*(0;\omega_I)$) is lower than the production cost:

$$\frac{T(0, q_I^*(0;\omega_I)) - T(q_E, q_I^*(q_E;\omega_I))}{q_I^*(0;\omega_I) - q_I^*(q_E;\omega_I)} = c_I - \frac{P(q_E) - P(0)}{q_I^*(0;\omega_I) - q_I^*(q_E;\omega_I)} < c_I,$$

because the penalty function is increasing, $P(q_E) > P(0)$, and the function q_I^* is decreasing, $q_I^*(0;\omega_I) > q_I^*(q_E;\omega_I)$. The above price-cost comparison is reminiscent of the "as-efficient competitor test". The precise form of the test advocated by the European Commission, which involves the notion of contestable demand, is more accurately described in a model with inelastic demand (see our companion article, Choné and Linnemer (2014)).

Quadratic example With $h(q_E, q_I; s_E) = q_E^2/2 + q_I^2/2 + \sigma q_E q_I$, the second-best quantities purchased from both suppliers under a conditional tariff are given by

$$q_E^c(\omega_E) = \max\left(\omega_E - \beta\frac{1 - F(\omega_E)}{f(\omega_E)} - \sigma q_I^c\,,\, 0\right) \quad \text{and} \quad q_I^c(\omega_E) = q_I^*(q_E^c),$$

The quantity purchased from the dominant firm is conditionally efficient while that purchased from the rival is distorted downwards. We get the allocation that would be efficient if the rival's efficiency index ω_E were artificially reduced by $\beta(1 - F(\omega_E))/f(\omega_E)$:

$$q_E^c(\omega_E) = q_E^{**}\left(\omega_E - \beta\frac{1 - F(\omega_E)}{f(\omega_E)}\right), q_I^c(\omega_E) = q_I^{**}\left(\omega_E - \beta\frac{1 - F(\omega_E)}{f(\omega_E)}\right)$$

where the efficient quantities q_E^{**} and q_I^{**} are given by (2). When ω_E is uniformly distributed over the interval $[\underline{\omega}_E, \overline{\omega}_E]$, the penalty function is quadratic, with its derivative being given by

$$P'(q_E) = \frac{\beta}{1+\beta}\left[q_E^{**}(\overline{\omega}_E) - q_E\right].$$

In the limiting case where the demands for the two goods are independent ($\sigma = 0$), the penalty is given by the formula above with $q_E^{**}(\overline{\omega}_E) = \overline{\omega}_E$, and hence is increasing in q_E. Rent-shifting appears here as pure extortion, and we now turn to more realistic price instruments.

4 Non-conditional price-quantity schedules

The analysis is more involved when the price schedule cannot freely depend on the quantity purchased from the rival, because buyer opportunism will materialize at the second-best allocation and the degree of buyer opportunism will depend on the magnitude of the disposal costs. To simplify the presentation, in this and the next section, we assume that the buyer must consume all the purchased units, $\gamma_E = \gamma_I = +\infty$, and hence $V(q_E, q_I) = v_E q_E + v_I q_I - h(q_E, q_I)$. The effect of disposal costs is examined Section 6.

Implementable quantity functions We now suppose that the buyer and the dominant firm are constrained to use a schedule of the form $T(q_I)$. As the schedule does not depend on q_E, the buyer and the rival trade the efficient quantity of good E given q_I, $q_E = q_E^*(q_I; \omega_E)$. Following Martimort and Stole (2009), we think of the buyer and rival joint utility as a *function* of the quantity purchased from the dominant firm:

$$\tilde{S}_{BE}(q_I; \omega_E) = \max_{q_E \geq 0} v_I q_I + \omega_E q_E - h(q_E, q_I) = v_I q_I + \omega_E q_E^*(q_I; \omega_E) - h(q_E^*(q_I; \omega_E), q_I).$$

The function $\tilde{S}_{BE}$ is concave in q_I as the marginal utility

$$\frac{\partial \tilde{S}_{BE}}{\partial q_I} = v_I - \frac{\partial h}{\partial q_I}(q_E^*(q_I; \omega_E), q_I) \tag{14}$$

decreases in q_I by convexity of h. It is nondecreasing in ω_E with derivative $q_E^*(q_I; \omega_E)$, and satisfies the single-crossing property:

$$\frac{\partial^2 \tilde{S}_{BE}}{\partial q_I \partial \omega_E} = \frac{\partial}{\partial q_I}\left(\frac{\partial \tilde{S}_{BE}}{\partial \omega_E}\right) = \frac{\partial q_E^*}{\partial q_I} \leq 0 \tag{15}$$

by substitutability of the two goods: the buyer and rival marginal utility for good I decreases with ω_E. For non super-efficient rival types, the isolines of $\tilde{S}_{BE}$ coincide with those of $v_I q_I - h(0, q_I)$ for large values of q_I, namely in the region where $q_E^*(q_I; \omega_E) = 0$; in this region, the marginal joint utility (14) does not depend on ω_E, and the Spence-Mirrlees inequality (15) is in fact an equality.

The chosen quantity of incumbent good, $q_I(\omega_E)$, is solution to

$$S_{BE}(\omega_E) = \max_{q_I \geq 0} \tilde{S}_{BE}(q_I; \omega_E) - T(q_I), \tag{16}$$

for some price schedule $T(q_I)$. Adapting usual arguments, we find that a quantity allocation $(q_E(\omega_E), q_I(\omega_E))$ is implementable under a non-conditional schedule if and only if the two conditions are satisfied: (i) $q_E = q_E^*(q_I; \omega_E)$; (ii) q_I is decreasing in ω_E where $q_E > 0$ and constant in ω_E where $q_E = 0$.

Constrained maximization of the virtual surplus We now maximize the virtual surplus under the two constraints listed above, namely $q_E = q_E^*(q_I;\omega_E)$ and q_I non-increasing in ω_E. To account for the former constraint, we define the constrained virtual surplus as

$$\tilde{S}^{\mathrm{v}}(q_I;\omega_E) = S^{\mathrm{v}}(q_E^*(q_I;\omega_E), q_I;\omega_E).$$

Then we maximize the constrained surplus subject to the monotonicity requirement imposed on the function $q_I(\omega_E)$. Following Martimort and Stole (2009)'s approach in multi-principal games, we make the following regularity assumption:

Assumption 2. *The constrained virtual surplus $\tilde{S}^{\mathrm{v}}(q_I;\omega_E)$ is strictly quasi-concave in q_I and has strict decreasing differences with respect to q_I and ω_E where $q_E^*(q_I;\omega_E) > 0$.*

Before proceeding to the maximization, we comment on the two conditions stated in Assumption 2. The concavity condition is equivalent to the following function being decreasing in q_I:

$$\frac{\partial \tilde{S}^{\mathrm{v}}(q_I;\omega_E)}{\partial q_I} = \omega_I - \frac{\partial h}{\partial q_I}(q_E^*(q_I;\omega_E), q_I) - \beta \frac{1-F(\omega_E)}{f(\omega_E)} \frac{\partial q_E^*(q_I;\omega_E)}{\partial q_I}. \tag{17}$$

The second term of the right-hand side is indeed decreasing in q_I by convexity of h, which tends to make the virtual surplus concave in q_I. The last term involves the slope of the conditionally efficient quantity, $\partial q_E^*/\partial q_I$. In the quadratic example, that slope is $-\sigma$, and hence the virtual surplus is concave. In general, however, the slope is equal to a ratio of second-order derivatives of h whose variations with q_I depend on properties of third-order derivatives of h.[8]

The second part of Assumption 2 is equivalent to the partial derivative $\partial \tilde{S}^{\mathrm{v}}(q_I;\omega_E)/\partial q_I$ being decreasing in ω_E. By substitutability, the second term at the right-hand side of (17) is decreasing in ω_E when $q_E^* > 0$ and constant when $q_E^* = 0$. The last term has two factors: by Assumption 1 the hazard rate $f/(1-F)$ also tends to make $\partial \tilde{S}^{\mathrm{v}}/\partial q_I$ decrease with ω_E (recall that $\partial q_E^*/\partial q_I$ is negative). The contribution of the last factor, however, is ambiguous as we do not know how the slope of the conditionally efficient quantity, $\partial q_E^*/\partial q_I$, varies with ω_E. In the quadratic case, the slope is constant and the first two forces yield the desired property.

Lemma 1. *Let $(\hat{q}_E^u, \hat{q}_I^u)$ denote the quantity allocation that maximizes the constrained virtual surplus for each value of ω_E. There exists $\hat{\omega}_E^u$ in $(\underline{\omega}_E, \overline{\omega}_E)$ such that $(\hat{q}_E^u, \hat{q}_I^u)$ satisfies*

$$\omega_I - \frac{\partial h}{\partial q_I}(\hat{q}_E^u, \hat{q}_I^u) = \beta \frac{1-F(\omega_E)}{f(\omega_E)} \frac{\partial q_E^*}{\partial q_I} \quad \textit{and} \quad \omega_E - \frac{\partial h}{\partial q_E}(\hat{q}_E^u, \hat{q}_I^u) = 0, \tag{18}$$

for $\omega_E \geq \hat{\omega}_E^u$ and $\hat{q}_E^u = q_E^*(\hat{q}_I^u;\omega_E) = 0$ for $\omega_E \leq \hat{\omega}_E^u$. The quantity $\hat{q}_I^u$ decreases (increases) with ω_E above (below) $\hat{\omega}_E^u$.

[8]The derivative of q_E^* with respect to q_I is $-\left(\partial^2 h/\partial q_E \partial q_I\right) / \left(\partial^2 h/\partial q_E^2\right)$ evaluated at $(q_E^*(q_I;\omega_E), q_I)$.

Proof. Under Assumption 2, the virtual surplus is concave and hence its maximum is determined by the first-order conditions. These conditions are given by (18) when $\hat{q}_E^u > 0$. The second part of Assumption 2 guarantees that $\hat{q}_I^u$ decreases with ω_E as long as $\hat{q}_E^u > 0$. It follows that $\hat{q}_E^u = q_E^*(\hat{q}_I^u; \omega_E)$ increases with ω_E in this region.

The existence of the threshold $\hat{\omega}_E^u$ follows from the assumption that $q_E^{**}(\omega_E^{**}) = 0 < q_E^{**}(\overline{\omega}_E)$ and the observation that q_E^u is equal to (lower than or equal to) q_E^{**} at $\overline{\omega}_E$ (at ω_E^{**}). Below that threshold, the maximum of the constrained virtual surplus is achieved at a point where $q_E^u = 0$ and the value of $\hat{q}_I^u$ is determined by conditional efficiency, i.e., by the condition $0 = q_E^*(\hat{q}_I^u; \omega_E)$, implying that $\hat{q}_I^u$ is increasing in ω_E in this region. $\square$

Figure 3 shows a situation in the quadratic example where the maximum of the constrained virtual surplus is achieved at a point where $q_E > 0$, i.e., the represented case corresponds to a value of ω_E higher than $\hat{\omega}_E^u$. For ω_E lower than that threshold, the point U lies on the vertical q_I-axis. The non-monotonicity of $\hat{q}_I^u$ is apparent on Figure 4b, see the thin dotted line.

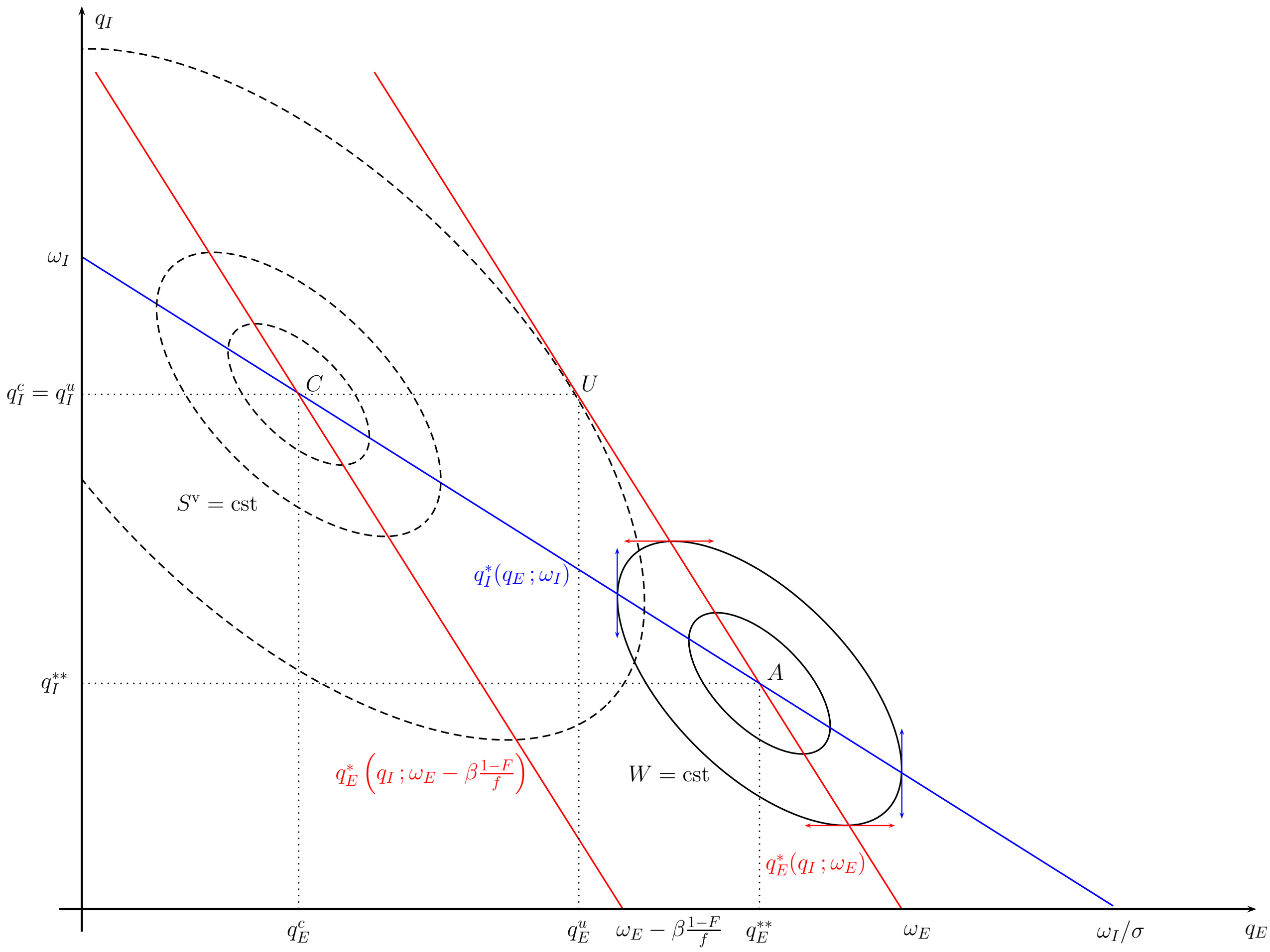

Figure 3: The constrained virtual surplus is maximal at U (quadratic example)

Lemma 1 shows that the pointwise maximization of the constrained virtual surplus yields a quantity of incumbent good that is not monotonic and hence not implementable. There must therefore be bunching at the bottom of the distribution of ω_E. The next proposition characterizes the optimal quantity allocation (q_E^u, q_I^u) under a non-conditional schedule.

Proposition 2. *Under Assumption 2, the optimal quantities implementable with a non-conditional schedule $T(q_I)$ satisfy the following properties:*

1. *There exists $\tilde{\omega}_E^u$ in $(\underline{\omega}_E, \overline{\omega}_E)$ such that $q_I^u(\omega_E)$ is constant up to $\tilde{\omega}_E^u$ and then equal to $\hat{q}_I^u(\omega_E)$.*

2. *The quantity purchased from the rival is efficient given q_I^u, $q_E^u = q_E^*(q_I^u; \omega_E)$, and distorted downwards relative to q_E^{**} for all $\omega_E < \overline{\omega}_E$. Exclusion is complete for $\omega_E \leq \omega_E^u$, with $\omega_E^{**} < \omega_E^u < \tilde{\omega}_E^u$.*

3. *The quantity purchased from the dominant firm is distorted upwards relative to the conditionally efficient quantity, $q_I^u > q_I^*(q_E^u; \omega_I)$ ("buyer opportunism") for all $\omega_E < \overline{\omega}_E$.*

Proof. The condition that determines the bunching threshold $\tilde{\omega}_E^u$ is

$$\int_{\underline{\omega}_E}^{\tilde{\omega}_E^u} \frac{\partial}{\partial q_I} \tilde{S}^{\mathrm{v}}(\bar{q}_I; \omega_E)\, \mathrm{d}F(\omega_E) = 0, \tag{19}$$

where $\bar{q}_I = \hat{q}_I^u(\tilde{\omega}_E^u)$ is the constant value of q_I over the interval $[\underline{\omega}_E, \tilde{\omega}_E^u]$, see Figure 4b. The derivative of the constrained virtual surplus $\partial \tilde{S}^{\mathrm{v}} / \partial q_I$ depends on whether q_E is positive or zero. Let ω_E^u be defined by $q_E^*(\bar{q}_I; \omega_E^u) = 0$. For $\omega_E < \omega_E^u$, $q_E^u = 0$, the derivative of the constrained virtual surplus is $w_I - \partial h / \partial q_I(0, \bar{q}_I)$, which is negative because $\bar{q}_I$ is above $\hat{q}_I^u(\omega_E)$. For $\omega_E^u < \omega_E < \tilde{\omega}_E^u$, $q_E^u > 0$, the derivative features the additional (positive) term $-\beta(1-F)/f \partial q_E^* / \partial q_I$; in this region it is positive because $\bar{q}_I$ is below $\hat{q}_I^u(\omega_E)$. The threshold $\tilde{\omega}_E^u$ is such that the positive and negative contributions offset each other. □

The quantity purchased from the rival is undistorted for $\omega_E = \overline{\omega}_E$ and is zero below a threshold ω_E^u that is strictly larger than ω_E^{**}. The quantity purchased from the dominant firm, being distorted upwards relative to $q_I^*(q_E; \omega_I)$, is a fortiori distorted upwards relative to q_I^{**}. In particular, $q_I^u(\omega_E)$ is above $q_I^*(0; \omega_I)$ for low values of ω_E. This can be seen on Figure 4c where the "trajectories" of the quantity pairs (q_E, q_I) are represented in various regimes. We see that q_I is efficient conditionally on q_E at the first-best allocation as well as under a conditional price-quantity schedule. In contrast, inefficiently many units of incumbent good given rival supply are purchased under a non-conditional schedule.

Shape of the price-quantity schedule Under Assumption 2 and assuming furthermore that h is twice continuously differentiable, $q_I^u = \hat{q}_I^u$ is differentiable outside the bunching region, i.e., for $\omega_E > \tilde{\omega}_E^u$, and its derivative is positive in that region. Now we observe that the surplus function $S_{BE}(\omega_E) = v_I q_I + \omega_E q_E^* - h(q_E^*, q_I) - T(q_I)$ is convex and hence differentiable at almost every value of ω_E. It follows that the price schedule T is almost everywhere differentiable over the range of $q_I^u(\omega_E)$. Differentiating S_{BE} and simplifying by $\mathrm{d}q_I^u/\,\mathrm{d}\omega_E$, we get $v_I - \partial h/\partial q_I(q_E^*, q_I) = T'(q_I)$, which, combined with (18), yields

$$T'(q_I^u(\omega_E)) = c_I + \beta \frac{1 - F(\omega_E)}{f(\omega_E)} \frac{\partial q_E^*}{\partial q_I} < c_I, \tag{20}$$

where the slope $\partial q_E^*/\partial q_I$ is evaluated at $(q_I^u(\omega_E); \omega_E)$. The monotonicity of the hazard rate tends to make the schedule concave in q_I. Indeed, as ω_E rises, the quantity q_I^u falls and the hazard rate pushes the the right-hand side of (20) upwards because $\partial q_E^*/\partial q_I$ is negative. There is, however, the additional effect that the derivative $\partial q_E^*/\partial q_I$ can itself move with ω_E; this effect is absent in the quadratic case where the derivative is constant. The following proposition presents sufficient conditions (derived from (20)) for the price schedule to be concave below the maximum quantity purchased from the dominant firm, $\bar{q}_I$, given by (19).

Proposition 3. *Suppose that the slope of the conditionally efficient quantity, $\partial q_E^*/\partial q_I$, is non-increasing (nondecreasing) in q_I (resp. ω_E). Then the optimal non-conditional price-quantity schedule $T(q_I)$ is concave in q_I up to $\bar{q}_I$ and has a convex kink at this point.*

The shape of the optimal non-conditional schedule is shown on Figure 5. The convex kink at $\bar{q}_I$ is due to complete exclusion and the associated bunching phenomenon at the bottom of the distribution. Indeed, the slope of the price schedule at the left of $\bar{q}_I$ is equal to the marginal rate of substitution for $v_I - \partial h/\partial q_I$ evaluated at $(q_E^*(\bar{q}_I; \tilde{\omega}_E^u), \bar{q}_I)$. This rate is higher for the agents with lower type, and is the highest for $\omega_E = \underline{\omega}_E$, because these types value the rival good less, and hence the incumbent good more. To prevent these agents from purchasing more than $\bar{q}_I$, the price schedule must lie above the iso-utility curve of the lowest type, hence a the convex kink. To be specific, the right derivative of the schedule at $\bar{q}_I$ must be greater than $v_I - \partial h/\partial q_I$ evaluated at $(0, \bar{q}_I)$.[9]

In the limiting case of two independent markets, the conditionally efficient quantity of rival good, q_E^*, does not depend on q_I, and Lemma 2 and Proposition 2 show that both quantities are fully efficient at the second-best allocation. In particular, the quantity of incumbent good

[9] The bunching at the bottom of the distribution of ω_E, and the corresponding non-concavity of $T(q_I)$ at $\bar{q}_I$, are present because $\hat{q}_E^u(\omega_E) = 0$ for low values of ω_E. This is due in particular to our assumption that q_E^{**} is zero at the bottom of the distribution ($\omega_E^{**} > \underline{\omega}_E$, see Assumption 1). We would have no bunching and a globally concave schedule if $q_E^*(q_I; \omega_E)$ were positive for all q_I and ω_E.

does not vary with the rival's efficiency index ω_E, so the range of $q_I^u(\omega_E)$ is the singleton $\{q_I^{**}\}$, which makes the above analysis inoperative. Here we see directly that $T' = c_I$ is necessary to induce efficiency.

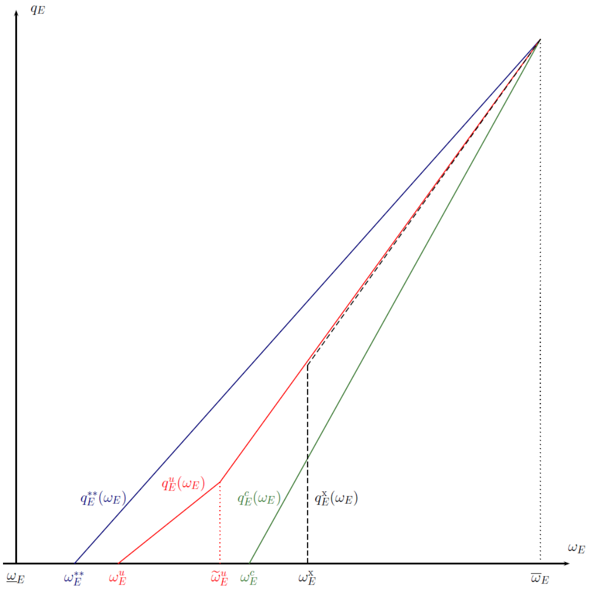

Figure 4a: Equilibrium quantities of good E under each type of price schedule

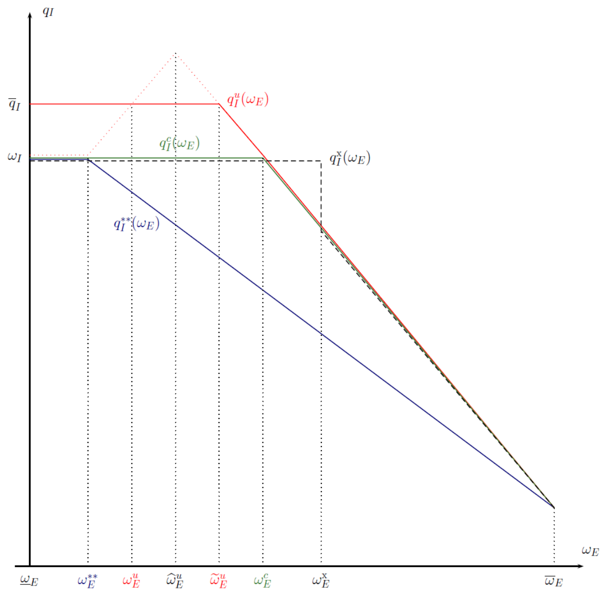

Figure 4b: Equilibrium quantities of good I under each type of price schedule

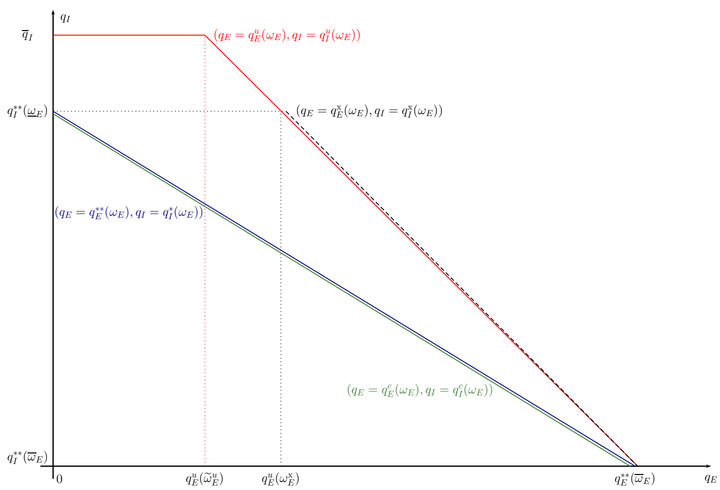

Figure 4c: Buyer opportunism in regimes u and x: q_I is above $q_I^*(q_E; \omega_I)$

Quadratic example When the buyer's utility is quadratic, the second-best quantities under a non-conditional schedule are given in the no-bunching region, i.e., for $\omega_E > \tilde{\omega}_E^u$, by

$$q_E^u(\omega_E) = q_E^*(q_I^u; \omega_E) \quad \text{and} \quad q_I^u(\omega_E) = \omega_I + \sigma\beta \frac{1 - F(\omega_E)}{f(\omega_E)} - \sigma q_E^u(\omega_E),$$

which yields

$$q_E^u(\omega_E) = q_E^{**}(\omega_E) - \beta \frac{\sigma^2}{1-\sigma^2} \frac{1 - F(\omega_E)}{f(\omega_E)} \quad \text{and} \quad q_I^u(\omega_E) = q_I^{**}(\omega_E) + \beta \frac{\sigma}{1-\sigma^2} \frac{1 - F(\omega_E)}{f(\omega_E)}.$$

When the distribution is uniform, the bunching condition (19) can be rewritten as:

$$(\omega_I - \bar{q}_I)(\omega_E^u - \underline{\omega}_E) + \int_{\omega_E^u}^{\tilde{\omega}_E^u} \left[\omega_I - (1-\sigma^2)\bar{q}_I - \sigma\omega_E + \beta\sigma(\overline{\omega}_E - \omega_E)\right] \mathrm{d}\omega_E = 0, \tag{21}$$

with $\omega_E^u = \sigma\bar{q}_I$ and q_I^u continuous at $\tilde{\omega}_E^u$. The quantities are represented as a function of ω_E on Figures 4a and 4b.

As already observed, the sufficient conditions of Proposition 3, that guarantee the concavity of the price schedule, are satisfied when the buyer's utility is quadratic. To illustrate, we compute the curvature of the schedule when the distribution of ω_E is uniform. Observing that $(1-F)/f = \overline{\omega}_E - \omega_E$ and combining (20) with the expression of $q_I^u(\omega_E)$, we find that the first derivative of the schedule is linear in q_I:

$$T'(q_I) = c_I - \frac{\beta}{1+\beta}(1-\sigma^2)\left[q_I - q_I^{**}(\overline{\omega}_E)\right] < c_I$$

for all q_I between $q_I^u(\overline{\omega}_E)$ and $\bar{q}_I$; the schedule is quadratic (and concave) in this region.

5 Exclusivity offer

In this section, we investigate the situation where the price of the incumbent good can be conditioned on the rival supply only through the events $q_E = 0$ and $q_E > 0$. This situation is somewhat intermediary between conditional and non-conditional schedules.

Let $(T(q_I), T^{\mathrm{x}}(q_I))$ be an exclusivity price scheme constituted of a pair of non-conditional schedules, where T^{x} is available to the buyer only if she supplies exclusively from the dominant firm. Under such a scheme, the buyer and the rival, should they settle on a positive quantity, choose $q_E = q_E^*(q_I; \omega_E)$ and q_I solution to (16). If they fail to find such an agreement, the buyer earns

$$S^{\mathrm{x}} = v_I q_I - h(0, q_I) - T^{\mathrm{x}}(q_I),$$

which does not depend on ω_E. It follows that the price schedule T^{x} consists in fact of a single price-quantity pair. The only difference with the non-conditional situation studied in Section 4 is the availability of an independent instrument to control the outside option.

Proposition 4. *There exists a threshold ω_E^{x} such that the quantities purchased by the buyer under an exclusivity scheme satisfy $(q_E^{\mathrm{x}}, q_I^{\mathrm{x}}) = (0, q_I^*(0))$ for $\omega_E \leq \omega_E^{\mathrm{x}}$ and $(q_E^{\mathrm{x}}, q_I^{\mathrm{x}}) = (q_E^u, q_I^u)$ for $\omega_E \geq \omega_E^{\mathrm{x}}$.*

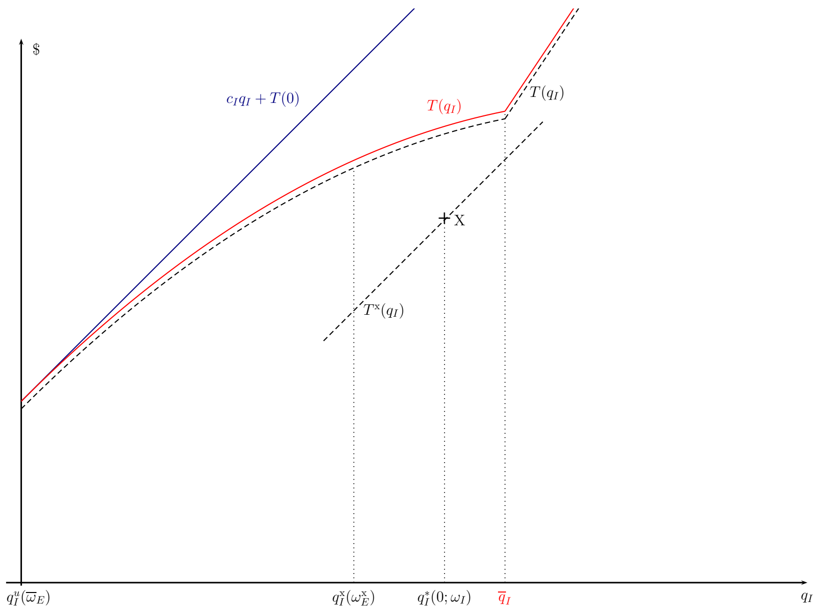

Figure 5: Price-quantity schedules in the three regimes

Next, we compare the magnitude of exclusionary effects in the three considered pricing regimes.

Assumption 3. *The nonlinear part of the buyer's utility, $h(q_E, q_I)$ satisfies*

$$\int_{q_I^0}^{q_I^1} \left[\frac{\partial^2 h}{\partial q_E^2}(q_E, q_I^1) \frac{\partial^2 h}{\partial q_I^2}(q_E, q_I) - \frac{\partial^2 h}{\partial q_E \partial q_I}(q_E, q_I^1) \frac{\partial^2 h}{\partial q_E \partial q_I}(q_E, q_I) \right] \mathrm{d}q_I \geq 0$$

for all q_E and $q_I^1 \geq q_I^0$.

Assumption 3 holds for any convex quadratic function because the term under the integral is then constant and nonnegative. If h is a convex function with positive second-order cross derivative as assumed in this article, the assumption is true for instance when $\partial^2 h / \partial q_I^2$ and $\partial^2 h / \partial q_E \partial q_I$ are respectively non-increasing and nondecreasing in q_I.

Recall ω_E^{**}, ω_E^u, ω_E^c and ω_E^{x} denote the maximum values of ω_E for which the rival is inactive at the first-best optimum and at the second-best optima under respectively a non-conditional schedule $T(q_E, q_I)$, a conditional schedule $T(q_I)$, and an exclusivity price scheme. We refer to these values as the exclusion thresholds in each regime.

Proposition 5. *Under Assumption 3, the quantities purchased from the rival firm in each regime are ordered as follows:*

$$0 = q_E^{\mathrm{x}}(\omega_E) \leq q_E^c(\omega_E) \leq q_E^u(\omega_E) < q_E^{**}(\omega_E) \tag{22}$$

for $\omega_E^{\mathrm{x}} < \omega_E$ *and*

$$q_E^c(\omega_E) \leq q_E^u(\omega_E) = q_E^{\mathrm{x}}(\omega_E) < q_E^{**}(\omega_E) \tag{23}$$

for $\omega_E > \omega_E^{\mathrm{x}}$. *The exclusion thresholds are ordered as follows:*

$$\omega_E^{**} \leq \omega_E^u \leq \omega_E^c \leq \omega_E^{\mathrm{x}}. \tag{24}$$

The above analysis implies that $q_I^c(\omega_E^{\mathrm{x}}) < q_I^c(\omega_E^c) = q_I^*(0;\omega_I)$ and hence the optimal quantity of incumbent good under an exclusivity scheme, q_I^{x}, admits a downward discontinuity at ω_E^{x}, see the dashed line on Figure 4b for an illustration.

Quadratic Example For $\omega_E = \omega_E^{\mathrm{x}}$, the buyer and the dominant firm are ex ante indifferent between the points $(0, q_I^*(0))$ and the point U on Figure 3. Geometrically, the same isoline of the virtual surplus contains the point $(0, q_I^*(0))$ and the non-conditional second-best allocation denoted by U (the dashed ellipsis passing through $(0, q_I^*(0))$ is tangent to the straight line $q_E = q_E^*(q_I; \omega_E^{\mathrm{x}})$).

The quantities sold by each suppliers in each of the three regimes are represented on Figures 4a and 4b. A specificity of the quadratic case is that when $q_E > 0$ the quantity of incumbent good is the same under the conditional and non-conditional regimes. (Geometrically the points C and U are on the same horizontal line on Figure 3.) The ordering of q_I across regimes is unclear in general.

Welfare analysis The welfare implications of the three pricing regimes involve two types of distortion. First, the quantity of rival good is distorted downwards, which reduces the social welfare. The best regime in this dimension is non-conditional pricing. Second, the quantity of incumbent good may be distorted upwards conditionally on the rival supply. The best regime in this dimension is conditional pricing because it completely eliminates buyer opportunism. In this respect, non-conditional schedules perform badly at the bottom of the distribution because q_I is larger than $q_I^*(0;\omega_I)$ in this region. Exclusivity schemes avoid the latter effect while behaving like unconditional schedules at the top of the distribution; they induce, however, the largest distortions for both goods in an intermediate range of values for the efficiency index ω_E. All these effects are summarized on Figure 6.

In the quadratic case with a uniform distribution, numerical simulations suggest that the non-conditional regime is socially preferred to the conditional and exclusivity regimes and that the exclusivity regime is preferred to the conditional regime for small values of β.

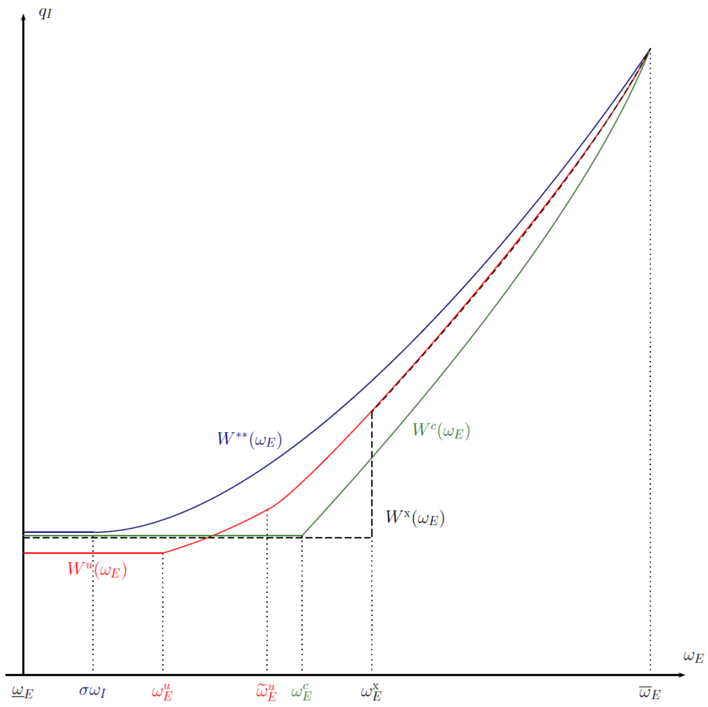

Figure 6: Welfare at the first-best and second-best allocations

6 Disposal costs

We now allow the buyer to dispose of unconsumed units at the unit cost $\gamma_I > -c_I$. We know that the total welfare and the virtual surplus linearly decrease in the region where the buyer indeed does not consume all of the purchased units of incumbent good. As noted in Section 3, the possibility of disposal is of no importance for the analysis of the conditional regime because the virtual surplus attains its maximum in the interior of the no-disposal region for all $\gamma_I > -c_I$.

In contrast, we have seen in Section 4 that when the buyer must consume all purchased units ($\gamma_I = \infty$) the second-best allocation under a non-conditional schedule, (q_E^u, q_I^u), is essentially determined by the maximum of the *constrained* virtual surplus, which is attained at $(\hat{q}_E^u, \hat{q}_I^u)$ as explained in Lemma 1. More precisely, (q_E^u, q_I^u) coincides with $(\hat{q}_E^u, \hat{q}_I^u)$ for $\omega_E \geq \tilde{\omega}_E^u$; below this threshold, the quantity purchased from the dominant firm, q_I^u, is constant and equal to $\bar{q}_I = \hat{q}_I^u(\tilde{\omega}_E^u)$, and the quantity purchased from the rival, q_E^u, declines to zero as ω_E falls to ω_E^u.

The possibility of disposal does not change the second-best allocation if and only if the solutions found for $\gamma_I = \infty$ remain in the no-disposal region for finite γ_I. This is the case if and only if the buyer is strictly better off consuming all the units purchased than disposing of some of them:

$$v_I - \frac{\partial h}{\partial q_I}(\hat{q}_E^u(\omega_E), \hat{q}_I^u(\omega_E)) = T'(\hat{q}_I^u(\omega_E)) > -\gamma_I, \tag{25}$$

for all ω_E greater than $\tilde{\omega}_E^u$. If this condition is violated, the maximum of the constrained virtual surplus lies on boundary of the no-disposal region. It is then determined as the intersection of that boundary, $v_I - \partial h/\partial q_I(q_E, q_I) = -\gamma_I$, and of the conditionally efficient curve, $q_E = q_E^*(q_I; \omega_E)$. The intersection point is denoted by B^γ on Figure 7 for the quadratic example.[10]

Proposition 6. *Suppose that the assumptions of Proposition 3 hold. Then the second-best allocation under a non-conditional schedule does not vary with the magnitude of disposal costs as long as γ_I remains above $-T'(q_I^u(\tilde{\omega}_E^u))$. As γ_I falls below these thresholds and tends to $-c_I$, the quantity purchased from the rival and the dominant firm respectively increases and decreases, tending to q_E^{**} and q_I^{**}; the slope of the price schedule tends to c_I; the welfare rises to its first-best optimum.*

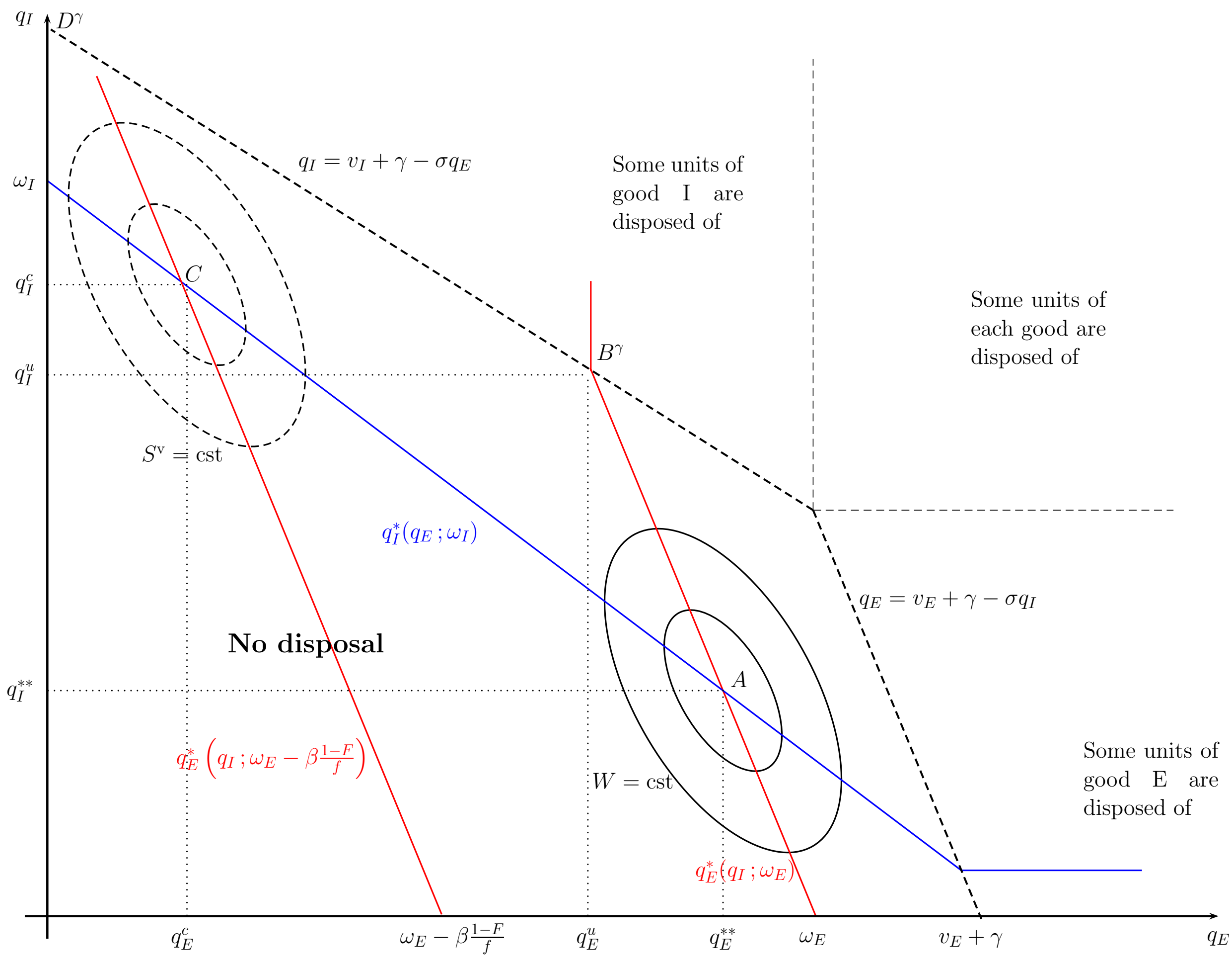

Figure 7: The constrained virtual surplus is maximal at B^γ

When the magnitude of the disposal cost is low, the purchased quantity of incumbent good

[10]In this case, the tangency point of the isoline of the virtual surplus (dashed ellipsis) to the straight line $q_E = \omega_E - \sigma q_I$, lies *above* the boundary of the no-disposal region, $q_I = v_I + \gamma_I - \sigma q_E$.

is not far away from the conditionally efficient quantity, $q_I^*(q_E;\omega_I)$. In other words, the possibility of disposing of unconsumed units of good I reduces the degree of buyer opportunism present at the second-best allocation. The intuition behind this result is best understood from the ex ante perspective. When designing the price schedule under lower disposal costs, the dominant firm and the buyer have less incentives to distort q_I upwards because the conditionally efficient quantity of rival good, $q_E^*(q_I;\omega_E)$ is less responsive to q_I (it is actually unresponsive in the disposal region, see the vertical part of q_E^* represented on Figures 1, 2 or 7). As γ_I declines to $-c_I$, an unconditional price schedule has less and less power to influence the rival supply policy because the rival understands that the buyer can always pocket rebates and dispose of or even resell unneeded units of incumbent good.

7 Extensions

We now consider a couple of variants in the timing of events and the instruments available to the parties.

We first consider the situation where the buyer and the dominant firm can wait for the uncertainty to be resolved before deciding on the price-quantity schedule and still enjoy the same commitment power at this point. (See Marx and Shaffer (2004) for a similar complete information framework.) They can then implement the efficient allocation and extract all the surplus from non super-efficient rival types through a non-conditional price-quantity schedule. Indeed, if $\bar{q}_I$ is such that $q_E^*(\bar{q}_I;\omega_E)=0$, the following non-conditional schedule yields the first-best outcome: $T(q_I)=c_Iq_I+T(0)$ for $q_I<\bar{q}_I$; $T(\bar{q}_I)$ is such that $V(q_E^{**},q_I^{**})-c_Eq_E^{**}-T(q_I^{**})=W(q_E^{**},q_I^{**})-T(0)$ is slightly above $V(0,\bar{q}_I)-T(\bar{q}_I)$; and $T(q_I)=+\infty$ beyond $\bar{q}_I$. (The constant $T(0)$ serves to share the surplus $W(q_E^{**},q_I^{**})$.) It is easy to check that the quantities purchased in equilibrium are q_E^{**} and q_I^{**}. If the rival and the buyer failed to agree on a price and a quantity for good E, the buyer would purchase $\bar{q}_I$ from the dominant firm. It follows from the definition of $T(\bar{q}_I)$ that the surplus ΔS_{BE} created by from the trade with the rival is negligible, and the rival profit can be made arbitrarily close to zero.[11] This timing, therefore, would be very favorable to the buyer and the dominant firm. In contrast, this article has assumed that the dominant firm and the buyer cannot wait for the resolution of uncertainty and at the same time keep their commitment power.

Next we discuss the perhaps intriguing feature of the model that the buyer is part of two successive coalitions. One might consider an interim stage where the buyer has learned the

[11] When the rival is super-efficient, dealing with the rival creates a positive surplus however large q_I becomes. Formally the decreasing function $\beta[V(q_E^*(q_I;\omega_E);q_I)-c_Eq_E^*(q_I;\omega_E)-V(0,q_I)]$ remains positive for all q_I. It can be shown that the rival's rent at the second-best optimum is equal to the lower bound of this function.

characteristics of the rival good but has not yet started negotiating a price and a quantity with the rival. It would then be natural to endow the buyer and the dominant firm with a more powerful instrument consisting of a menu of price-quantity schedules, $(T(q_I;\hat{\omega}_E))_{\hat{\omega}_E}$, and to consider the following game: (i) the buyer and the dominant firm agree on such a menu; (ii) the buyer learns ω_E and announces $\hat{\omega}_E$; (iii) the buyer and the rival negotiate under the price-quantity schedule $T(q_I;\hat{\omega}_E)$. At the interim stage, the buyer pursues her own interest and may therefore try to cheat on the dominant firm by manipulating $\hat{\omega}_E$. The menu should be designed to maintain truthfulness.

A fundamental observation is that at the interim stage the buyer is weakly better off colluding with the rival firm on the announcement $\hat{\omega}_E$. In other words, it is in the buyer's interest to agree with the rival not only on the quantities of both goods and the price of the rival good *but also on the announcement*. Indeed, negotiating on all variables under control weakly increases the surplus to be shared with the rival, and hence the part that goes to the buyer. We believe that in practice collusion on the announcement is unavoidable, and for this reason we have not included such an interim stage in our modeling framework.

Suppose, for the sake of the discussion, that the buyer and the rival can be prevented from colluding on the announcement. We now show that if the rival is never super-efficient and has all the bargaining power vis-à-vis the buyer ($\beta = 1$), then there exists a menu of schedules $T(q_I;\hat{\omega}_E)$ that yields the first-best outcome. Let $\bar{q}_I$ be such that $q_E^*(\bar{q}_I;\omega_E) = 0$ for all ω_E. We define $T(q_I;\hat{\omega}_E)$ for each $\hat{\omega}_E$ in the same way as in the complete information case presented above. The only difference with that case is that we choose $T(0;\hat{\omega}_E) = W(q_E^{**}(\hat{\omega}_E), q_I^{**}(\hat{\omega}_E);\hat{\omega}_E) - \bar{V}$, where $\bar{V}$ is a constant. The latter equality ensures that $T(\bar{q}_I;\hat{\omega}_E)$, and hence the buyer's outside option, does not depend on ω_E or $\hat{\omega}_E$. Since $\beta = 1$, the buyer, at the interim stage, gets utility $\bar{V}$ irrespective of her announcement. If we assume that she declares the true value of ω_E to the dominant firm, then the first-best allocation obtains.[12] Yet, as explained above, the mechanism is not collusion-proof because the rival would like $\hat{\omega}_E$ to be as low as possible and is ready to bribe the buyer in return for such an announcement. Since an arbitrarily small bribe is sufficient to break the buyer's indifference, truthful revelation is unrealistic.

8 Concluding remarks

Market dominance, almost by definition, implies long-term business relationships and repeated interactions. In a given market, buyers have extensive experience in dealing with the dominant

[12] The problem with $\beta < 1$, under the assumption that the buyer decides on $\hat{\omega}_E$ without colluding with the rival, is open. We only know that the first-best allocation cannot be achieved.

operator and expect the dominant firm to remain an important trading partner in the foreseeable future. The history of past interactions and the prospect of future interactions give the dominant firm a form of market power, which we have modeled as a first-mover advantage in a sequential game à la Aghion and Bolton (1987).[13] Our setting with variable quantities allows to study a number of pricing regimes depending on whether and how the price set by the dominant firm depends on the rival supply. Our findings support standard antitrust doctrine that conditional schemes allow dominant firms to manipulate buyer-rival relationships and lead to anticompetitive exclusion. We discuss policy implications concerning non-conditional schemes in greater length.

In the absence of uncertainty, i.e., when the dominant firm can let the offered price schedule depend on the rival characteristics, it can often extract all the rival surplus without having to distort the buyer's supply policy (see Marx and Shaffer (1999) and Section 7). We have focused in this article on a situation that is less favorable to the dominant firm, namely the case where it can exert its commitment power only before rivals characteristics are known. We have found quantity distortions, both at the extensive and intensive margins, in this circumstance. It is therefore important for competition authorities to assess the degree of uncertainty in the negotiation process between buyers and dominant firms. Uncertainty is likely to be severe in industries with rapid pace of innovation and high product complexity because the dominant firm is then under constant competitive threat and may find it difficult to anticipate the commercial success of upcoming rival products. It may respond to uncertainty with price schemes that foreclose the market.

Another key parameter for competition analysis is the magnitude of disposal costs. By granting generous quantity rebates, the dominant firm can induce the buyer to purchase (inefficiently too) many units of incumbent good and thereby discourage her to supply from the rival. The dominant firm's indirect control over the buyer's supply policy, however, is weakened when the buyer can dispose of or resell unconsumed units of incumbent good at a low cost. In the limiting case where the buyer can resell units on a secondary market at a price close to c_I (negative disposal cost), non-conditional quantity rebates lack any power to alter the bargaining process with the rival supplier as the latter understands that the buyer can always pocket the rebates without consuming the corresponding units. It follows that higher disposal costs are associated to more inefficient exclusion and lower values of the expected total welfare. Antitrust authorities, therefore, should pay close attention to contracting provisions that restrict the buyers' ability to dispose of or resell unconsumed units of incumbent good.

The role of product differentiation is also worth noticing. In this article, we have found that

[13]Modeling exclusionary nonlinear pricing in a dynamic setting is out of the scope of this study.

the quantity sold by the dominant firm, given the quantity supplied from the rival, is excessive. This phenomenon, which we call buyer opportunism, actually occurs when the substitutability between the two goods is positive but imperfect, $0 < \sigma < 1$. Because the conditionally efficient quantity of rival good is not perfectly responsive to the purchased quantity of incumbent good, the dominant firm and the buyer have an ex ante incentive to distort the latter quantity upwards. As σ declines to zero, the dominant firm has less and less control over the rival supply, hence less scope for rent extraction or exclusion. As σ rises to one, the rival supply responds better and better to the quantity purchased from the dominant firm, hence less need to distort the latter quantity and less buyer opportunism in equilibrium.[14] Irrespective of the degree of substitutability, the extreme form of buyer opportunism whereby the buyer would purchase units from the dominant firm without consuming them is never seen in equilibrium.

Finally, a crucial aspect of the pricing strategies studied in this article is below-cost pricing. Under each of the price regimes we have considered, the dominant firm sells some product units below cost to extract rents from competitors. Such price patterns can be thought of as a static form of predatory pricing, i.e., predation that does not involve future recoupment of present losses. Predation, here, is "at the margin" in the sense that the marginal price is below cost whereas the average price is above cost. To prevent inefficient exclusion, competition agencies should check that prices implemented by dominant firms cover incremental costs –a difficult and costly task in practice.

To provide insights about price-cost tests in the context of nonlinear pricing, our companion article Choné and Linnemer (2014) abstracts away from the buyer opportunism issue and assumes that the rival firm cannot address the buyer's entire demand. In this perspective, the size of the contestable demand is the relevant quantity range over which price-cost comparisons should be run. When the contestable demand is uncertain, concave price-quantity schedules can generate full exclusion, and many other shapes of nonlinear schedules, including retroactive rebates, can be observed in equilibrium.

A Appendix

Proof of Proposition 1 We start by introducing a function $P(q_E)$ whose derivative is given by (13) on the interval $[0, q_E^c(\overline{\omega}_E)]$ and is zero above $q_E^c(\overline{\omega}_E)$. The value of $P(0)$ determines the sharing of the surplus between the buyer and the dominant firm. It is straightforward to

[14] In the limiting case $\sigma = 1$, buyer opportunism does not materialize in equilibrium, see Choné and Linnemer (2014).

verify that the function P is globally concave. Moreover, by definition of q_E^c, we have

$$\frac{\partial h}{\partial q_E}(q_E^c(\omega_E), q_I^*(q_E^c(\omega_E))) + P'(q_E^c(\omega_E)) = \omega_E, \tag{A.1}$$

for all ω_E between $\underline{\omega}_E$ and $\overline{\omega}_E$. We now check that the function $h(q_E, q_I^*(q_E))+P(q_E)-\omega_I q_I^*(q_E)$ is convex in q_E. Indeed its derivative at some $q_E = q_E^c(\omega_E)$ in the interval $[0, q_E^c(\overline{\omega}_E)]$ is the left-hand side of (A.1) and, therefore is increasing with ω_E and q_E. Hence the convexity result.

Under the two-part tariff $T(q_E, q_I) = c_I q_I + P(q_E)$, the buyer and the rival choose the efficient quantity of good I given q_E, $q_I^*(q_E;\omega_I)$. Replacing q_I with $q_I^*(q_E;\omega_E)$ in their common objective (3), we find that the buyer and the rival choose the quantity q_E that maximizes the function $\omega_E q_E + \omega_I q_I^*(q_E) - h(q_I, q_I^*(q_E)) - P(q_E)$. This function is concave in q_E from the above analysis. The quantity of good E, therefore, is determined by the first-order conditions, and is thus $q_E^c(\omega_E)$ for any ω_E.

Proof of Proposition 4 Maximizing the virtual surplus under the constraint $q_E = 0$ yields the conditionally efficient quantity of incumbent good, $q_I^*(0;\omega_I)$. If $q_E > 0$, we maximize as above the *constrained* virtual surplus to account for the constraint $q_E = q_E^*(q_I;\omega_E)$. The virtual surplus is $S^{\text{v}}(0, q_I^*(0;\omega_I)) = W(0, q_I^*(0;\omega_I))$ in the former situation, $\max_{q_I} S^{\text{v}}(q_E^*(q_I;\omega_E), q_I;\omega_E)$ in the latter. Let ω_E^{x} be the type for which the buyer and the dominant firm are ex ante indifferent between these alternatives:

$$\max_{q_I} S^{\text{v}}(q_E^*(q_I;\omega_E^{\text{x}}), q_I;\omega_E^{\text{x}}) = W(0, q_I^*(0;\omega_I)). \tag{A.2}$$

Since $\max_{q_I} S^{\text{v}}(q_E^*(q_I;\omega_E), q_I;\omega_E)$ increases with ω_E, the quantity allocation $(q_E^{\text{x}}, q_I^{\text{x}})$ defined in the statement of the proposition maximizes the virtual surplus.

We now explain how to implement this allocation with a pair (T, T^{x}). Regarding T, we use the same schedule as in Section 4. We define T^{x} as a two-part tariff with slope c_I to ensure that $q_I = q_I^*(0;\omega_I)$. The intercept of this tariff, and hence the price $T^{\text{x}}(q_I^*(0;\omega_I))$ associated to the exclusivity offer, is adjusted so that the buyer and the rival, for $\omega_E = \omega_E^{\text{x}}$, are ex post indifferent between $(q_E^u(\omega_E^{\text{x}}), q_I^u(\omega_E^{\text{x}}))$ and $(0, q_I^*(0;\omega_I))$. Then the types above the threshold ω_E^{x}, who value the incumbent good less than ω_E^{x}, prefer a non-exclusive arrangement and pick a point in the nonlinear schedule T. In contrast, the types below ω_E^{x}, who value the incumbent good more than ω_E^{x}, are attracted by the exclusivity offer. That offer is represented by the point X on Figure 5.

Proof of Proposition 5 We first observe that the bunching procedure at the bottom of the distribution leads to increase the quantity of good E, i.e., $q_E^u \geq \hat{q}_E^u$, where $\hat{q}_E^u$ maximizes the constrained virtual surplus, see Lemma 1. This follows from $\hat{q}_E^u = q_E^*(\hat{q}_I^u;\omega_E)$ and $q_E^u = q_E^*(\bar{q}_I;\omega_E)$, together with $\bar{q}_I \leq \hat{q}_I^u$ when $\hat{q}_E^u > 0$. It is therefore sufficient to prove that $\hat{q}_E^u \geq q_E^c$.

Let $\tilde{q}_I$ be defined by $q_E^*(\tilde{q}_I;\omega_E) = q_E^c$. By concavity of the modified virtual surplus, the ordering $q_E^c(\omega_E) \leq \hat{q}_E^u(\omega_E;\gamma)$ is equivalent to

$$\omega_I - \frac{\partial h}{\partial q_I}(q_E^c, \tilde{q}_I) - \beta\frac{1-F(\omega_E)}{f(\omega_E)}\frac{\partial q_E^*}{\partial q_I}(q_E^c, \tilde{q}_I) \leq 0. \tag{A.3}$$

This inequality is indeed equivalent to the modified virtual surplus reaching its maximum for $q_I < \tilde{q}_I$, and hence $q_E > q_E^c$. We have, using $q_I^c = q_I^*(q_E^c)$

$$\omega_I - \frac{\partial h}{\partial q_I}(q_E^c, \tilde{q}_I) = -\int_{q_I^c}^{\tilde{q}_I} \frac{\partial^2 h}{\partial q_I^2}(q_E^c, q_I)\,\mathrm{d}q_I \tag{A.4}$$

and, using $q_E^c = q_E^*(\tilde{q}_I)$

$$\beta\frac{1-F(\omega_E)}{f(\omega_E)} = \omega_E - \frac{\partial h}{\partial q_E}(q_E^c, q_I^c) = \int_{q_I^c}^{\tilde{q}_I} \frac{\partial^2 h}{\partial q_E \partial q_I}(q_E^c, q_I)\,\mathrm{d}q_I \tag{A.5}$$

Finally recall that

$$\frac{\partial q_E^*}{\partial q_I}(q_E^c, \tilde{q}_I) = -\left.\frac{\partial^2 h}{\partial q_E \partial q_I}(q_E^c, \tilde{q}_I)\right/\frac{\partial^2 h}{\partial q_E^2}(q_E^c, \tilde{q}_I) \tag{A.6}$$

We get (A.3) by combining (A.4), (A.5), and (A.6) and applying the inequality of Assumption 3 with $q_E = q_E^c$, $q_I^0 = q_I^c$ and $q_I^1 = \tilde{q}_I$.

The left two inequalities in (24) follow directly. We now prove the right inequality. From the analysis of Section 3, we have:

$$W(0, q_I^*(0;\omega_I)) = \max_{q_E, q_I} S^{\mathrm{v}}(q_E, q_I; \omega_E^c).$$

Imposing the constraint that q_E must be efficient conditional on q_I reduces the maximum value of the virtual surplus:

$$\max_{q_I} S^{\mathrm{v}}(q_E^*(q_I;\omega_E^c), q_I; \omega_E^c) < W(0, q_I^*(0;\omega_I)) = \max_{q_E, q_I} S^{\mathrm{v}}(q_E, q_I; \omega_E^c). \tag{A.7}$$

The right inequality in (24) follows from the comparison of (A.2) and (A.7), combined with the observation that $\max_{q_I} S^{\mathrm{v}}(q_E^*(q_I;\omega_E), q_I; \omega_E)$ increases with ω_E.

Proof of Proposition 6 Under the assumptions of Proposition 3, the optimal non-conditional schedule is concave, i.e., $T'(\hat{q}_I^u(\omega_E))$ increases with ω_E. The condition imposed by (25), therefore, is stronger for lower values of ω_E or equivalently higher values of $\hat{q}_I^u$. If (25) holds for $\omega_E = \tilde{\omega}_E^u$, it holds for all ω_E above the threshold.

Otherwise, if (25) is violated at $\tilde{\omega}_E^u$, it is violated for all values of ω_E below a new threshold that is greater than $\tilde{\omega}_E^u$ and that rises as γ_I falls. As ω_E falls from this threshold to $\underline{\omega}_E$, the maximum of the constrained virtual surplus is first located on the boundary of the no-disposal region (point B^γ on Figure 7); at some point, q_E reaches zero (point D^γ on Figure 7); for lower values of ω_E, the maximum of the constrained virtual surplus is determined by $q_E = 0$ and $q_E^*(q_I;\omega_E) = 0$, which gives rise to bunching as in Section 4.

When γ_I falls to $-c_I$, the boundary of the no-disposal region, $v_I - \partial h/\partial q_I = -\gamma_I$, moves closer to the conditional efficiency line $q_I = q_I^*(q_E;\omega_I)$. On Figure 7, the point B^γ tends to the first-best optimum A. At the same time, the slope of the price schedule, $T'(q_I)$, which lies between $-\gamma_I$ and c_I, tends to c_I.

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